Modular Invariance and the Strong CP problem

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$$\tau \to \frac{a\,\tau + b}{c\,\tau + d}$$

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Workshop on the Standard Model and Beyond

in collaboration with:
Matteo Parricciatu (Rome III), Alessandro Strumia and Arsenii Titov (Pisa)
& Robert Ziegler, in preparation

[2305.08908,2406.01689]

the strong CP problem

$$\mathcal{L}_{QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{4g_3^2}\mathcal{G}^a_{\mu\nu}\mathcal{G}^{a\mu\nu} + \frac{\theta_{QCD}}{32\pi^2}\mathcal{G}^a_{\mu\nu}\tilde{\mathcal{G}}^{a\mu\nu}$$

$$\bar{\theta} = \theta_{QCD} + \arg\det m \qquad d_n \approx 1.2\times 10^{-16}\,\bar{\theta}\; e\cdot cm$$

$$|\bar{\theta}| \lesssim 10^{-10} \quad \& \quad \delta_{CKM} \approx \mathcal{O}(1)$$

solutions

1. $\bar{\theta}$ promoted to a field, the axion, pseudoGB of a global, anomalous $U(1)_{PQ}$ symmetry
VEV dynamically relaxed to zero by QCD dynamics

2. CP (or P) is a symmetry of the UV

- CP ➔ $\theta_{QCD} = 0$
- CP spontaneously broken such that $\arg\det m = 0$
- $|\bar{\theta}| \lesssim 10^{-10}$ & $\delta_{CKM} \approx \mathcal{O}(1)$

our solution: CP symmetry of the UV theory

- no heavy VL quarks as in
  A.E. Nelson, 'Naturally Weak CP Violation', Phys.Lett.B 136 (1984) 387.
  S.M. Barr, 'Solving the Strong CP Problem Without the Peccei-Quinn Symmetry', Phys.Rev.Lett. 53 (1984) 329.

  no extra Higgs doublets as in
  H. Georgi, Hadronic J. **1**, 155 (1978).
  L. Hall, C. A. Manzari, and B. Noether (2024).
  Ferro-Hernandez, Morisi, Peinado 2407.18161

- in its minimal version, SM extended by a gauge singlet complex scalar field: the modulus

$\arg\det m = 0$

$$m = m(z)$$

z gauge-invariant (dimensionless) complex scalar fields

in string theory $z = z(\tau)$ are moduli describing e.g. sizes and shapes of the compactified dimensions

in d and u sectors

$$m(z) = \begin{pmatrix} 0 & 0 & c_{13} \\ 0 & c_{22} & \cdot \\ c_{31} & \cdot & \cdot \end{pmatrix}$$

$$\det m = -\, c_{13}\, c_{13}\, c_{13}$$

c_{13}, c_{13}, c_{13} real constants

$\arg\det m = 0$

$$m = m(z)$$

z gauge-invariant (dimensionless) complex scalar fields

in string theory $z = z(\tau)$ are moduli describing e.g. sizes and shapes of the compactified dimensions

same pattern in d and u sectors

$$m(z) = \begin{pmatrix} 0 & 0 & c_{13} \\ 0 & c_{22} & \cdot \\ c_{31} & \cdot & \cdot \end{pmatrix}$$

$$\det m = -\, c_{13}\, c_{13}\, c_{13}$$

c_{13}, c_{13}, c_{13} real constants

CKM phase generated by this block

$$m(z) = \begin{pmatrix} 0 & 0 & c_{13} \\ 0 & c_{22} & c_{23}(z) \\ c_{31} & c_{32}(z) & c_{33}(z) \end{pmatrix}$$

picks up a nontrivial phase from the z - VEV this pattern can be generated by two simple requirements

1. assign a weight to each field

field	D_i^c	Q_i	z_a	$\cdots$
weight	$k_{D_i^c}$	k_{Q_i}	$k_{z_a} > 0$	$\cdots$

require $$\sum_i (k_{D_i^c} + k_{Q_i}) = 0$$

example

$$k_{D_i^c} = (-1, 0, +1) \qquad k_{z_1} = +1$$

$$k_{Q_i} = (-1, 0, +1) \qquad k_{z_2} = +2$$

2. require $m_{ij}(z)$ to be a polynomial in z_a of weight $(k_{D_i^c}+k_{Q_j})$

in previous example $(k_{D_i^c}+k_{Q_j}) =$

$$\begin{array}{cc} & \overbrace{\begin{array}{ccc} -1 & 0 & +1 \end{array}}^{k_{Q_j}} \\ k_{D_i^c}\left\{\begin{array}{c} -1 \\ 0 \\ +1 \end{array}\right. & \begin{pmatrix} -2 & -1 & 0 \\ -1 & 0 & +1 \\ 0 & +1 & +2 \end{pmatrix} \end{array}$$

$$m_{ij}(z) = \begin{pmatrix} 0 & 0 & c_{13} \\ 0 & c_{22} & c_{23}\, z_1 \\ c_{31} & c_{31}\, z_1 & c_{33}\, z_1^2 + c'_{33}\, z_2 \end{pmatrix}$$

so far only a mathematical trick…

physics

above rules mandatory in a CP-invariant gauge theory

- modular-invariant ⇒ a weight to each field
- anomaly-free ⇒ sum of the weights should vanish
- supersymmetric ⇒ $Y^q_{ij}(z)$ a polynomial in z_a

(other realizations are also possible)

string-theory motivated

ST has no free parameters. Yukawa couplings are field-dependent quantities

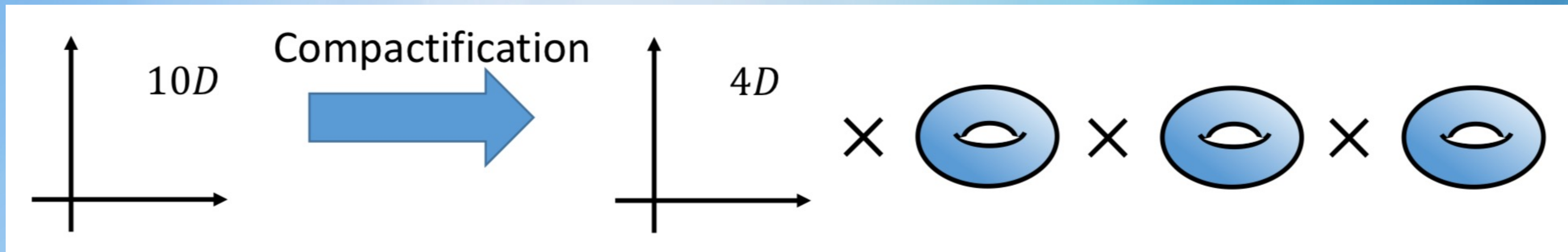
4D CP symmetry is a gauge symmetry in ST compactifications

modular invariance is a key aspect of most ST compactifications

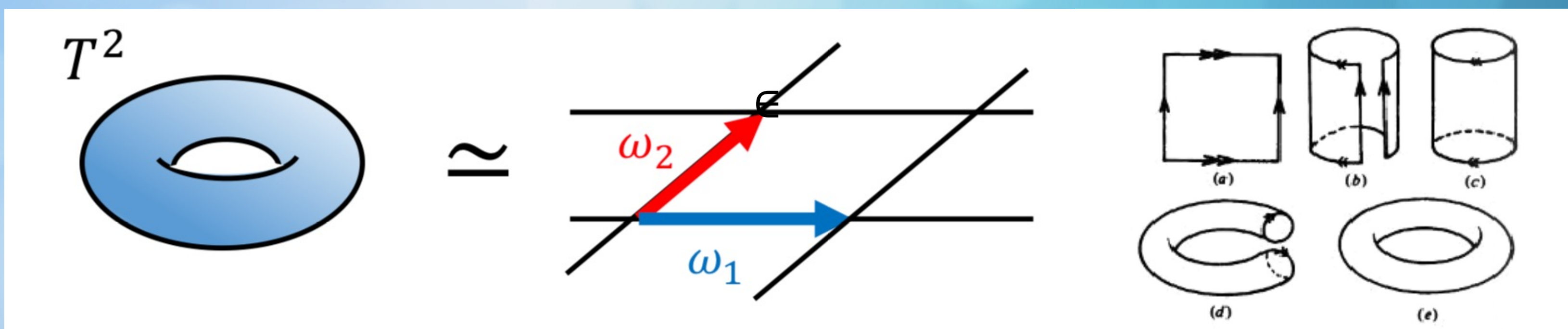
modular invariance

[see H.P. Nilles talk]

string theory in d=10 need 6 compact dimensions



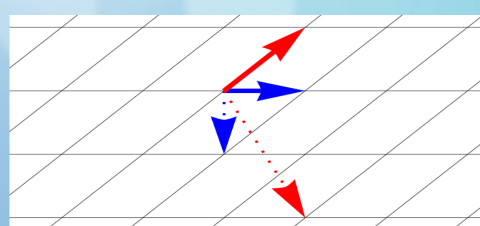
simplest compactification: 3 copies of a torus T^2



tori parametrized by

$$\mathcal{M} = \left\{ \tau = \frac{\omega_2}{\omega_1} \quad Im(\tau) > 0 \right\}$$

lattice left invariant by modular transformations:

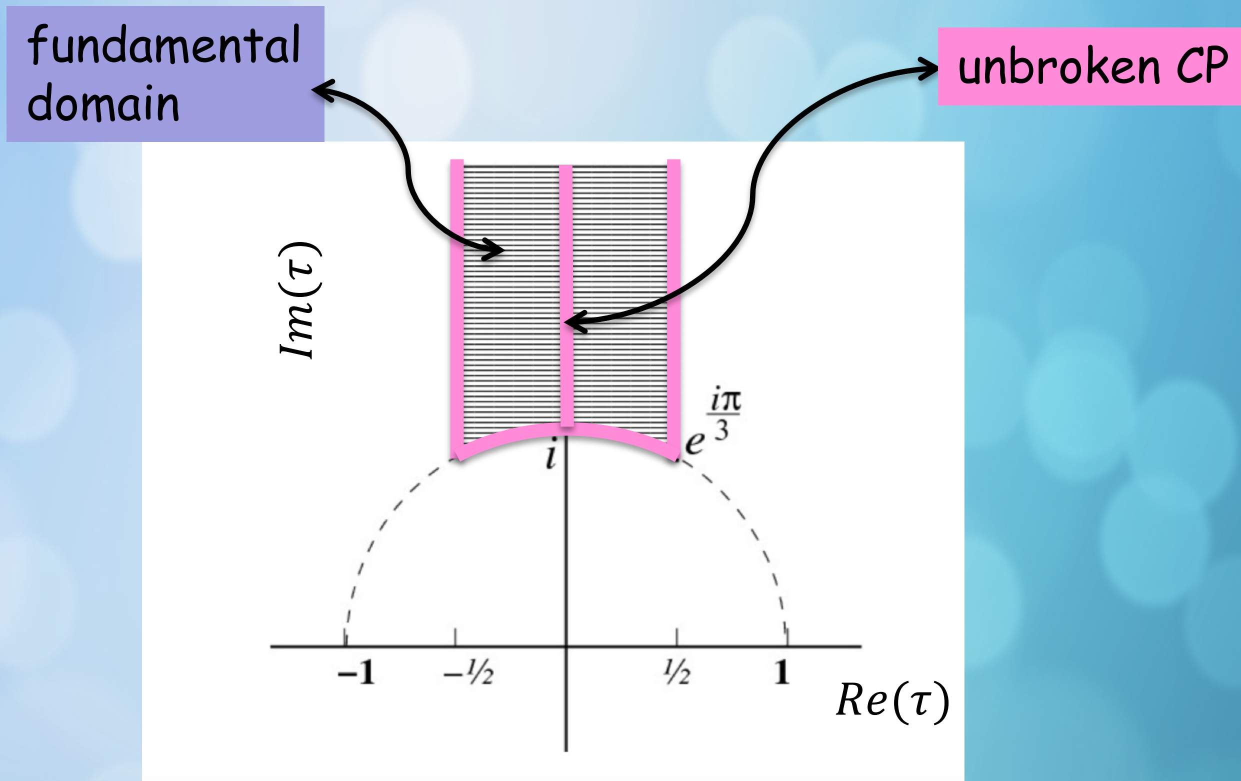


$$\tau \to \frac{a\tau + b}{c\tau + d} \in SL(2,Z)$$

a, b, c, d integers

$ad - bc = 1$

- τ promoted to a field. Through a gauge choice we can restrict τ to the fundamental domain



- CP

$$\tau \to -\tau^*$$

[up to modular transformations]

[Novichkov, Penedo, Petcov and Titov 1905.11970
Baur, Nilles, Trautner and Vaudrevange, 1901.03251]

$\mathcal{N}$=1 SUSY CP & modular-invariant theories

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\, K(\tau, e^{2V}\varphi, \bar{\tau}, \bar{\varphi}) + \left[\int d^2\theta\, w(\tau,\varphi) + \frac{1}{16}\int d^2\theta\, f\, WW + h.c\right]$$

kinetic terms

Yukawa couplings $\mathcal{Y}(\tau)$

gauge kinetic function

$$f_3 = \frac{1}{g_3^2} - i\frac{\theta_{QCD}}{8\pi^2}$$

$$\arg\det m(\tau) = \arg\det Y(\tau) v$$

G. Hiller, M. Schmaltz, 'Solving the Strong CP Problem with Supersymmetry', Phys.Lett.B 514 (2001) 263 [arXiv:hep-ph/0105254].

$$\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y(\tau) v$$

no dependence on K

$\bar{\theta}$ holomorphic

A note on the predictions of models with modular flavor symmetries

Mu-Chun Chen (UC, Irvine), Saúl Ramos-Sánchez (Mexico U. and Munich, Tech. U.), Michael Ratz (UC, Irvine)

15, 2019)

Published in: *Phys.Lett.B* 801 (2020) 135153 • e-Print: 1909.06910 [hep-ph]

CP & modular-invariance

- $CP \leftrightarrow$ real coupling constants
- modular invariance

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$\varphi \to (c\,\tau + d)^{-k_\varphi}\ \varphi$$ matter multiplets

$$V \to V$$ vector multiplets

- $f_3 = \dfrac{1}{g_3^2}$ $\quad (\theta_{QCD} = 0)$
- $w(\tau, \varphi) = U_i^c\, Y_{ij}^u(\tau)\, Q_j H_u + D_i^c\, Y_{ij}^d(\tau)\, Q_j H_d + E_i^c\, Y_{ij}^e(\tau)\, L_j H_d + \cdots$

$$Y_{ij}^q(\tau) \to (c\tau + d)^{k_{ij}^q}\, Y_{ij}^q(\tau)$$

$$k_{ij}^u = k_{Q_j} + k_{U_i^c} + k_{H_u}$$

$$k_{ij}^d = k_{Q_j} + k_{D_i^c} + k_{H_d}$$

assuming no singularities: $Y_{ij}^q(\tau)$ are modular forms of weight k_{ij}^q

$k_{ij}^{q} < 0$: no modular forms

$k_{ij}^{q} = 0$: modular forms are constants

$k_{ij}^{q} > 0$: modular forms polynomials in $E_4(\tau), E_6(\tau)$

Modular weight k	0	2	4	6	8	10	12	14
Number of forms	1	0	1	1	1	1	2	1
Modular forms	1	–	E_4	E_6	$E_8 = E_4^2$	$E_{10} = E_4 E_6$	E_4^3, E_6^2	$E_{14} = E_4^2 E_6$

$$\det Y(\tau) \equiv \det Y^u(\tau) \det Y^d(\tau)$$

$$\det Y(\tau) \to (c\tau + d)^{k_{\det}} \quad \det Y(\tau)$$

$$k_{\det} = \sum_{i=1}^{3} \left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) + 3k_{H_u} + 3k_{H_d}$$

$k_{\det} = 0$ ⇒ $\det Y(\tau) =$ (real) constant

cancellation of modular anomalies

$$\psi_{can} \to \left(\frac{c\tau + d}{c\tau^{+} + d}\right)^{-\frac{k_\varphi}{2}} \psi_{can}$$

conditions for gauge-modular anomaly cancellation

$SU(3)$
$$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) = 0$$

$SU(2)$
$$\sum_{i=1}^{3}\left(3k_{Q_i} + k_{L_i}\right) + k_{H_u} + k_{H_d} = 0$$

$U(1)$
$$\sum_{i=1}^{3}\left(k_{Q_i} + 8k_{U_i^c} + 2k_{D_i^c} + 3k_{L_i} + 6k_{E_i^c}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

cancellation of modular anomalies

$$\psi_{can} \to \left(\frac{c\tau + d}{c\tau^+ + d}\right)^{-\frac{k_\varphi}{2}} \psi_{can}$$

conditions for gauge-modular anomaly cancellation

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$U(1)$ $$\sum_{i=1}^{3}\left(k_{Q_i} + 8k_{U_i^c} + 2k_{D_i^c} + 3k_{L_i} + 6k_{E_i^c}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

simplest solution:

$$k_{H_u} + k_{H_d} = 0$$

$$k_{Q_i} = k_{U_i^c} = k_{D_i^c} = k_{L_i} = k_{E_i^c} = (-k, 0, k)$$

$$k_{\det} = 0$$

$\det Y(\tau)$ & f real constants

$$\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y = 0$$

holds also after SUSY breaking, if no new phases in SUSY breaking sector

Example $k_{Q_i} = k_{U_i^c} = k_{D_i^c} = (-6, 0, +6)$

$$Y_q(\tau) = \begin{pmatrix} 0 & 0 & c_{13}^q \\ 0 & c_{22}^q & c_{23}^q E_6 \\ c_{31}^q & c_{32}^q E_6 & c_{33}^q E_4^3 + c_{33}^{q\prime} E_6^2 \end{pmatrix}$$

$\tan\beta = 10 \quad \tau = 0.125 + i$

$$c_{ij}^u \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.56 \\ 0 & -1.86 & 0.87 \\ 1.29 & 4.14 & 3.51, 1.40 \end{pmatrix}, \qquad c_{ij}^d \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.55 \\ 0 & -2.59 & 4.59 \\ 0.378 & 0.710 & 0.734, 1.76 \end{pmatrix}$$

reproduce quark masses, mixing angles and CKM phase

$\delta_{CKM} \neq 0 \quad \Leftrightarrow \quad Im\,\det[Y_u^+ Y_u, Y_d^+ Y_d] \neq 0$ non-holomorphic

Leptons: $k_{L_i} = k_{E_i^c} = (-6, 0, +6)$

$$c_{ij}^e = 10^{-3} \begin{pmatrix} 0 & 0 & 1.29 \\ 0 & 5.95 & 0.35 \\ -2.56 & 1.47 & 1.01, 1.32 \end{pmatrix}, \qquad c_{ij}^\nu = \frac{1}{10^{16}\,\text{GeV}} \begin{pmatrix} 0 & 0 & 3.4 \\ 0 & 7.1 & 1.2 \\ 3.4 & 1.2 & 0.19, 0.95 \end{pmatrix}$$

deviations from $\bar{\theta} = 0$

SUSY unbroken

no corrections from K
no corrections from nonrenormalizable operators: $SL(2, \mathbb{Z})$
no corrections from additional moduli/singlets under reasonable assumptions

SUSY breaking corrections

potentially big if soft terms violate flavour in a generic way

minimized if $\Lambda_{CP} \gg \Lambda_{SUSY}$ (as e.g. in gauge mediation)

and soft breaking terms respect the flavour structure of the SM

$$\bar{\theta} \lesssim \frac{M_t^4 M_b^4 M_c^2 M_s^2}{v^{12}} J_{\mathrm{CP}} \tan^6 \beta \sim 10^{-28} \tan^6 \beta .$$

SM corrections

negligible: $\bar{\theta} \leq 10^{-18}$ at four loops

J.R. Ellis, M.K. Gaillard, 'Strong and Weak CP Violation', Nucl.Phys.B 150 (1979) 141.

I.B. Khriplovich, 'Quark Electric Dipole Moment and Induced θ Term in the Kobayashi-Maskawa Model', Phys.Lett.B 173 (1986) 193.

variants

Solving the strong CP problem without axions #1
Ferruccio Feruglio (INFN, Padua), Matteo Parriciatu (INFN, Rome and Rome III U.), Alessandro Strumia (Pisa U.), Arsenii Titov (Pisa U.) (Jun 3, 2024)
e-Print: 2406.01689 [hep-ph]

- higher levels, smaller weight

  modular forms associated with subgroups of $SL(2,Z)$

  $$k_{Q_i} = k_{U_i^c} = k_{D_i^c} = (-1,0+1) \; or \; (-2,0+2)$$

  perhaps easier to occur in string theory

- With heavy vector-like quarks

  anomaly of IR theory canceled by a nontrivial gauge kinetic function

  $$f_{IR} = f_{UV} - \frac{1}{8\pi^2} \log \det Y_{Heavy}(\tau)$$

  many more viable patterns of quark mass matrices

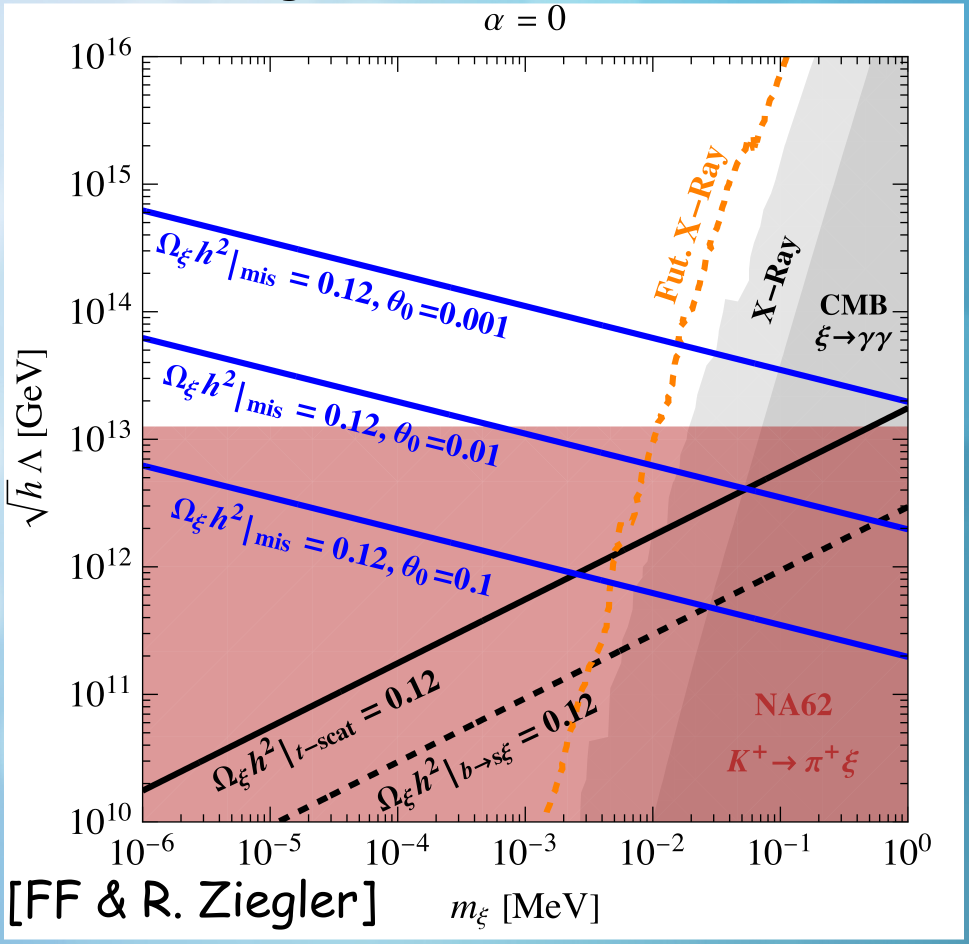
- can be extended to supergravity

Modular invariance and the QCD angle #3
Ferruccio Feruglio (INFN, Padua), Alessandro Strumia (Pisa U.), Arsenii Titov (Pisa U.) (May 15, 2023)
Published in: *JHEP* 07 (2023) 027 • e-Print: 2305.08908 [hep-ph]

phenomenology

- couplings to matter suppressed by $1/\Lambda$ $(1/M_{Pl}$ in SUGRA)
- difficult to test if modulus heavy
- if light, modulus ≈ CP-violating ALP [see G. Levati talk at this meeting]

DM candidate if modulus mass below 1 MeV



[FF & R. Ziegler]

Ingredients

1. CP in the UV

2. Yukawa couplings are field-dependent quantities

3. the vacuum has a redundant description: vacua related by $SL(2,\mathbb{Z})$ are equivalent

4. CP and $SL(2,\mathbb{Z})$ are unified in a gauge flavour symmetry

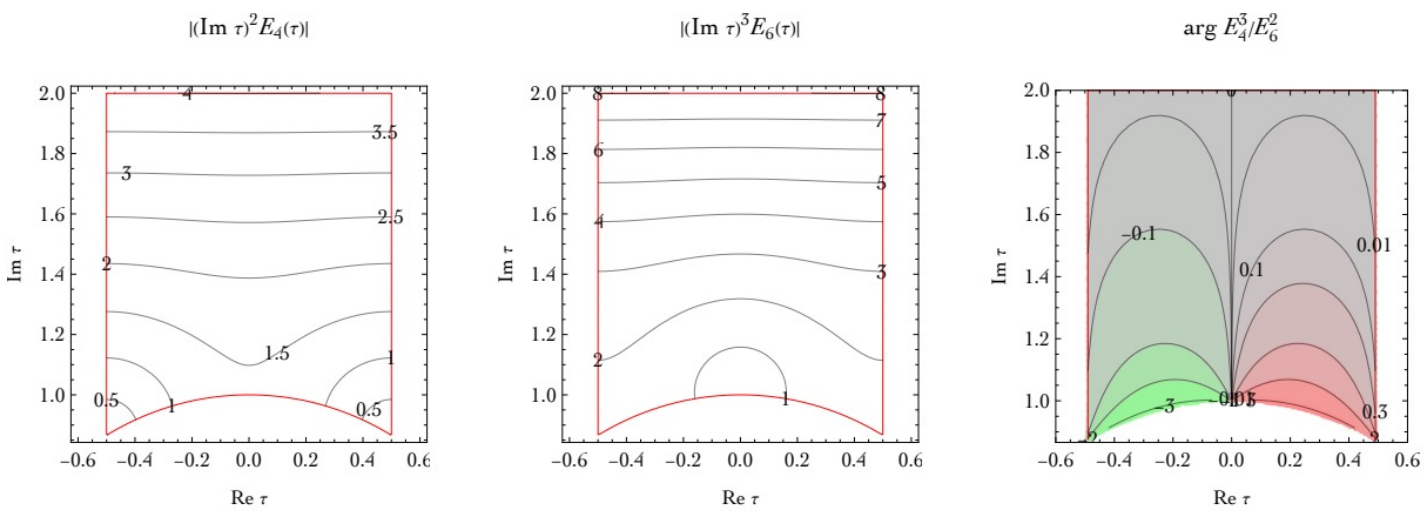
5. absence of anomalies

6. no singularities in the UV theory

Ingredients		String Theory
1.	CP in the UV	the four-dimensional CP symmetry is a gauge symmetry in most string theory compactifications.
2.	Yukawa couplings are field-dependent quantities	string theory has no free parameters anc Yukawa couplings are set by moduli VEVs
3.	the vacuum has a redundant description: vacua related by $SL(2,\mathbb{Z})$ are equivalent	modular invariance is a key ingredient of string theory compactifications
4.	CP and $SL(2,\mathbb{Z})$ are unified in a gauge flavour symmetry	**Unification of Flavor, CP, and Modular Symmetries** Alexander Baur (Munich, Tech. U.), Hans Peter Nilles (Bonn U. and Bonn U., HISK Trautner (Heidelberg, Max Planck Inst.), Patrick K.S. Vaudrevange (Munich, Tech Published in: *Phys.Lett.B* 795 (2019) 7-14 • e-Print: 1901.03251 [hep-th]
5.	absence of anomalies	mandatory in string theory
6.	no singularities in the UV theory	string theory is free of singularities. These arise in the IR when some UV modes become massless


THANK
YOU!

back-up slides



$|(\mathrm{Im}\,\tau)^2 E_4(\tau)|$
$|(\mathrm{Im}\,\tau)^3 E_6(\tau)|$
$\arg E_4^3/E_6^2$
Im τ
Re τ

axion solution

- $\bar{\theta}$ dynamically relaxed to zero by the axion, would-be GB of a global, anomalous $U(1)_{PQ}$ symmetry

- provides a candidate for DM
- many axion candidates in e.g. superstring theories

- axion quality problem

minimum of $V(a)$ should be at $a = 0$

$$V(a) = V_{QCD}(a) - M^4 e^{-S} \cos(\frac{a}{f_a} + \delta)$$

$M = M_P$
$\delta = \mathcal{O}(1)$

$\Rightarrow$ $S \geq 200$

- axion undetected, so far

Nelson-Barr solution | our solution

CP ia a symmetry of the UV,
SB to get $\bar{\theta} = 0$ & $\delta_{CKM} = \mathcal{O}(1)$

CP $\Rightarrow$ $\theta_{QCD} = 0$

Nelson-Barr solution	our solution
heavy vector-like quark sector	no extra matter
CP spontaneously broken by $\langle \eta_a \rangle$ complex [one is not enough]	CP spontaneously broken by τ alone
$\mu \approx \lambda_a \eta_a$ [tuning]	no tuning

$$\begin{array}{c|c} Q & q \end{array}$$

$$m = \begin{pmatrix} \mu & \lambda_a \eta_a \\ 0 & y\, v \end{pmatrix}$$

Yukawa matrices $Y_{u,d}$	Modular weights $(u_L, d_L)_{1,2,3}$	$u_{R1,2,3}$	$d_{R1,2,3}$	Alternative bigger weights $(u_L, d_L)_{1,2,3}$	$u_{R1,2,3}$	$d_{R1,2,3}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_6 \\ 1 & E_6 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ -2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4^2 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4 E_6 \\ 1 & E_6 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^3 + E_6^2 \\ 1 & E_4 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$

Table 2: *Simplest modular weights that lead to Yukawa matrices such that $\bar{\theta} = 0$ and $\delta_{\rm CKM} \neq 0$. The list is complete up to permutations and transpositions, and assumes vanishing modular weights of the Higgs doublets and of the super-potential. Real constants c_{ij}^q are here omitted.*

fixed point

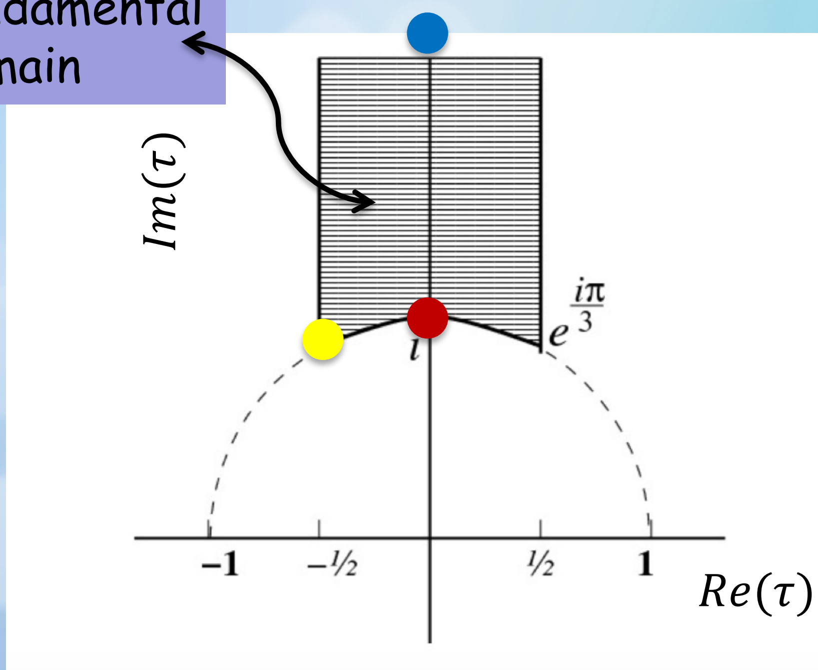
$\tau = i$ $\quad \tau \xrightarrow{S} -\frac{1}{\tau}$ $\quad \mathbb{Z}_4^S$

$\tau = e^{i\,2\pi/3}$ $\quad \tau \xrightarrow{ST} -\frac{1}{\tau+1}$ $\quad \mathbb{Z}_2^{ST} \times \mathbb{Z}_2^{S^2}$

$\tau = i\,\infty$ $\quad \tau \xrightarrow{T} \tau + 1$ $\quad \mathbb{Z}^T \times \mathbb{Z}_2^{S^2}$

residual symmetry

fundamental domain



modular invariance completely broken everywhere but at three fixed points

$SL(2,Z)$ generated by

$$S : \tau \to -\frac{1}{\tau} \quad , \qquad T : \tau \to \tau + 1$$

heavy quarks and singularities

- heavy quarks not needed, but they can exist in the UV

  example $$k_\varphi = (\underbrace{-6,-2,0,}_{\text{chiral}}\underbrace{+2,+6)}_{\text{heavy vector-like quark}}$$ $$k_{H_u} + k_{H_d} = 0$$

- UV theory $$\bar{\theta} = -8\pi^2 Im\, f_{UV} + \arg\det Y_{UV} = 0$$

- IR theory has an anomalous field content, anomaly cancelled by: $$f_{IR} = f_{UV} - \frac{1}{8\pi^2}\log\det Y_{Heavy}(\tau)$$

$$\bar{\theta} = -8\pi^2 Im\, f_{IR}(\tau) + \arg\det Y_{Light}(\tau) =$$

$$= +\arg\det Y_{Heavy}(\tau) + \arg\det Y_{Light}(\tau)$$

$$= \arg\det Y_{UV} = 0$$

- $Y_{Light}(\tau)$ is singular at τ values such that $\det Y_{Heavy}(\tau) = 0$

$\mathcal{N} = 1$ supergravity

$$K = -h^2 \log(-i\tau + i\tau^+) + \cdots$$

$$k_W = \frac{h^2}{M_{Pl}^2} \to 0$$

back to the rigid case

corrections of $\mathcal{O}(k_W)$?

- K and w no more independent

$$\mathcal{G} = \frac{K}{M_{Pl}^2} + \log\left|\frac{w}{M_{Pl}^3}\right|^2$$

$$w(\tau) \to (c\tau + d)^{-k_W}\, w(\tau)$$

$$k_W > 0$$

no negative weight modular forms, $w(\tau)$ singular somewhere

- modular-QCD anomaly modified into

$$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c} - 2k_W\right) + 3k_W$$

quarks

gluino

can be rotated away
if gluino is massless

V. Kaplunovsky, J. Louis, 'On Gauge couplings in string theory', Nucl.Phys.B 444 (1995) 191 [arXiv:hep-th/9502077].

J.P. Derendinger, S. Ferrara, C. Kounnas, F. Zwirner, 'On loop corrections to string effective field theories: Field dependent gauge couplings and sigma model anomalies', Nucl.Phys.B 372 (1992) 145.

L.J. Dixon, V. Kaplunovsky, J. Louis, 'Moduli dependence of string loop corrections to gauge coupling constants', Nucl.Phys.B 355 (1991) 649.

spontaneously broken supergravity

- $\bar{\theta} = -8\pi^2 Im\, f + \arg\det M_{quark} + 3\arg M_3 = 0$

$$\arg\det M_{quark} = 0 \Leftarrow \sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c} - 2k_W\right) = k_{H_u} + k_{H_d} = 0$$

$\arg M_3 = -\arg w \Leftarrow$ if no other phases from SUSY breaking

$$M_3 = \frac{1}{2} e^{\frac{K}{2M_{Pl}^2}} K^{i\bar{j}} D_{\bar{j}} w^{+} f_i$$

- assume unique singularity at $\tau = i\infty$

$$w(\tau) = \cdots + c_0 M_{Pl}^3\, \eta(\tau)^{-2k_W}$$

$$f = \cdots + 3\frac{k_W}{4\pi^2}\log\eta(\tau)$$

$\eta(\tau)$ Dedekind eta function

H. Rademacher, H.S. Zuckeman, 'On the Fourier coefficients of certain modular forms of positive dimensions', Annals of Mathematics 39 (1938) 433.

cancels the gluino anomaly

$$\bar{\theta} = -8\pi^2 Im\, f + 3\arg M_3 = 0$$