Noncommutative geometry and higher spin gravity/symmetry

Workshop on noncommutative and generalized geometry, Corfu

Evgeny Skvortsov, UMONS

September 21, 2024

European Research Council

Established by the European Commission

UMONS

Université de Mons

fnrs

FREEDOM TO RESEARCH

Main Messages/Plan

- **Higher spin gravity** (HiSGRA) is one of ideas to construct/solve models of quantum gravity, which is heavily based on ∞-dim symmetry
- **Higher spin symmetry** is an unusual interpretation/application of the usual Deformation Quantization, also present in simple CFT's, but HiSGRA require much more ...
- **Chiral HiSGRA** is a $4d$ model of quantum gravity that via AdS/CFT is related to $3d$ CFT's aka second order phase transitions (Ising). Equations of motion $=$ Poisson sigma-model that originates from a pre-Calabi-Yau A_∞ algebra aka NC Poisson structure. It owns its existence to **fuzzy-sphere**.

Why higher spins?

How high is higher spin?

Standard context: quantum gravity problem

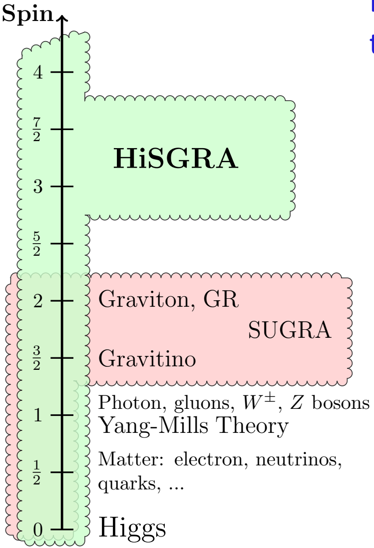
Standard assumption: to "solve all problems" without going too far from the well-established concept of particles/fields. Massless fields have helicity $\lambda = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm\mathbf{2}, \ldots$

For any triplet of helicities λ_i, $\lambda_1 + \lambda_2 + \lambda_3 > 0$ there is a unique interaction vertex (Brink, Bengtsson[2], Linden, 1983-7):

$$V_3 \sim C_{\lambda_1,\lambda_2,\lambda_3}[12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[31]^{\lambda_3+\lambda_1-\lambda_2}$$

Questions: what are the options to have interactions? Which ones incorporate gravity? Which ones are quantum consistent? What the right math to describe them? Relation to actual physics? ...

Spin by spin



Different spins lead to very different types of theories/physics:

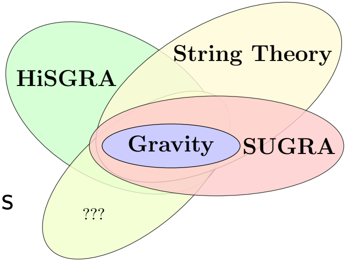
- $s = 0$: Higgs
- $s = 1/2$: Matter

- $s = 1$: Yang-Mills, Lie algebras
- $s = 3/2$: SUGRA and supergeometry, graviton $\in$ spectrum
- $s = 2$: GR and Riemann Geometry, no color (Boulanger et al)

- $\boldsymbol{s > 2}$: HiSGRA and String theory, ∞ states, graviton is there too!

Why massless higher spins?

- string theory
- acausality of higher der. corrections to gravity (Camanho et al.)
- divergences in (SU)GRA's
- Quantum Gravity via AdS/CFT



$\rightarrow$ quantization of gravity $\rightarrow$

- unbounded spin $\rightarrow \infty$ many fields
- UV $\rightarrow$ massless

HiSGRA $=$ the smallest extension of gravity by massless, i.e. gauge, higher spin fields. Vast gauge symmetry should render it finite.

Quantizing Gravity via HiSGRA $\sim$ Classical HiSGRA?

HiSGRA that survived

Quantizing Gravity via HiSGRA $\sim$ Classical HiSGRA

HiSGRA can be good probes of the Quantum Gravity Problem

Local HiSGRA in 2024

3d massless, conformal and partially-massless (Blencowe; Bergshoeff, Stelle; Campoleoni, Fredenhagen, Pfenninger, Theisen; Henneaux, Rey; Gaberdiel, Gopakumar; Grumiller; Grigoriev, Mkrtchyan, E.S.; Pope, Townsend; Fradkin, Linetsky; Lovrekovic; ...), $S = S_{CS}$ for a HS extension of $sl_2 \oplus sl_2$ or $so(3,2)$

$$S = \int \omega\, d\omega + \tfrac{2}{3}\omega^3$$

4d conformal (Tseytlin, Segal; Bekaert, Joung, Mourad; Adamo, Tseytlin; Basile, Grigoriev, E.S.; ...), higher spin extension of Weyl gravity, local Weyl symmetry tames non-localities

$$S = \int \sqrt{g}\,(C_{\mu\nu,\lambda\rho})^2 + \ldots$$

4d massless chiral (Metsaev; Ponomarev, E.S.; Ponomarev; E.S., Tran, Tsulaia, Sharapov, Van Dongen, ...). The smallest HiSGRA with propagating fields.

IKKT model for fuzzy H_4 (Steinacker, Sperling, Fredenhagen, Tran)

The theories avoid all no-go's, as close to Field Theory as possible

Chiral HiSGRA: M-theory of all self-dual ones

It is easy to get SDYM as "truncation" of YM ($\boldsymbol{A_\mu} \rightarrow \boldsymbol{\Phi^\pm}$)

$$\boldsymbol{\mathcal{L}}_{\text{YM}} = \mathbf{tr}\, \boldsymbol{F_{\mu\nu} F^{\mu\nu}}$$
$$\wr\wr$$
$$\boldsymbol{\mathcal{L}}_{\text{SDYM}} = \boldsymbol{\Phi^- \Box \Phi^+} + \boldsymbol{V^{++-}} + \boldsymbol{V^{--+}} + \boldsymbol{V^{++--}}$$

SDYM can also be described covariantly with the help of

$$F \wedge F = F^2_{AB} - F^2_{A'B'} \qquad\qquad F^2_{\mu\nu} = F^2_{AB} + F^2_{A'B'}$$

where F_{AB}, $F_{A'B'}$ are the (anti)self-dual components in the $sl(2,\mathbb{C})$-language. Next, a couple of tricks due to (Chalmers, Siegel)

$$S_{YM} = \int F^2_{\mu\nu} \sim \int F^2_{AB} \sim \int \boldsymbol{C^{AB} F_{AB}} - \boldsymbol{\frac{g'}{2} C^2_{AB}}\,,$$

Self-dual gravity (SDGR)

Starting from Plebansky formulation of Einstein-Hilbert action, but with a bit more tricks one can arrive at (Krasnov)

$$S_{EH} = \int F^{AB} \wedge F_{AB} + \int \boldsymbol{C^{ABCD} F_{AB} \wedge F_{CD}} + \boldsymbol{\mathcal{O}(C^2)}$$

where C^{ABCD} looks like the SD part of the Weyl tensor and

$$F^{AB} = d\omega^{AB} + \omega^{A}{}_{C} \wedge \omega^{CB}$$

is the SD part of the Riemann two-form.

SDGR is a UV-finite theory of massless spin-two field (Krasnov)

One can easily "draw a line" that goes through SDYM, SDGR and extends to higher-spin fields ...

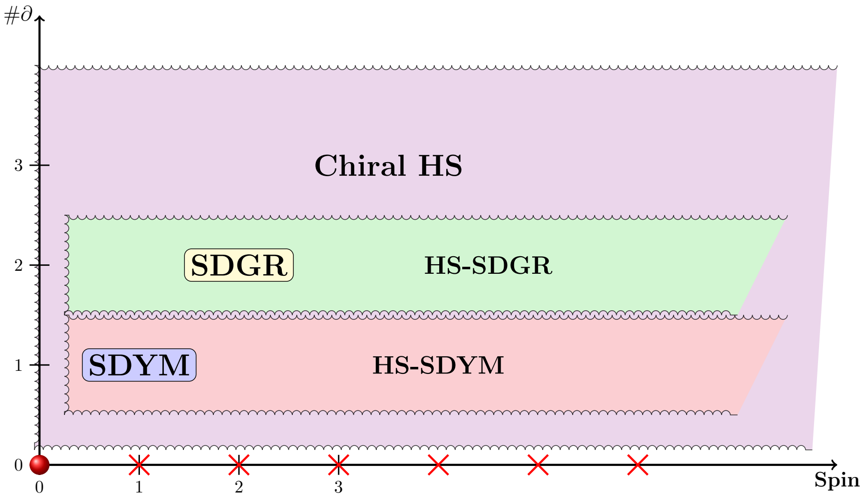
Self-dual Theories

- actions are not real in Minkowski space
- actions are simpler than the complete theories
- integrability, instantons (Atiyah, Hitchin, Drinfeld, Manin; ...)
- **SD theories are consistent truncations**, so anything we can compute will be a legitimate observable in the full theory; Unitary physics from nonunitary theories!

In general: amplitudes (MHV, BCFW, double-copy, ...), strings, QFT, Twistors, ... encourage to go outside Minkowski

In higher spins: little explored (Adamo, Hähnel, McLoughlin; E.S., Ponomarev; Ponomarev; Tran; Krasnov, Herfray, E.S.), can be the only reasonably local theories

M-theory of self-dual theories



Twistors treat positive and negative helicities differently:

$-s$: $C^{A(2s)}$ (Penrose, 1965)

$+s$: $\delta\omega^{A(2s-2)} = \nabla\xi^{A(2s-2)} + \dots$ (Hitchin, 1980; Krasnov, E.S.)

The simplest free action for higher-spin fields reads

$$\boldsymbol{S = \int \Psi^{A(2s)} \wedge H_{AA} \wedge \nabla\omega_{A(2s-2)}}$$

where $H^{AB} \equiv e^{A}{}_{C'} \wedge e^{BC'}$ is the basis of self-dual two-forms built from vierbein $e^{AA'}$.

It reproduces the free limits of SDYM and SDGR. Interactions?

HS-SDYM, HS-SDGR

Chiral HiSGRA admits two contractions (Ponomarev) to higher spin extensions of SDYM and SDGR

They can be covariantized (Krasnov, E.S., Tran). Firstly, we pack

$$\omega(y) = \sum_k \omega_{A_1\ldots A_k}\, y^{A_1}\ldots y^{A_k} \qquad C(y) = F^{AB} y_A y_B + \ldots$$

$\nabla\omega$ and H can be replaced with

$$F = d\omega + \tfrac{1}{2}[\omega,\omega]$$

where $[f,g]$ is either due to Lie algebra (HS-SDYM) or due to Poisson bracket on $\mathbb{R}^2$ (HS-SDGR, $\Lambda \neq 0$), which is the same as $w_{1+\infty}$:

$$\{f,g\} = \epsilon^{AB}\,\partial_A f(y)\,\partial_B g(y)$$

Chiral Higher Spin Gravity

(what can be said without NC-geometry)

Chiral HiSGRA: down to Earth approach

Chiral HiSGRA (Metsaev; Ponomarev, E.S.) is a 'higher spin extension' of SDYM/SDGR. It has fields of all spins $s = 0, 1, 2, 3, \dots$:

$$\boldsymbol{\mathcal{L}} = \sum_{\boldsymbol{\lambda}} \boldsymbol{\Phi}^{-\boldsymbol{\lambda}} \Box \boldsymbol{\Phi}^{+\boldsymbol{\lambda}} + \sum_{\boldsymbol{\lambda}_i} \frac{\boldsymbol{\kappa}\, \boldsymbol{l}_{\mathsf{Pl}}^{\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2+\boldsymbol{\lambda}_3-1}}{\boldsymbol{\Gamma}(\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2+\boldsymbol{\lambda}_3)} \boldsymbol{V}^{\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\boldsymbol{\lambda}_3} + \mathcal{O}(\boldsymbol{\Lambda})$$

light-cone gauge is very close to the spinor-helicity language

$$\boldsymbol{V}^{\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\boldsymbol{\lambda}_3} \sim [\mathbf{12}]^{\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2-\boldsymbol{\lambda}_3} [\mathbf{23}]^{\boldsymbol{\lambda}_2+\boldsymbol{\lambda}_3-\boldsymbol{\lambda}_1} [\mathbf{13}]^{\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_3-\boldsymbol{\lambda}_2}$$

Locality + Lorentz invariance + genuine higher spin interaction result in a unique completion. Smooth in the cosmological constant.

This is the smallest higher spin theory and it is unique. Graviton and scalar field belong to the same multiplet

No UV Divergences! One-loop finiteness

Tree amplitudes vanish. The interactions are naively non-renormalizable, the higher the spin the more derivatives:

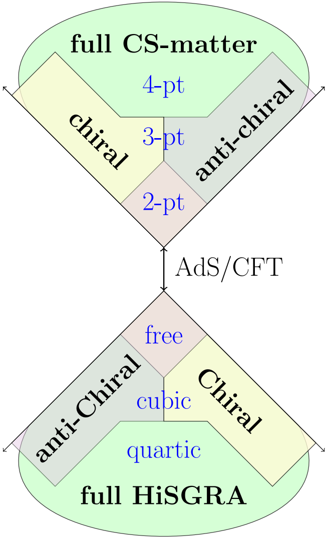
$$\boldsymbol{V^{\lambda_1,\lambda_2,\lambda_3} \sim \partial^{|\lambda_1+\lambda_2+\lambda_3|}\Phi^3}$$

there are **no UV divergences!** (E.S., Tsulaia, Tran). Some loop momenta factor out, just as in $\mathcal{N}=4$ SYM, but ∞-many times.

At one loop we find three factors: (1) SDYM or all-plus 1-loop QCD; (2) higher spin dressing to account for λ_i; (3) total number of d.o.f.:

$$\boldsymbol{A}^{\text{1-loop}} = \boldsymbol{A}^{++\ldots+}_{\text{QCD,1-loop}} \times \boldsymbol{D_{\lambda_1,\ldots,\lambda_n}} \times \sum_\lambda 1 \to 0$$

$\#$ d.o.f.$= \sum_\lambda 1 = 1+2\sum_{\lambda>0} 1 = 1+2\zeta(0) = 0$ to comply with no-go's, (Beccaria, Tseytlin) and agrees with many results in AdS, where $\neq 0$



The existence of Chiral HiSGRA implies: there are two closed subsectors of Chern-Simons vector models

One can define them holographically, but it would be interesting to identify them on the CFT side (Aharony, Kalloor, Kukolj; Jain, Dhruva, E.S.);

All 3pt-functions, bosonization up to 4pt (Yin, E.S.)

There are two new CFTs!

Chiral Higher Spin Gravity

(NC-geometry)

Chiral HiSGRA: Poisson sigma-model

Covariant equations of motion are those of $\boldsymbol{4d}$ Poisson Sigma Model

$$dC^i = \pi^{ij}(C)\,\omega_j\,, \qquad\qquad d\omega_k = \tfrac{1}{2}\partial_k\pi^{ij}(C)\,\omega_i\,\omega_j\,.$$

$\omega_i = \omega + \omega^{AB}y_Ay_B + ...$ and $C^i = \phi + F^{AB}y_Ay_B + ...$

It is not topological since C is ∞-dimensional!

If we lived in $2d$ the action would be just PSM

$$S_{PSM} = \int_\Sigma C^i d\omega_i + \tfrac{1}{2}\omega_i\omega_j\,\pi^{ij}(C)$$

The same structure works in $3d$ (Sharapov, E.S., Sukhanov) and the action is a Courant sigma-model. In $4d$ one can write down a presymplectic (Grigoriev et al) action (Sharapov, E.S.)

Poisson structure awakens

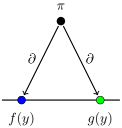
The Poisson structure is on $C^i = C(y)$, which is ∞-dim.

$\pi^{ij} = 0 + \pi^{ij}_k C^k + \ldots$ and π^{ij}_k is the Moyal-Weyl commutator

Any linear Poisson structure is just a Lie algebra

$$\begin{array}{ccccccccc} \mathbb{R}^2 & & f(y) \in C[\mathbb{R}^2] & & \boldsymbol{A_1} & & \mathrm{Lie}(\boldsymbol{A_1}) & & \\ & \to & & \to & & \to & & \to & \pi^{ij}_k \\ \epsilon^{AB} & & \{f,g\} & & f \star g & & [f,g]_\star & & \end{array}$$

$$\{f,g\} = \partial_A f\, \epsilon^{AB}\, \partial_B g$$

$$f \star g = \exp$$ 

What about higher orders? The PSM equations is a particular case of

$$\boldsymbol{d\Phi = l_2(\Phi, \Phi) + l_3(\Phi, \Phi, \Phi) + ... = Q(\Phi)}$$

where $\boldsymbol{l_n}$ define an L_∞-algebra. Our L_∞ originates from an A_∞ of pre-Calabi-Yau type (cyclic), which is NC Poisson structure (Kontsevich et al). $\boldsymbol{m_2}(\bullet, \bullet)$ defines an associative algebra, Weyl algebra $\boldsymbol{A_1}$.

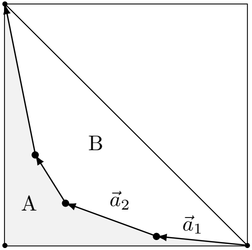
Master Hamiltonian $S = \frac{1}{2}\pi^{ij}(C)\,\omega_i\,\omega_j$ obeys the master equation

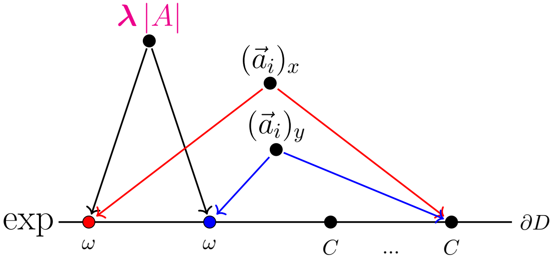
$$\{S, S\} = \frac{\partial S}{\partial C^i}\frac{\partial S}{\partial \omega_i} = 0 \qquad\qquad \Omega = dC^i \wedge d\omega_i$$

pre-Calabi-Yau is a non-commutative Poisson structure, $[\boldsymbol{S}, \boldsymbol{S}]_{nc} = 0$,

$$\boldsymbol{S} = \sum \langle \boldsymbol{m_n}(\omega, \omega, C, ..., C)|C\rangle$$

Explicit answer for all maps, e.g. $\boldsymbol{S}(\omega, \omega, C, ..., C)$:





(Sharapov, E.S., Van Dongen) the configuration space is of convex polygons B or swallowtails A, related to Grassmannian of two-planes.

The proof of $\boldsymbol{A}_\infty$ is by Stokes theorem: $\int_C d\Omega = \int_{\partial C} \Omega$

Deformations of Poisson Orbifold: Weyl Algebra

Weyl algebra $\boldsymbol{A_1}$ gives a **fuzzy-sphere** at a particular radius

$$x^i = (q^2, p^2, \tfrac{1}{2}[qp+pq]) \qquad\qquad C_2 = r^2 = -\tfrac{3}{4}$$

$\boldsymbol{A_1}$ is a rigid ∞-dim associative algebra that contains $sp(2)$.

Orbifold $\mathbb{R}^2/\mathbb{Z}_2$ admits 'second' quantization on top of the Moyal-Weyl $\star$-product, (Wigner; ...; Pope et al; Vasiliev; Madore; Bieliavsky et al; Joung, Mrtchyan; Korybut; Basile et al; Sharapov et al), freeing radius r

$$[q,p] = i\hbar + i\nu \boldsymbol{R} \qquad\qquad \boldsymbol{R}\, y_A \boldsymbol{R} = -y_A\,, \quad y_A = (q,p)$$

The first deformation is given by some two-cocycle ϕ_1. This is thanks to $\boldsymbol{HH^2(A_1, A_1^*) = \mathbb{C}}$ and the cocycle was obtained (Feigin, Felder, Shoikhet) from Shoikhet-Tsygan-Kontsevich formality

What else?

- Relation to tensionless strings on $AdS_4 \times \mathbb{CP}^3$?
- Slightly-broken HS $=$ new physical symmetry, L_∞. Uniqueness of invariants suggests $3d$ bosonization
- Chiral HiSGRA is detected via celestial OPE (Ren, Spradlin, Srikant, Volovich)
- Instantons/exact solutions (E.S, Yin): the simplest (BPST) instanton can be uplifted to a solution of Chiral HiSGRA, perhaps all ADHM as well ...
- Chiral description for massive HS interactions (Ochirov, E.S.), Black-Hole scattering (Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, E.S.)

Summary, Remarks

- **Higher spin symmetries** originate from Deformation Quantization. However, theories/applications (Chiral HiSGRA/Chern-Simons vector models) require A_∞/L_∞-extensions thereof, which originate from **fuzzy-sphere**
- Chiral HiSGRA is a nice model, which is supposed to be UV finite at all loops. It is related to second order phase transitions via AdS/CFT and sheds more light on the $3d$ bosonization duality. It requires sophisticated NC-techniques that are not yet covered by math results.
- Bigger formality? 1st layer = Kontsevich; 2nd layer Shoikhet-Tsygan-Kontsevich. Deformation Quantization of Poisson (Orbifolds): formality is yet to be understood, but $\mathbb{R}^2/\mathbb{Z}_2$ is related to **fuzzy-sphere**

Thank you for your attention!

That's all!

... backup slides ...

S-matrix constraints from the higher spin symmetry

S-matrix summary

We see that **asymptotic higher spin symmetries** (HSS)

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\dots\mu_s}$$

seem to completely fix (holographic) S-matrix to be

$$S_{\text{HiSGRA}} = \begin{cases} 1^{***}, & \text{flat space, (Weinberg)} \\ \text{free CFT}, & \text{asymptotic AdS, unbroken HSS} \\ \textbf{Chern-Simons Matter}, & \textbf{asymptotic AdS}_4\textbf{, SB HSS} \end{cases}$$

Most interesting applications are to vector models, (Klebanov, Polyakov; Sezgin, Sundell; Maldacena, Zhiboedov; Giombi et al; ...)

Both Minkowski and AdS cases reveal certain non-localities since HSS mixes ∞ spins and derivatives, invalidating the local QFT approach

S-matrix summary

For HS-SDYM we have

$$S = \sum_n \mathrm{tr} \int \Psi^{A(2s)} H_{AA} \wedge F_{A(2s-2)}$$

For HS-SDGR in flat space ($m = n = 2 \leftrightarrow$ SDGR)

$$S = \sum_{m,n} \int \Psi^{A(n+m)}\, d\omega_{A(n)} \wedge d\omega_{A(m)}$$

For HS-SDGR with $\Lambda \neq 0$ we need Poisson bracket in F

$$S = \sum_{m,n} \int \Psi^{A(n+m)}\, F_{A(n)} \wedge F_{A(m)}$$

The flat limit is smooth! $F = d\omega + \frac{1}{2}[\omega, \omega] \to d\omega$

All these are truncations of Chiral HiSGRA (Ponomarev)

Higher spin symmetry $\in$ Deformation Quantization

Ex 1. Moyal-Weyl star-product, $[Y^A, Y^B] = \pi^{AB}$, Weyl algebra $\boldsymbol{A_n}$

$$(f \star g)(Y) = f(Y) \exp \tfrac{1}{2} \left[\overleftarrow{\partial}_A \pi^{AB} \overrightarrow{\partial}_B\right] g(Y)$$

$so(2,1) \in \boldsymbol{A_1}$, $so(3,2) \in \boldsymbol{A_2} \rightarrow$ **Chiral HiSGRA**, $sp(2n) \in \boldsymbol{A_n}$.

Ex 2. Fuzzy 2-sphere/hyperboloid, $[x^i, x^j] = \epsilon^{ijk} x^k \sim sp(2)$

$$\boldsymbol{A_1}: \qquad [q,p] = i\hbar \qquad x^i = (q^2, p^2, \tfrac{1}{2}\{q,p\})$$

whenever radius is quantized $gl_n \ni sl_n \rightarrow 3d$ **HiSGRA**

Ex 3. Fuzzy H_4, DQ of the minimal coadjoint orbit of $so(4,2)$

$$[P_a, P_b] = L_{ab} \qquad [L_{ab}, L_{cd}] = \eta_{bc} L_{ad} + \ldots \qquad [L_{ab}, P_c] = \ldots$$

IKKT (Steinacker et al), also **conformal HiSGRA**

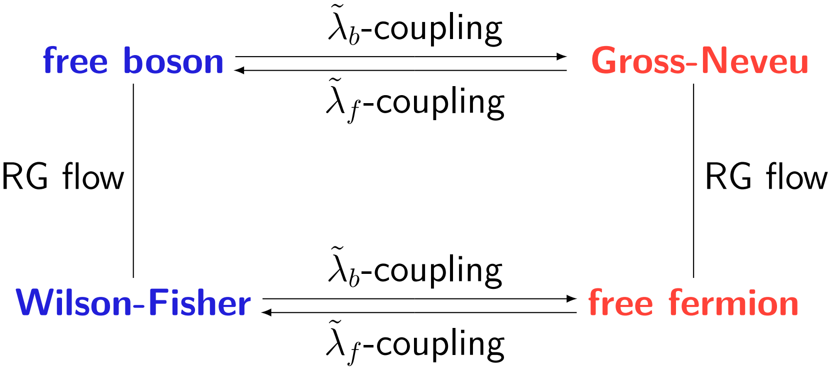
Chern-Simons Matter theories and dualities

In AdS_4/CFT_3 one can do much better — there exists a large class of models, Chern-Simons Matter theories (extends to ABJ(M))

$$\frac{k}{4\pi}S_{CS}(A) + \text{Matter}\begin{cases}(D\phi^i)^2 & \textbf{free boson}\\ (D\phi^i)^2 + g(\phi^i\phi^i)^2 & \textbf{Wilson-Fisher (Ising)}\\ \bar{\psi}\not{D}\psi & \textbf{free fermion}\\ \bar{\psi}\not{D}\psi + g(\bar{\psi}\psi)^2 & \textbf{Gross-Neveu}\end{cases}$$

- describe physics (Ising, quantum Hall, ...)
- break parity in general (Chern-Simons)
- two parameters $\lambda = N/k$, $1/N$ (λ continuous for N large)
- exhibit remarkable dualities, e.g. ***3d* bosonization duality** (Aharony, Alday, Bissi, Giombi, Karch, Maldacena, Minwalla, Prakash, Seiberg, Tong, Witten, Yacobi, Yin, Zhiboedov, ...)

Chern-Simons Matter theories and dualities



The simplest gauge-invariant operators are $J_s = \phi D...D\phi$ or $J_s = \bar{\psi}\gamma D...D\psi$, which are dual to higher spin fields.

Currents are slightly non-conserved $\partial \cdot J = \frac{1}{N}[JJ]$

$\gamma(J_s)$ at order $1/N$ (Giombi, Gurucharan, Kirillin, Prakash, E.S.) confirm the duality. Many other tests!

Higher spin symmetry and bosonization duality

Unbroken Higher spin symmetry

In free theories we have ∞-many conserved $J_s = \phi\partial ... \partial\phi$ tensors.

Free CFT = Associative (higher spin) algebra

Conserved tensor $\rightarrow$ current $\rightarrow$ symmetry $\rightarrow$ invariants=correlators.

$$\partial \cdot J_s = 0 \qquad \Longrightarrow \qquad Q_s = \int J_s \qquad \Longrightarrow \qquad [Q,Q] = Q \quad \& \quad [Q,J] = J$$

HS-algebra (free boson) = HS-algebra (free fermion) in $3d$.

Correlators are given by invariants (Sundell, Colombo; Didenko, E.S.; ...)

$$\langle J ... J \rangle = \mathrm{Tr}(\Psi \star ... \star \Psi) \qquad\qquad \Psi \leftrightarrow J$$

where Ψ are coherent states representing J in the higher spin algebra

$$\langle JJJJ \rangle_{F.B.} \sim \cos(Q_{13}^2 + Q_{24}^3 + Q_{31}^4 + Q_{43}^1)\cos(P_{12})\cos(P_{23})\cos(P_{34})\cos(P_{41}) + ...$$

Slightly-broken Higher spin symmetry is new Virasoro?

In large-N Chern-Simons vector models (e.g. Ising) higher spin symmetry does not disappear completely (Maldacena, Zhiboedov):

$$\partial \cdot J = \frac{1}{N}[JJ] \qquad\qquad [Q, J] = J + \frac{1}{N}[JJ]$$

What is the right math? We should deform the algebra together with its action on the module, so that the currents can 'backreact':

$$\delta_\xi J = l_2(\xi, J) + l_3(\xi, J, J) + \ldots\,, \qquad\qquad [\delta_{\xi_1}, \delta_{\xi_2}] = \delta_\xi\,,$$

where $\xi = l_2(\xi_1, \xi_2) + l_3(\xi_1, \xi_2, J) + \ldots$. This leads to L_∞-algebra.

Correlators = invariants of L_∞-algebra and are unique (Gerasimenko, Sharapov, E.S.), **which proves $3d$ bosonization duality at least in the large-N**.
Without having to compute anything one prediction is

$$\langle J \ldots J \rangle = \sum \langle \text{fixed} \rangle_i \times \text{params}$$