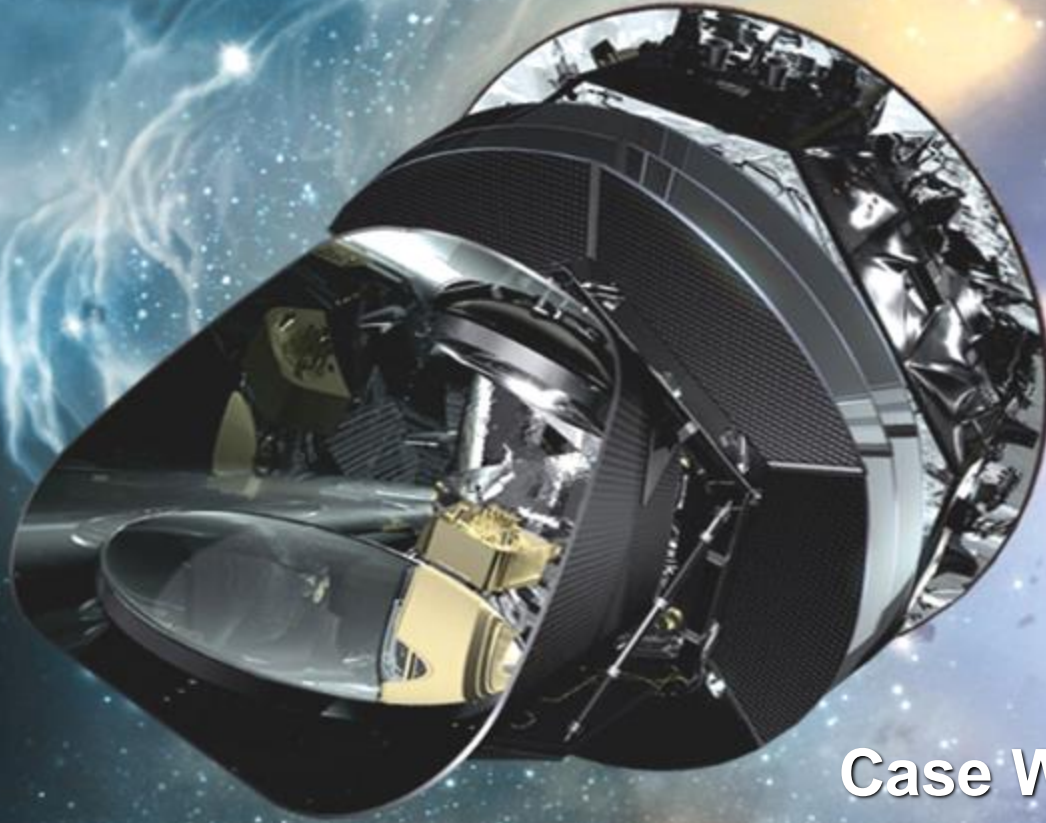


# The Universe is Not Isotropic



Glenn Starkman  
Case Western Reserve University  
Dark Side of the Universe  
Sept 2024



# The Universe is Not Isotropic



Joann Jones, C. Copi, Y. Akrami  
A. Jaffe, A. Kosowsky, T. Pereira  
Stefano Anselmi, Fernando Cornet, Deyan Mihaylov, Andrius Tomasiunas, Javier Carron  
Ananda Smith, Mikel Martin, Samanta Saha, Amirhossein Samandar, Quinn Taylor  
A. Bernui, N. Cornish, F. Ferrer, D. Huterer, L. Knox, D. Schwarz, D. Spergel  
S. Aiola, M. O'Dwyer, O. Gungor, J. Gurian, J. Oskilt, P. Petersen, V. Vardanyan, P. Vaudrevangé, A. Yoho,



Mikel  
MARTIN



Amirhossein  
SAMANDAR



Stefano  
ANSELMINI



Deyan  
MIHAYLOV



Andrius  
TAMOŠIŪNAS



Javier  
CARRON DUQUE



Yashar  
AKRAMI



Craig  
COPI



Fernando  
CORNET-GOMEZ



Andrew  
JAFFE



Arthur  
KOSOWSKY

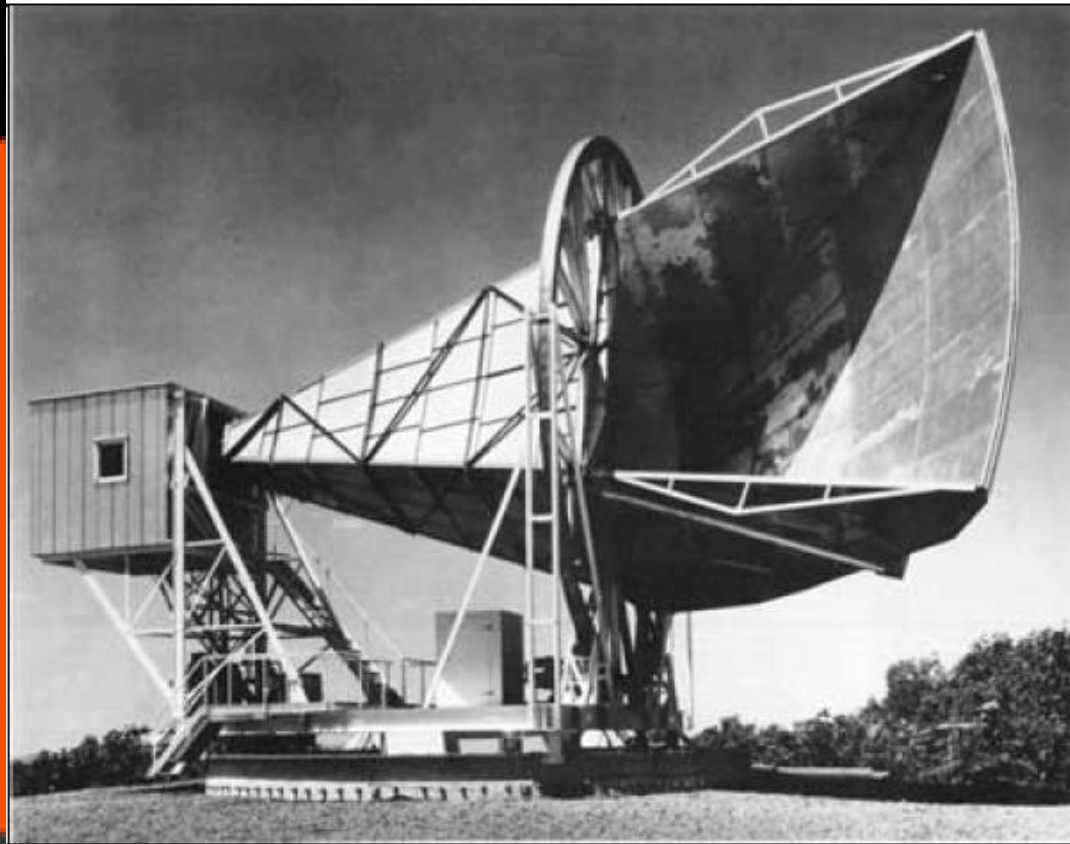


Thiago  
PEREIRA



Glenn  
STARKMAN

# *Penzias & Wilson (1965)*



Horn Antenna — Holmdel, New Jersey.  
Horn Antenna, circa 1960.  
*(Photo Credit: Bell Labs)*

**$T=2.728K$**



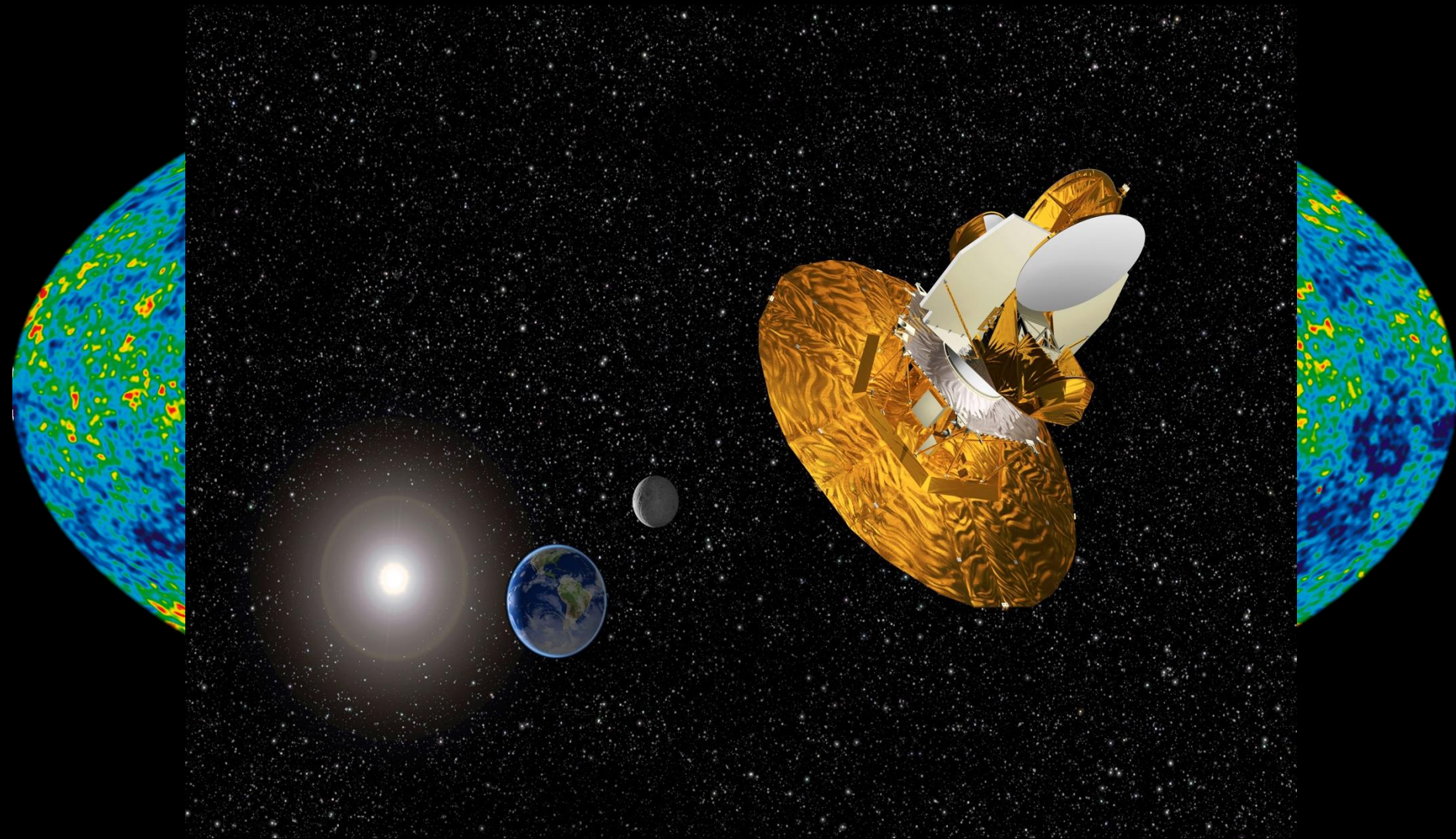
# *COBE - DMR*



NASA/COBE-DMR science team

$T=18\mu\text{K}$

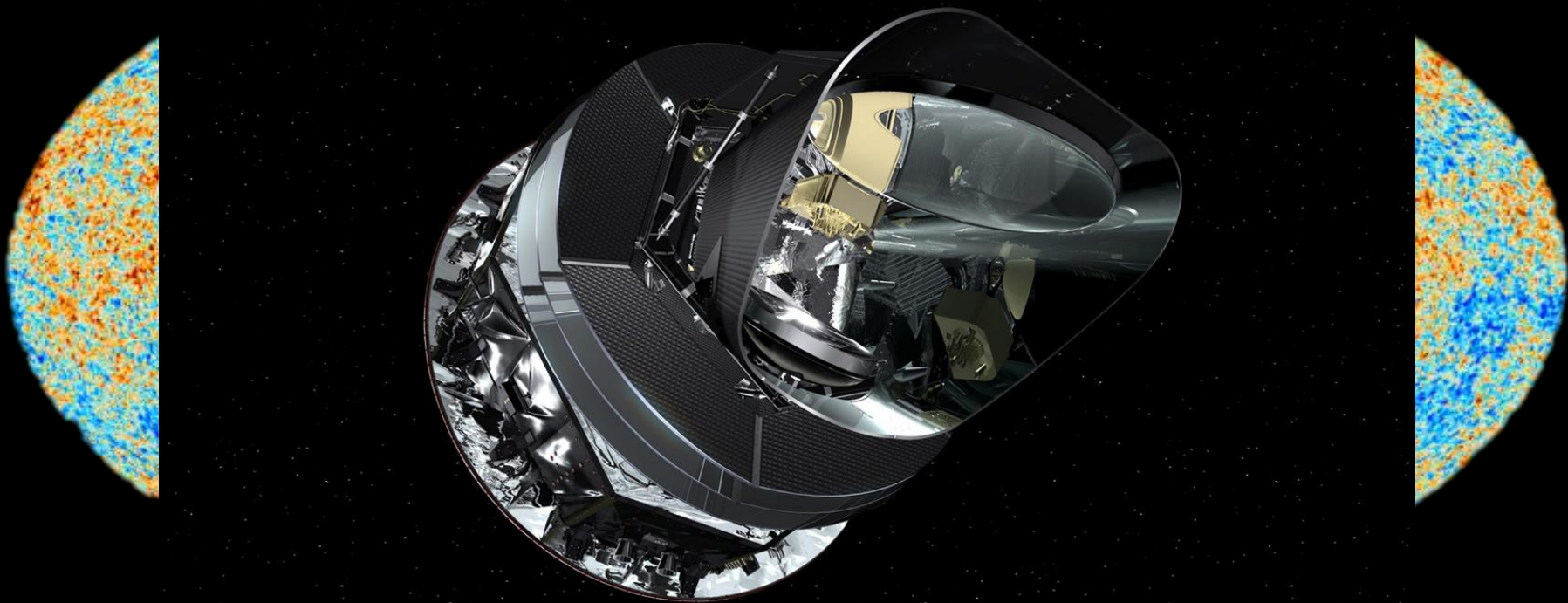
# *WMAP*



NASA/WMAP Science team



# *Planck*



ESA/Planck Science team

# *Angular Power Spectrum*

$$\Delta T = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \varphi)$$



# Angular Power Spectrum

$$\Delta T = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \varphi)$$

Standard model for the fluctuations (inflation):

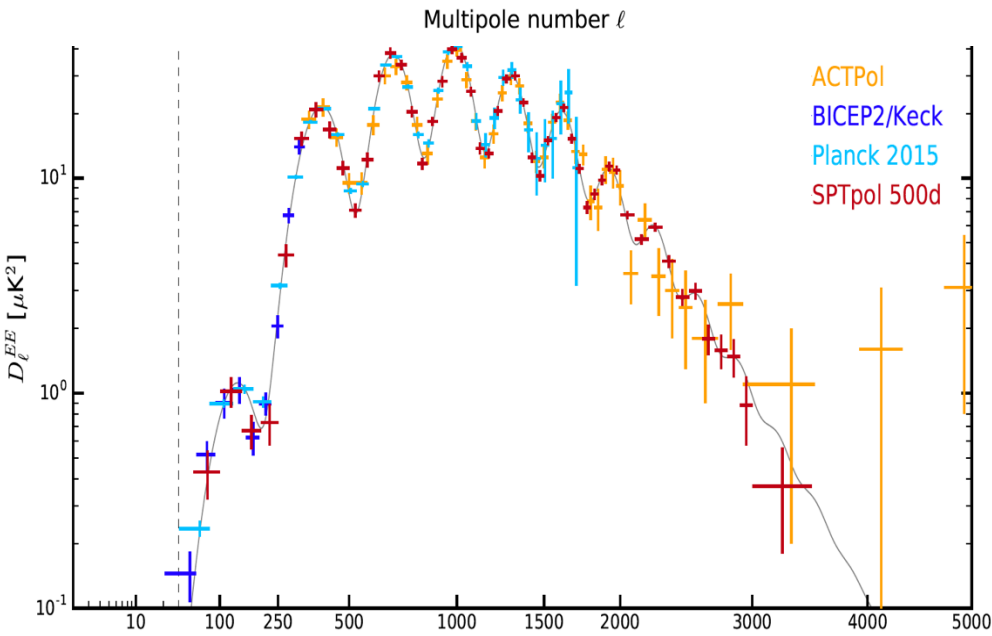
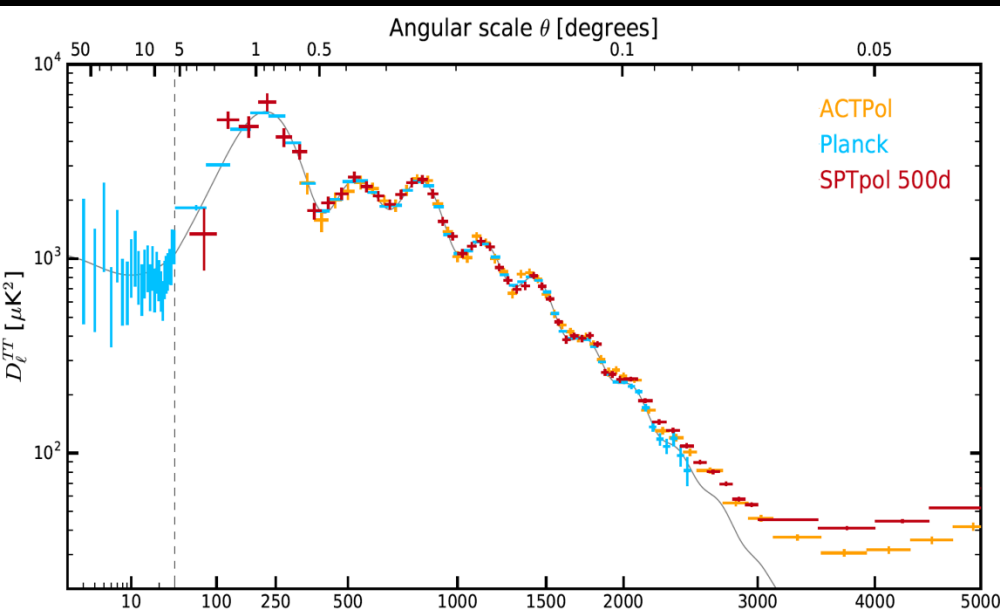
- Sky is statistically isotropic
- $a_{\ell m}$  are independent Gaussian random variables

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_{\ell} \delta_{\ell \ell'} \delta_{m m'}$$

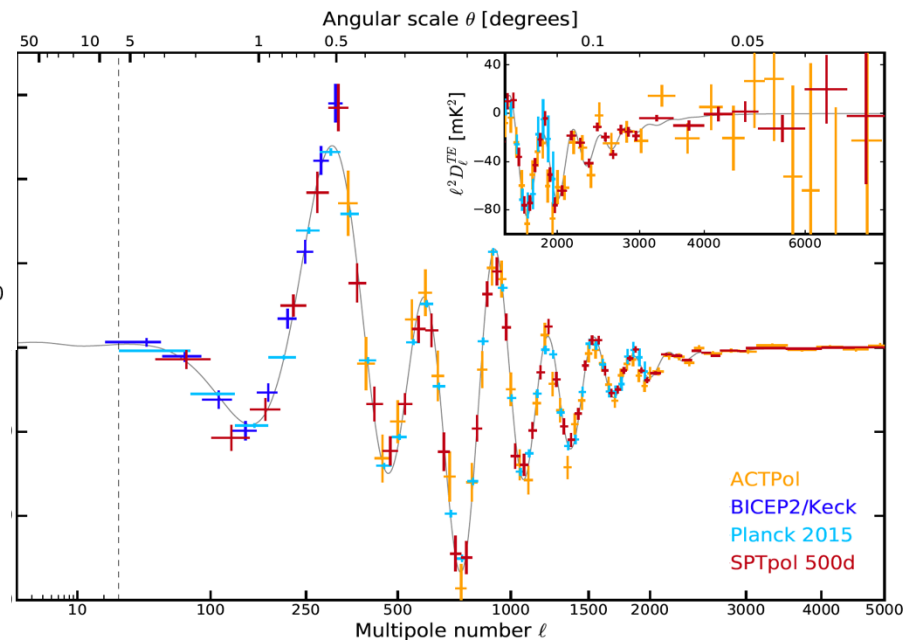
ALL interesting information is contained in:

$$C_{\ell} = (2\ell + 1)^{-1} \sum_m |a_{\ell m}|^2$$

# Angular Power Spectrum



6 or 7 parameter fit  
to  $\gg 7$  points



SPTPol arXiv:1707.09353

- **Astonishing**  
experimental accomplishment
- **Remarkable**  
agreement with theory

**BUT !**



## Standard Model for fluctuations (inflation):

- Sky is statistically isotropic
- $a_{\ell m}$  are independent (very nearly) Gaussian random variables

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_{\ell} \delta_{\ell \ell'} \delta_{m m'}$$

ALL interesting information is contained in:  $C_{\ell}$

*Shouldn't we check?!*

# Outline

## Troubles in (iso)tropical paradise:

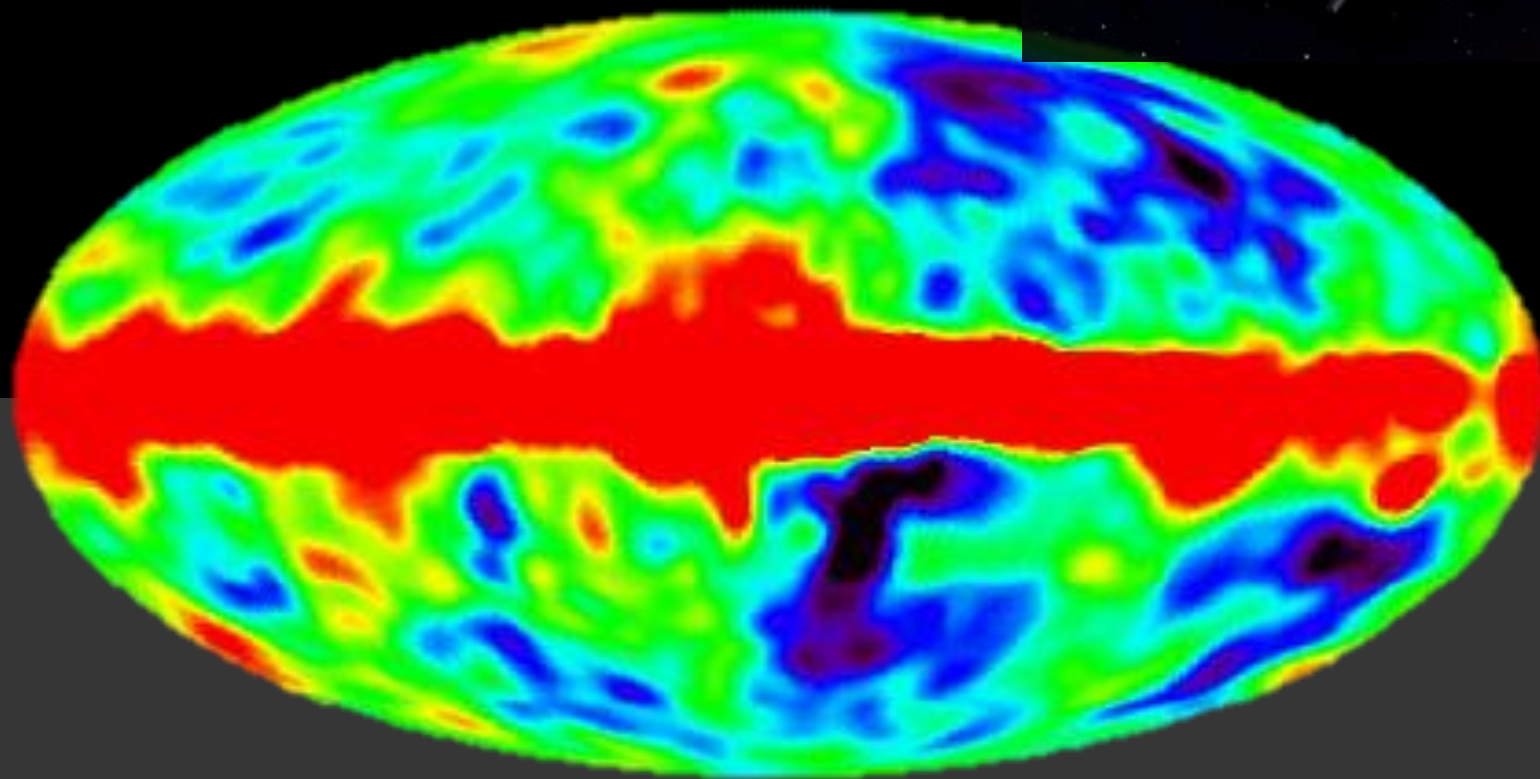
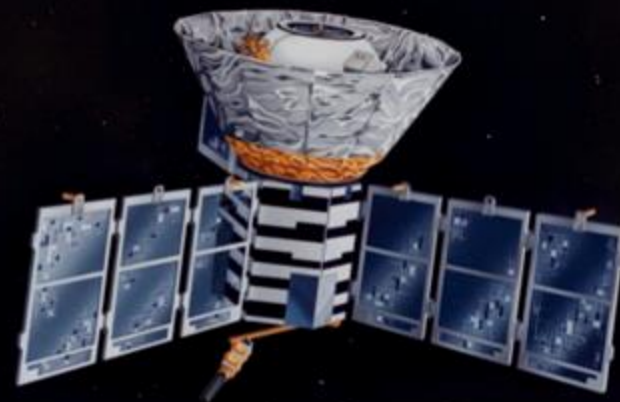
- (low- $\ell$ ) large-angle problem:  $C(\theta > 60^\circ) \simeq 0$
- low- $\ell$  alignments
- hemispheres
- parity
- *etcetera*

## Bottom line:

**The Universe is not statistically isotropic**

**Promises of topological musings**

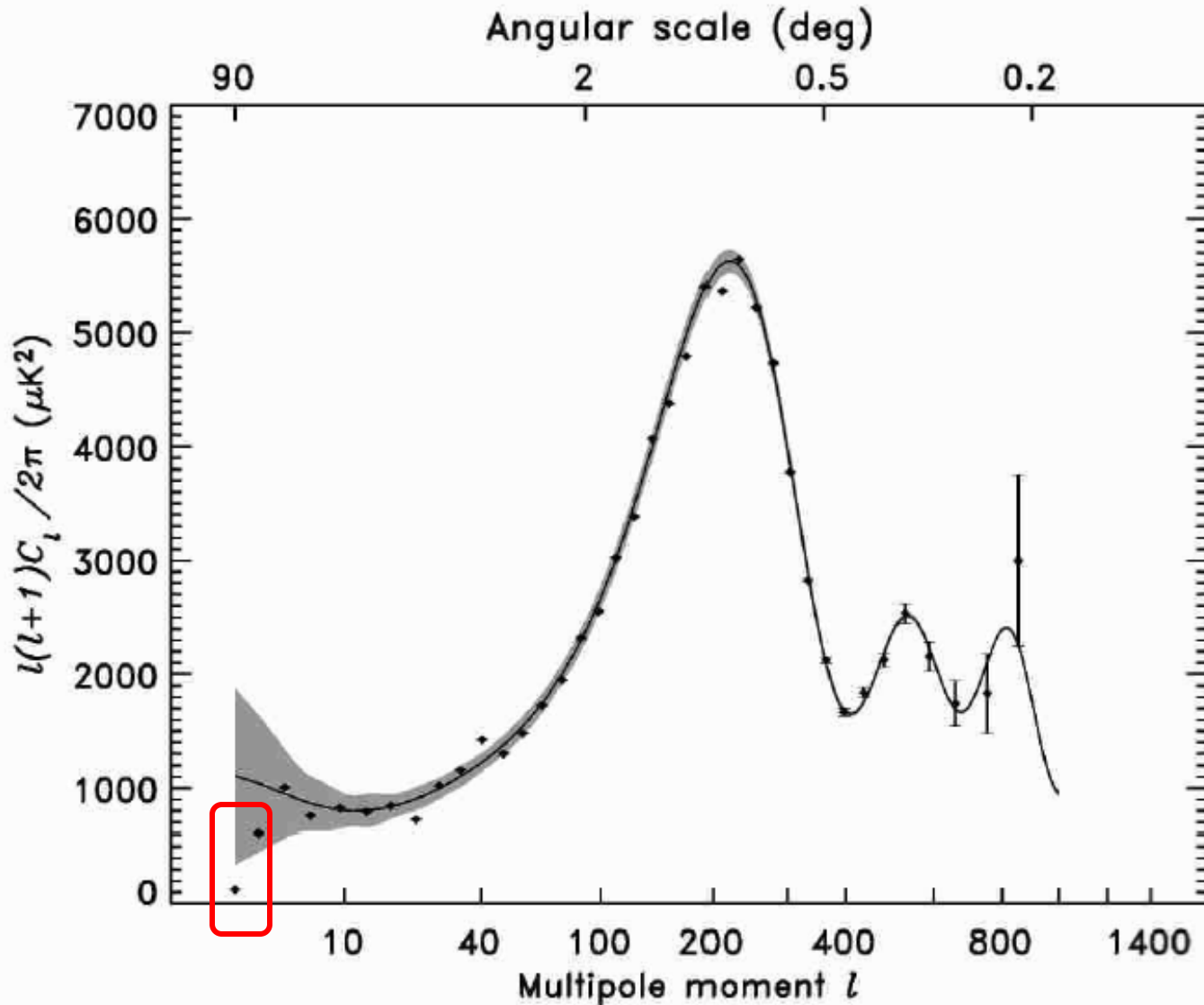
# ***COBE - DMR***



NASA/COBE-DMR science team



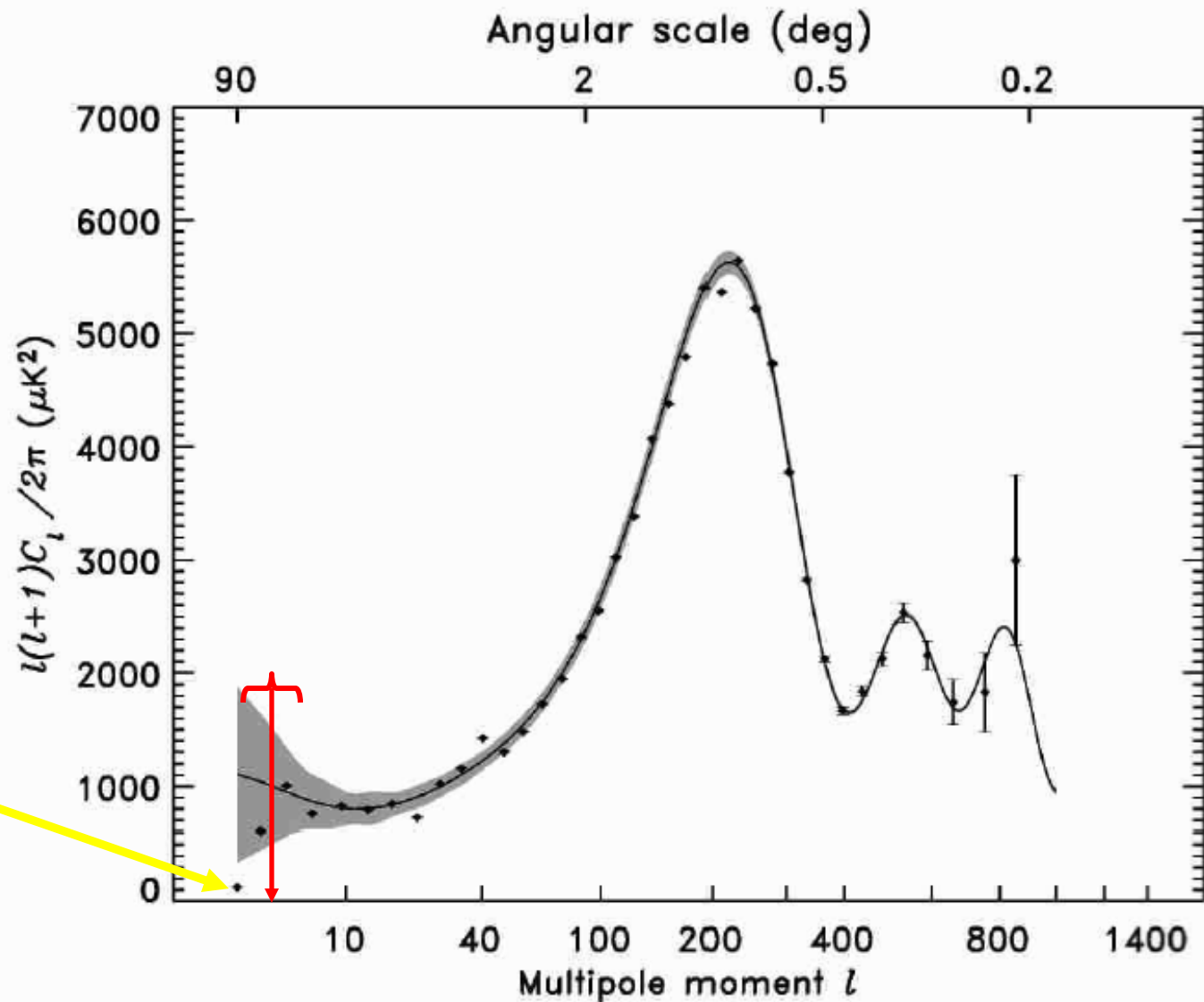
# *The first hint: “The low- $\ell$ Anomaly”*



NASA WMAP  
Science Team  
WMAP 1

The uncorrelation ...

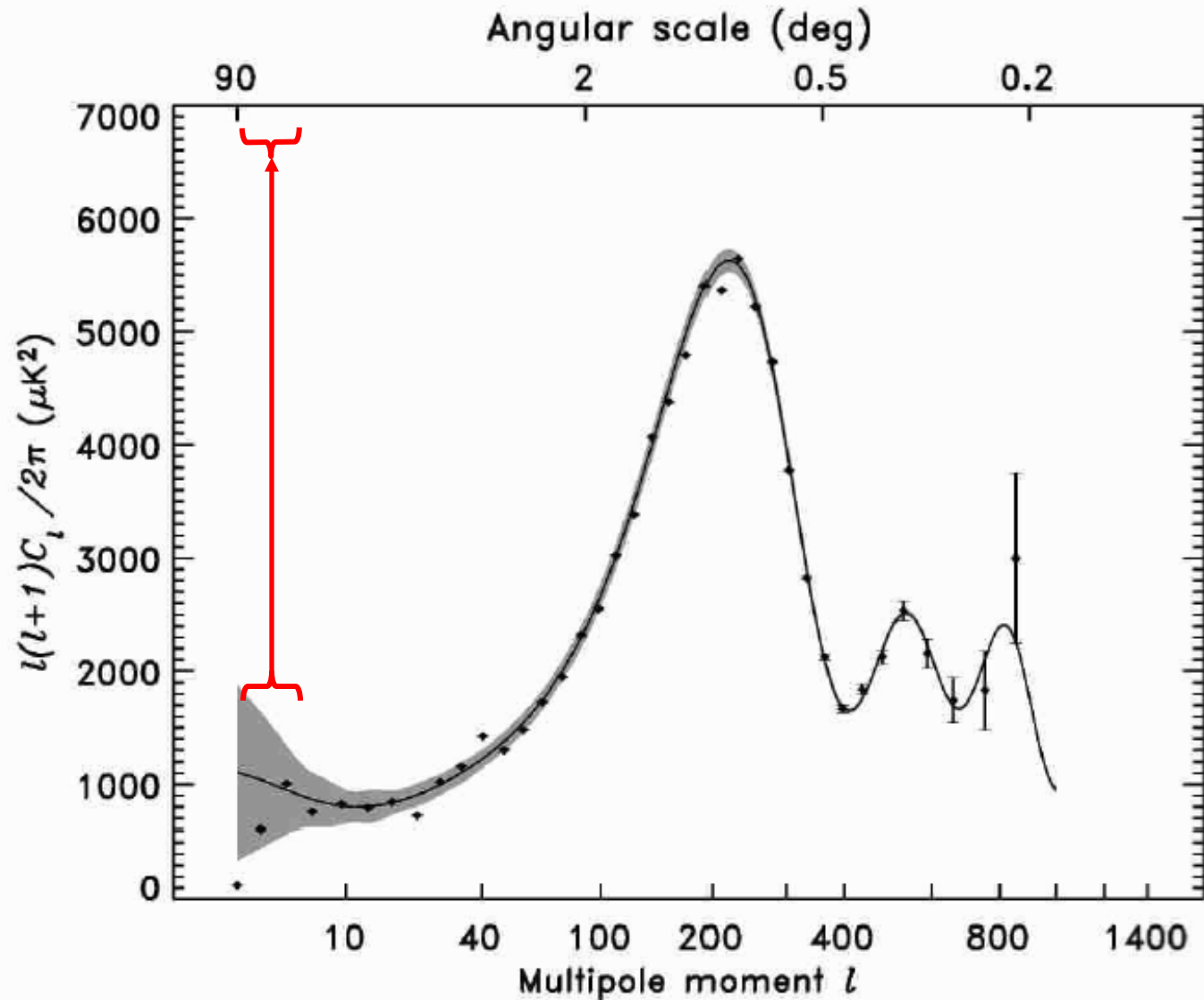
# “The Low- $l$ Anomaly”



The low quadrupole



# “The Large-Angle Anomaly”



# Angular Correlation Function $C(\theta)$

$$C(\theta) = \langle T(\Omega_1)T(\Omega_2) \rangle_{\Omega_1 \cdot \Omega_2 = \cos\theta}$$

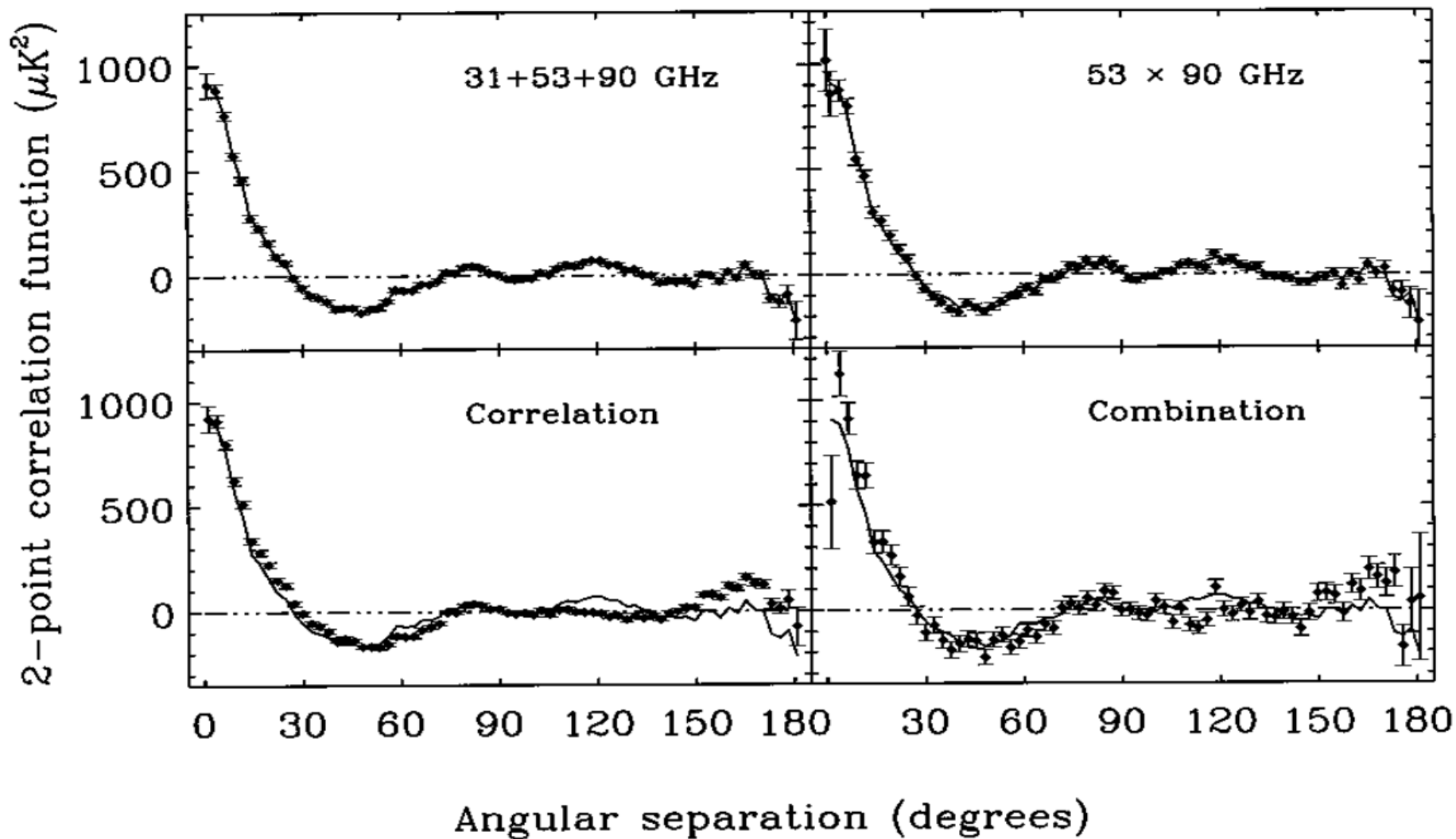
But  $C(\theta) = \sum_l C_l P_l(\cos(\theta))$

$\Rightarrow$  Same information as  $C_l$ , just differently organized

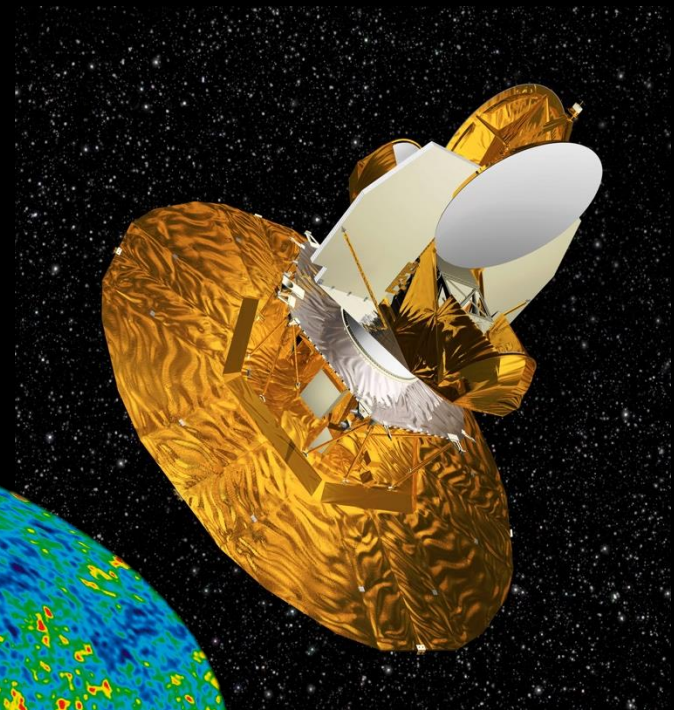
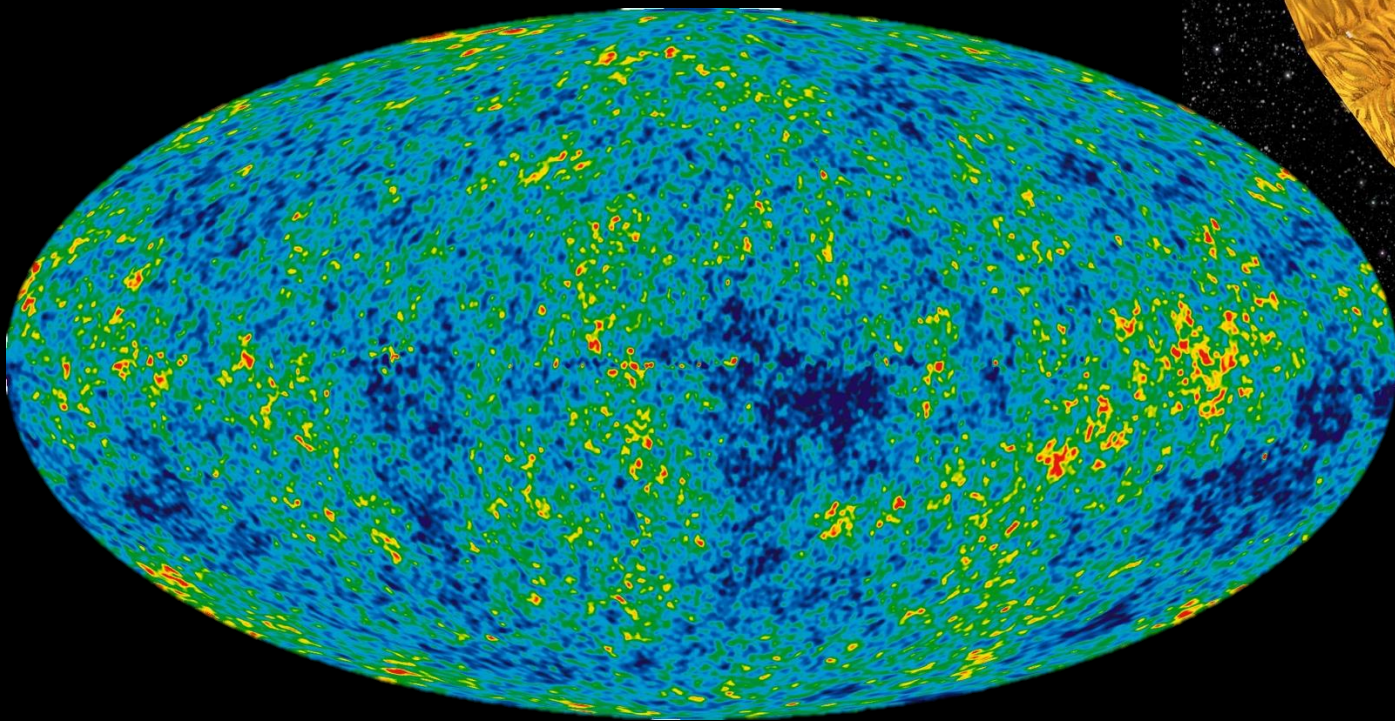
## TWO-POINT CORRELATIONS IN THE *COBE*<sup>1</sup> DMR FOUR-YEAR ANISOTROPY MAPS

G. HINSHAW,<sup>2,3</sup> A. J. BANDAY,<sup>2,4</sup> C. L. BENNETT,<sup>5</sup> K. M. GÓRSKI,<sup>2,6</sup> A. KOGUT,<sup>2</sup> C. H. LINEWEAVER,<sup>7</sup> G. F. SMOOT,<sup>8</sup> AND E. L. WRIGHT<sup>9</sup>

Received 1996 January 9; accepted 1996 March 21

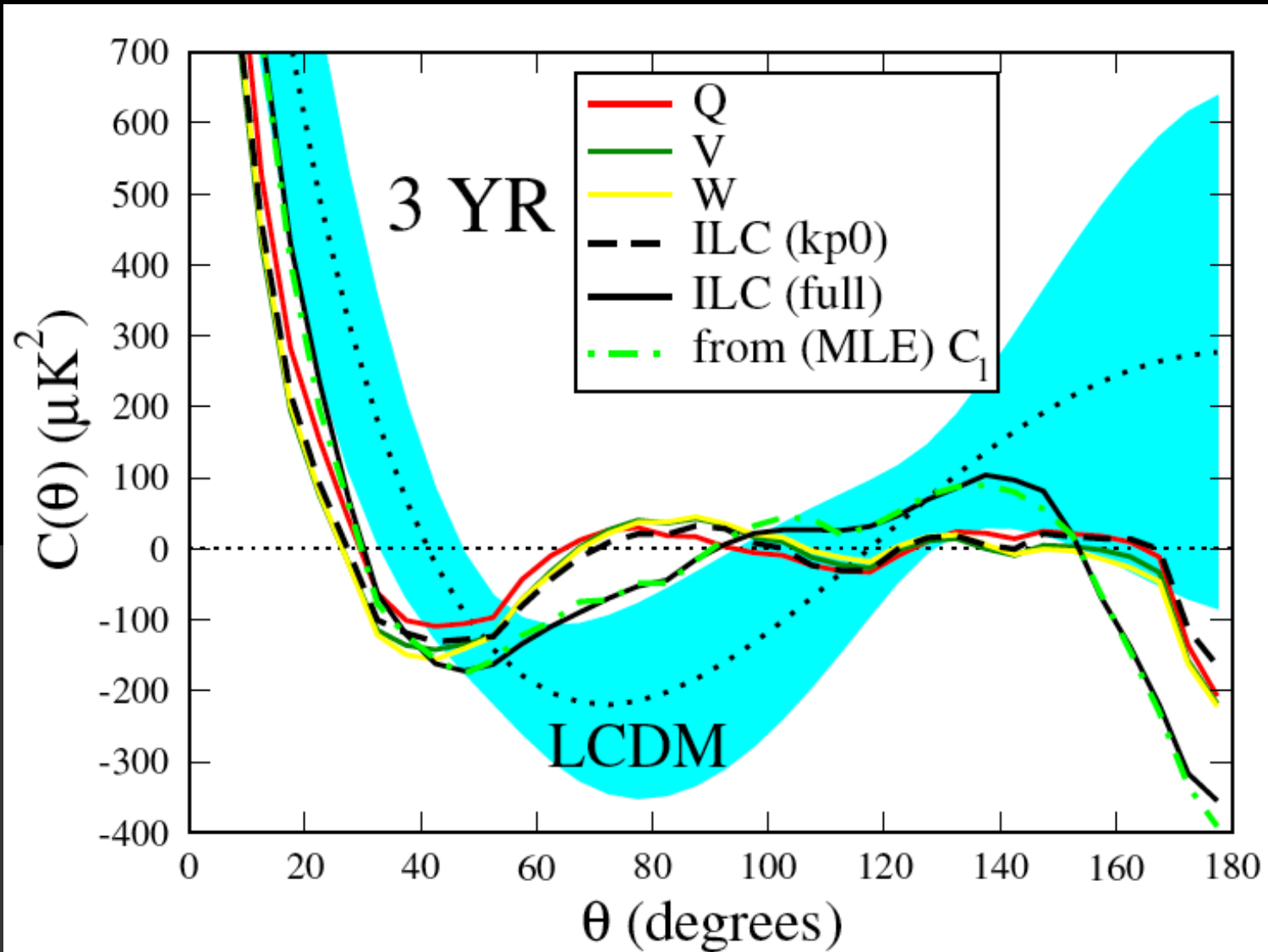


# *WMAP*



NASA/WMAP Science team

# Two-point angular correlation function





# Is the Large-Angle Anomaly Significant?

One measure (WMAP1):

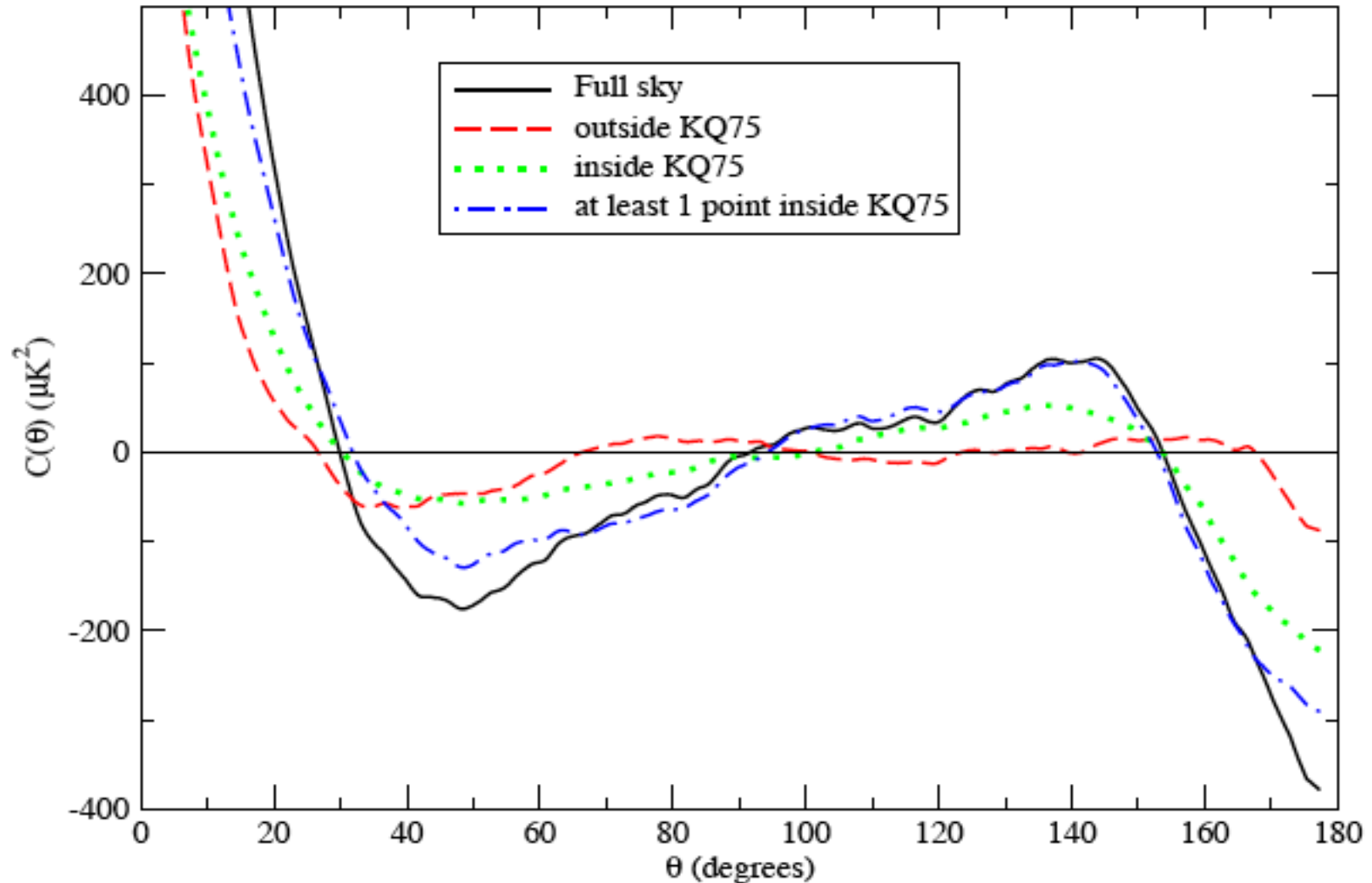
$$S_{1/2} = \int_{-1}^{1/2} [C(\theta)]^2 d \cos \theta$$

# WMAP statistics of $C(\theta)$

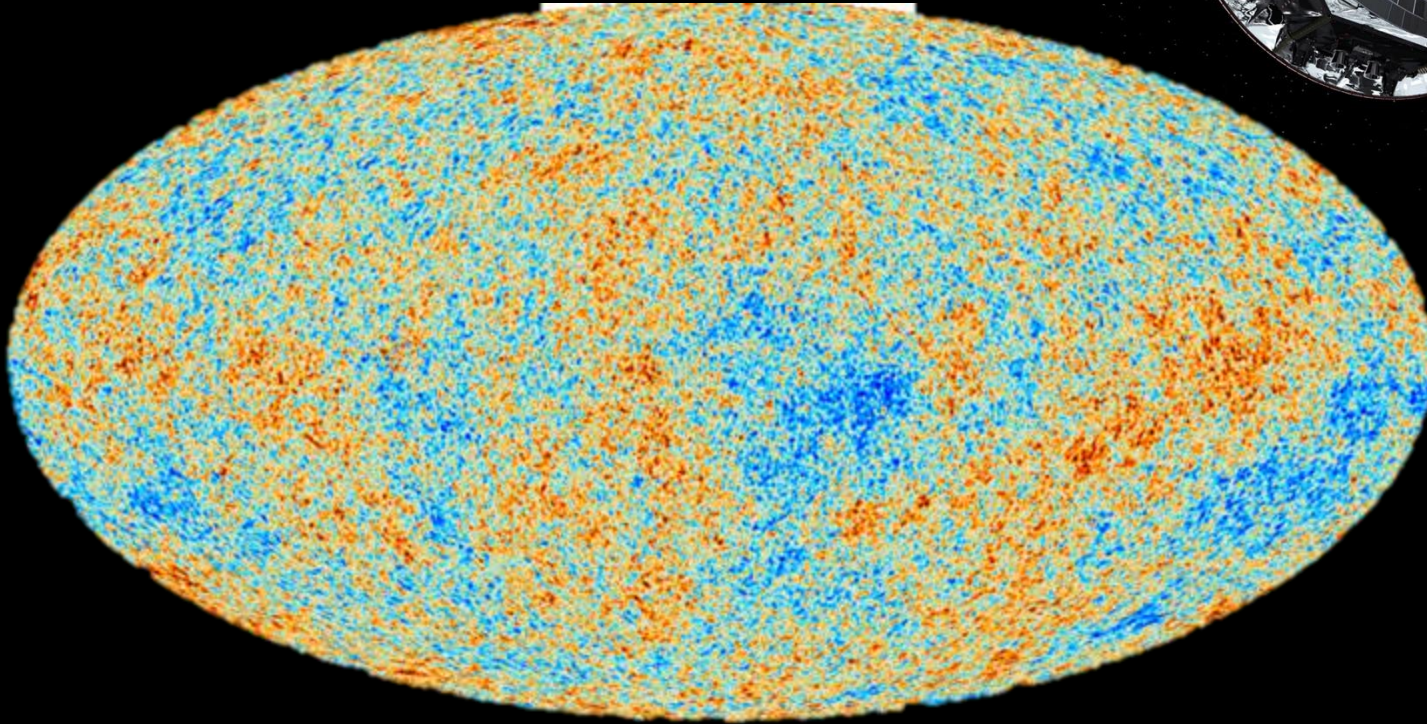
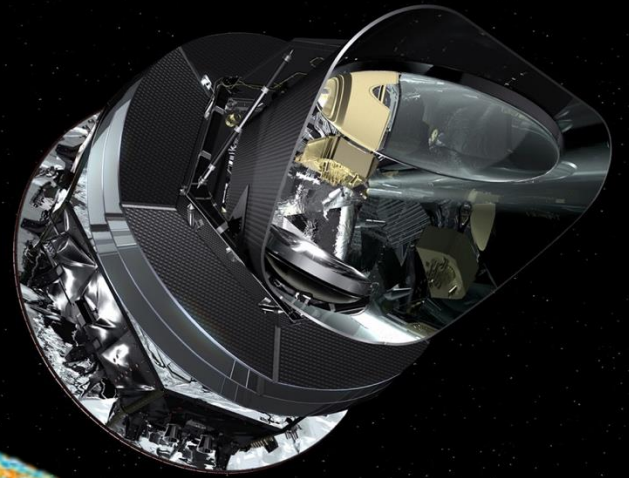
Table 1. The  $C_\ell$  calculated from  $C(\theta)$  for the various data maps. The WMAP (pseudo and reported MLE) and best-fit theory  $C_\ell$  are included for reference in the bottom five rows.

Data Source	$S_{1/2}$ ( $\mu\text{K}$ ) <sup>4</sup>	$P(S_{1/2})$ (per cent)	$6C_2/2\pi$ ( $\mu\text{K}$ ) <sup>2</sup>	$12C_3/2\pi$ ( $\mu\text{K}$ ) <sup>2</sup>	$20C_4/2\pi$ ( $\mu\text{K}$ ) <sup>2</sup>	$30C_5/2\pi$ ( $\mu\text{K}$ ) <sup>2</sup>
V3 (kp0, DQ)	1288	0.04	77	410	762	1254
W3 (kp0, DQ)	1322	0.04	68	450	771	1302
ILC3 (kp0, DQ)	1026	0.017	128	442	762	1180
ILC3 (kp0), $C(> 60^\circ) = 0$	0	—	84	394	875	1135
ILC3 (full, DQ)	8413	4.9	239	1051	756	1588
V5 (KQ75)	1346	0.042	60	339	745	1248
W5 (KQ75)	1330	0.038	47	379	752	1287
V5 (KQ75, DQ)	1304	0.037	77	340	746	1249
W5 (KQ75, DQ)	1284	0.034	59	379	753	1289
ILC5 (KQ75)	1146	0.025	81	320	769	1156
ILC5 (KQ75, DQ)	1152	0.025	95	320	768	1158
ILC5 (full, DQ)	8583	5.1	253	1052	730	1590
WMAP3 pseudo- $C_\ell$	2093	0.18	120	602	701	1346
WMAP3 MLE $C_\ell$	8334	4.2	211	1041	731	1521
Theory3 $C_\ell$	52857	43	1250	1143	1051	981
WMAP5 $C_\ell$	8833	4.6	213	1039	674	1527
Theory5 $C_\ell$	49096	41	1207	1114	1031	968

# Origin of $C(\theta)$

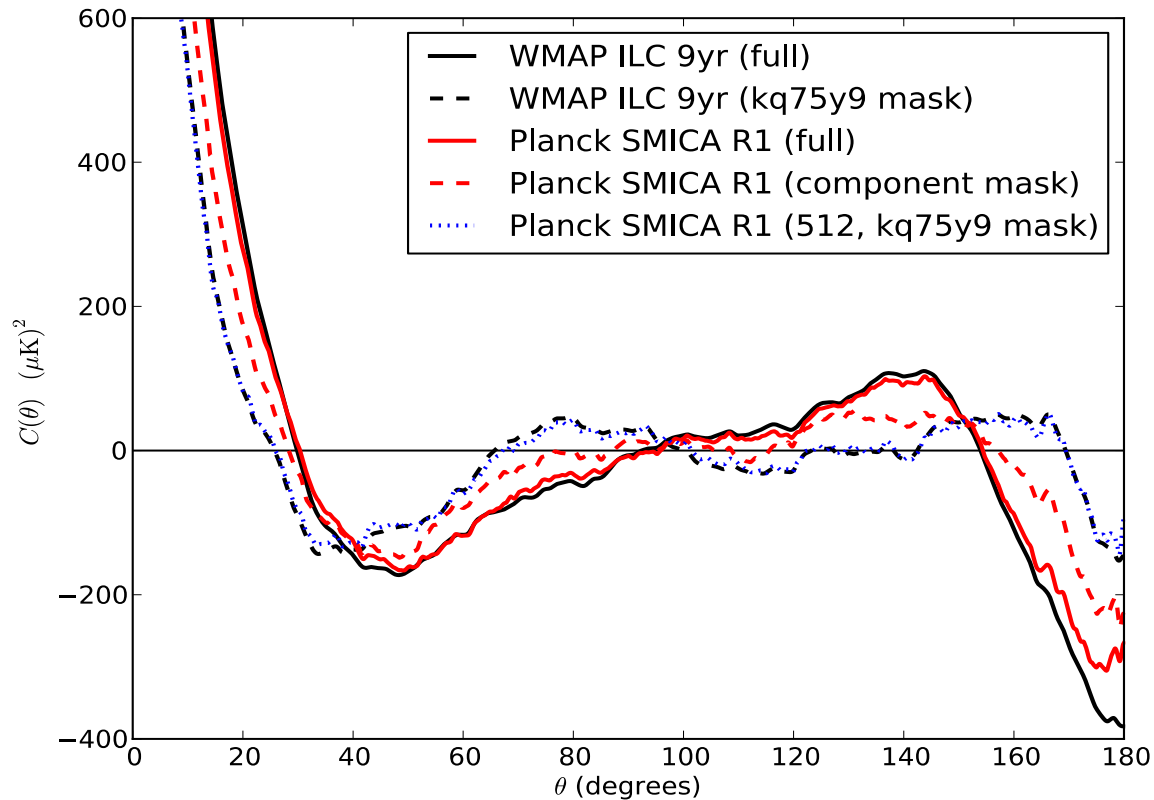


# *Planck*



**Did this change in Planck?**





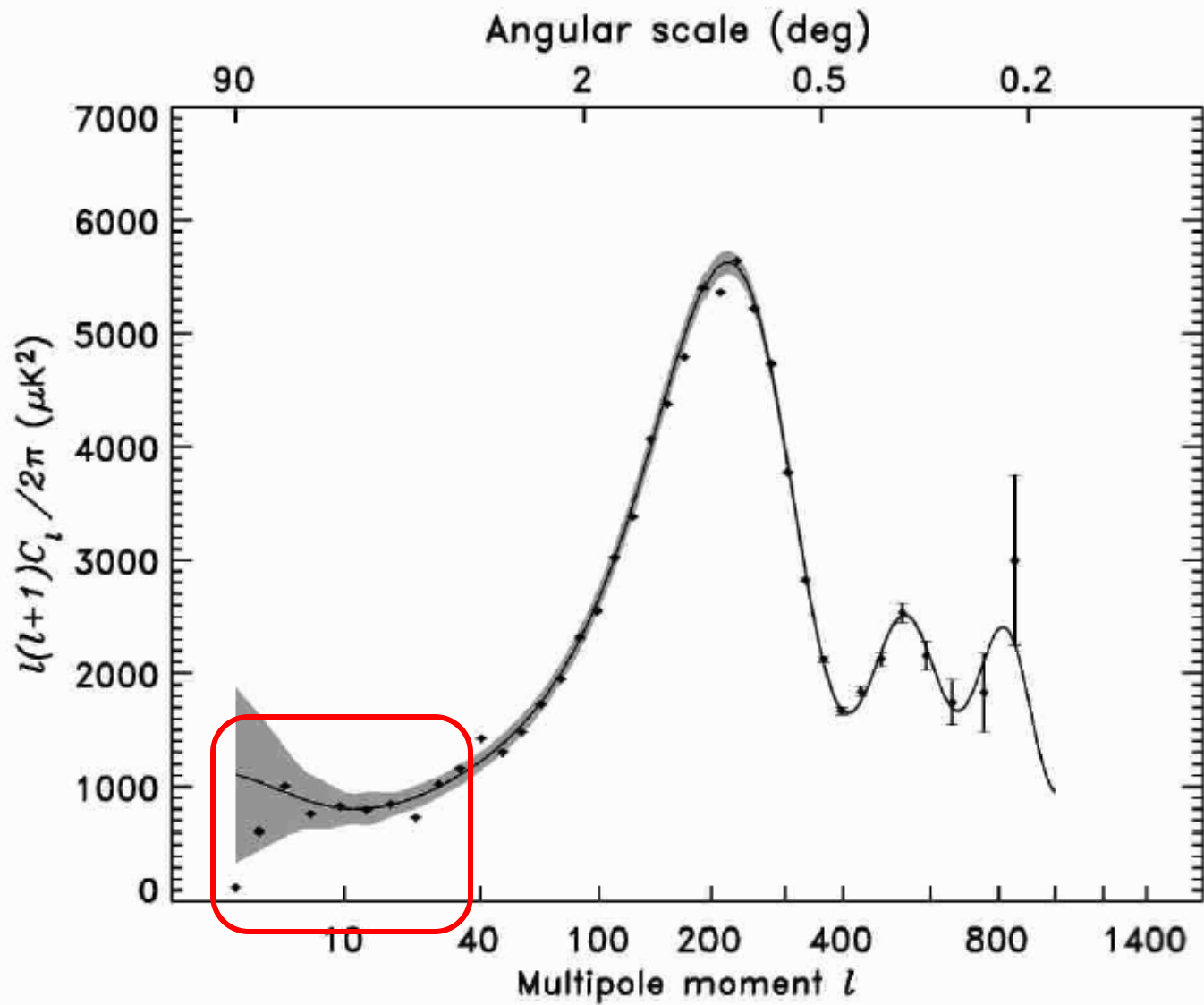
Planck 2018  
A&A 641, A7  
(2020)

Statistic	$S_{1/2}^{XY} [\mu K^4]$			
	Comm.	NILC	SEVEM	SMICA
$TT$ . . . . .	1209.2	1156.6	1146.2	1142.4
	Probability [%]			
	Comm.	NILC	SEVEM	SMICA
	>99.9	>99.9	>99.9	>99.9

# Statistics of $C(\theta)$

- 0.03-0.1% of realizations of the concordance model of inflationary  $\Lambda$ CDM have so little cut sky large-angle correlation !

and most of those have all low- $\ell$   $C_\ell$  small



# The Conspiracy theory: minimizing $S_{1/2}$

To obtain  $S_{1/2} < 1000$  with the observed  $C_\ell$   
requires correlating  $C_2, C_3, C_4$  &  $C_5$ !

# Violation of GRSI

Even if we replaced all the theoretical  $C_\ell$  by their measured values up to  $\ell=20$ , cosmic variance would give only a 3% chance of recovering so little correlation in a particular realization...

and most of those would be much poorer fits to that theory than is the current data



# Explaining small $S_{1/2}$

1. “Didn’t that go away?”
2. “I never believe *a posteriori* statistics.”
3. Cosmic variance -- “I never believe anything less than a (choose one:)  $5\sigma$   $10\sigma$   $20\sigma$  result.”
4. “Inflation can do that”

## 5. New physics that correlates $C_l$ 's

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle \not\propto \delta_{\ell\ell'}$$

$$\Rightarrow \langle a_{\ell m} a_{\ell' m'}^* \rangle \neq C_\ell \delta_{\ell\ell'} \delta_{mm'}$$

# Beyond $C_\ell$ :

## Searching for Departures from Gaussianity/Statistical Isotropy

- angular momentum dispersion axes (da Oliveira-Costa, *et al.*)
- genus curves (Park)
- spherical Mexican-hat wavelets (Vielva *et al.*)
- bipolar spherical harmonics (BiPoSH) (Souradeep *et al.*)
- **north-south asymmetries (Eriksen *et al.*, Hansen *et al.*)**
- dipolar modulations
- cold hot spots, hot cold spots (Larson and Wandelt)
- Land & Magueijo scalars/vectors
- **even/odd  $C_\ell$  anomaly**
- your favourite technique/anomaly that I missed
- **multipole vectors (Copi, Huterer, Schwarz, GDS;  
Weeks; Seljak and Slosar; Dennis)**

# *Alignments ...*



# *Multipole Vectors*

Q: What directions are associated w the  $\ell^{\text{th}}$  multipole:

$$\Delta T_{\ell}(\theta, \phi) \equiv \sum_m a_{\ell m} Y_{\ell m}(\theta, \phi) ?$$

Dipole ( $\ell = 1$ ) :

$$\sum_m a_{1m} Y_{1m}(\theta, \phi) = A^{(1)} \hat{u}_x^{(1,1)} \cdot (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$$

Advantages:

1)  $\hat{u}^{(1,1)}$  is a vector,  $A^{(1)}$  is a scalar

2) Only  $A^{(1)}$  depends on  $C_1$

# Multipole Vectors

General  $\ell$ , write:

$$\sum_m a_{\ell m} Y_{\ell m}(\theta, \phi) \approx A^{(\ell)} [(\hat{u}^{(\ell, 1)} \cdot \hat{e}) \dots (\hat{u}^{(\ell, \ell)} \cdot \hat{e}) - \text{all traces}]$$

$$\{\{a_{\ell m}, m = -\ell, \dots, \ell\}, \ell = (0, 1, 2, \dots)\} \Rightarrow \{A^{(\ell)}, \{\hat{u}^{(\ell, i)}, i = 1, \dots, \ell\}, \ell = (0, 1, 2, \dots)\}$$

Advantages: 1)  $\hat{u}^{(\ell, i)}$  are vectors,  $A^{(\ell)}$  is a scalar

2) Only  $A^{(\ell)}$  depends on  $C_\ell$



# Maxwell Multipole Vectors

$$\sum_m a_{\ell m} Y_{\ell m}(\theta, \phi) = \left[ (\mathbf{u}^{(\ell, 1)} \cdot \nabla) \dots (\mathbf{u}^{(\ell, \ell)} \cdot \nabla) r^{-1} \right]_{r=1}$$

J.C. Maxwell,

*A Treatise on Electricity and Magnetism*, v.1, 1873 (1<sup>st</sup> ed.)

# *Area Vectors*

Notice:

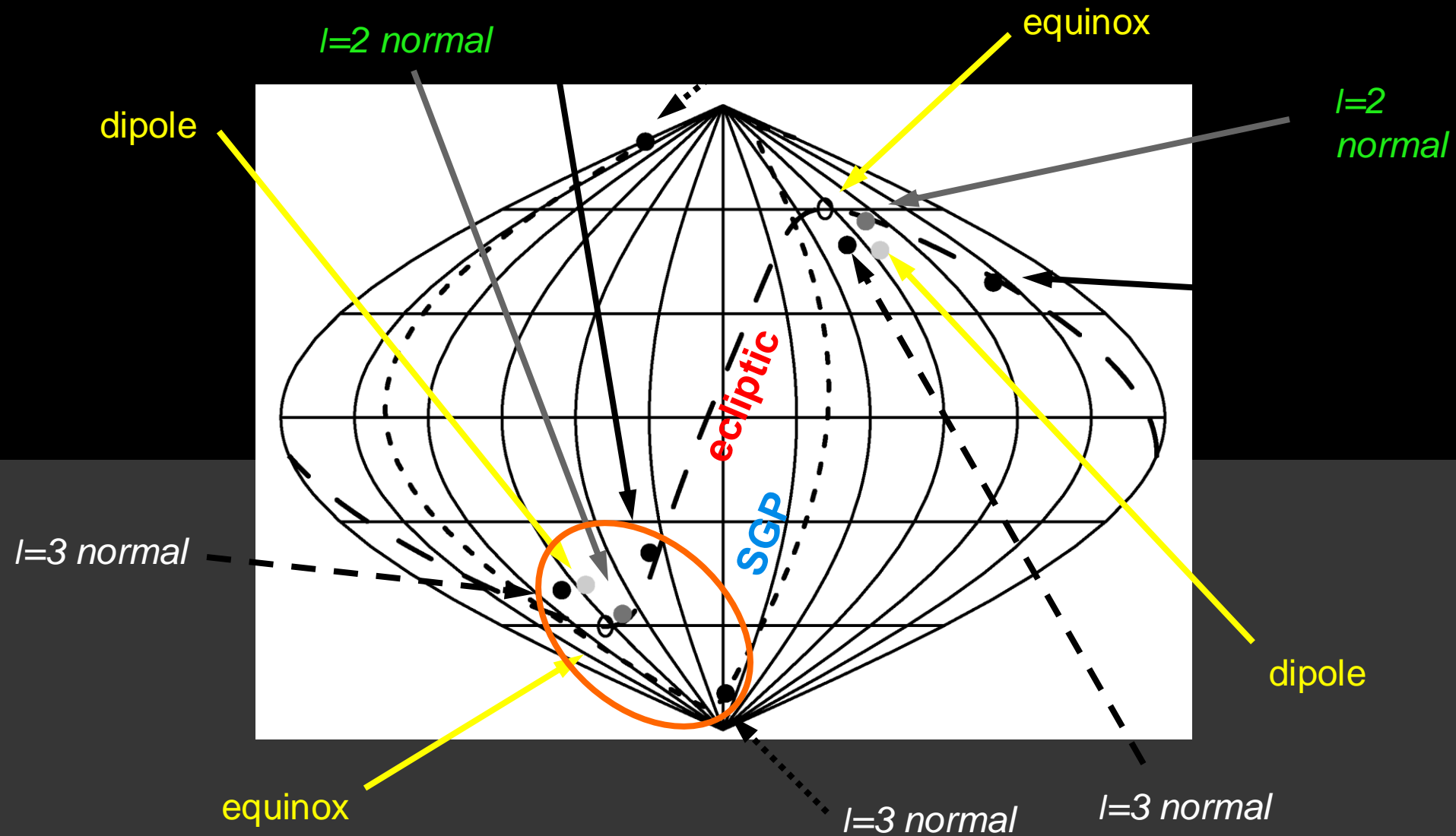
- Quadrupole has 2 vectors,  
*i.e.* quadrupole is a plane
- Octopole has 3 vectors,  
*i.e.* octopole is 3 planes

Suggests defining:

$$\mathbf{w}^{(\ell,i,j)} \equiv (\hat{\mathbf{u}}^{(\ell,i)} \times \hat{\mathbf{u}}^{(\ell,j)}) \quad \text{“area vectors”}$$

Carry some, but not all, of the information

# $\ell = 2 \& 3$ Area Vectors



**Quadrupole plane &  
3 octopole planes are  
aligned with one another**

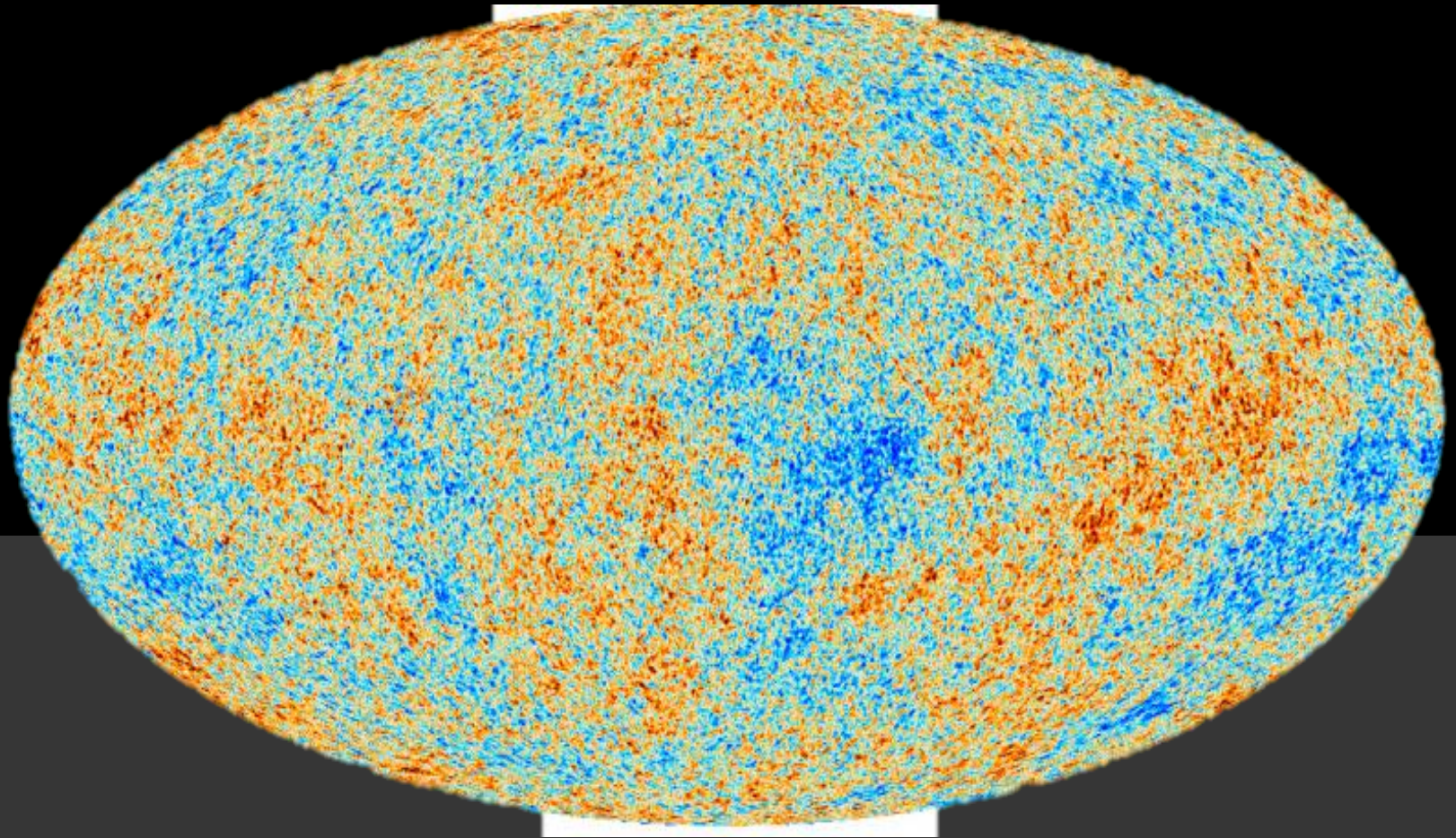
p-value of the quadrupole & octopole  
planes being so aligned: (0.1-0.6)%

Power asymmetry

Dipole modulation

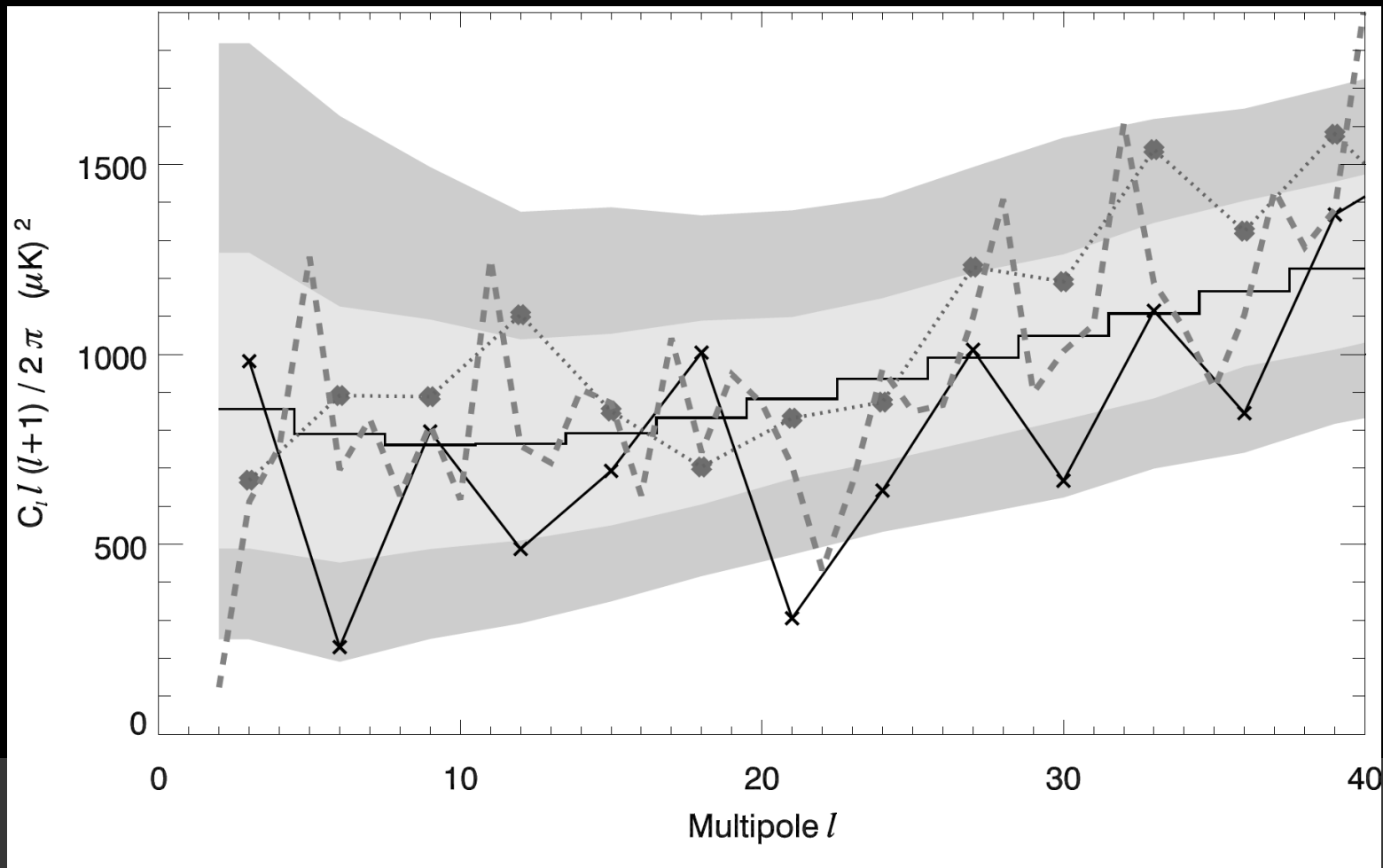
Low Northern Variance





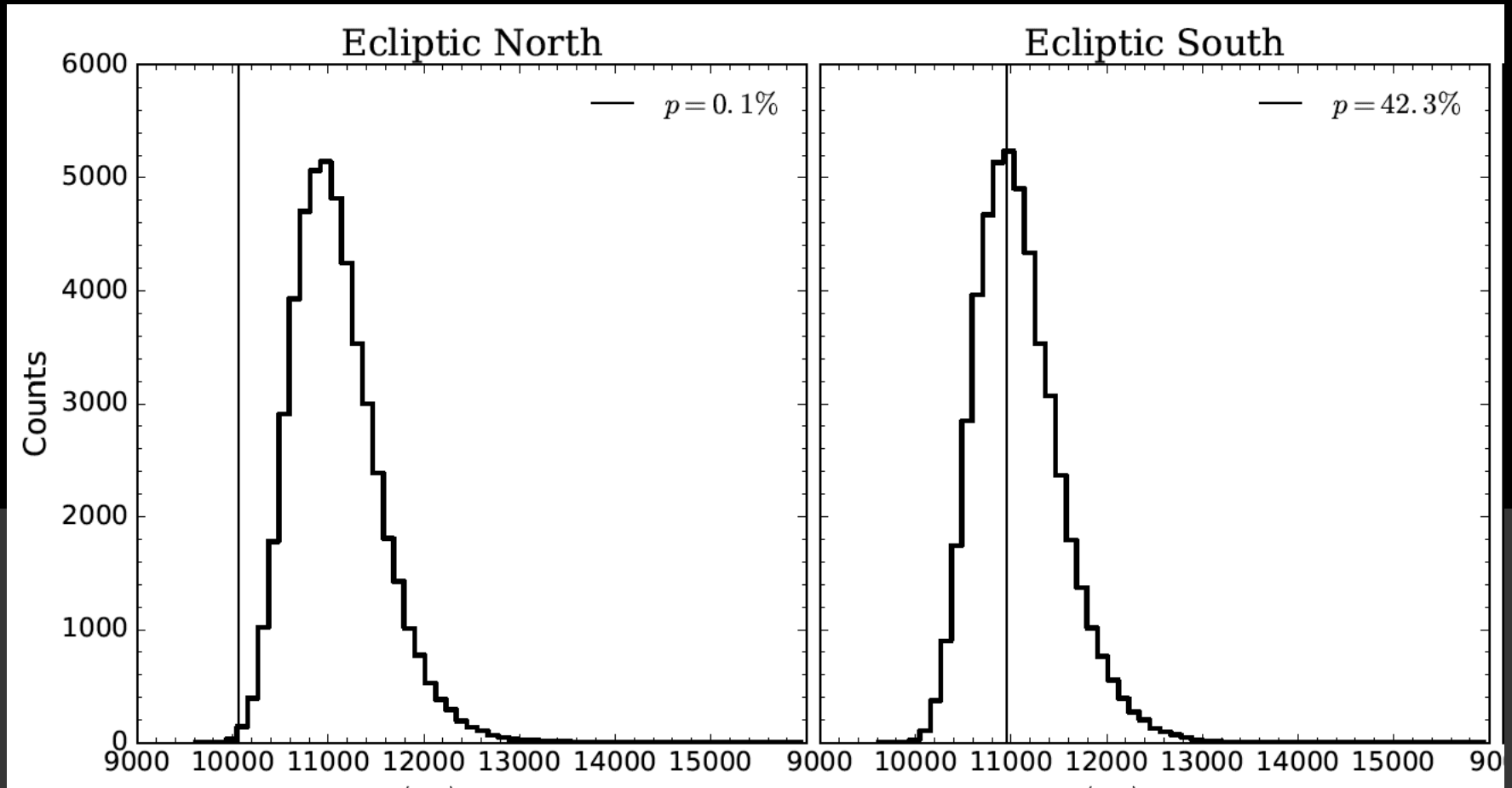
Bennett et al 2003

Eriksen et al 2004, and many others



Eriksen, Hansen, Banday, Gorski and Lilje 2004 *Astrophys. J.* 605 , 114  
 binned angular power spectrum over the whole unmasked sky (dashed),  
 northern hemisphere (solid line), and southern hemisphere (dotted line).  
 Optimized N vs. S is approximately ecliptic

# SMICA N vs S variance

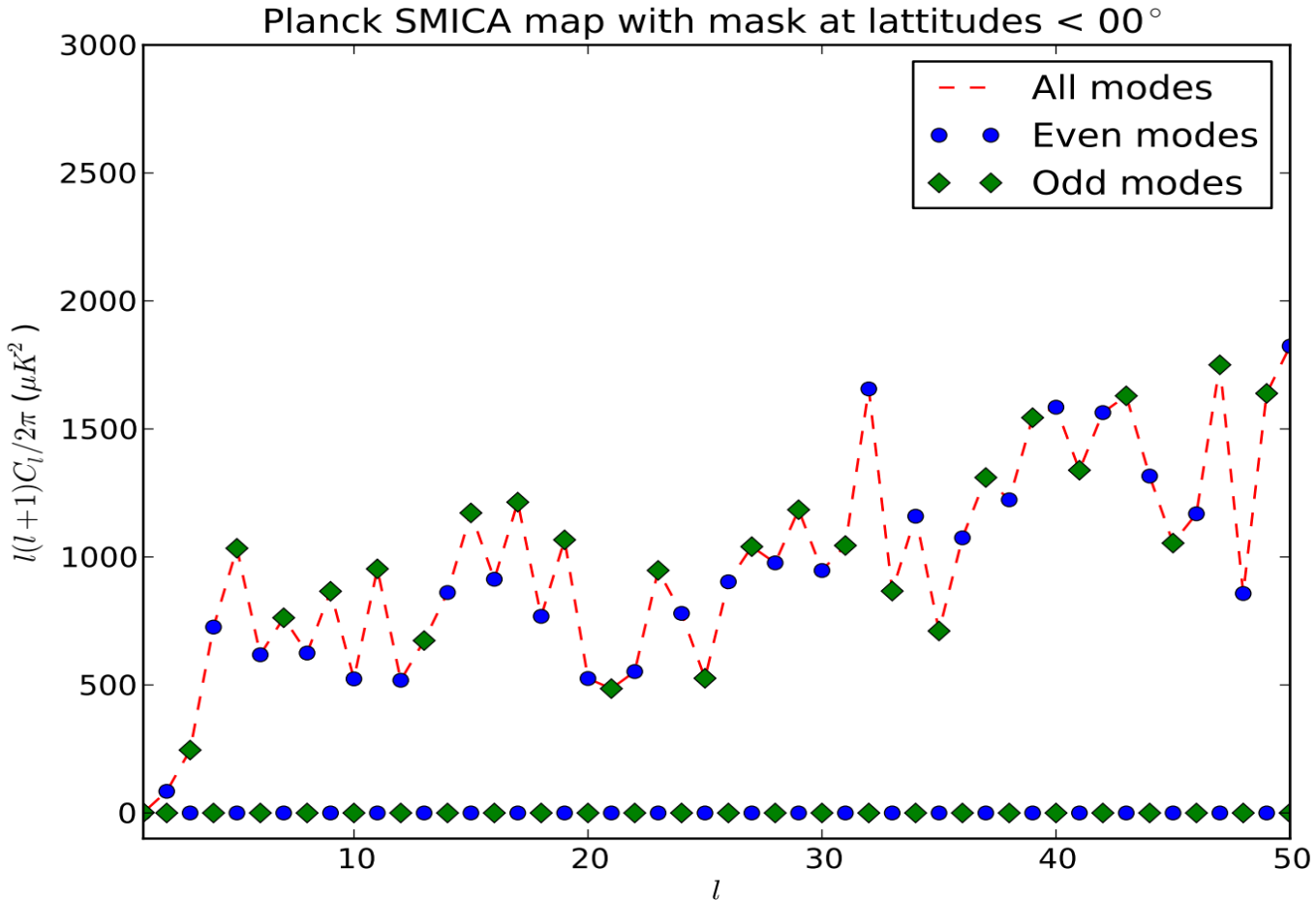


$p \sim 0.001$

Parity

Parity

# Parity anomaly



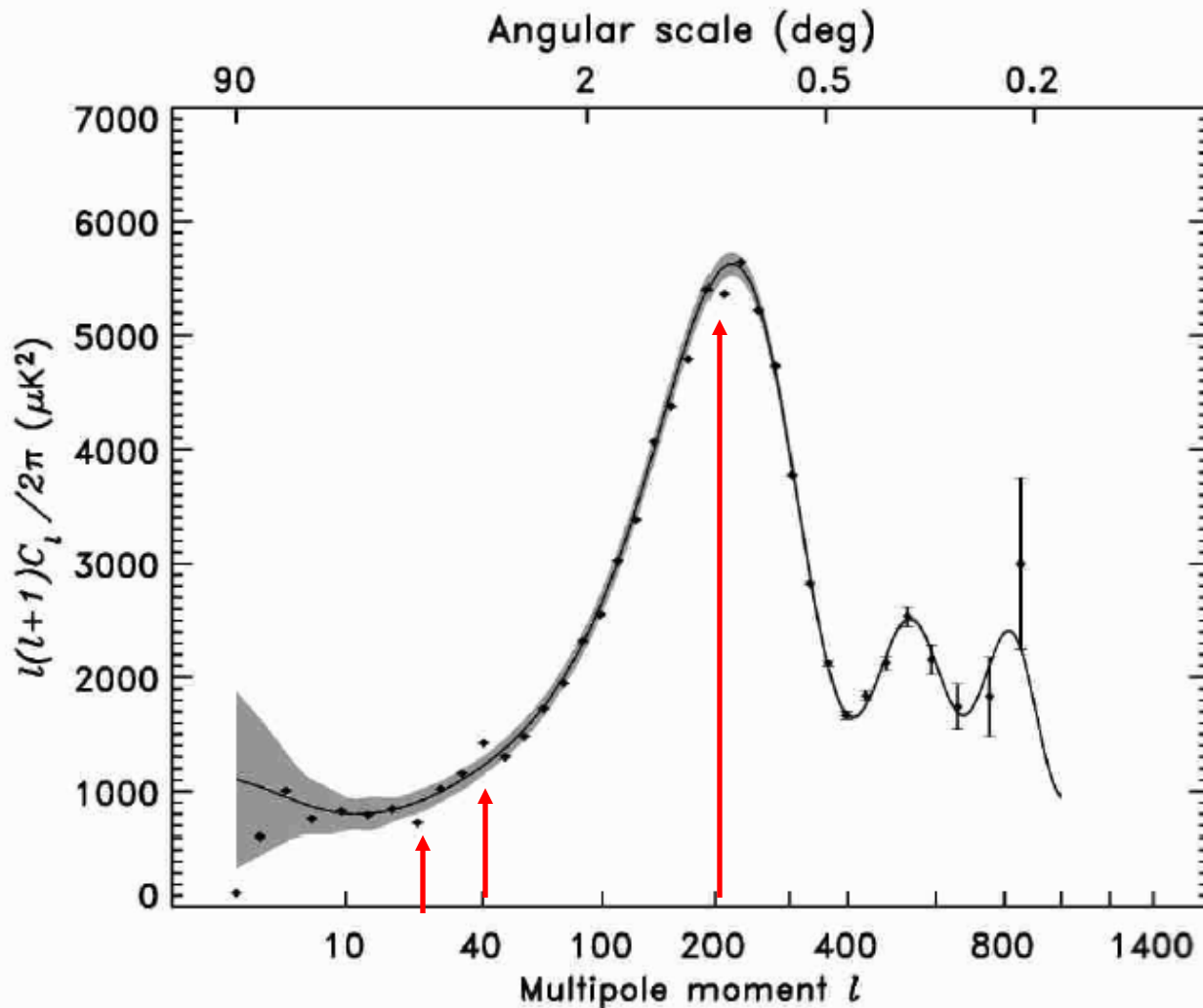
Plot by J. Muir (then U Michigan)



And ...

(N) ecliptic polar excursions

# Angular Power Spectrum



At least 3 other major deviations in the  $C_l$  in 1st year WMAP data

# Power spectrum: ecliptic plane vs. poles

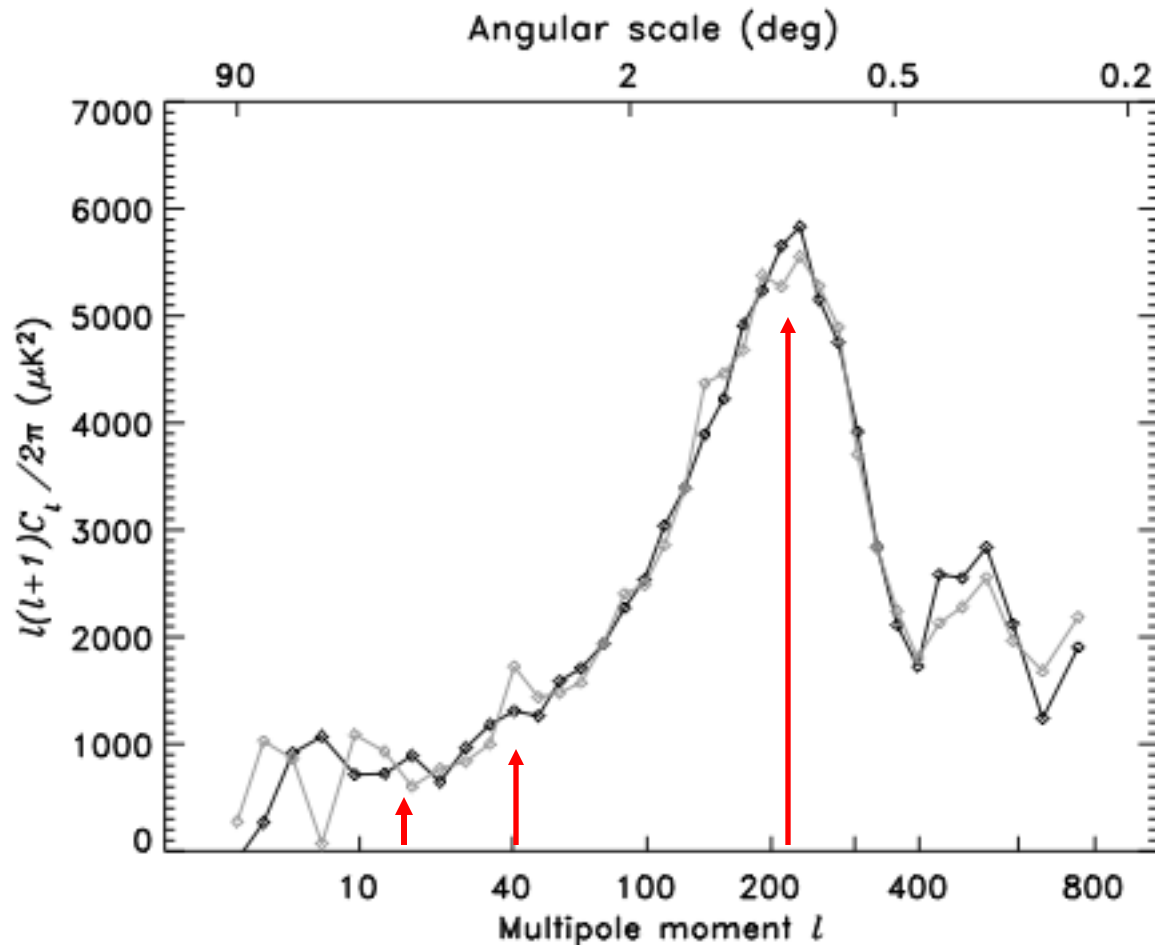


Fig. 7.— A comparison of the power spectrum computed with data from the ecliptic plane (black) vs. data from the ecliptic poles (grey). Note that some of the “bite” features that appear in the combined spectrum are not robust to data excision. There is also no evidence that beam ellipticity, which would be more manifest in the plane than in the poles, systematically biases the spectrum. This is consistent with estimates of the effect given by Page et al. (2003a).

All 3 other major deviations are in the ecliptic polar  $C_l$  only!!

# With so many anomalies, what do we do?

In preparation: Large-angle anomalies of the CMB  
and the evidence against statistical isotropy,  
Physics Reports

[Submitted on 19 Oct 2023]

# The Universe is not statistically isotropic

Joann Jones, Craig J. Copi, Glenn D. Starkman, Yashar Akrami

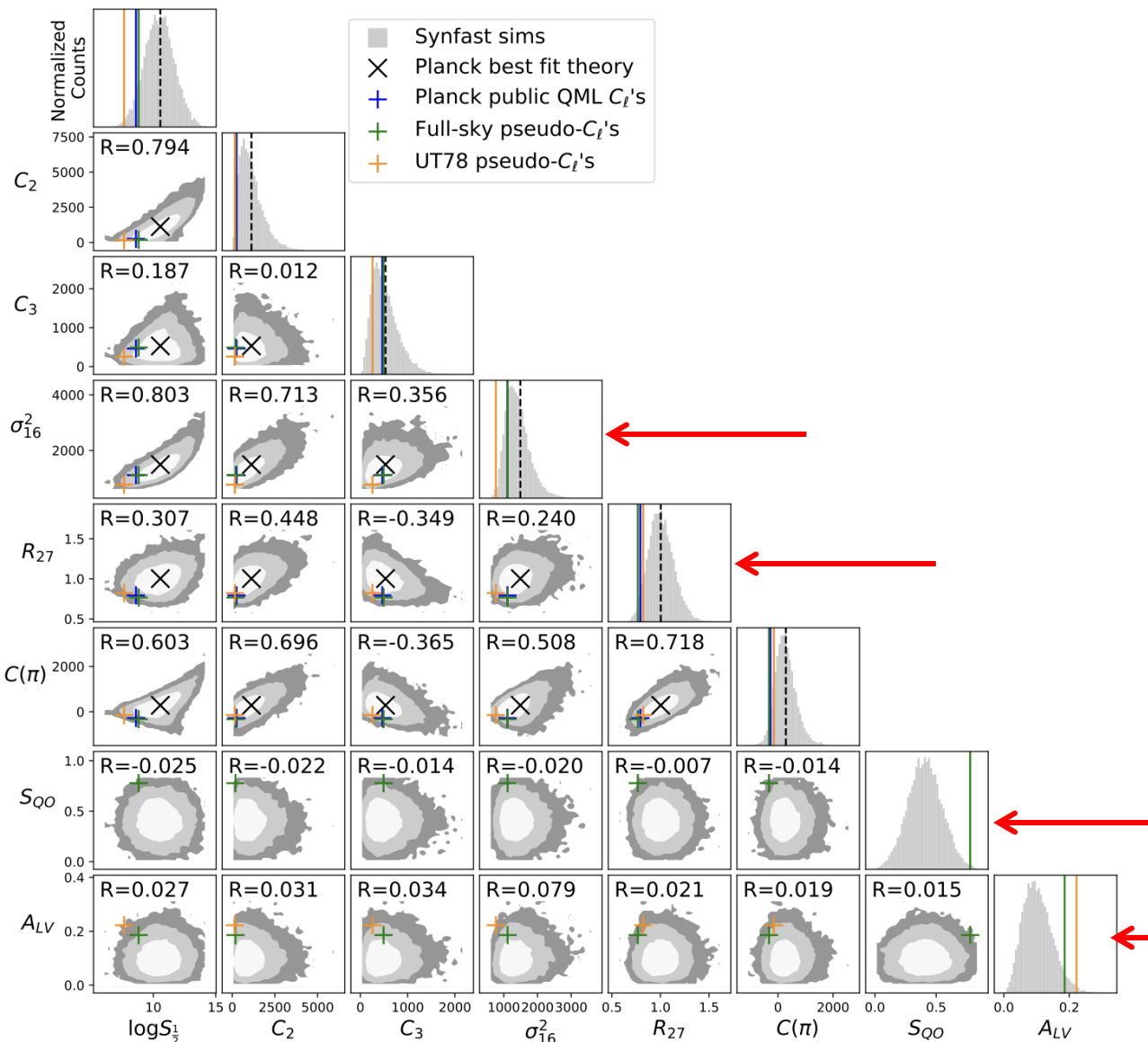
The standard cosmological model predicts statistically isotropic cosmic microwave background (CMB) fluctuations. However, several summary statistics of CMB isotropy have anomalous values, including: the low level of large-angle temperature correlations,  $S_{1/2}$ ; the excess power in odd versus even low- $\ell$  multipoles,  $R^{TT}$ ; the (low) variance of large-scale temperature anisotropies in the ecliptic north, but not the south,  $\sigma_{16}^2$ ; and the alignment and planarity of the quadrupole and octopole of temperature,  $S_{QO}$ . Individually, their low  $p$ -values are weak evidence for violation of statistical isotropy. The correlations of the tail values of these statistics have not to this point been studied. We show that the joint probability of all four of these happening by chance in  $\Lambda$ CDM is likely  $\leq 3 \times 10^{-8}$ . This constitutes more than  $5\sigma$  evidence for violation of statistical isotropy.

## Four “representative” anomaly statistics:

- $\mathbf{S}_{1/2}$  – lack of large-angle correlations,  $p \simeq 10^{-3}$
  - $\mathbf{R}_{\text{TT}}$  – odd-parity preference,  $p \simeq 0.01 - 0.05$
  - $\sigma_{16}^2$  – low northern variance,  $p \simeq (2 - 4) \times 10^{-3}$
  - $\mathbf{S}_{\text{QO}}$  – quadrupole-octupole alignment,  $p \simeq 4 \times 10^{-(2-4)}$
- in Planck 2018 Commander, NILC, SEVEM, SMICA

But are these statistics  
correlated in LCDM?





Muir, Adikhari,  
 Huterer (PRD  
 98 (2018),  
 023521) :  
 $S_{1/2}$  &  $\sigma_{16}^2$   
 are somewhat  
 correlated  
 Others, not  
 really



# But are the anomalies (tails) correlated?

- $10^8$  realizations of CMB in best fit LCDM

Stat.	Value	$S_{1/2}$	$R^{TT}$	$\sigma_{16}^2$	$S_{QO}$
<b>Commander</b>					
$S_{1/2}$	1272	$1.5 \times 10^{-3}$	$\times 0.6$	$\times 27$	$\times 1.3$
$R^{TT}$	0.7896	$2.8 \times 10^{-5}$	$3.0 \times 10^{-2}$	$\times 1.1$	$\times 1.0$
$\sigma_{16}^2$	617.6	$1.2 \times 10^{-4}$	$1.0 \times 10^{-4}$	$3.1 \times 10^{-3}$	$\times 1.7$
$S_{QO}$	0.7630	$8.3 \times 10^{-6}$	$1.3 \times 10^{-4}$	$2.3 \times 10^{-5}$	$4.4 \times 10^{-3}$
<b>NILC</b>					
$S_{1/2}$	1218	$1.3 \times 10^{-3}$	$\times 0.4$	$\times 29$	$\times 1.3$
$R^{TT}$	0.7448	$4.8 \times 10^{-6}$	$1.0 \times 10^{-2}$	$\times 1.0$	$\times 1.0$
$\sigma_{16}^2$	605.9	$9.2 \times 10^{-5}$	$2.4 \times 10^{-5}$	$2.5 \times 10^{-3}$	$\times 1.9$
$S_{QO}$	0.8203	$6.3 \times 10^{-7}$	$3.8 \times 10^{-6}$	$1.8 \times 10^{-6}$	$3.9 \times 10^{-4}$
<b>SEVEM</b>					
$S_{1/2}$	1215	$1.3 \times 10^{-3}$	$\times 0.8$	$\times 33$	$\times 1.2$
$R^{TT}$	0.8194	$5.6 \times 10^{-5}$	$5.4 \times 10^{-2}$	$\times 1.2$	$\times 1.0$
$\sigma_{16}^2$	583.4	$6.5 \times 10^{-5}$	$1.0 \times 10^{-4}$	$1.6 \times 10^{-3}$	$\times 1.5$
$S_{QO}$	0.6547	$6.3 \times 10^{-5}$	$2.2 \times 10^{-3}$	$9.8 \times 10^{-5}$	$4.1 \times 10^{-2}$
<b>SMICA</b>					
$S_{1/2}$	1257	$1.4 \times 10^{-3}$	$\times 0.6$	$\times 25$	$\times 1.3$
$R^{TT}$	0.7906	$2.8 \times 10^{-5}$	$3.0 \times 10^{-2}$	$\times 1.1$	$\times 1.0$
$\sigma_{16}^2$	631.0	$1.4 \times 10^{-4}$	$1.3 \times 10^{-4}$	$3.9 \times 10^{-3}$	$\times 1.8$
$S_{QO}$	0.8048	$1.7 \times 10^{-6}$	$2.9 \times 10^{-5}$	$6.6 \times 10^{-6}$	$9.2 \times 10^{-4}$

← pairwise correlations

triplet correlations



	$S_{1/2}$ and $\sigma_{16}^2$	$S_{QO}$
<b>Commander</b>		
$S_{1/2}$ and $\sigma_{16}^2$	$1.2 \times 10^{-4}$	$\times 1.7$
$S_{QO}$	$9.1 \times 10^{-7}$	$4.4 \times 10^{-3}$
<b>NILC</b>		
$S_{1/2}$ and $\sigma_{16}^2$	$9.2 \times 10^{-5}$	$\times 0.6$
$S_{QO}$	$2.0 \times 10^{-8}$	$3.9 \times 10^{-4}$
<b>SEVEM</b>		
$S_{1/2}$ and $\sigma_{16}^2$	$6.5 \times 10^{-5}$	$\times 1.3$
$S_{QO}$	$3.6 \times 10^{-6}$	$4.1 \times 10^{-2}$
<b>SMICA</b>		
$S_{1/2}$ and $\sigma_{16}^2$	$1.4 \times 10^{-4}$	$\times 2.1$
$S_{QO}$	$2.7 \times 10^{-7}$	$9.2 \times 10^{-4}$

# Are the anomalies correlated in LCDM?

Map	$p_4$	Correlation Factor
Commander	$3 \times 10^{-8}$	51
NILC	$< 1 \times 10^{-8}$	N/A
SEVEM	$18 \times 10^{-8}$	40
SMICA	$1 \times 10^{-8}$	64

Answer: only weakly

Conclusion:

Statistical isotropy is falsified at  $>5\sigma$   
in CMB TT correlations!

The Universe is not  
Statistically Isotropic

# Conversation points:

1. You can't believe data without a model;  
i.e., you can't falsify a model without an alternative
2. Look elsewhere penalties  
i.e., you can always find anomalous statistics

The Universe is NOT  
Statistically Isotropic



The End?

# Making Progress

1. Find a fundamental physics model, make testable predictions.
2. Make reasonable phenomenological extrapolations and test them.
3. Continue to test the “fluke hypothesis.”  
i.e. test LCDM!

# Testing the fluke hypothesis

Philosophy: assume  $\Lambda$ CDM is correct, predict how measured anomalies affect predictions for other observables, test them.

Note:  
this is a fundamentally frequentist approach.

# Phenomenological extrapolations

## Philosophy:

1. Assume statistical isotropy is violated, identify generic predictions
2. Assume each anomaly is “physical” and guess what that implies for other observables

Testing the fluke hypothesis

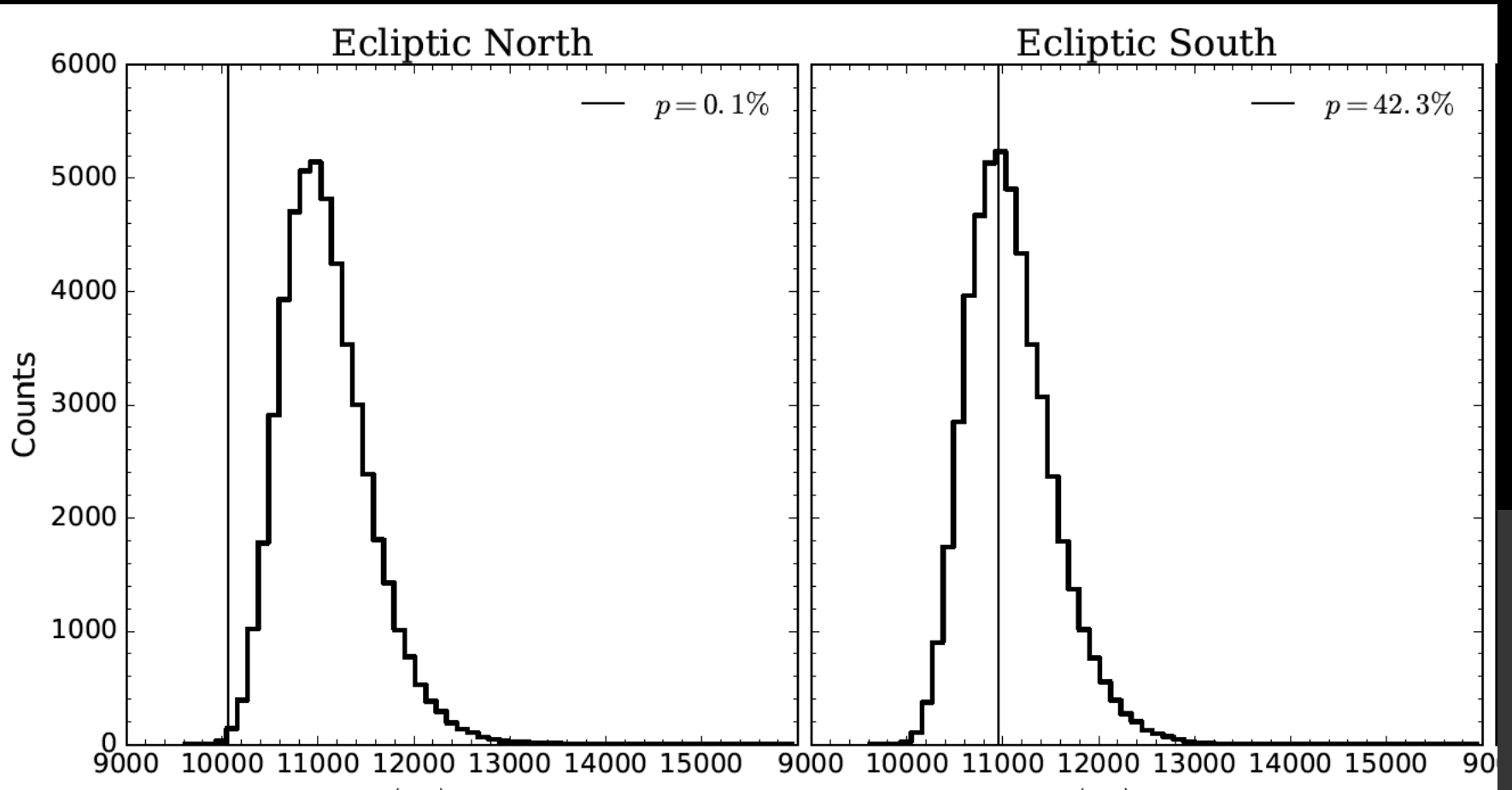
Phenomenological extrapolations

go hand in hand

Example 1: Low N variance in E



# Low N variance in T



# Phenomenological guess

Low variance in  $T$   
over a large region of the sky



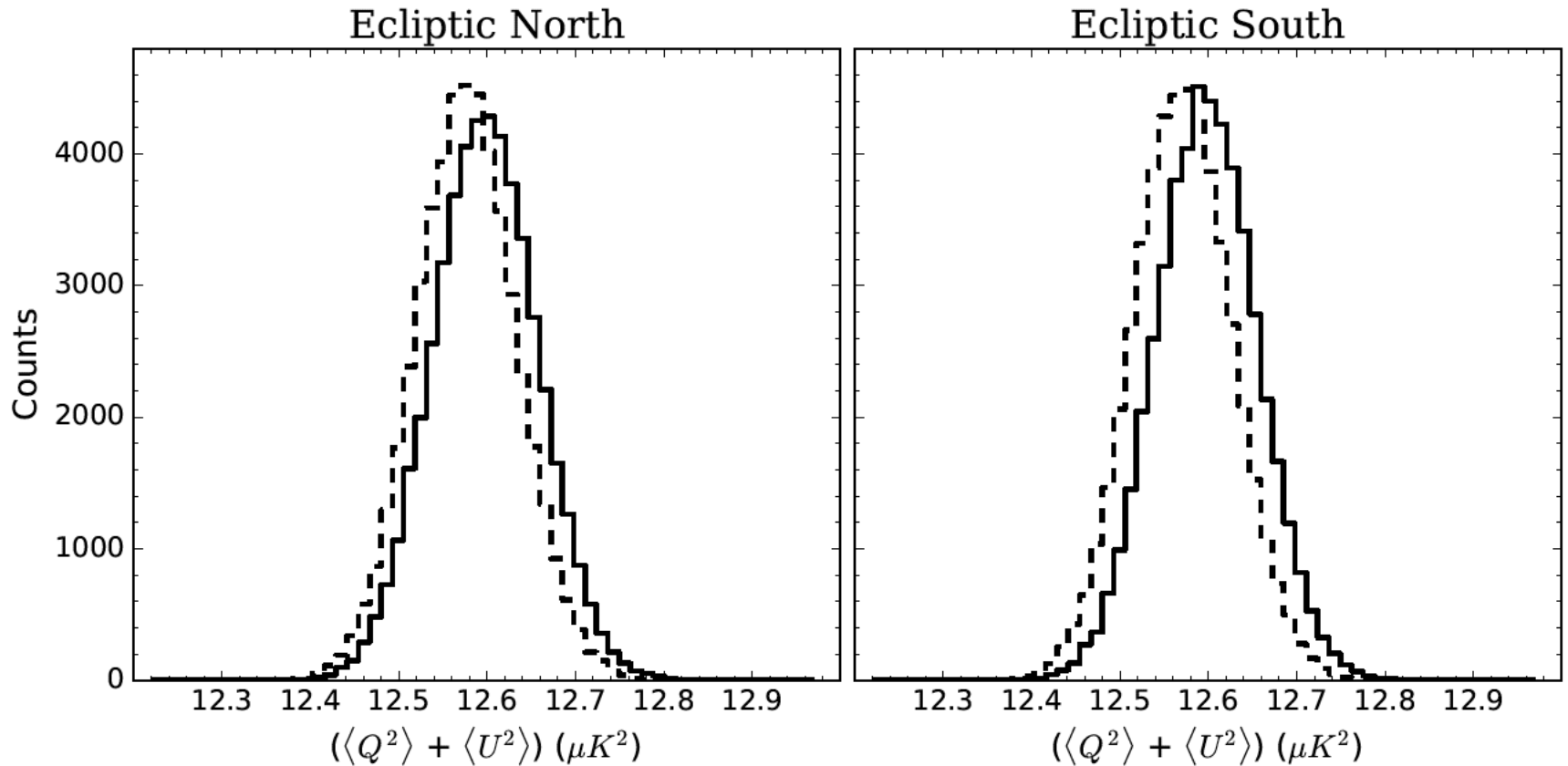
Low variance in underlying curvature fluctuations



low variance in E-mode polarization  
in that same region

What does  $\Lambda$ CDM say?

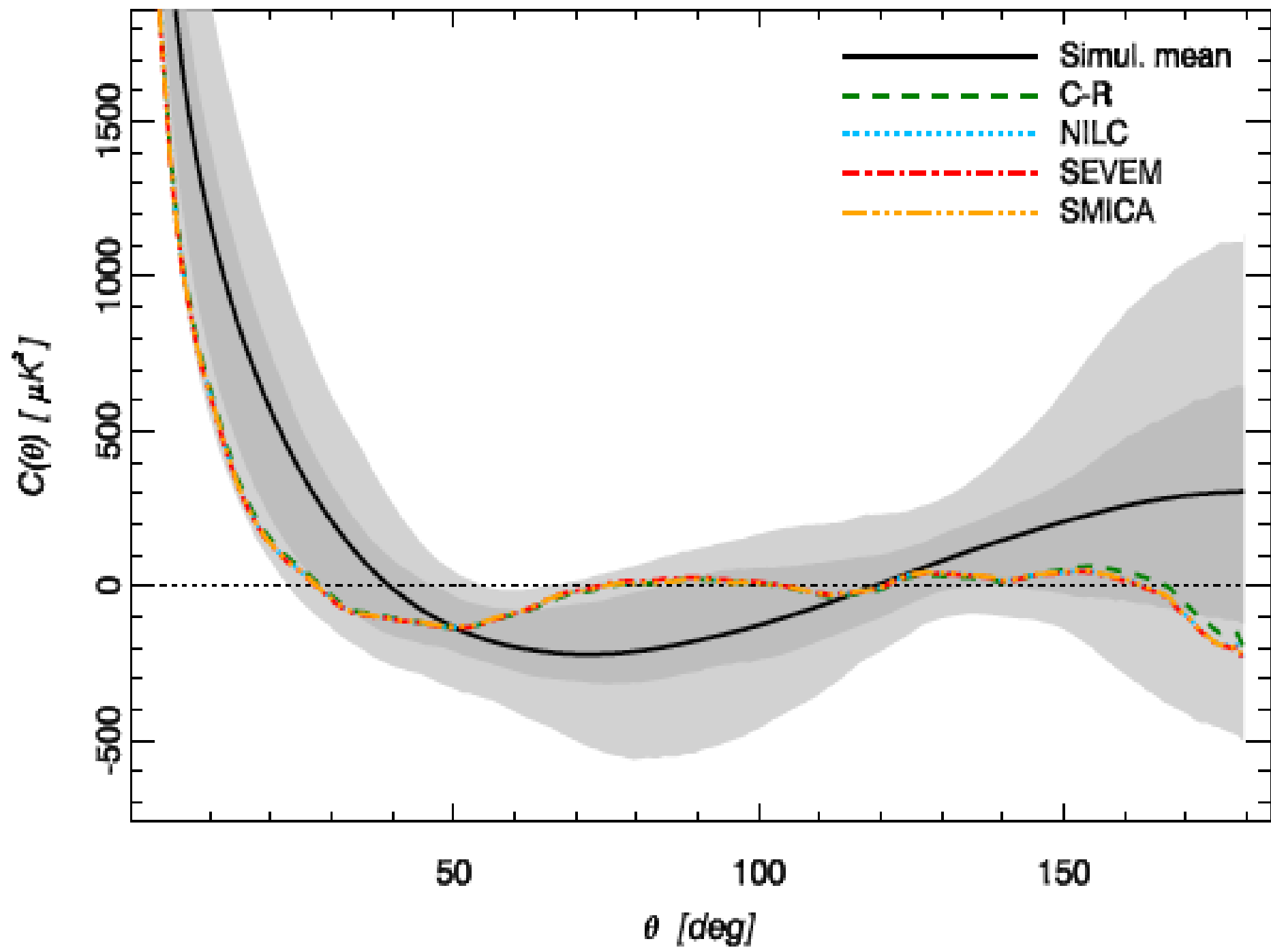
After all, T-E are correlated!



Observe:

- a small reduction in the polarization variance
- approximately equal in N and S

Example 2: Absence of  
angular correlation in Q,U,E



# Phenomenological guess

Low correlation in T



Low correlation in curvature fluctuations



low correlation in Q,U,E polarization



# Constrained-LCDM QQ and UU correlations

A. Yoho, A. Aiola, C. Copi, A. Kosowsky, GDS (PRD91 (2015) 12, 123504)

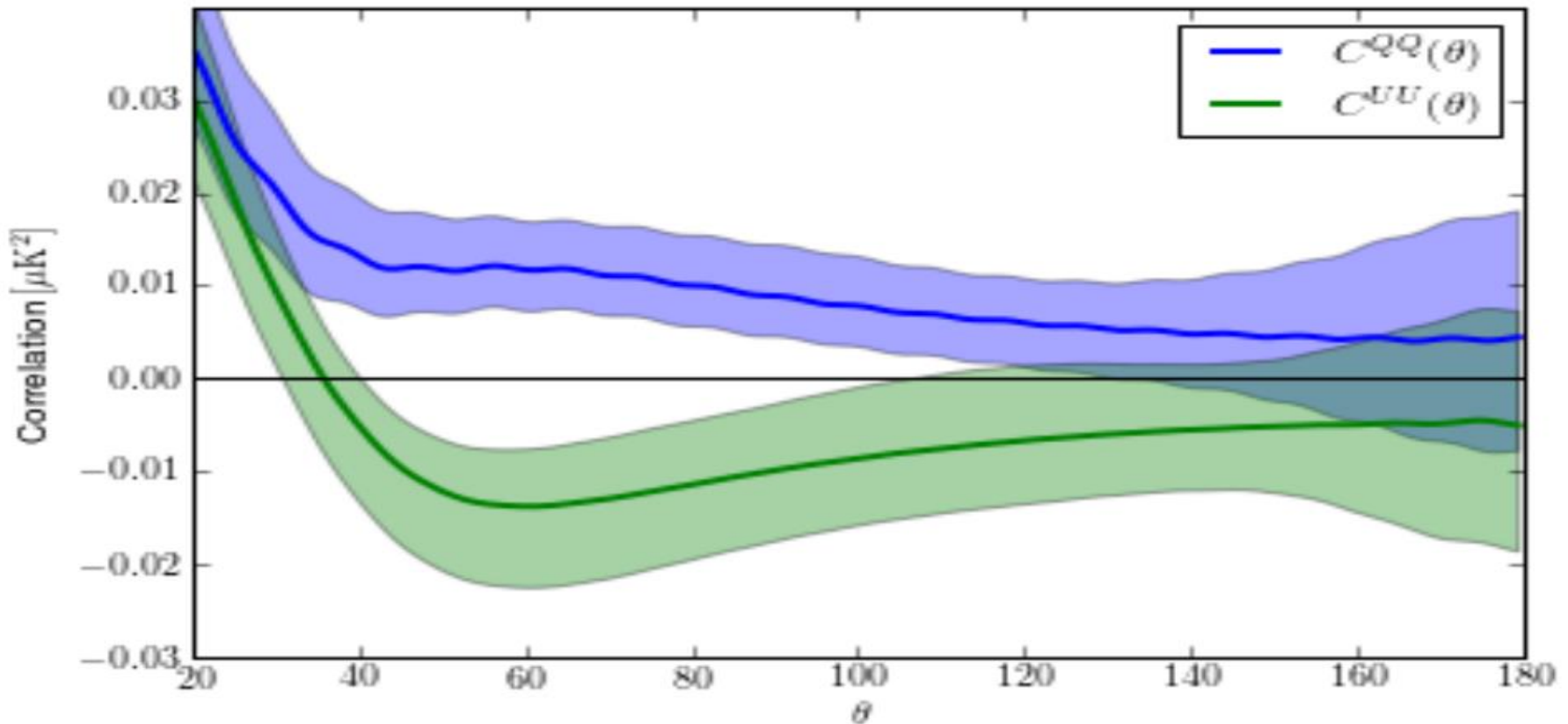


FIG. 5: Angular correlation function of  $Q$  and  $U$  polarizations with  $r = 0.1$ . The shaded regions correspond to the 68% C.L. errors. The ranges include instrumental noise for a future generation PIXIE-like experiment and cosmic variance.

# Constrained- $\Lambda$ CDM EE correlations

A. Yoho, A. Aiola, C. Copi, A. Kosowsky,  
GDS (PRD91 (2015) 12, 123504)

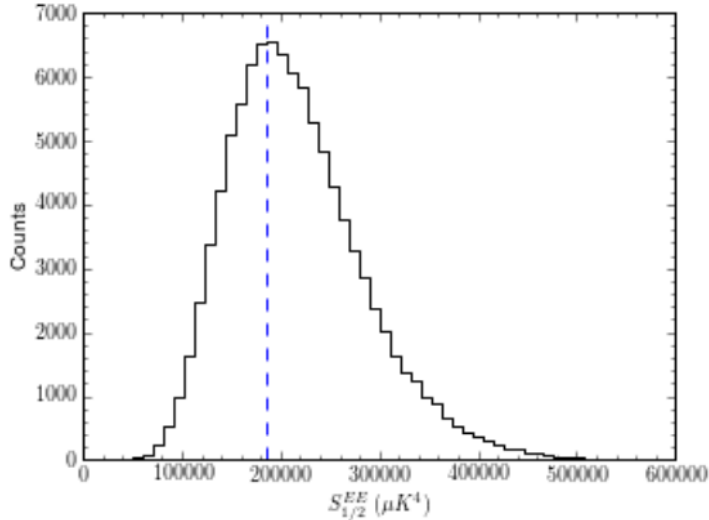
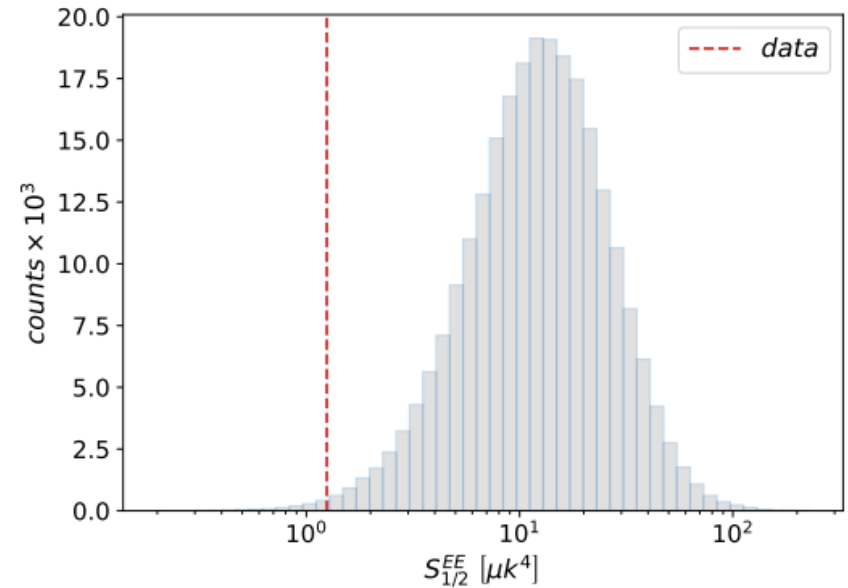


FIG. 3:  $S_{1/2}$  statistic distribution for the angular correlation function of E-modes  $r = 0.1$  with  $\sigma_{\text{beam}} = 2.7^\circ$  radian smoothing. The blue dashed line marks the  $\Lambda$ CDM prediction for the ensemble average.



$S_{1/2}$  statistic for the angular correlation function of E-modes in Planck 2018 HFI 100

*Lack-of-correlation anomaly in CMB large scale polarisation maps, C.Chiochetta, A. Gruppuso, M.Lattanzi, P. Natoli, L. Pagano arXiv:2012.00024*

# Consequence 1

A broken symmetry  
offers no protection

# Statistical isotropy

- Forbids off-diagonal TT, EE, BB correlations:

$$\langle a_{\ell m}^X a_{\ell' m'}^{X*} \rangle = C_\ell^X \delta_{\ell\ell'} \delta_{mm'} , X=T,E,B$$

- Forbids TB, EB correlations:

$$\langle a_{\ell m}^X a_{\ell' m'}^{B*} \rangle = 0 , X=T,E$$

Common misconception:

this is due to parity invariance

# Statistical anisotropy $\Rightarrow$

## General Correlation matrix

$$\langle a_{\ell m}^X a_{\ell' m'}^{Y*} \rangle = C_{\ell m \ell' m'}^{XY}$$

- Some terms require parity violation
- Statistical anisotropy tied to statistical inhomogeneity
- Statistical inhomogeneity  $\Rightarrow$  parity violation
- the anomalies are parity violating

COMPACT: A. Samandar *et al.* (2407.09400)

Consequence 2

Anisotropy is not  
non-Gaussianity

Violation of SI does not imply  
(or preclude) NG;

but it may make it much harder to  
measure

# NG statistical tests (often?) assume SI

$$\hat{f}_{\text{NL}} = \frac{1}{N} \sum_{X_i, X'_i} \sum_{\ell_i, m_i} \sum_{\ell'_i, m'_i} \mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3} b_{\ell_1 \ell_2 \ell_3}^{X_1 X_2 X_3, \text{th}} \left\{ \left[ \left( \mathbf{C}_{\ell_1 m_1, \ell'_1 m'_1}^{-1} \right)^{X_1 X'_1} a_{\ell'_1 m'_1}^{X'_1} \right. \right. \\ \times \left. \left( \mathbf{C}_{\ell_2 m_2, \ell'_2 m'_2}^{-1} \right)^{X_2 X'_2} a_{\ell'_2 m'_2}^{X'_2} \left( \mathbf{C}_{\ell_3 m_3, \ell'_3 m'_3}^{-1} \right)^{X_3 X'_3} a_{\ell'_3 m'_3}^{X'_3} \right] \\ \left. - \left[ \left( \mathbf{C}_{\ell_1 m_1, \ell_2 m_2}^{-1} \right)^{X_1 X_2} \left( \mathbf{C}_{\ell_3 m_3, \ell'_3 m'_3}^{-1} \right)^{X_3 X'_3} a_{\ell'_3 m'_3}^{X'_3} + \text{cyclic} \right] \right\}, \quad (25)$$

$$\hat{f}_{\text{NL}} = \frac{1}{N} \sum_{X_i, X'_i} \sum_{\ell_i, m_i} \mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3} (\mathbf{C}^{-1})_{\ell_1}^{X_1 X'_1} (\mathbf{C}^{-1})_{\ell_2}^{X_2 X'_2} (\mathbf{C}^{-1})_{\ell_3}^{X_3 X'_3} b_{\ell_1 \ell_2 \ell_3}^{X_1 X_2 X_3, \text{th}} \\ \times \left[ a_{\ell_1 m_1}^{X'_1} a_{\ell_2 m_2}^{X'_2} a_{\ell_3 m_3}^{X'_3} - \mathbf{C}_{\ell_1 m_1, \ell_2 m_2}^{X'_1 X'_2} a_{\ell_3 m_3}^{X'_3} - \mathbf{C}_{\ell_1 m_1, \ell_3 m_3}^{X'_1 X'_3} a_{\ell_2 m_2}^{X'_2} \right. \\ \left. - \mathbf{C}_{\ell_2 m_2, \ell_3 m_3}^{X'_2 X'_3} a_{\ell_1 m_1}^{X'_1} \right]. \quad (27)$$

Optimal estimator given SI

diagonal covariance approximation, given SI

From Planck 2018 IX Constraints on PNG

Violation of SI affects the bispectrum/ $f_{\text{NL}}$ , and  $\sigma_{f_{\text{NL}}}$

$$\langle a_{\ell m}^X a_{\ell' m'}^Y a_{\ell'' m''}^Z \rangle \neq \mathcal{G}_{m m' m''}^{\ell \ell' \ell''} b_{\ell \ell' \ell''}^{XYZ}$$

Q: how would we even estimate  $f_{\text{NL}}$ ?!



$$\langle a_{\ell m} a_{\ell' m'} a_{\ell'' m''} \rangle \neq 0$$

implies NG distribution for  $a_{\ell m}$

but how do you construct a summary statistic  
and an estimator if

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle \neq C_{\ell} \delta_{\ell \ell'} \delta_{m m'} ?$$

And if  $\langle a a a a a a \rangle \neq \langle a a a a a a \rangle_{SI} ?$

# Nearly all NG statistical results assume SI

NG is not just about the bispectrum —  
e.g. features in the tail of the distribution of ...  $a_{\ell m}$ ?

But the tails of what distribution if not SI?

What if  $a_{\ell m} = 0$  for certain  $m$ ? and  $a_{\ell m} = a_{\ell m'}$  for other  $m$ ?

# New Models

# New Models

List of new fundamental physics models known to explain all/most/several anomalies:

Requirements:

- Break statistical isotropy

- Affect scales that were causally disconnected until recently

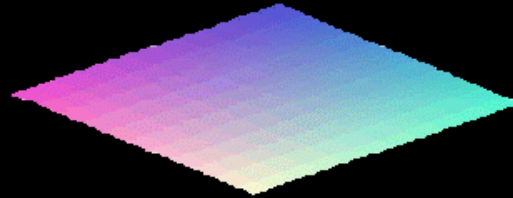
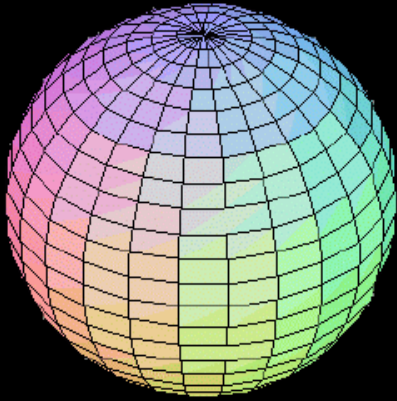
# New Models

Fundamental physics phenomena that break isotropy and are already in our theory:

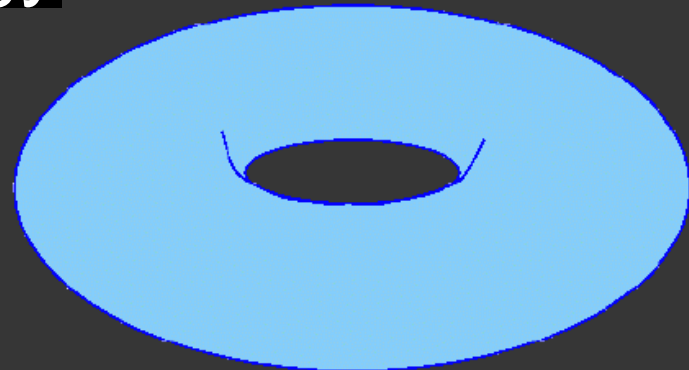
Non-trivial cosmic topology

# Cosmic Geometry

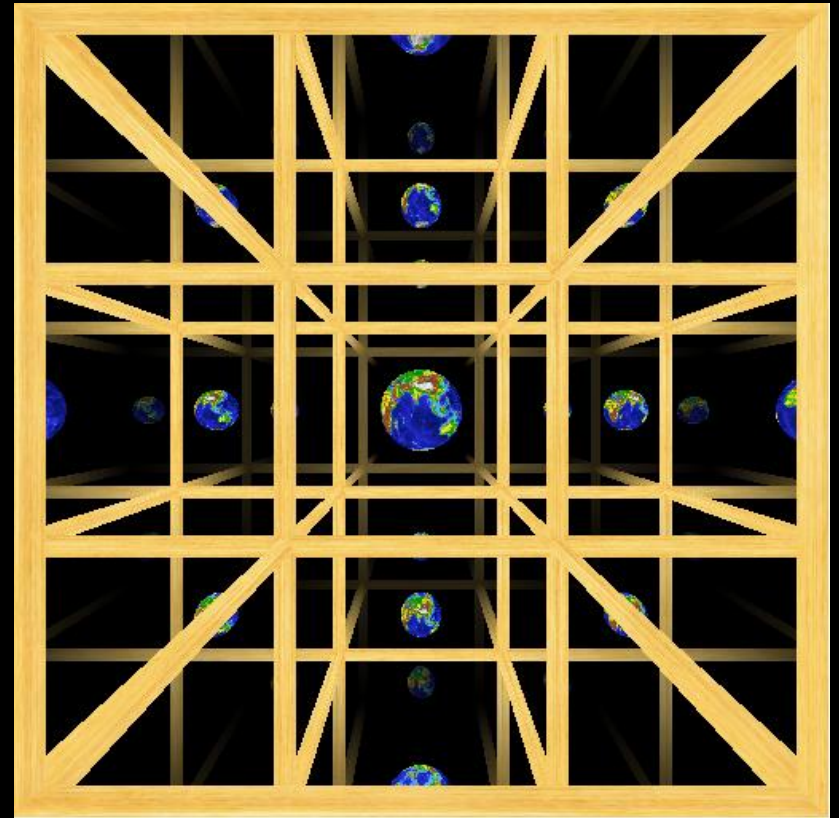
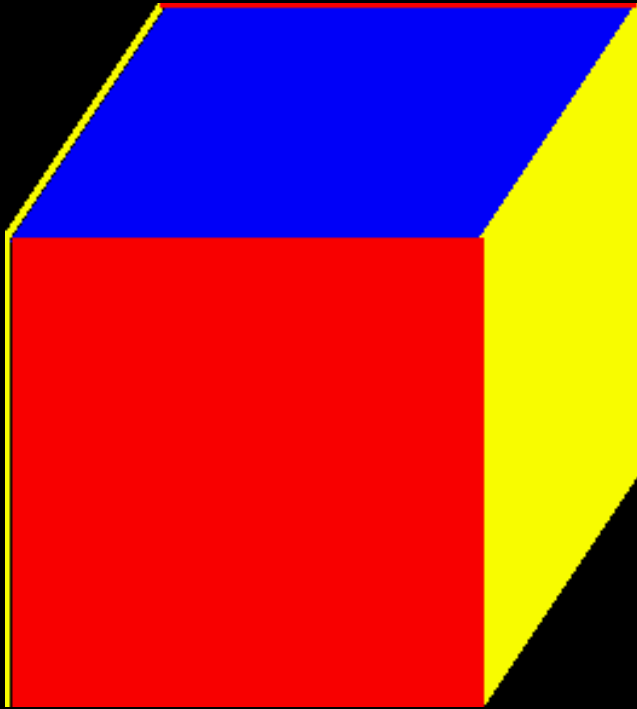
## Curvature



## Topology



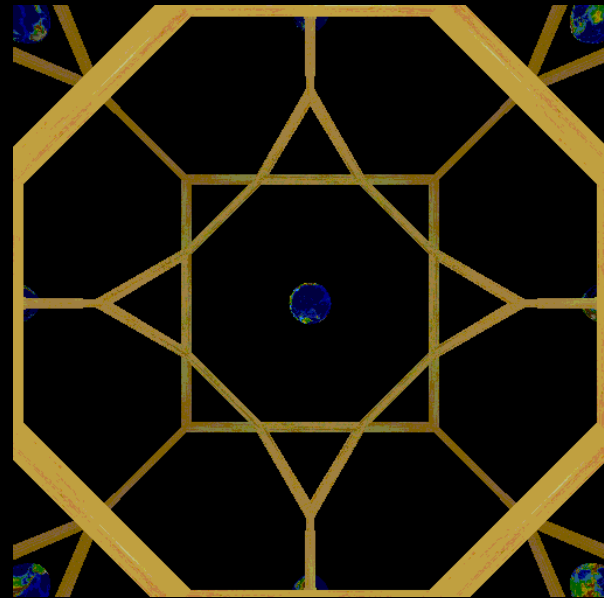
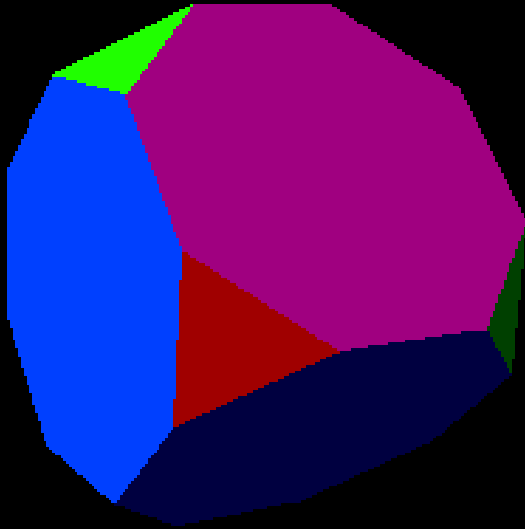
# Three Torus



Same idea works in three space dimensions

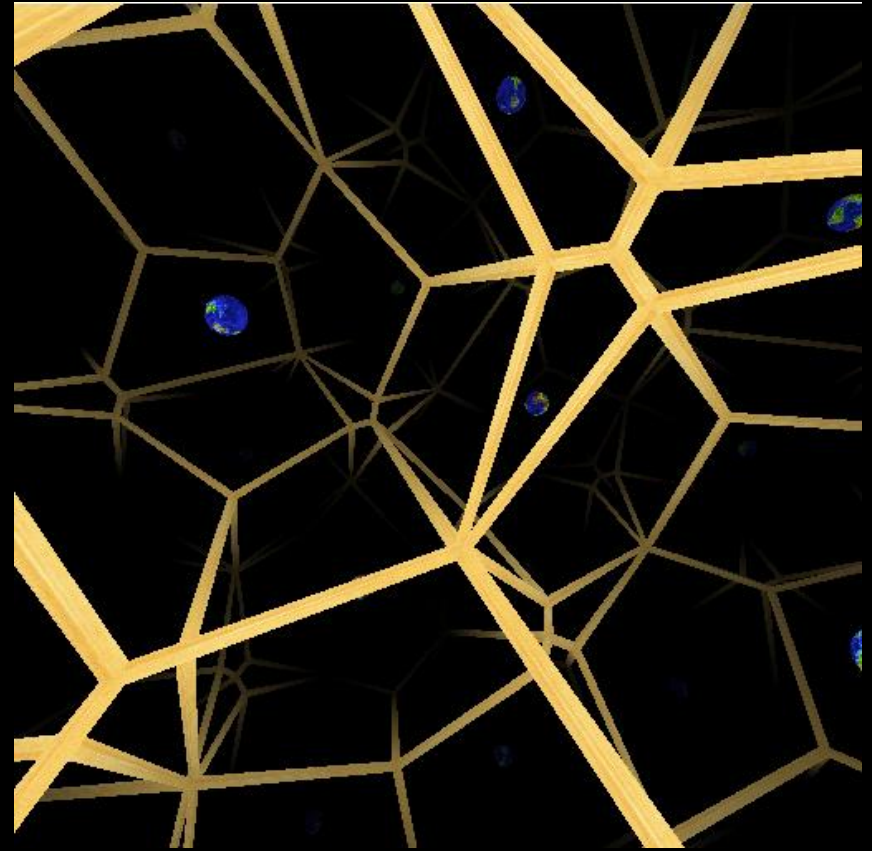
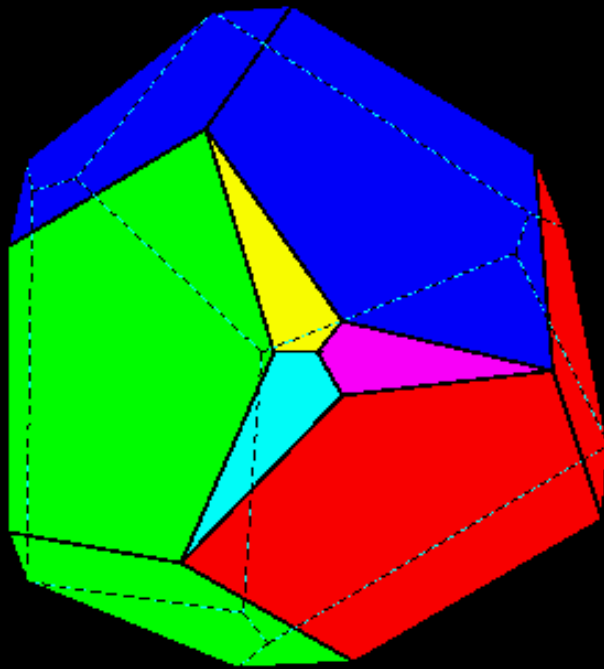
# Spherical Topologies

This example only works in spherical space





# Infinite number of tiling patterns



This one only works in hyperbolic space

*[Submitted on 20 Oct 2022 (v1), last revised 5 Mar 2024 (this version, v3)]*

# The Promise of Future Searches for Cosmic Topology

Yashar Akrami, Stefano Anselmi, Craig J. Copi, Johannes R. Eskilt, Andrew H. Jaffe, Arthur Kosowsky, Pip Petersen, [Glenn D. Starkman](#), Kevin González-Quesada, Özenç Güngör, Deyan P. Mihaylov, Samanta Saha, Andrius Tamosiunas, Quinn Taylor, Valeri Vardanyan (COMPACT Collaboration)

The shortest distance around the Universe through us is unlikely to be much larger than the horizon diameter if microwave background anomalies are due to cosmic topology. We show that observational constraints from the lack of matched temperature circles in the microwave background leave many possibilities for such topologies. We evaluate the detectability of microwave background multipole correlations for sample cases. Searches for topology signatures in observational data over the large space of possible topologies pose a formidable computational challenge.

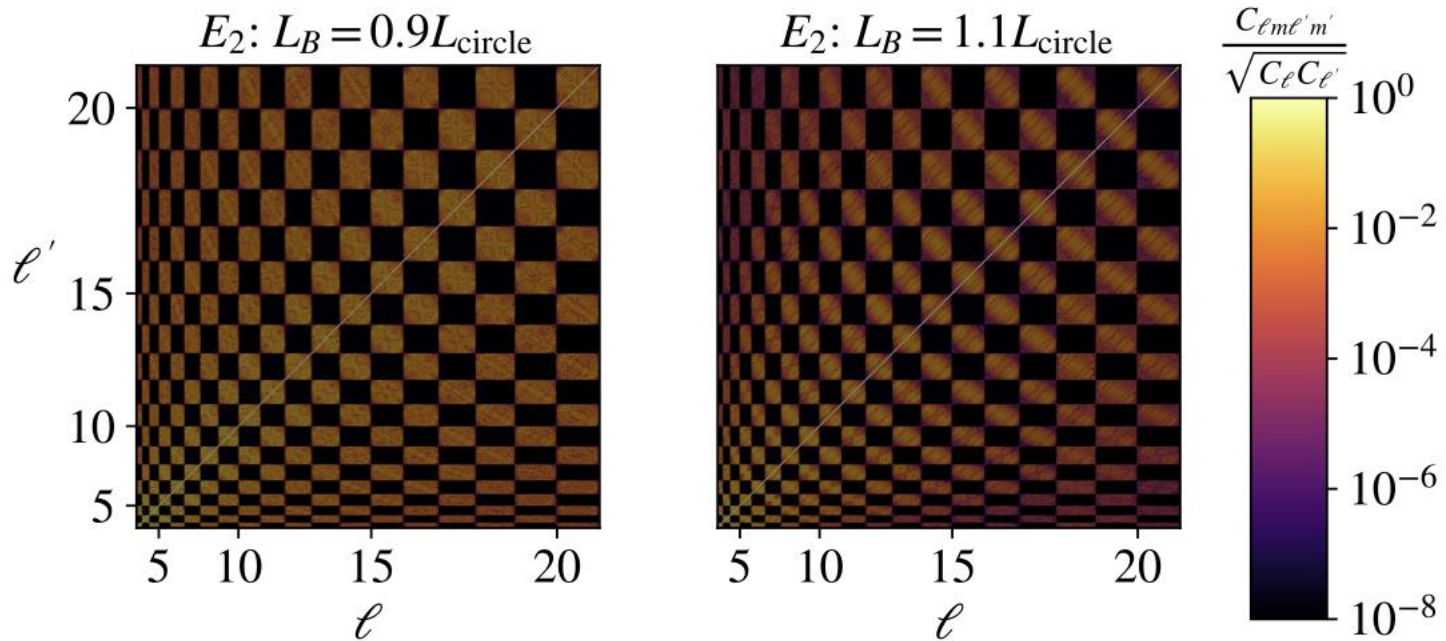


FIG. 1. Portions of rescaled CMB temperature correlation matrices for a half-turn space ( $E_2$ ).  $L_{\text{circle}}$  is the length scale below which matched circles would be detected;  $L_B = \{0.9, 1.1\} L_{\text{circle}}$  is the length along the corkscrew axis;  $L_A = 1.4 L_{\text{LSS}}$  is the other topological length scale. The observer is off-axis at  $x_0 = (0.35, 0, 0) L_{\text{LSS}}$

# SUMMARY

The CMB is NOT the realization of a Gaussian random statistically isotropic field.

The Universe is  
NOT Statistically Isotropic

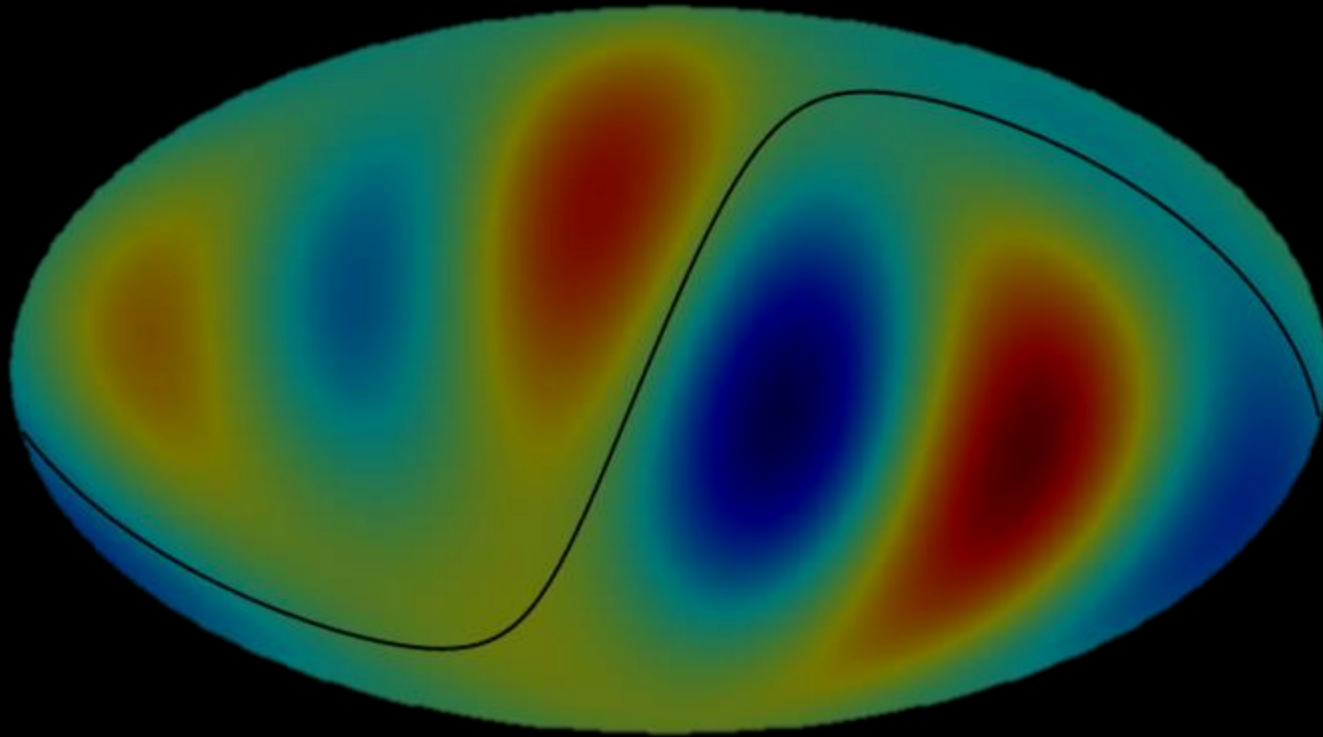
No proven “model” so far:

- ~~Systematics~~

- ~~Foregrounds~~

- Cosmology – topology?

The cosmic orchestra may be playing a LCDM symphony,  
But somebody gave the bass and tuba the wrong score.  
They tried hard to keep it quiet. They failed.



**We must find an explanation**

