Entanglement in flavored scalar scattering

Enrico Maria Sessolo

National Centre for Nuclear Research (NCBJ)
Warsaw, Poland

Based on

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in collaboration with

Kamila Kowalska

National
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Poland

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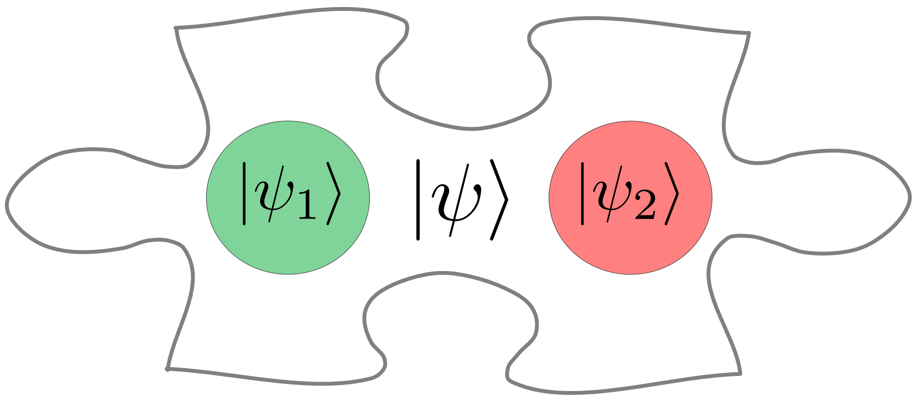
Outline

- Quantum entanglement in high-energy scattering and the perturbative *S*-matrix

- Properties of the density matrix an their consequences

- Application to the 2HDM potential and constraints

Quantum entanglement



$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

entanglement = non-separability

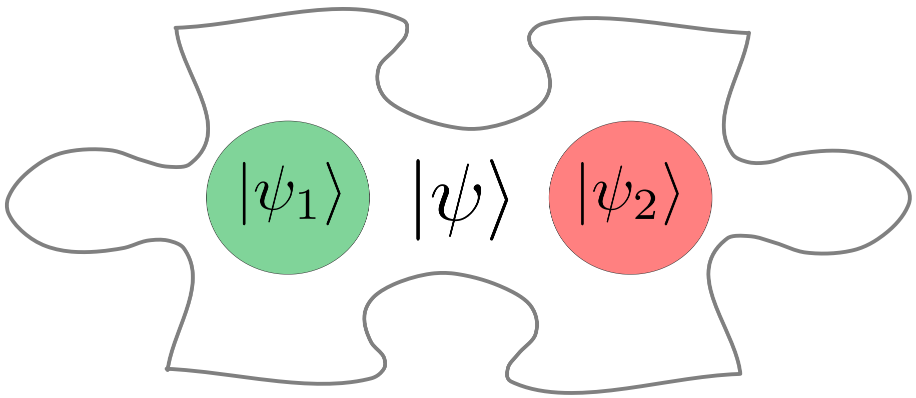
Example 1 Qubit space: $|1\rangle,\ |2\rangle \in \mathbb{C}^2 \to \mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2$

A) $a_1b_1|11\rangle + a_1b_2|12\rangle + a_2b_1|21\rangle + a_2b_2|22\rangle = (a_1|1\rangle + a_2|2\rangle) \otimes (b_1|1\rangle + b_2|2\rangle)$ (not entangled)

B) $\frac{1}{\sqrt{2}}|11\rangle + \frac{1}{\sqrt{2}}|22\rangle$ $\qquad$ $\frac{1}{\sqrt{2}}|11\rangle - \frac{1}{\sqrt{2}}|22\rangle$

$\frac{1}{\sqrt{2}}|12\rangle + \frac{1}{\sqrt{2}}|21\rangle$ $\qquad$ $\frac{1}{\sqrt{2}}|12\rangle - \frac{1}{\sqrt{2}}|21\rangle$ (entangled)

Quantum entanglement



$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

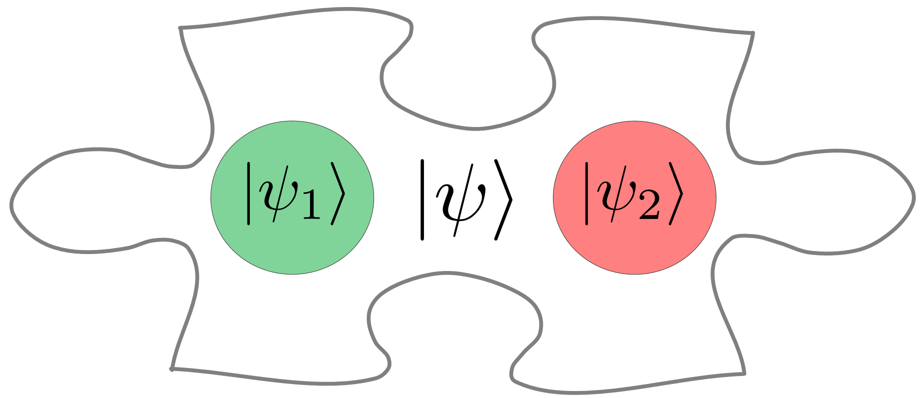
entanglement = non-separability

Example 2 $\qquad \mathcal{H}_{\text{comp}} = L^2(\mathbb{R}) \otimes \mathbb{C}^2 \qquad |\psi\rangle_{\text{comp}} = ?$

A) $$\sum_{i=1}^{2} \int_{-\infty}^{\infty} dx\, \psi(x) \epsilon_i |x\rangle |i\rangle$$ (not entangled)

B) $$\sum_{i=1}^{2} \int_{-\infty}^{\infty} dx\, \psi_i(x) |x\rangle |i\rangle$$ (entangled)

Quantum entanglement



$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

for mixed states:

$$\rho \neq \sum_i p_i \rho_1^i \otimes \rho_2^i$$

entanglement = non-separability

Why should we care?

- Measured experimentally (Bell inequalities violation) Clauser et al., ‘72; Aspect et al., ‘82; Zeilinger et al., ‘98;
- Can be tested in colliders Afik, Muñoz de Nova, ‘21 and many following studies; ATLAS Collab., ‘23; CMS Collab., ‘24

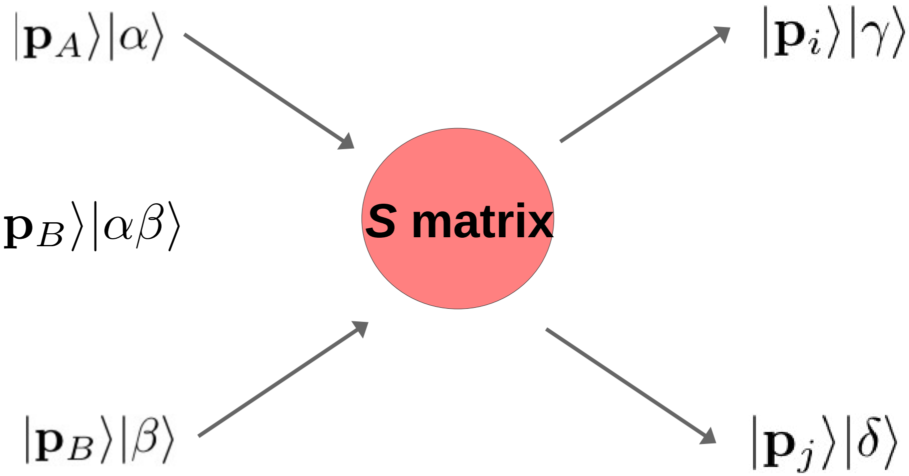
  K. Sakurai’s talk **P. Lamba’s talk**
- Quantum information dense coding (Bennett, Wiesner, ‘92), teleportation (Bennett et at., ‘93), etc.
- Emergence of space and time Moreva et al., ‘13; Van Raamsdonk, ‘10, Ryu and Takayanagi, ‘06; Maldacena, Susskind, ‘13
- **Scattering / symmetries** Cervera-Lierta et al., ‘17; Fedida, Serafini, ‘23; Beane et al., ‘19; Liu et. al., ‘23; Carena et al., ‘23; Blasone et al., ‘24

  This talk **B. Micciola’s talk**

Entanglement in scattering

2 → 2 scattering particles *A*, *B* with internal "qubit" quantum number: $|\mathbf{p}_A\rangle|\alpha\rangle, \; |\mathbf{p}_B\rangle|\beta\rangle$

$$|\text{in}\rangle = \frac{1}{\sqrt{V}} \sum_{\alpha,\beta=1,2} a_{\alpha\beta} |\mathbf{p}_A \mathbf{p}_B\rangle |\alpha\beta\rangle$$



$$|\text{out}\rangle = S\,|\text{in}\rangle$$

Hilbert space: **momentum + flavor (qubit)**

$$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^3 \otimes \mathbb{R}^3) \otimes \mathbb{C}^4$$

<u>In perturbation theory:</u>

$$S^{ijab}_{\gamma\delta\alpha\beta} = (\mathcal{I} + iT)^{ijab}_{\gamma\delta\alpha\beta}$$

$$= (2\pi)^6\, 4\, E_i E_j\, \delta^{ijab}_{\gamma\delta\alpha\beta} + (2\pi)^4 \delta^4(p_a + p_b - p_i - p_j)\, i\mathcal{M}_{\gamma\delta,\alpha\beta}(p_a, p_b \to p_i, p_j)$$

The final-state density matrix: $\boxed{\rho = |\text{out}\rangle\langle\text{out}|}$ encodes all the properties of a quantum system **(entanglement)**

Perturbative density matrix

$$\boxed{\rho = |\mathrm{out}\rangle\langle\mathrm{out}|}$$

Shao et al., '08; Seki et al., '15; Peschanski, Seki, '16, '19; Carney et al., '16; Rätzel et al., '17; Fan et al., '17; Araujo et al., '19; Fan et al., '21; Fonseca et al., '22; Shivashankara '23; Aoude et al., '24

Properties:

$$1)\ \mathrm{Tr}(\rho) = 1 \Longrightarrow$$ unitarity of the *S*-matrix

optical theorem

$$\langle \mathrm{out}|\mathrm{out}\rangle = 1 + \Delta\left(i \sum_{\alpha\beta,\gamma\delta} a^*_{\alpha\beta} \mathcal{M}_{\alpha\beta,\gamma\delta}(p_A, p_B \to p_A, p_B) a_{\gamma\delta} + \mathrm{c.c.}\right)$$
$$+ \Delta \int\int \frac{d^3 p_i}{(2\pi)^3}\frac{1}{2E_i}\frac{d^3 p_j}{(2\pi)^3}\frac{1}{2E_j}(2\pi)^4\delta^4(p_A + p_B - p_i - p_j)$$
$$\times \sum_{\alpha\beta,\rho\epsilon,\sigma\tau} \mathcal{M}_{\alpha\beta,\rho\epsilon}(p_A, p_B \to p_i, p_j) a_{\rho\epsilon} \mathcal{M}^*_{\alpha\beta,\sigma\tau}(p_A, p_B \to p_i, p_j) a^*_{\sigma\tau}$$

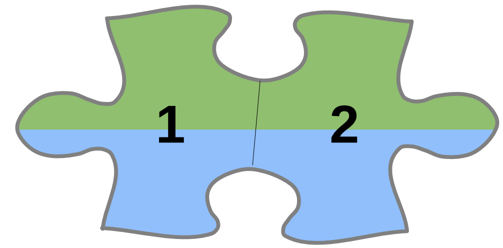
$$\Delta = \frac{(2\pi)^4\delta^4(p_A + p_B - p_A - p_B)}{4E_A E_B\left[(2\pi)^3\,\delta^3(0)\right]^2}$$

(indeterminate normalization)

$$2)\ \mathrm{Tr}(\rho^2)\begin{cases} = 1 & \text{pure state} \\ < 1 & \text{mixed state}\end{cases} \Longrightarrow$$ **need different entanglement measures**

Entanglement in the final state

$$\rho = |\mathrm{out}\rangle\langle\mathrm{out}|$$ is <u>pure</u>

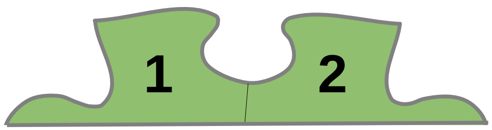


$$\mathcal{H}_{\mathrm{tot}} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

Tracing out momentum

bipartitions:

Tracing out subsystem 2



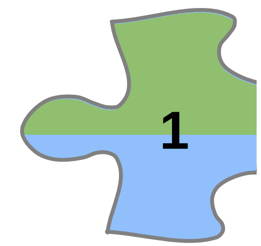
$$\mathcal{H}_{\mathrm{red}} = \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$\boxed{\tilde{\rho} = \mathrm{Tr}_{\mathbf{p}}(\rho)}$$

$$\text{basis}: \ |\alpha\beta\rangle\langle\gamma\delta|$$

$$\boxed{\tilde{\rho} = \mathrm{Tr}_2(\rho)}$$

$$\text{basis}: \ |\mathbf{p}_i\alpha\rangle\langle\mathbf{p}_\mathbf{j}\gamma|$$



$$\mathcal{H}_{\mathrm{red}} = L^2(\mathbb{R}^3) \otimes \mathbb{C}^2$$

- entanglement between bipartite states:

  von Neumann entropy

$$S_N(\tilde{\rho}) = -\mathrm{Tr}(\tilde{\rho}\,\log_2 \tilde{\rho})$$

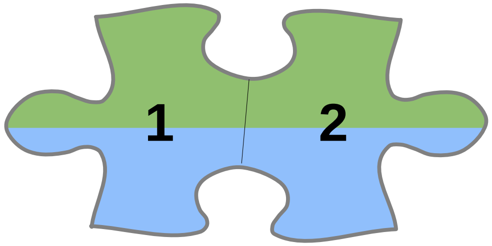
Entanglement monotone:

$S_N = 0$ no entanglement

$S_N = 1$ maximal entanglement

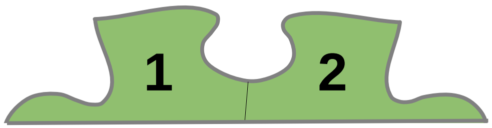
Entanglement in the final state

$$\rho = |\text{out}\rangle\langle\text{out}|$$ is <u>pure</u>



$$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

Tracing out momentum



$$\mathcal{H}_{\text{red}} = \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$\boxed{\tilde{\rho} = \mathrm{Tr}_{\mathbf{p}}(\rho)}$$

$$\text{basis}: \ |\alpha\beta\rangle\langle\gamma\delta|$$

$\tilde{\rho}_{\alpha\beta,\gamma\delta}$ in general is <u>mixed</u>

- entanglement between 2 qubits:

  Concurrence

$$C(\tilde{\rho}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

λ: eigenvalues of $\tilde{\rho}(\sigma_y \otimes \sigma_y)\tilde{\rho}^*(\sigma_y \otimes \sigma_y)$

Entanglement monotone:

$C = 0$ no entanglement

$C = 1$ maximal entanglement

Perturbative density matrix

$$2)\ \mathrm{Tr}(\rho^2) \begin{cases} = 1 & \text{pure state} \\ < 1 & \text{mixed state} \end{cases}$$

$$\begin{aligned}
\mathrm{Tr}(\tilde\rho^2) = \sum_{\alpha\beta,\gamma\delta} \tilde\rho_{\alpha\beta,\gamma\delta}\,\tilde\rho_{\gamma\delta,\alpha\beta} = 1 + 2\,\Delta \Bigg[& i \sum_{\alpha\beta,\epsilon\rho} \mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A, p_B \to p_A, p_B)\, a^*_{\alpha\beta} a_{\epsilon\rho} + \text{c.c.} \\
& + \int\!\!\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\frac{d^3p_j}{(2\pi)^3}\frac{1}{2E_j}(2\pi)^4\delta^4(p_A + p_B - p_i - p_j) \\
& \times \sum_{\epsilon\rho,\gamma\delta,\tau\sigma,\alpha\beta} \mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A, p_B \to p_i, p_j)\mathcal{M}^*_{\gamma\delta,\tau\sigma}(p_A, p_B \to p_i, p_j)\, a_{\gamma\delta}\, a^*_{\alpha\beta}\, a_{\epsilon\rho}\, a^*_{\tau\sigma} \Bigg] \\
- \Delta^2 \Bigg[& \sum_{\alpha\beta,\gamma\delta,\epsilon\rho,\tau\sigma} \mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A, p_B \to p_A, p_B)\mathcal{M}_{\gamma\delta,\tau\sigma}(p_A, p_B \to p_A, p_B)\, a^*_{\alpha\beta}\, a_{\epsilon\rho}\, a^*_{\gamma\delta}\, a_{\tau\sigma} + \text{c.c.} \\
& - 2 \sum_{\alpha\beta,\epsilon\rho,\tau\sigma} \mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A, p_B \to p_A, p_B)\,\mathcal{M}^*_{\alpha\beta,\tau\sigma}(p_A, p_B \to p_A, p_B) a^*_{\tau\sigma} a_{\epsilon\rho} \Bigg]. \qquad (2.18)
\end{aligned}$$

$$\boxed{\Delta \leq \frac{1}{16\pi}}$$

Its origin ...

$$\begin{aligned}
|\mathrm{in}\rangle &= \left(\prod_{i=1,2}\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{\sqrt{2E_i}}\right)\phi_A(\mathbf{p}_1)\phi_B(\mathbf{p}_2)\,|\mathbf{p}_1\mathbf{p}_2\rangle \sum_{\alpha,\beta=1,2} a_{\alpha\beta}|\alpha\beta\rangle \\
&\approx \frac{1}{\sqrt{V}} \sum_{\alpha,\beta=1,2} a_{\alpha\beta}|\mathbf{p}_A\mathbf{p}_B\rangle|\alpha\beta\rangle\,,
\end{aligned}$$

$$\phi_{A,B}(\mathbf{p}) = \sqrt{\frac{(2\pi)^3}{\delta^3(0)}}\,\delta^3(\mathbf{p} - \mathbf{p}_{A,B})$$

It does not belong to the Hilbert space

We should specify the full wave packet

The model: 2HDM

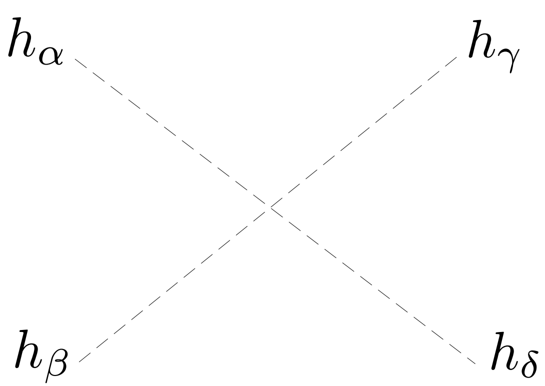
inert SU(2) doublets: $H_\alpha = \begin{pmatrix} h_\alpha^+ \\ h_\alpha^0 \end{pmatrix}_{Y=\frac{1}{2}} \qquad \alpha = 1,2 \;\; \rightarrow |1\rangle, |2\rangle$

cf. Carena, Low, Wagner, Xiao, 2307.08112

scalar potential: $$V(H_1,H_2) = \mu_1^2\, H_1^\dagger H_1 + \mu_2^2\, H_2^\dagger H_2 + \left(\mu_3^2\, H_1^\dagger H_2 + \text{H.c.}\right)$$
$$+ \lambda_1\,(H_1^\dagger H_1)^2 + \lambda_2\,(H_2^\dagger H_2)^2 + \lambda_3\,(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4\,(H_1^\dagger H_2)(H_2^\dagger H_1)$$
$$+ \left(\lambda_5\,(H_1^\dagger H_2)^2 + \lambda_6\,(H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7\,(H_2^\dagger H_2)(H_1^\dagger H_2) + \text{H.c.}\right)$$

high energy limit *p^2 >> μ^2*

contact interactions



$\boxed{i\,\mathcal{M}_{\gamma\delta,\alpha\beta}}$

$$i\mathcal{M}^{(0)}(h^0h^0 \to h^0h^0) = -i \begin{pmatrix} 4\lambda_1 & 2\lambda_6 & 2\lambda_6 & 4\lambda_5 \\ 2\lambda_6 & \lambda_3+\lambda_4 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 2\lambda_6 & \lambda_3+\lambda_4 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 4\lambda_5 & 2\lambda_7 & 2\lambda_7 & 4\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^+h^0 \to h^+h^0) = -i \begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & 2\lambda_5 \\ \lambda_6 & \lambda_3 & \lambda_4 & \lambda_7 \\ \lambda_6 & \lambda_4 & \lambda_3 & \lambda_7 \\ 2\lambda_5 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^+h^- \to h^+h^-) = -i \begin{pmatrix} 4\lambda_1 & 2\lambda_6 & 2\lambda_6 & \lambda_3+\lambda_4 \\ 2\lambda_6 & 4\lambda_5 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 2\lambda_6 & \lambda_3+\lambda_4 & 4\lambda_5 & 2\lambda_7 \\ \lambda_3+\lambda_4 & 2\lambda_7 & 2\lambda_7 & 4\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^0h^0 \to h^+h^-) = i\mathcal{M}^{(0)}(h^+h^- \to h^0h^0) = -i \begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & \lambda_3 \\ \lambda_6 & \lambda_4 & 2\lambda_5 & \lambda_7 \\ \lambda_6 & 2\lambda_5 & \lambda_4 & \lambda_7 \\ \lambda_3 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

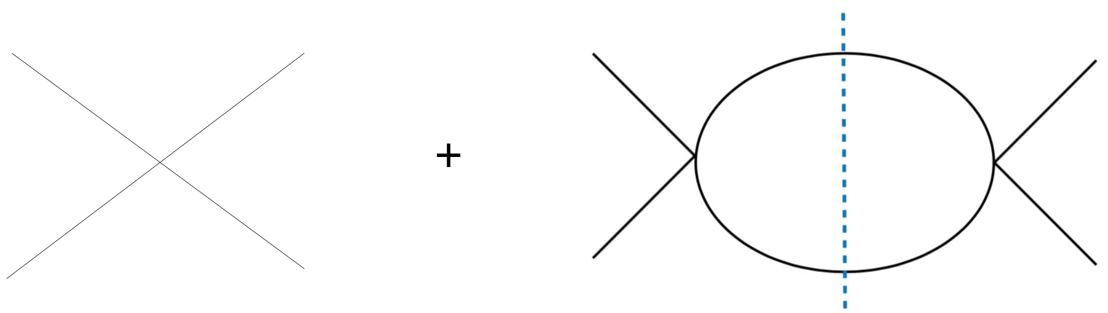
Question: any constraints on λ from entanglement?

The model: 2HDM

inert SU(2) doublets: $H_\alpha = \begin{pmatrix} h_\alpha^+ \\ h_\alpha^0 \end{pmatrix}_{Y=\frac{1}{2}} \qquad \alpha = 1,2 \ \rightarrow |1\rangle, |2\rangle$

scalar potential: $$V(H_1, H_2) = \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 + \left(\mu_3^2 H_1^\dagger H_2 + \text{H.c.}\right)$$
$$+ \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1)$$
$$+ \left(\lambda_5 (H_1^\dagger H_2)^2 + \lambda_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7 (H_2^\dagger H_2)(H_1^\dagger H_2) + \text{H.c.}\right)$$

We work at 1 loop order



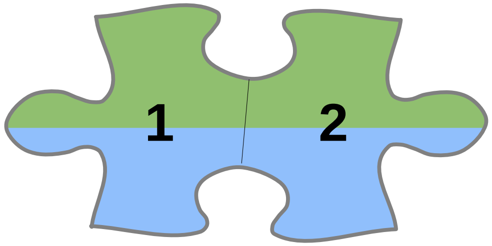
optical theorem OK! $$\frac{1}{2}\int d\Pi_2 \left(\mathcal{M}^\dagger \mathcal{M}\right)_{11,11} = \frac{\lambda_1^2}{\pi} + \frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi} + \mathcal{O}(\lambda^3) = 2\,\text{Im}\,\mathcal{M}_{11,11}^{(0+1)} + \mathcal{O}(\lambda^3)$$

Entanglement generation

$|\mathrm{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$ (separable)

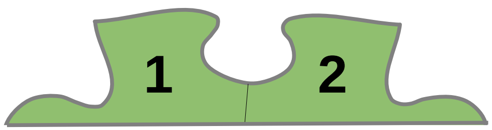
$|\mathrm{out}\rangle = S\,|\mathrm{in}\rangle$

$$\rho = |\mathrm{out}\rangle\langle\mathrm{out}|$$ is pure



$$\mathcal{H}_{\mathrm{tot}} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

Tracing out momentum



$$\mathcal{H}_{\mathrm{red}} = \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$\boxed{\tilde{\rho} = \mathrm{Tr}_{\mathbf{p}}(\rho)}$$

basis : $|\alpha\beta\rangle\langle\gamma\delta|$

$\tilde{\rho}(h^0 h^0 \to h^0 h^0)$

$$\begin{aligned}
\tilde{\rho}_{11,11} &= 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right),\\
\tilde{\rho}_{11,12} = \tilde{\rho}_{11,21} = \tilde{\rho}^*_{12,11} = \tilde{\rho}^*_{21,11} &= \Delta\left(2\,i\,\lambda_6 + \frac{2\lambda_1\lambda_6 - \lambda_3\lambda_6 - \lambda_4\lambda_6 - 2\lambda_5\lambda_7}{8\pi}\right),\\
\tilde{\rho}_{11,22} = \tilde{\rho}^*_{22,11} &= \Delta\left(4\,i\,\lambda_5 + \frac{2\lambda_1\lambda_5 - 2\lambda_2\lambda_5 - \lambda_6\lambda_7}{4\pi}\right),\\
\tilde{\rho}_{12,12} = \tilde{\rho}_{12,21} = \tilde{\rho}^*_{21,12} = \tilde{\rho}^*_{21,21} &= \Delta\,\frac{\lambda_6^2}{4\pi},\\
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\tilde{\rho}_{22,22} &= \Delta\,\frac{\lambda_5^2}{\pi}.
\end{aligned}$$

+ all other channels

- **entanglement of bipartition:**

  von Neumann entropy $S_N(\tilde{\rho}) = -\mathrm{Tr}(\tilde{\rho}\,\log_2\tilde{\rho})$

$$\begin{aligned}
\theta_1 &= 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) + 16\,\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right),\\
\theta_2 &= \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) - 16\,\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right),
\end{aligned}$$

$h^0 h^0 \to h^0 h^0$

generates entanglement between
flavor and momentum

Entanglement generation

$|\mathrm{in}\rangle = \frac{1}{\sqrt{V}}|\mathbf{p}_A\mathbf{p}_B\rangle|11\rangle$ (separable)

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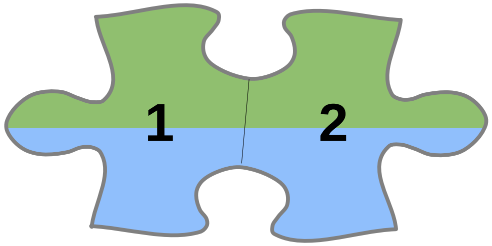
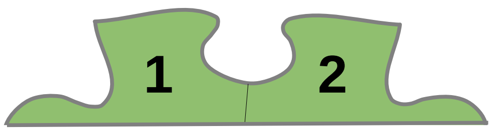


$$\mathcal{H}_{\mathrm{tot}} = L^2(\mathbb{R}^3)\otimes L^2(\mathbb{R}^3)\otimes\mathbb{C}^2\otimes\mathbb{C}^2$$

Tracing out momentum



$$\mathcal{H}_{\mathrm{red}} = \mathbb{C}^2\otimes\mathbb{C}^2$$

$$\boxed{\tilde{\rho} = \mathrm{Tr}_{\mathbf{p}}(\rho)}$$

basis : $|\alpha\beta\rangle\langle\gamma\delta|$

$\tilde{\rho}(h^0h^0 \to h^0h^0)$

$$\tilde{\rho}_{11,11} = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right),$$

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$$\tilde{\rho}_{11,22} = \tilde{\rho}^*_{22,11} = \Delta\left(4\,i\,\lambda_5 + \frac{2\lambda_1\lambda_5 - 2\lambda_2\lambda_5 - \lambda_6\lambda_7}{4\pi}\right),$$

$$\tilde{\rho}_{12,12} = \tilde{\rho}_{12,21} = \tilde{\rho}^*_{21,12} = \tilde{\rho}^*_{21,21} = \Delta\,\frac{\lambda_6^2}{4\pi},$$

$$\tilde{\rho}_{12,22} = \tilde{\rho}_{21,22} = \tilde{\rho}^*_{22,12} = \tilde{\rho}^*_{22,21} = \Delta\,\frac{\lambda_5\lambda_6}{2\pi},$$

$$\tilde{\rho}_{22,22} = \Delta\,\frac{\lambda_5^2}{\pi}.$$

+ all other channels

- **entanglement of bipartition:**

  von Neumann entropy $S_N(\tilde{\rho}) = -\mathrm{Tr}(\tilde{\rho}\,\log_2\tilde{\rho})$

$$\theta_1 = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) + 16\,\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right),$$

$$\theta_2 = \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) - 16\,\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right),$$

$h^0h^0 \to h^0h^0$

$$\Delta \le \frac{1}{16\pi}$$

0 < eigenvalues < 1 (physicality!)

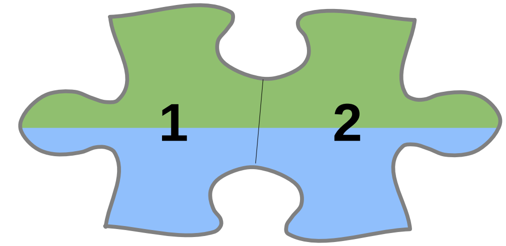
entanglement is perturbatively small !

Entanglement generation

$|\text{in}\rangle = \frac{1}{\sqrt{V}}|\mathbf{p}_A\mathbf{p}_B\rangle|11\rangle$ (separable)

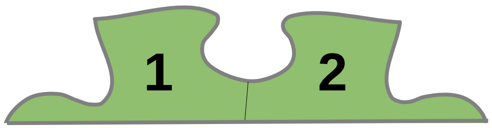
$|\text{out}\rangle = S\,|\text{in}\rangle$

$$\rho = |\text{out}\rangle\langle\text{out}|$$ is <u>pure</u>



$$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^3)\otimes L^2(\mathbb{R}^3)\otimes\mathbb{C}^2\otimes\mathbb{C}^2$$

Tracing out momentum



$$\mathcal{H}_{\text{red}} = \mathbb{C}^2\otimes\mathbb{C}^2$$

$$\boxed{\tilde\rho = \text{Tr}_{\mathbf{p}}(\rho)}$$

basis : $|\alpha\beta\rangle\langle\gamma\delta|$

$\tilde\rho(h^0h^0\to h^0h^0)$

$$\tilde\rho_{11,11} = 1-\Delta\left(\frac{\lambda_5^2}{\pi}+\frac{\lambda_6^2}{2\pi}\right),$$

$$\tilde\rho_{11,12} = \tilde\rho_{11,21} = \tilde\rho^*_{12,11} = \tilde\rho^*_{21,11} = \Delta\left(2\,i\,\lambda_6+\frac{2\lambda_1\lambda_6-\lambda_3\lambda_6-\lambda_4\lambda_6-2\lambda_5\lambda_7}{8\pi}\right),$$

$$\tilde\rho_{11,22} = \tilde\rho^*_{22,11} = \Delta\left(4\,i\,\lambda_5+\frac{2\lambda_1\lambda_5-2\lambda_2\lambda_5-\lambda_6\lambda_7}{4\pi}\right),$$

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$$\tilde\rho_{22,22} = \Delta\,\frac{\lambda_5^2}{\pi}.$$

\+ <u>all other channels</u>

- **entanglement of bipartition:**

  von Neumann entropy $S_N(\tilde\rho) = -\text{Tr}(\tilde\rho\,\log_2\tilde\rho)$

Repeating for |12>, |21>, |22> (all channels):

$\lvert\text{in}\rangle_F$	momentum-flavor space
$\lvert 11\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6$
$\lvert 12\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7$
$\lvert 21\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7$
$\lvert 22\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_7$

<u>comp. basis:</u>

MinEnt condition: No entanglement only if interaction is not flavored

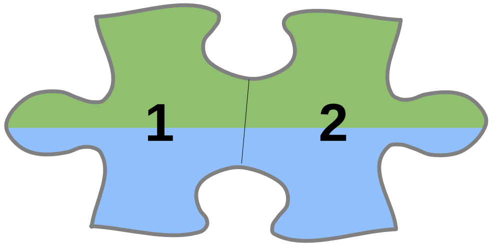
$$S\left(|\mathbf{p}_i\mathbf{p}_j\rangle|\alpha\alpha\rangle\right) = |\alpha\alpha\rangle S(\lambda_\alpha)|\mathbf{p}_i\mathbf{p}_j\rangle \qquad \alpha = 1,2$$

Entanglement generation

$|\mathrm{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$ (separable)

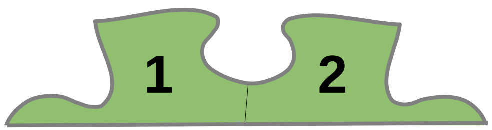
$|\mathrm{out}\rangle = S\,|\mathrm{in}\rangle$

$$\rho = |\mathrm{out}\rangle\langle\mathrm{out}|$$ is <u>pure</u>



$$\mathcal{H}_{\mathrm{tot}} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

Tracing out momentum



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+ <u>all other channels</u>

- **entanglement of bipartition:**

  von Neumann entropy $S_N(\tilde{\rho}) = -\mathrm{Tr}(\tilde{\rho}\,\log_2 \tilde{\rho})$

  Generic linear comb. in C^4 (all channels)

No mom/flav entanglement <u>for all separable states</u> iff

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$$

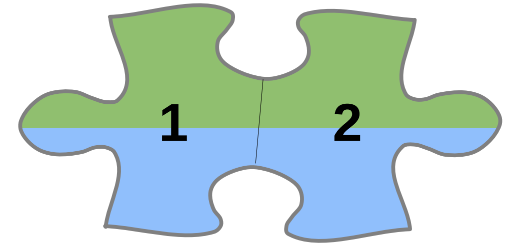
Easy to construct an “in” state that will end up entangled

Entanglement generation

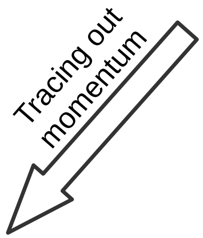
$|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$ (separable)

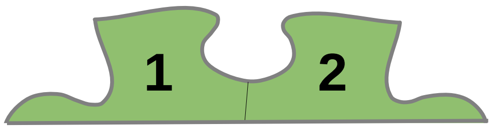
$|\text{out}\rangle = S\,|\text{in}\rangle$

$\rho = |\text{out}\rangle\langle\text{out}|$ is <u>pure</u>



$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$





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$$\tilde{\rho}_{22,22} = \Delta\,\frac{\lambda_5^2}{\pi}.$$

+ <u>all other channels</u>

- **entanglement of 2 flavor qubits:**

  Concurrence $C(\tilde{\rho}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$

No qubit entanglement <u>for all separable states</u> iff

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0\,, \quad \lambda_6 = \lambda_7$$

Can't confirm symmetry from MinEnt

cf. Carena *et al.* '23

Entanglement transformation

separable in L^2 x C^4 (mom / flav)

Initial state: $$|\mathrm{in}\rangle = \frac{1}{\sqrt{V}}\,|\mathbf{p}_A\mathbf{p}_B\rangle\,\frac{1}{\sqrt{2}}\left(|11\rangle + |22\rangle\right)$$

maximally entangled in C^4 (2 qubits)

von Neumann entropy

$$S_N(\tilde{\rho}_{\mathrm{in}}) = 0$$
$$S_N(\tilde{\rho}_{\mathrm{out}}) > 0$$

with

$$\theta_1 = 1 - \Delta\frac{(\lambda_1-\lambda_2)^2 + (\lambda_6+\lambda_7)^2}{4\pi}$$
$$\theta_2 = \Delta\frac{(\lambda_1-\lambda_2)^2 + (\lambda_6+\lambda_7)^2}{4\pi}$$

concurrence

$$C(\tilde{\rho}_{\mathrm{in}}) = 1$$
$$C(\tilde{\rho}_{\mathrm{out}}) < 1$$

with

$$C(\tilde{\rho}_{\mathrm{out}}) = \sqrt{1 - \Delta\frac{(\lambda_1-\lambda_2)^2 + (\lambda_6+\lambda_7)^2}{2\,\pi}}$$

Entanglement "flows"
from
one Hilbert space to another

	Entanglement transformation flavor space $\to$ full Hilbert space
$\frac{1}{\sqrt{2}}(\|11\rangle + \|22\rangle)$	$\lambda_1-\lambda_2,\ \lambda_6+\lambda_7$
$\frac{1}{\sqrt{2}}(\|11\rangle - \|22\rangle)$	$\lambda_1-\lambda_2,\ \lambda_6-\lambda_7$
$\frac{1}{\sqrt{2}}(\|12\rangle + \|21\rangle)$	$\lambda_6+\lambda_7$
$\frac{1}{\sqrt{2}}(\|12\rangle - \|21\rangle)$	none

2HDM symmetries? … cf. *e.g.* Ferreira *et al.* '23

Conclusions

- Can post-scattering entanglement provide a **complementary way of constraining** the interactions of BSM models?

- Scattering interactions **inject** entanglement in a separable system, but this is **perturbatively small** in λ, Δ

- 2HDM: **any quartic coupling** can potentially create entanglement between momentum and “flavor” dof's

- 2HDM: entanglement can be **transformed** by some coupling combinations... may lead to symmetries?

Backup

Final state with measured momenta

Project the "out" state along a choice of momentum:

$$|\text{proj}\rangle \equiv |f\rangle\langle f|\text{out}\rangle \qquad |f\rangle = \left(\prod_{i=1,2}\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{\sqrt{2E_i}}\right)\phi_C(\mathbf{p}_1)\phi_D(\mathbf{p}_2)\,|\mathbf{p}_1\mathbf{p}_2\rangle \approx \frac{1}{\sqrt{V}}\,|\mathbf{p}_C\mathbf{p}_D\rangle$$

Density matrix: $$\tilde{\rho}_p = \frac{|\text{proj}\rangle\langle\text{proj}|}{\langle\text{proj}|\text{proj}\rangle} = \sum_{\alpha\beta,\gamma\delta}(\tilde{\rho}_p)_{\alpha\beta,\gamma\delta}|\alpha\beta\rangle\langle\gamma\delta|$$

$$(\tilde{\rho}_p)_{\alpha\beta,\gamma\delta} = \frac{\sum_{\epsilon\rho,\tau\sigma}\mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A,p_B\to p_C,p_D)\mathcal{M}^*_{\gamma\delta,\tau\sigma}(p_A,p_B\to p_C,p_D)\,a_{\epsilon\rho}\,a^*_{\tau\sigma}}{\sum_{\gamma\delta,\epsilon\rho,\tau\sigma}\mathcal{M}_{\gamma\delta,\epsilon\rho}(p_A,p_B\to p_C,p_D)\mathcal{M}^*_{\gamma\delta,\tau\sigma}(p_A,p_B\to p_C,p_D)\,a_{\epsilon\rho}\,a^*_{\tau\sigma}}$$

No dependence on Δ

2HDM results

$\lvert\text{in}\rangle_F$	minimal entanglement	maximal entanglement
$\lvert 11\rangle$	$2\lambda_1\lambda_3 = \lambda_6^2,\ \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$	$\lambda_1 = \lambda_3 = \lambda_4 = \lambda_5 = 0$ or $\lambda_6 = 0,\ \lambda_1 = \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$
$\lvert 12\rangle, \lvert 21\rangle$	$\lambda_6\lambda_7 = \lambda_3^2,\ \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$	$\lambda_6 = \lambda_7,\ \lambda_3 = \lambda_4 = \lambda_5 = 0$ or $\lambda_6 = \lambda_7 = 0,\ \lambda_3 = \lambda_4 = 2\lambda_5$
$\lvert 22\rangle$	$2\lambda_2\lambda_3 = \lambda_7^2,\ \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$	$\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0$ or $\lambda_7 = 0,\ \lambda_2 = \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$
Total	1) $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$ 2) $\lambda_3^2 = \lambda_4^2 = 4\lambda_5^2 = \lambda_6\lambda_7 = 2\lambda_1\lambda_2$	1) $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0,$ $\lambda_6 = \lambda_7$ 2) $\lambda_1 = \lambda_2 = \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4,$ $\lambda_6 = \lambda_7 = 0$