Higher-derivative deformations of the ModMax theory

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Introduction

- In 2020, a new model for nonlinear ED was proposed as the **unique conformal and duality-invariant extension** of Maxwell theory

$$\mathcal{L}_{\mathrm{ModMax}}(F) = -\frac{1}{4}\cosh(\gamma)\ F^2 + \frac{1}{4}\sinh(\gamma)\ \sqrt{(F^2)^2 + (F\tilde{F})^2}\ , \quad \gamma \geq 0$$

Bandos, Lechner, Sorokin & Townsend (2020)

- Much interest has been directed towards understanding its dynamics, but studies of its quantum properties are limited due to computational difficulty
  - Non-minimal operator $\implies$ standard HK techniques not applicable!
- $\mathcal{L}_{\mathrm{ModMax}}(F)$ is **not** generated as a one-loop quantum correction

Pinelli (2021)

- Loop quantum corrections to the theory must be higher-derivative!
- **Goal:** Identification of consistent higher-derivative deformations of ModMax which may contribute to a low-energy effective action

Review of electromagnetic duality

- We will study **duality** as a continuous symmetry of the equations of motion

  Gaillard & Zumino (1981,1997), Gibbons & Rasheed (1995)

- Maxwell electrodynamics in a vacuum is the best known example of a self-dual theory (also conformal in four dimensions)

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= 0 \qquad & \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 \\ \vec{\nabla} \cdot \vec{B} &= 0 \qquad & \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} &= 0 \end{aligned}$$

- Invariant under the continuous U(1) deformation

$$\vec{E} + \mathrm{i}\vec{B} \longrightarrow \mathrm{e}^{\mathrm{i}\varphi}(\vec{E} + \mathrm{i}\vec{B}) \ , \qquad \varphi \in \mathbb{R}$$

- It is an interesting area of research to see what nonlinear U(1) duality invariant systems may be constructed e.g. Born-Infeld theory

  Born & Infeld (1934), Schrödinger (1935)

Duality in Maxwell electrodynamics

- Electrodynamics is described by a gauge field A_b

$$\delta_\zeta A_b = \mathcal{D}_b \zeta \ , \qquad [\mathcal{D}_a, \mathcal{D}_b] = \frac{1}{2}\mathcal{R}_{ab}{}^{cd} M_{cd}$$

- The corresponding gauge-invariant field strength is

$$F_{ab} = \mathcal{D}_a A_b - \mathcal{D}_b A_a \quad \longrightarrow \quad \mathcal{L}_{\rm Maxwell}(F) = -\frac{1}{4} F^{ab} F_{ab}$$

- The Bianchi identity (BI) and the equation of motion (EoM) read

$$\mathcal{D}^b \tilde{F}_{ab} := \mathcal{D}^b \left(\frac{1}{2} \varepsilon_{abcd} F^{cd} \right) = 0 \ , \qquad \mathcal{D}^b F_{ab} = 0$$

- BI and EoM are preserved by the U(1) transformations

$$\delta_\varphi F_{ab} = \varphi \tilde{F}_{ab} \ , \qquad \delta_\varphi \tilde{F}_{ab} = -\varphi F_{ab} \ , \qquad \varphi \in \mathbb{R}$$

- Remarkably, the energy-momentum (EM) tensor is U(1)-invariant

$$T^{ab} = \frac{1}{2}(F + {\rm i}\tilde{F})^{ac}(F - {\rm i}\tilde{F})^{bd}\eta_{cd} = F^{ac}F^{bd}\eta_{cd} - \frac{1}{4}\eta^{ab}F^{cd}F_{cd}$$

- The scalings $\delta_\lambda F_{ab} = \lambda F_{ab}$ preserve EoM and BI but not the EM tensor!

Duality in nonlinear electrodynamics

- Consider the nonlinear Lagrangian $\mathcal{L}(F) = -\frac{1}{4} F^{ab} F_{ab} + \mathcal{O}(F^4)$

$$\tilde{G}_{ab}(F) \ := \ \frac{1}{2} \varepsilon_{abcd} \, G^{cd}(F) \ = \ 2 \frac{\partial \mathcal{L}(F)}{\partial F^{ab}} \ , \qquad G(F) = \tilde{F} + \mathcal{O}(F^3)$$

- The Bianchi identity and the equation of motion read

$$\mathcal{D}^b \tilde{F}_{ab} = 0 \ , \qquad \mathcal{D}^b \tilde{G}_{ab} = 0$$

- Preserved by the U(1) transformations

$$\delta_\varphi F_{ab} = \varphi G_{ab} \ , \qquad \delta_\varphi G_{ab} = -\varphi F_{ab} \ , \qquad \varphi \in \mathbb{R}$$

Duality in nonlinear electrodynamics

- Lagrangian is **not invariant** under U(1) duality rotations, instead

$$\delta_\varphi\Big(\mathcal{L}(F) - \frac{1}{4}F^{ab}\tilde{G}_{ab}\Big) = 0\ , \qquad \delta_\varphi\mathcal{L}(F) = \delta_\varphi F_{ab}\frac{\partial\mathcal{L}(F)}{\partial F_{ab}}$$

- Duality invariance leads to the fundamental constraint on $\mathcal{L}(F)$

$$G^{ab}\,\tilde{G}_{ab} \;+\; F^{ab}\,\tilde{F}_{ab} \;=\; 0\ ,$$

known as the **self-duality** equation

Gibbons & Rasheed (1995), Gaillard & Zumino (1997)

- Important properties of U(1) duality-invariant models
  1. Given an invariant parameter g, $\partial\mathcal{L}(F;g)/\partial g$ is duality-invariant. Implies duality-invariance of energy-momentum tensor!
  2. Self-duality under Legendre transformations

$$\mathcal{L}_D(F_D) := \Big(\mathcal{L}(F) - \frac{1}{2}F^{ab}\tilde{F}^D_{ab}\Big)\Big|_{F=F(F_D)}\ , \qquad F^D_{ab} = \partial_a A^D_b - \partial_b A^D_a$$

$$\mathcal{L}_D(F) = \mathcal{L}(F)$$

Ivanov-Zupnik (auxiliary variable) formulation

Ivanov & Zupnik (2001, 2002)

- In general, the equation $G^{ab}\tilde{G}_{ab} + F^{ab}\tilde{F}_{ab} = 0$ is very difficult to solve!
- Consider instead the model with auxiliary variables $V_{ab} = -V_{ba}$

$$\mathfrak{L}(F, V) = \frac{1}{4}F^{ab}F_{ab} + \frac{1}{2}V^{ab}V_{ab} - V^{ab}F_{ab} + \mathfrak{L}_{\rm int}(V) \ .$$

- Equation of motion for V_{ab} is algebraic

$$V_{ab} = F_{ab} - \frac{\partial \mathfrak{L}_{int}(V)}{\partial V^{ab}} \quad \Longrightarrow \quad \mathfrak{L}(F, V) \to \mathcal{L}(F)$$

- Condition of U(1) duality invariance:

$$\mathfrak{L}_{\rm int}(V) = \mathfrak{L}_{\rm int}(\nu, \bar{\nu}) \ , \qquad \nu := V_{+}^{ab}V_{+ab} \ ,$$

$$V_{\pm}^{ab} = \frac{1}{2}\left(V^{ab} \pm {\rm i}\tilde{V}^{ab}\right) \ , \quad \tilde{V}_{\pm} = \mp{\rm i}V_{\pm} \ , \quad V = V_{+} + V_{-}$$

$$G^{ab}\,\tilde{G}_{ab} + F^{ab}\,\tilde{F}_{ab} = 0 \quad \Longrightarrow \quad \mathfrak{L}_{\rm int}(\nu, \bar{\nu}) = \mathfrak{L}_{\rm int}(\nu\bar{\nu})$$

ModMax electrodynamics

- Maxwell electrodynamics is also conformal; action is Weyl invariant

$$\delta_\sigma \mathcal{D}_a = \sigma \mathcal{D}_a - (\mathcal{D}^b \sigma) M_{ab} \ , \quad \delta_\sigma F_{ab} = 2\sigma F_{ab}$$

- What conformal & U(1) invariant models for electrodynamics exist beyond the free case?
- **Solution:** Unique one parameter family of models

Bandos, Lechner, Sorokin & Townsend (2020)
Kosyakov (2020)

$$\mathcal{L}_{\mathrm{ModMax}}(F) = -\frac{1}{4}\cosh(\gamma)\ F^2 + \frac{1}{4}\sinh(\gamma)\ \sqrt{(F^2)^2 + (F\tilde{F})^2}$$

$\gamma \geq 0$ is necessary as superluminal propagation is possible for $\gamma < 0$

- Auxiliary variable formulation (unique conformal interaction):

Kuzenko (2021)

$$\mathfrak{L}_{\mathrm{int}}(\nu\bar{\nu}) = \kappa\sqrt{\nu\bar{\nu}} \ , \qquad \sinh(\gamma) = \frac{\kappa}{1-(\kappa/2)^2} \ , \qquad \kappa \in \mathbb{R}$$

Quantum corrections to ModMax theory

$$\mathcal{L}_{\rm ModMax}(F) = -\frac{1}{4}\cosh(\gamma)\ F^2 + \frac{1}{4}\sinh(\gamma)\ \sqrt{(F^2)^2 + (F\tilde{F})^2}$$

- Lagrangian cannot be perturbatively expanded about $F_{\mu\nu} = 0$
- Linearise about a solution to ModMax equations of motion

$$\partial^\nu F_{\mu\nu} - \partial^\nu \left[(\omega\bar{\omega})^{-1/2} \Big(\mathrm{Re}(\omega) F_{\mu\nu} + \mathrm{Im}(\omega) \tilde{F}_{\mu\nu} \Big) \right] \tanh(\gamma) = 0\ .$$

Here $\omega = \alpha + \mathrm{i}\beta$, $\alpha = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ and $\beta = \frac{1}{4}\tilde{F}^{\mu\nu}F_{\mu\nu}$.

- Quantisation proves to be very difficult about a generic solution
- Significantly simplifies by splitting about constant background $F_{\mu\nu}$

$$F_{\mu\nu} = F^{\rm B}_{\mu\nu} + F^{\rm Q}_{\mu\nu}\ , \qquad \mathcal{D}_\rho F^{\rm B}_{\mu\nu} = 0\ .$$

- At the one-loop level no quantum corrections arise!

Pinelli (2021)

Quantum corrections to ModMax theory

- Quantum corrections absent for $\mathcal{D}_\rho F^{\mathrm{B}}_{\mu\nu} = 0$, but higher-derivative corrections are possible
- One-loop log. divergences should respect Weyl and duality invariance

  Fradkin & Tseytlin (1985)

  Roiban & Tseytlin (2012)
- Computing higher-derivative deformations of ModMax maintaining Weyl and U(1) symmetry is easier than obtaining one-loop corrections
- Such deformations are also valid higher-derivative extensions of the model as they maintain its defining properties/symmetries

Duality rotations for higher-derivative electrodynamics

- Need to generalise the GZGR formalism to higher-derivative models
  Kuzenko & Theisen (2001)

  1. Work with the action $\mathcal{S}[F]$ instead of $\mathcal{L}(F)$ to simplify expressions
  2. Definition of $\tilde{G}_{ab}$

$$\tilde{G}^{ab}[F] = 2\frac{\delta \mathcal{S}[F]}{\delta F^{ab}}$$

  3. Self-duality equation

$$\int \mathrm{d}^4x\, e \left(\tilde{G}^{ab}\tilde{G}_{ab} + \tilde{F}^{ab}F_{ab} \right) = 0$$

  where $F_{ab} = -F_{ba}$, but otherwise unconstrained

- Action $\mathcal{S}[F]$ is unambiguously defined as a functional of an unconstrained two-form F_{ab}; no dependence on $\mathcal{D}_b\tilde{F}^{ab}$

Higher-derivative extension of IZ formulation

- The IZ reformulation is obtained by replacing the $\mathcal{S}[F]$ with

$$\mathfrak{S}[F,V] = \int \mathrm{d}^4x\, e\, \left\{\frac{1}{4}F^{ab}F_{ab} + \frac{1}{2}V^{ab}V_{ab} - V^{ab}F_{ab}\right\} + \mathfrak{S}_{\mathrm{int}}[V]\ ,$$

- Imposing the equation of motion reduces auxiliary action to $S[F]$

$$\frac{\delta}{\delta V_{ab}}\mathfrak{S}[F,V] = 0 \quad \Longrightarrow \quad \mathfrak{S}[F,V] \to \mathcal{S}[F]$$

- The self-duality equation turns into

$$\int \mathrm{d}^4x\, e\, \tilde{V}_{ab}\frac{\delta \mathfrak{S}_{\mathrm{int}}[V]}{\delta V_{ab}} = 0$$

- **<u>Note:</u>** if interaction takes the form $\mathfrak{S}^{\mathrm{int}}[\nu,\bar{\nu}]$, $\nu = V_+^{ab}V_{+ab}$, then the SD equation reduces to

$$\mathfrak{S}_{\mathrm{int}}[\mathrm{e}^{2\mathrm{i}\varphi}\nu, \mathrm{e}^{-2\mathrm{i}\varphi}\bar{\nu}] = \mathfrak{S}_{\mathrm{int}}[\nu,\bar{\nu}]\ , \qquad \varphi \in \mathbb{R}$$

Higher-derivative deformations of ModMax theory

Kuzenko & ER (2024)

- Need to identify Weyl-invariant HD functionals $\mathfrak{S}_{\rm HD}[V]$

$$\mathfrak{S}[F,V] = \int \mathrm{d}^4x\, e\, \left\{ \frac{1}{4} F^{ab}F_{ab} + \frac{1}{2} V^{ab}V_{ab} - V^{ab}F_{ab} + \kappa\sqrt{\nu\bar{\nu}} \right\} + \mathfrak{S}_{\rm HD}[V] ~,$$

which solve the self-duality equation

- The space of solutions is quite large. Specifically:

$$\mathfrak{S}_{\rm HD}[V] = \int \mathrm{d}^4x\, e\, \sqrt{\nu\bar{\nu}}\, \mathfrak{H}(\Sigma, \Upsilon, \bar{\Upsilon}, \Xi_n, \bar{\Xi}_n)$$

where we have defined the Weyl-invariant fields:

$$\Sigma = \frac{\Box_c (\nu\bar{\nu})^{1/8}}{(\nu\bar{\nu})^{3/8}} ~, \quad \Upsilon = \frac{\bar{\nu}^{1/4} \Box_c \nu^{1/4}}{\sqrt{\nu\bar{\nu}}} ~, \quad \Xi_n = \frac{\bar{\Psi}^n \Delta_0 \Psi^n}{\sqrt{\nu\bar{\nu}}} ~, \quad \Psi = \frac{\nu}{\bar{\nu}} ~,$$

$$\Box_c = \left(\mathcal{D}^2 - \frac{1}{6}\mathcal{R} \right) ~, \qquad \Delta_0 = (\mathcal{D}^a\mathcal{D}_a)^2 + 2\mathcal{D}^a\left(\mathcal{R}_{ab}\,\mathcal{D}^b - \tfrac{1}{3}\mathcal{R}\,\mathcal{D}_a\right) ~.$$

Note that Δ_0 is the Fradkin-Tseytlin operator Fradkin & Tseytlin (1982)

Example of elimination of auxiliary variables

Kuzenko & ER (2024)

- Consider, as an example, the following deformation of ModMax

$$\mathfrak{S}_{\text{int}}[V] = \int \mathrm{d}^4x\, e\, \left\{ \kappa\sqrt{\nu\bar{\nu}} + g(\nu\bar{\nu})^{-1/4}\left[\Box_c(\nu\bar{\nu})^{1/8}\right]^2 \right\} , \qquad g \in \mathbb{R}$$

- Eliminating the auxiliary fields to quadratic order in g gives

$$\begin{aligned} \mathcal{S} = \mathcal{S}_{\text{MM}} + \int \mathrm{d}^4x\, e \Big\{ & g\Omega^{-\frac{1}{2}}(\Box_c\Omega^{\frac{1}{4}})^2 + \frac{g^2\Omega^{-\frac{3}{2}}}{4(1-(\kappa/2)^2)(1+(\kappa/2)^2)^2}\Big(\Box_c(\Omega^{-\frac{1}{2}}\Box_c\Omega^{\frac{1}{4}}) - \Omega^{-\frac{3}{4}}(\Box_c\Omega^{\frac{1}{4}})^2\Big)^2 \\ & \times \Big\{ \Big(3 - 12(\kappa/2)^2 + 20(\kappa/2)^4\Big)(\omega+\bar{\omega}) - 4(\kappa/2)\Big(2 + \kappa/2 - 5(\kappa/2)^2 + 2(\kappa/2)^3 \\ & + 9(\kappa/2)^4 + (\kappa/2)^5\Big)\Omega\Big\}\Big\} + \mathcal{O}(g^3) \end{aligned}$$

where we have defined

$$\Omega = \frac{\big(1+(\kappa/2)^2\big)(\omega\bar{\omega})^{\frac{1}{2}} - (\kappa/2)(\omega+\bar{\omega})}{\big(1-(\kappa/2)^2\big)^2} = \frac{1}{2}(\cosh\gamma + 1)\frac{\partial L_{\text{MM}}}{\partial\gamma} .$$

- Note that Ω is manifestly invariant under ModMax duality rotations!

Duality-invariant observables

- The leading contribution to the deformation was manifestly invariant under ModMax duality rotations

$$\Omega = \frac{\big(1 + (\kappa/2)^2\big)(\omega\bar{\omega})^{\frac{1}{2}} - (\kappa/2)(\omega + \bar{\omega})}{\big(1 - (\kappa/2)^2\big)^2} \quad \Longrightarrow \quad \delta_\varphi \Omega = 0$$

- In perturbation theory, the leading contribution to the deformation of any self-dual theory must be duality-invariant!
- **Theorem:** Any two duality-invariant local observables are functionally dependent

Ferko, Smith, Kuzenko & Tartaglino-Mazzucchelli (2024)

- **Way out:** Consider functionals involving derivatives of F_{ab}

$$\mathcal{I} = \sqrt{\omega}(1 + \cosh\gamma) - \sqrt{\bar{\omega}}\sinh\gamma \ , \qquad \delta_\varphi \mathcal{I} = \mathrm{i}\varphi\mathcal{I} \ ,$$

$$\mathcal{J} = \mathcal{I}\Big(\Box_c\sqrt{\bar{\mathcal{I}}}\Big)^2 \quad \Longrightarrow \quad \delta_\varphi \mathcal{J} = 0$$

Kuzenko, ER (2024)

In-out vacuum amplitude for ModMax

- The family of higher-derivative deformations of ModMax is very big!
- Want to single out those deformations of ModMax which may contribute to a **low-energy effective action** of the theory
- Consider the in-out vacuum amplitude

$$Z = \int [\mathfrak{D}A_a][\mathfrak{D}V_{ab}]\,\delta\big[\nabla_a A^a - \xi\big]\mathrm{Det}(\nabla^2)\ \exp\left\{\frac{\mathrm{i}}{\hbar}\mathfrak{S}_{\mathrm{MM}}[F,V]\right\}\ ,$$

$$\mathfrak{S}_{\mathrm{MM}}[F,V] = \int \mathrm{d}^4x\, e\,\left\{\frac{1}{4}F^{ab}F_{ab} + \frac{1}{2}V^{ab}V_{ab} - V^{ab}F_{ab} + \kappa\sqrt{\nu\bar{\nu}}\right\}$$

- The functional $\hbar^{-1}\mathfrak{S}_{\mathrm{MM}}[F,V]$ is invariant under rescalings

$$\hbar \to \lambda^2\hbar\ , \qquad F_{ab}(x) \to \lambda F_{ab}(x)\ , \qquad V_{ab}(x) \to \lambda V_{ab}(x)$$

- The effective action $\Gamma_{\mathrm{MM}}[F,V]$ is expected to share this symmetry

ModMax effective action

Kuzenko & ER (2024)

- Posit that (a local part of) the effective action has the form

$$\Gamma_{\rm MM}[F,V] = \mathfrak{S}_{\rm MM}[F,V] + \sum_{n=1}^{\infty} \hbar^n \Gamma^{(n)}[V]$$

and possesses the following properties:

1. $\hbar^{-1}\Gamma_{\rm MM}[F,V]$ is invariant under the rescalings
2. each functional $\Gamma^{(n)}[V]$ is Weyl invariant
3. each functional $\Gamma^{(n)}[V]$ obeys the self-duality equation

$$\int {\rm d}^4x\, e\, \tilde{V}_{ab} \frac{\delta \Gamma^{(n)}[V]}{\delta V_{ab}} = 0$$

- Implies that the ModMax coupling $\int {\rm d}^4x\, e\, \sqrt{\nu\bar{\nu}}$ **cannot** be generated as a one loop quantum correction!
- Possible solution for general n

$$\Gamma^{(n)}[V] = g_n \int {\rm d}^4x\, e\, \frac{\left[\Box_c (\nu\bar{\nu})^{1/8}\right]^{2n}}{(\nu\bar{\nu})^{(3n-2)/4}}\,, \quad g_n \in \mathbb{R}$$

ModMax one-loop effective action

Kuzenko & ER (2024)

- Keeping in mind these arguments, our ansatz for $\Gamma^{(1)}[V]$ is:

$$\Gamma^{(1)}[V] = \int \mathrm{d}^4x\, e\, \sqrt{\nu\bar{\nu}} \Big\{ g_1 \Upsilon^2 + \bar{g}_1 \bar{\Upsilon}^2 + g_2 \Upsilon \bar{\Upsilon} + \sum_{n=1}^{4} g_3^{(n)} \Xi_n + g_4 \Sigma^2 \Big\}$$
$$= \hbar \int \mathrm{d}^4x\, e \Big\{ \frac{g_1 \bar{\nu}^{\frac{1}{2}} (\Box_c \nu^{\frac{1}{4}})^2 + \bar{g}_1 \nu^{\frac{1}{2}} (\Box_c \bar{\nu}^{\frac{1}{4}})^2}{(\nu\bar{\nu})^{\frac{1}{2}}} + g_2 \frac{\Box_c \nu^{\frac{1}{4}} \Box_c \bar{\nu}^{\frac{1}{4}}}{(\nu\bar{\nu})^{\frac{1}{4}}}$$
$$+ \sum_{n=1}^{4} g_3^{(n)} \bar{\Psi}^n \Box_0 \Psi^n + g_4 \frac{(\Box_c (\nu\bar{\nu})^{\frac{1}{8}})^2}{(\nu\bar{\nu})^{\frac{1}{4}}} \Big\} ,$$

  where $g_1 \in \mathbb{C}$ and $g_2, g_3^{(n)}, g_4 \in \mathbb{R}$.

- Rescaling symmetry has lead to **significant restrictions** on the structure of the one-loop deformation!

ModMax one-loop effective action

Kuzenko & ER (2024)

- Eliminating the auxiliary fields to leading order in $\hbar$ leads to the higher-derivative action

$$\Gamma_{\rm MM}[F] = \mathcal{S}_{\rm MM}[F] + \hbar \int {\rm d}^4x\, e \Bigg\{ \frac{g_1 \bar{\mathcal{I}} (\Box_c \sqrt{\mathcal{I}})^2 + \bar{g}_1 \mathcal{I} (\Box_c \sqrt{\bar{\mathcal{I}}})^2}{2\Omega} + g_2 \frac{\Box_c \sqrt{\mathcal{I}} \Box_c \sqrt{\bar{\mathcal{I}}}}{\sqrt{2\Omega}} + \sum_{n=1}^{4} g_3^{(n)} \frac{\mathcal{I}^{2n}}{\bar{\mathcal{I}}^{2n}} \Delta_0 \frac{\bar{\mathcal{I}}^{2n}}{\mathcal{I}^{2n}} + g_4 \Omega^{-\frac{1}{2}} \left(\Box_c \Omega^{\frac{1}{4}}\right)^2 \Bigg\} \Bigg\} + \mathcal{O}(\hbar^2)$$

- Should be emphasised that the sector linear in $\hbar$ is **duality invariant**!
- All structures may contribute to the one-loop effective action for ModMax, but explicit calculations remain to be completed

On-shell vanishing structures

- We excluded a priori structures containing the primary vector fields

$$\chi_a^{(1)} = \mathcal{D}^b V_{ab} \ , \qquad \chi_a^{(2)} = \mathcal{D}^b \tilde{V}_{ab}$$

- Considering these contributions as deformations to Maxwell theory, they lead to trivial contributions once auxiliaries are eliminated

$$\chi_a^{(1)} = \mathcal{D}^b F_{ab} + \dots \ , \qquad \chi_a^{(2)} = \cancelto{0}{\mathcal{D}^b \tilde{F}_{ab}} + \dots$$

Recall that $\mathcal{D}^b F_{ab} = 0$ on-shell for Maxwell electrodynamics

- These structures could potentially arise at the one-loop level, but further analysis is required

Concluding comments

- **Outcomes:**
  - Classified consistent higher-derivative deformations of ModMax
  - Identified new duality-invariant local observables
  - Provided general ansatz for one-loop deformation
- **Future work:**
  - Study of on-shell vanishing structures
  - Extension to $\mathcal{N} = 1$ super ModMax

    Bandos, Lechner, Sorokin & Townsend (2021)

    Kuzenko (2021)
  - Existence of $\mathcal{N} = 2$ super ModMax?

    Kuzenko & ER (2021)
  - Explicit computation of one-loop effective action (Bosonic and $\mathcal{N} = 1$)