Extending JT/SYK duality via $\mathfrak{so}(2,2)$ Poisson Sigma Model

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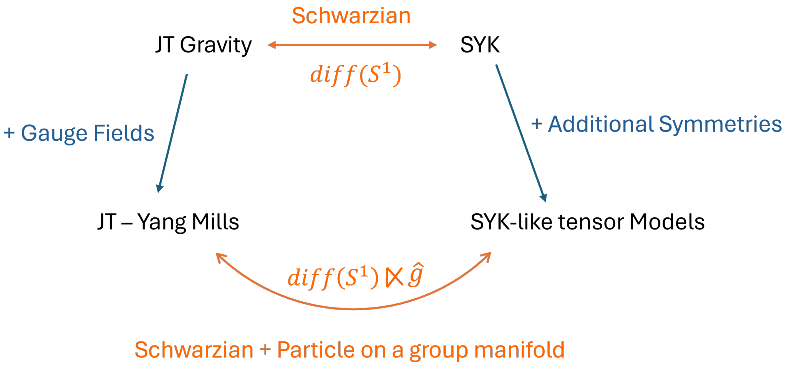
JT/SYK: gateway btw many body quantum chaos and gravity

Outline



idea: realize this scheme with a $SO(2,2)$-Poisson Sigma Model: extended Schwarzian as a coadjoint orbit of the semidirect product Virasoro-Kac-Moody

2D dilaton gravity MODELS general near horizon geometry

- 2D dilaton gravity models describe near extremal black holes, or more generally, nearly AdS_2 spacetimes. (KK-like derivation from arbitrary stationary black holes) [Carlip, Yoon]

$$I[g,\Phi] = -\frac{1}{16\pi G_N}\int_\Sigma dx^2\sqrt{-g}(\Phi R + V(\Phi)) + \ldots \text{KK matter fields},$$

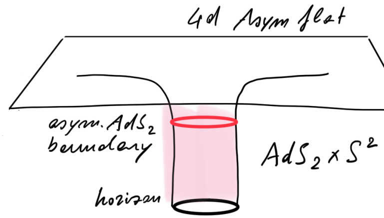
- topological: no propagating degrees of freedom
- dilaton (scalar field) Φ = parameter determines the classical geometry
- Jackiw–Teitelboim (JT) gravity corresponds to a linear choice of dilaton potential $V(\Phi) = -\Lambda\Phi$ with action [Teitelboim, Jackiw, Almheiri, Polchinski]

$$I_{JT}[g,\Phi] = -\frac{1}{16\pi G_N}\int_\Sigma \sqrt{-g}\,\Phi(R-\Lambda)$$

- solutions are spacetimes with constant curvature $R = \Lambda$. We have: AdS ($\Lambda < 0$) and dS ($\Lambda > 0$) versions JT gravity

Jackiw-Teitelboim (JT) gravity model: focus on AdS_2

- $\Lambda = -2$ gives AdS_2: JT solutions well approximate the AdS-factor in the near horizon geometry of near-extremal black holes in GR ($AdS_2 \times S^2$)



- symmetries in perfect AdS_2:
  - SL(2,R) isometry group
  - asymptotic symmetries are by definition the subgroup of the 2D diffeos that leaves the metric asymptotically invariant: the group of time-translations $t \to \tilde{t}(t)$: conformal symmetry spontaneously broken to SL(2,R)

- replace asymptotically AdS boundary by a finite boundary = cutting off AdS_2 along a trajectory $\gamma = (t(u), z(u))$
(e.g. Euclidean setting, H_2 topology) [Maldacena, Stanford,Yang]

- fix the proper length of γ: $g|_\gamma = 1/\epsilon^2 \Rightarrow z(u) = \epsilon t'(u) + O(\epsilon^3)$
(asymptotically AdS_2 for $\epsilon \to 0$)
- the full set of different interior geometries is given by the set of all solutions $t(u)$ (zero modes, Goldstone bosons, boundary gravitons) up to the PSL(2,R) (Moebius) transformation of the original AdS_2 :

$$t(u) \to \tilde{t}(u) = \frac{at(u) + b}{ct(u) + d}, \quad \text{with } ad - cb = 1 \quad \text{(same cutout shape)}$$

> PSL(2,R) symmetry of AdS_2 is promoted to an infinite dimensional reparametrization on the boundary [Maldacena, Mertens, Cadoni]

NEARLY AdS_2 setting

JT= approximate (low dim) AdS_2: Φ measures deviations from pure AdS_2.
Since Φ is diverging near the boundary (eoms): set $\Phi_\partial = \Phi_r(u)/\epsilon$

- the bulk action induces a boundary Gibbons-Hawking-York (GHY) action

$$I_{JT}[g,\Phi] = -\frac{1}{16\pi G_N}\int_\Sigma \sqrt{-g}\,\Phi(R+2) - \frac{1}{8\pi G_N}\int_{\partial\Sigma}\sqrt{-h}\,\Phi_\partial(K-1)$$

- eom's for Φ imposes $R=-2$ (AdS_2)
- on-shell, gravitational dynamics only involves the location of the regularized boundary, depending on the boundary value of Φ_∂

$$I_{JT}[g,\Phi_\partial] = -\frac{1}{8\pi G_N}\int_{\partial\Sigma}\frac{du}{\epsilon}\,\frac{\Phi_r}{\epsilon}(K-1)$$

- the extrinsic curvature is computed as

$$K = \frac{t'(t'^2+z'^2+zz'') - zz't''}{(t'^2+z'^2)^{\frac{3}{2}}} = 1+\epsilon^2\mathrm{Sch}(t,u)$$

[Maldacena, Stanford,Yang, Mertens 23]

Schwarzian theory on the S^1 boundary

> the GHY term can written as a Schwarzian action,

$$I_S = -\frac{1}{8\pi G_N}\int_{\partial\Sigma} du\, \Phi_r(u)\mathrm{Sch}(t,u)$$

with $\mathrm{Sch}(t,u) \equiv \{t,u\} := \frac{t'''}{t'} - \frac{3}{2}\left(\frac{t''}{t'}\right)^2$ is the Schwarzian derivative, $\Phi_r(u)$ is an external coupling and the reparametrization $t(u)$ the field variable

- the boundary gravitons (zero modes) get an effective action determined by the Schwarzian

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!!! Schwarzian = gateway btw many-body quantum chaos and gravity

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- Sachdev–Ye–Kitaev (SYK) 1D QM model: N Majorana fermions at finite temperature $T = \beta^{-1}$ interacting via 4-Fermi interactions with random couplings (J).

SYK model in the low energy limit

- at low temperatures, $T \ll J$, in the large N limit, $N \gg 1$, the system develops conformal symmetry, which is spontaneously broken to SL(2,R) due to finite temperature effects
- in this limit the SYK model is effectively controlled by a field $f(\tau)$ whose dynamics is governed by the Schwarzian action

$$I_S = -\frac{N}{\beta J} \int_0^\beta d\tau \, \{f, \tau\}$$

- the Schwarzian action is not invariant under all reparametrizations of τ, but realizes non-linearly the SL(2, R) transformations due to the invariance of the Schwarzian derivative:

$$\mathrm{Diff}(S^1) \to \mathrm{SL(2,R)} \quad \text{(nearly conformal)}$$

=> nearly-JT/nearly-SYK holographic duality (AdS_2 boundary gravitons $\sim$ SYK Nambu-Goldstone bosons)

$$N \sim 1/G_N \quad J \sim 1/\Phi_r$$

Question

perspective: spacetime arises as a low energy limit of a well defined UV theory. What besides SYK?

? It possible to extend JT/SYK within a larger theory-space of possible holographic relationships?

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method: symmetry

- generalize of JT gravity in the BF formulation starting from generalized symmetries

  JT = SL(2,R)-BF theory = (linear) SL(2,R)-Poisson Sigma Model (PSM)

- boundary degrees of freedom are elements of $\mathcal{M} = \mathrm{Diff}(S^1)/\mathrm{SL}(2,\,\mathrm{R})$

> Schwarzian action arises as the coadjoint orbit action (Kirillov) of $\mathrm{Diff}(S^1)/\mathrm{SL}(2,\mathrm{R})$

> properly generalize $\mathcal{M}$ and look for its coadjoint orbit action.

JT $=$ SL(2,R)-BF theory

- bulk JT action in a first order formalism can be written in terms of a 2D BF model with SL(2,R) gauge symmetry [Jackiv, Fukuyama]

$$S_{BF} = \int_\Sigma \mathrm{Tr}(XF)$$

  where F is the curvature of the connection 1-form $A = A^a_\mu \, dx^\mu J_a$, $X = X^a J_a$ a Lie algebra valued scalar field and J_a the $\mathfrak{sl}(2, R)$ generators.
- eom's wrt X give $\epsilon^{\mu\nu} F_{\mu\nu}{}^k = 0$
- JT gravity action recovered by identifying the components of the gauge connection with the Einstein-Cartan variables: $A^{0,1}_\mu = e^{0,1}_\mu$, the zweibein on Σ and $A^2_\mu = \omega_\mu$ the Lorentz (spin) connection.
- the dilaton field $X^2 = \Phi$ and the Ricci curvature as $F_{\mu\nu}{}^2 = R_{\mu\nu}$, $F_{\mu\nu}{}^k = T_{\mu\nu}{}^k$ (torsion for $k = 0, 1$)
- $F = 0 \rightarrow \omega_\mu = -e^{-1} \epsilon^{\gamma\delta} \partial_\gamma e^k_\delta \, e_{k\mu}$ with $e = \det\{e^k_\mu\}$, $k = 0, 1$.
- mapping back by: $g_{\alpha\beta} = e^h_\alpha \, e^k_\beta \, \eta_{hk}$, and spin connection $\omega_\mu{}^{ab} = \omega_\mu \epsilon^{ab}$

BF= (linear) $SL(2,R)$-Poisson Sigma Model

- Poisson sigma model (PSM): 2D topological field theory on Σ, with target space a finite dimensional Poisson manifold (M,Π) [Ikeda, Strobl & Schaller]

$$S_{PSM}(X,A) = \int_\Sigma A_i \wedge dX^i + \frac{1}{2}\Pi^{ij}(X) A_i \wedge A_j,$$

- (X,A) real fields, w/ $X:\Sigma \mapsto M$ embedding maps and $A \in \Omega^1(\Sigma, X^*(T^*M))$ one-forms on Σ w/ values in the pull-back of the cotangent bundle over M.
- contact with the BF model requires linear Poisson tensor of Lie algebra type:

$$\Pi^{ij}(X) = f_k^{ij} X^k$$

  with f_k^{ij} $\mathfrak{sl}$(2,R) structure constants
- integrating by parts the linear *PSM* action one gets

$$S_{PSM} = S_{BF} - \int_{\partial\Sigma} X^i A_i$$

$SL(2,R)$-Poisson Sigma Model

RMK the boundary term of PSM breaks the gauge invariance and one should restrict to the gauge transformations that satisfy $\delta_g A|_{\partial\Sigma} = 0$

! such restriction is responsible for the rise of dynamical boundary degrees of freedom (PSM formalism makes it explicit)

- variation of the action wrt X and A yields

$$\delta S_{PSM} = \int_\Sigma (E.L.)\,\delta X^i + (E.L.)\,\delta A_i - \int_{\partial\Sigma} \delta X^i A_i$$

with eoms

$$D_A A = 0, \qquad dX + [X, A] =: \delta_X A = 0$$

- $D_A A = 0 \quad => $ A pure gauge: $A = g^{-1} dg$
- $dX + [X, A] =: \delta_X A = 0 \quad =>$ X is a stabilizer of A (on-shell)

$>$ dilaton dynamics $\sim$ infinitesimal gauge transformation preserving A along X on-shell corresponds to gauge transformations that satisfy $\delta_g A|_{\partial\Sigma} = 0$

$SL(2,R)$-Poisson Sigma Model

- given

$$\delta S_{PSM} = \int_{\Sigma} (E.L.)\, \delta X^i + (E.L.)\, \delta A_i - \int_{\partial\Sigma} \delta X^i A_i$$

a well-defined variational principle requires:

either fixing the boundary values of the fields => no boundary dynamics (bad)

or adding counter boundary term => matching GHY (good - how?)

- natural solution: boundary Casimir function

$$S_{(\Sigma+\partial\Sigma)} = S_{PSM} + \frac{1}{2}\int_{\partial\Sigma} X^i X_i \,\mathrm{d}u$$

& extra condition: $X_i|_{\partial\Sigma}\,\mathrm{d}u = A_i|_{\partial\Sigma}$

why the X fields, restricted at the boundary, generate the Poisson algebra of currents associated with $\mathfrak{sl}(2,R)$, which admits a natural class of quadratic functions, the Sugawara tensors, which close the Virasoro algebra

- boundary dynamics is related with the Schwarzian action <=> GHY
[Mertens, Turiaci, Verlinde]

How does such correspondence come about?

- PSM bulk action is invariant under the SL$(2, R)$ gauge group and under diffeomorphisms.
- PSM boundary action, nonvanishing on-shell, explicitly breaks the Diff(S^1) invariance, since the boundary condition of the fields are not invariant under reparametrisation of S^1.
- the SL$(2, R)$ gauge symmetry is preserved because of the stabilizerness condition $\delta_\lambda A|_{\partial\Sigma} = 0$, with $\lambda \in \mathfrak{sl}(2, R)$.
- PSM boundary action must then depend on fields transforming in some representation of the coset space Diff(S^1)/SL(2,R)

> Schwarzian action arises as the coadjoint orbit action (Kirillov) of the Virasoro group Diff(S^1)/SL(2,R)

Yang–Mills extensions of JT gravity

GOAL generalized models of dilaton gravity based on a gauge group G, with suitable extension of the Schwarzian dynamics governed by one-dimensional actions located at the boundary of the space-time.

Recipe:

- generalized JT gravity possible any time the Lie algebra of symmetries contains an $\mathfrak{sl}(2, R)$ sub-algebra and another sub-algebra which is ad-invariant under the first one
- bulk: $SL(2, R) \subset G$: the $\mathfrak{sl}(2, R)$ subalgebra allows for an identification of this sl(2)-part with Cartan variables (zweibein and dualized Lorentz connection).
- boundary: single out residual degrees of freedom, which results in non-abelian gauge fields minimally coupled with gravity
- e.g. non-abelian BF-theories with gauge group G= SL(2,R) $\times$ K [Grumiller18, ...]

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our take: $\mathfrak{so}(2, 2)$-Poisson sigma model over a 2D manifold $\Sigma = R \times S^1$

$\mathfrak{so}(2,2)$ - Poisson Sigma Model

Fields: decomposition pattern is crucial [Jackiv 92]

- Ω be the $\mathfrak{so}(2,2)$-valued connection 1-form over Σ.
- $\mathfrak{so}(2,2)$ algebra: $[J_i, J_j] = \epsilon_{ij}^k J_k, \qquad [P_i, P_j] = \alpha\epsilon_{ij}^k J_k, \qquad [J_i, P_j] = \epsilon_{ij}^k P_k$
- def. new generators $L_i(+), R_i(-)$ which transforms like vectors under J_i's action:

$$L_i = \frac{1}{2}(J_i + P_i) \longrightarrow [L_i, L_j] = \epsilon_{ij}^k L_k \quad \text{close inv. subalg.}$$

s.t. $\mathfrak{so}(2,2) \simeq \mathfrak{sl}(2,R)_R \oplus \mathfrak{sl}(2,R)_L$

or decompose $\mathfrak{so}(2,2)$ non-chiral basis

$$[J_i, J_j] = \epsilon_{ij}^k J_k, \qquad [L_i, L_j] = \epsilon_{ij}^k L_k, \qquad [J_i, L_j] = \epsilon_{ij}^k L_k.$$

> $\mathfrak{so}(2,2) \neq \mathfrak{sl}(2,R)_J \oplus \mathfrak{sl}(2,R)_L$: two $\mathfrak{sl}(2,R)_{J,L}$ sub-algebras.

- refer to the $\mathfrak{sl}(2,R)_J$ sub-algebra as the gravitational sector and to the $\mathfrak{sl}(2,R)_L$ as the Yang-Mills sector. Hence

$$\Omega = A_i J^i + B_i L^i$$

$\mathfrak{so}(2,2)$- Poisson Sigma Model

- $\mathfrak{Z}^i$ embedding maps $\mathfrak{Z}^i : \Sigma \to \mathfrak{so}(2,2)^*$ with linear Poisson brackets

$$\{\mathfrak{Z}^i, \mathfrak{Z}^j\} = \Pi^{ij}(\mathfrak{Z}) = f^{ij}_k \mathfrak{Z}^k$$

- similar "non-chiral" decomposition (dual basis):$\mathfrak{Z} = \mathfrak{X}^i J_i + \mathfrak{Y}^i L_i$
- Poisson Sigma model:

$$S_{PSM}(\mathfrak{Z}, \Omega) = \int_\Sigma d\Omega_i \wedge \mathfrak{Z}^i + \frac{1}{2} \Pi^{ij}(\mathfrak{Z})\, \Omega_i \wedge \Omega_j$$

- eom: $\mathfrak{D}_A A = 0, \qquad \mathfrak{D}_\Omega B = \epsilon_i^{hk} B_h A_k L^i,$
  $\delta_{\mathfrak{X}} A = 0, \qquad \delta_{\mathfrak{Y}} \Omega = -\epsilon_i^{hk} \mathfrak{X}_h B_k L^i$

> on-shell A is pure gauge wrt $SL(2,R)_J$

> on-shell $\mathfrak{X}$ field is stabilizer for A ($\mathfrak{Y}$ stabilizer for Ω for suitable b.c.)

$\mathfrak{D}_\Omega B$ A-sector of the model equivalent to the ordinary JT gravity, while B behaves like a gauge field minimally coupled with gravity.

Boundary action: coadjoint orbits of $(\mathrm{Diff}(S^1) \ltimes \mathrm{LG})/\mathrm{SL}(2,\mathrm{R})$

- As it is the case for the $\mathfrak{sl}(2,R)$-PSM, if we insert a boundary Casimir counter-term in $\mathfrak{Z}^2$ and set the same boundary condition $\Omega|_{S^1} = \mathfrak{Z}|_{S^1}\mathrm{d}u$, we get a particle on a group action
- on-shell reduces to :

$$S_{PSM}|_{S^1} = \frac{1}{2}\int_{S^1}\left(\mathfrak{X}^i\mathfrak{X}_i + \mathfrak{X}^i\mathfrak{Y}_i + \mathfrak{Y}^i\mathfrak{Y}_i\right)\mathrm{d}u$$

RMK expect boundary action to comprise:

- a Schwarzian for the $\mathfrak{sl}(2,R)_J$-PSM sector ($\mathfrak{X}^2$)
- a particle-on-a-group term for the YM sector ($\mathfrak{Y}^2$)
- plus interactions

! picture realised via a strong stabilizerness condition: $\mathfrak{X}_i|_{S_1} d\tau = -B_i|_{S_1}$

- together with $\Omega|_{S^1} = \mathfrak{Z}|_{S^1}\mathrm{d}u$ implies $\mathfrak{Y}_i = -\mathfrak{X}_i$

=> $\mathfrak{Z}|_{S_1}$ is no longer $\mathfrak{so}(2,2)$-valued, rather is $\mathfrak{sl}(2,R)$-valued: residual SL(2R) symmetry on S^1

=> boundary action given by a coadjoint orbit of a product $\mathrm{Diff}(S^1)\ltimes\widehat{SL(2,R)}$

Boundary action: coadjoint orbits of $(\mathrm{Diff}(S^1) \ltimes \mathrm{LG})/\mathrm{SL}(2,\mathrm{R})$

What's left?

- compute the extended Schwarzian action as the coadjoint orbit action of the Virasoro-Kac-Moody semidirect product group $(\mathrm{Diff}(S^1) \ltimes \mathrm{LG})/\mathrm{SL}(2,\mathrm{R})$
  - the Kac-Moody sector reflects the residual gauge symmetry at the boundary
  - the Virasoro group has the exact same role played in JT gravity

!!! details in LUCIO VACCHIANO'S TALK! 

Insigths from $\mathfrak{so}$(2,2)- Poisson Sigma Model:

- partial breaking of the $\mathfrak{so}(2,2)$ gauge symmetry is responsible for the rising of extra edge modes on the boundary
- the model provides a gravitational dual for SYK-like generalization with internal symmetries, whose low energy dynamics is characterized by a $\mathfrak{diff}(S^1) \ltimes \hat{\mathfrak{g}}$ symmetry [Yoon]
- the $\mathfrak{so}(2,2)$ algebra connect the model with 3D gravity: 3D Chern-Simons theories with WZW term at the boundary, once dimensionally reduced, give 2D BF theories with the particle on a group action at the 1D boundary [Mertens 18]
- Chern-Simons-WZW theory whose dimensional reduction gives the $\mathfrak{so}(2,2)$-PSM is the 3D topological theory describing AdS_3 geometry.

Thank You