EISA
European Institute for Sciences and Their Applications

Corfu Summer Institute
Hellenic School and Workshops on Elementary Particle Physics and Gravity
Corfu, Greece

Exceptional symmetry as a symmetry principle for sigma models

David Osten (University of Wrocław)

Workshop on Noncommutative and Generalized Geometry
in String theory, Gauge theory and Related Physical Models,

Corfu, September 22th

based on: 2402.10269, 2306.11093, 2103.03267

Uniwersytet Wrocławski

POLONEZ BIS

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(Non-linear) σ-models: big class of theories for model building & fundamental physics

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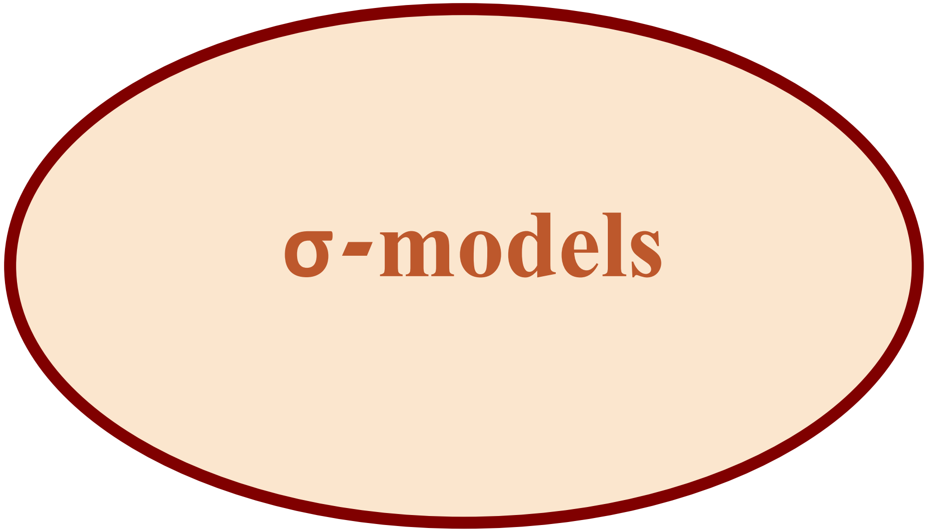

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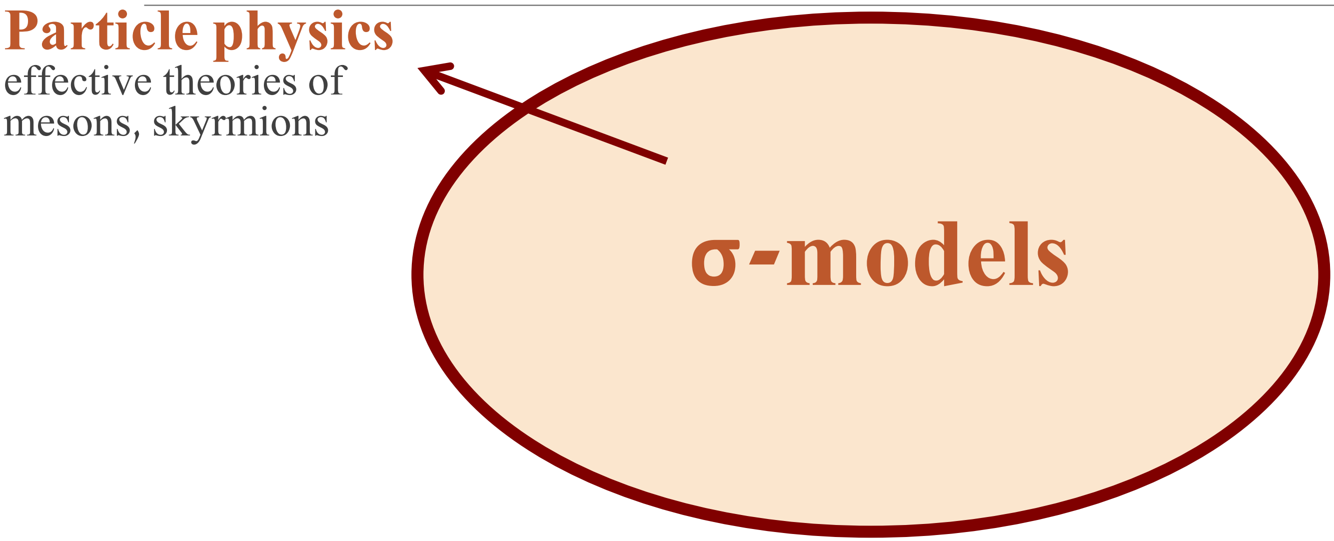

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effective theories of
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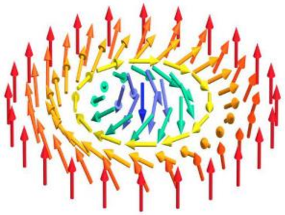

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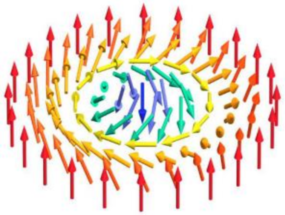

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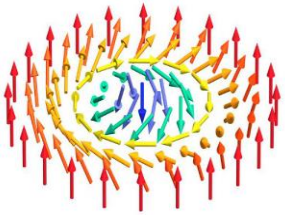

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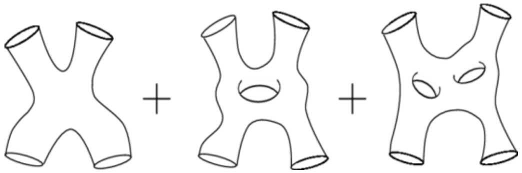

String and M-theory
description of fundamental objects

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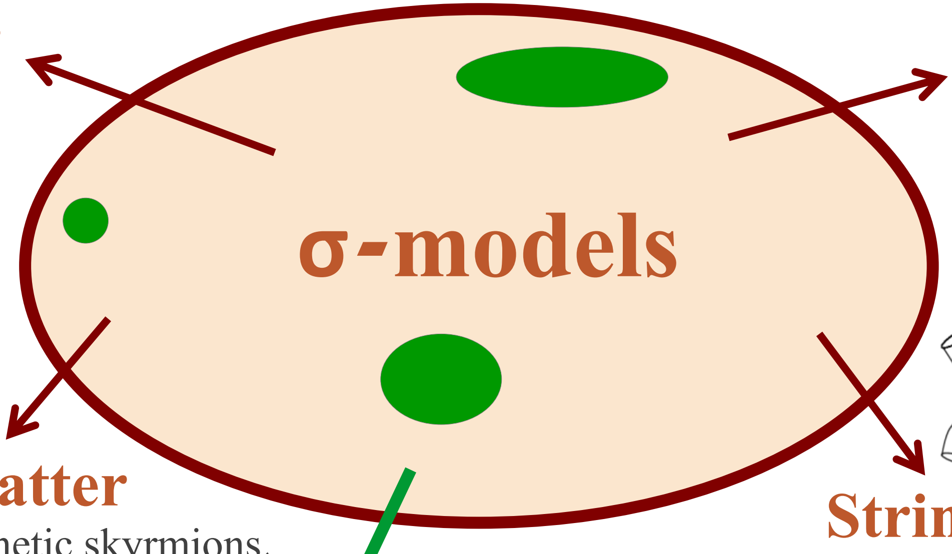

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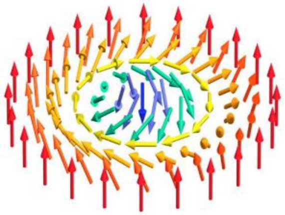

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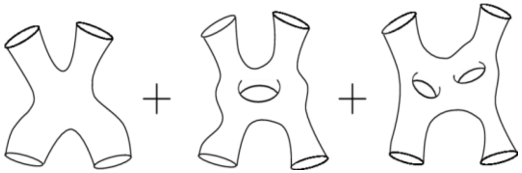

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integrability *(exactly solvable models),*
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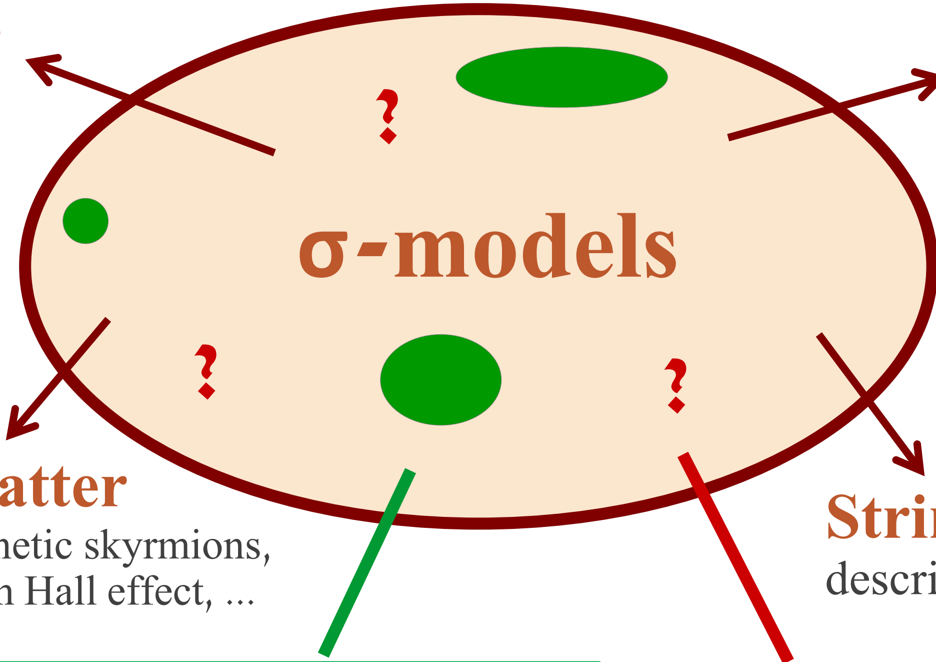

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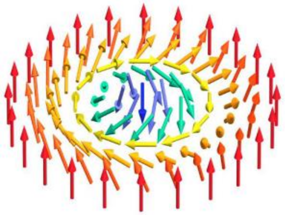

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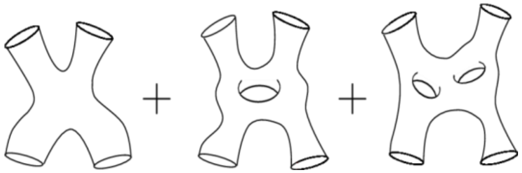

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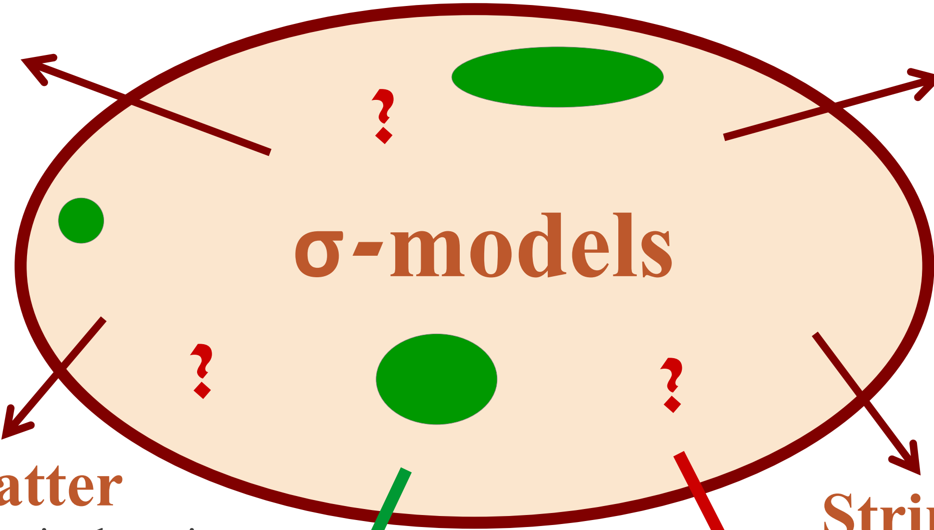

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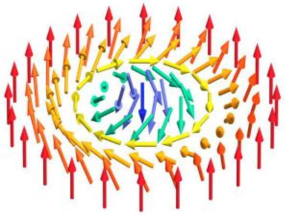

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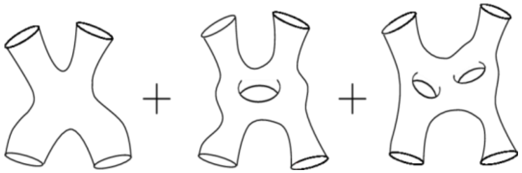

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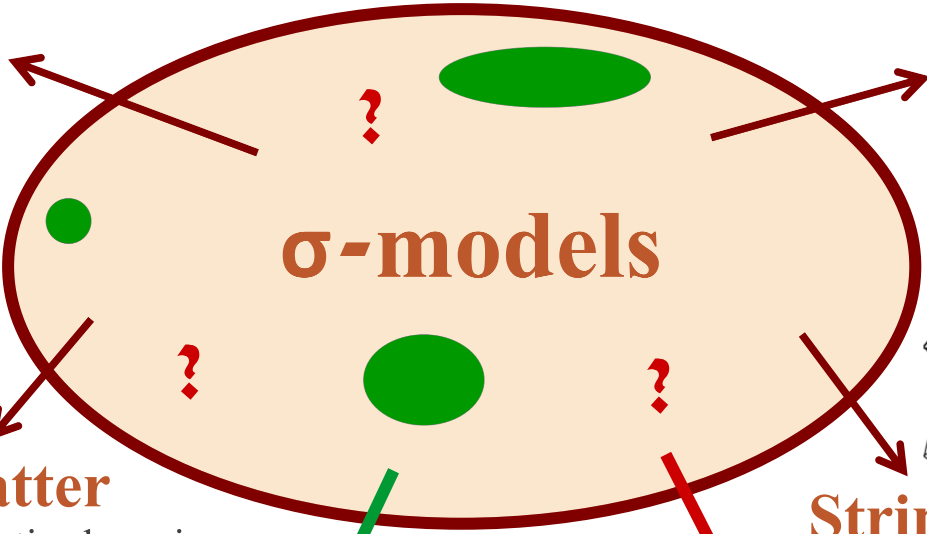

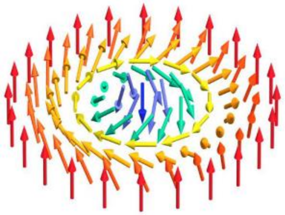

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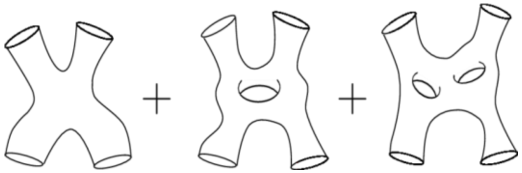

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(new symmetry principle (exceptional covariance) for special class of σ-models (sigma models with diff-invariance))

Programme

1. Sigma models
2. (Exceptional) generalised geometry
3. exceptional covariance as a symmetry property

(Non-linear) σ-models

$$L \sim g_{mn}(x)\partial_\alpha x^m \partial^\alpha x^n \left(+C_{m_1\ldots m_p}\right)\partial_{\alpha_1} x^{m_1} \ldots \partial_{\alpha_p} x^{m_p} \epsilon^{\alpha_1\ldots\alpha_p}\right)$$

- fields $x : \Sigma \to M$

σ^α: *coordinates of* Σ *,* $p = \dim\Sigma - 1$

x^m: *coordinates on field (target) space M,*

- **'coupling constants': g** is a metric tensor on M, **C** a *(p+1)*-form potential

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p	**physical model**	***Σ***	***M***	**C?**
1	point particle coupling to GR and EM	world-line	space-time	A_μ
2	2d materials (continuum limit of spin chains, ...)	material	O(3),O(N),...	-
	(first quantised) string	world-sheet	space-time	yes
	Geroch model (GR with 2 Killing vectors)	eff. space-time	SL(2,R)	-
3	relativistic membrane (e.g. in M-theory)	world-volume	space-time	yes
4	effective theory for pions	space-time	SU(N)	-
	original Skyrme model	space-time	$\frac{SU(N) \times SU(N)}{SU(N)}$	Skyrme term

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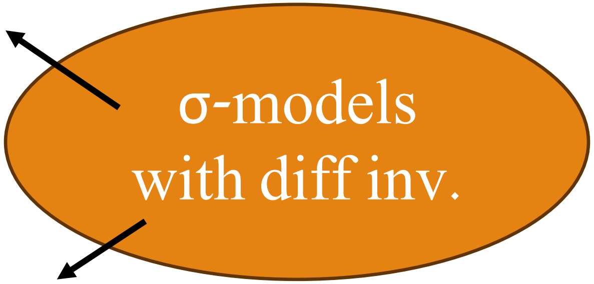

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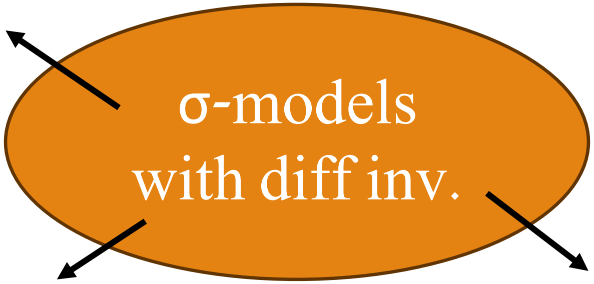

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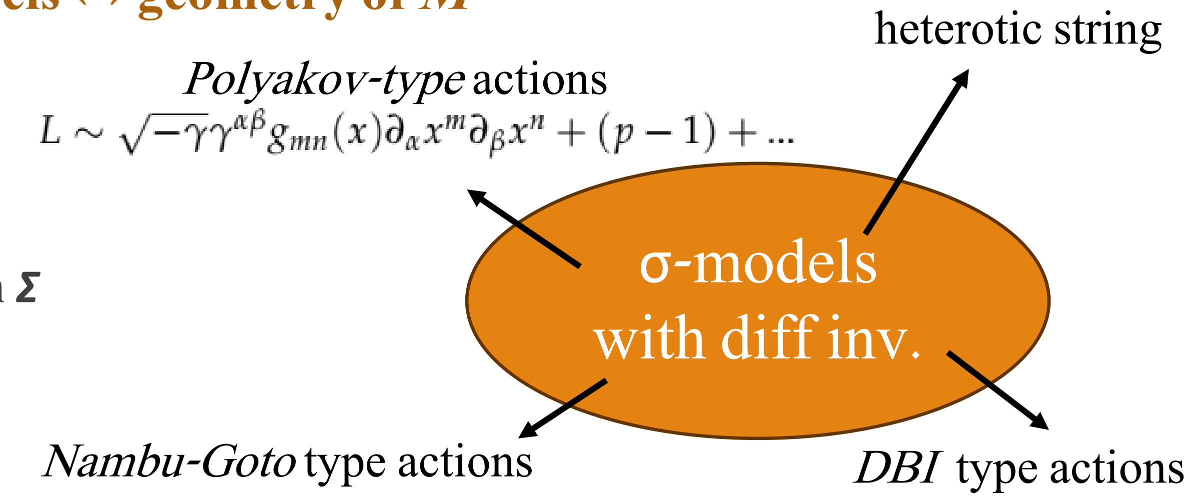

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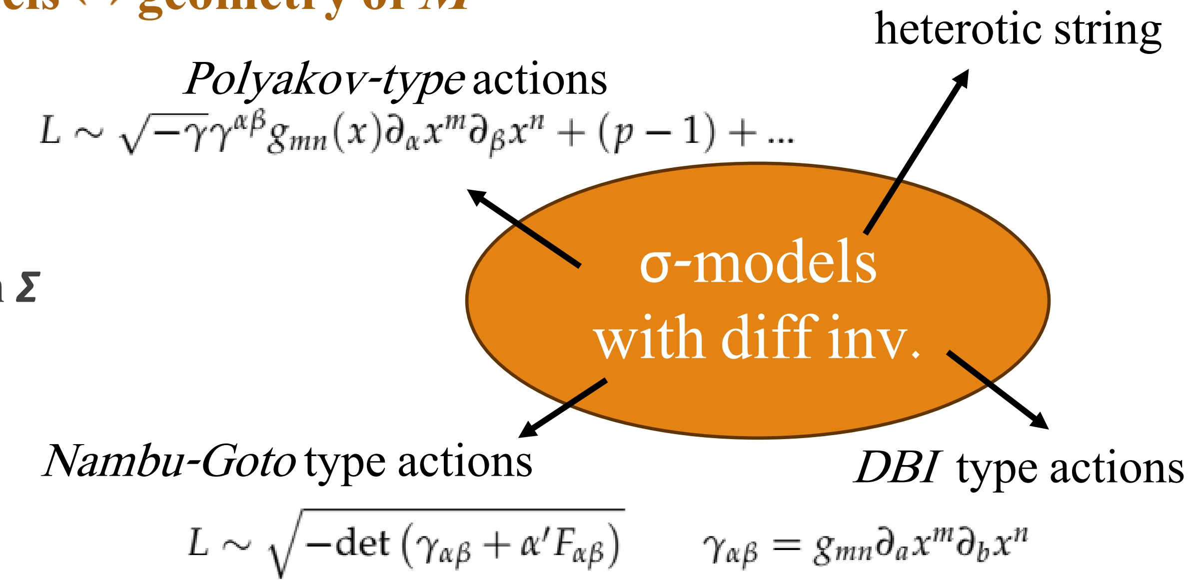

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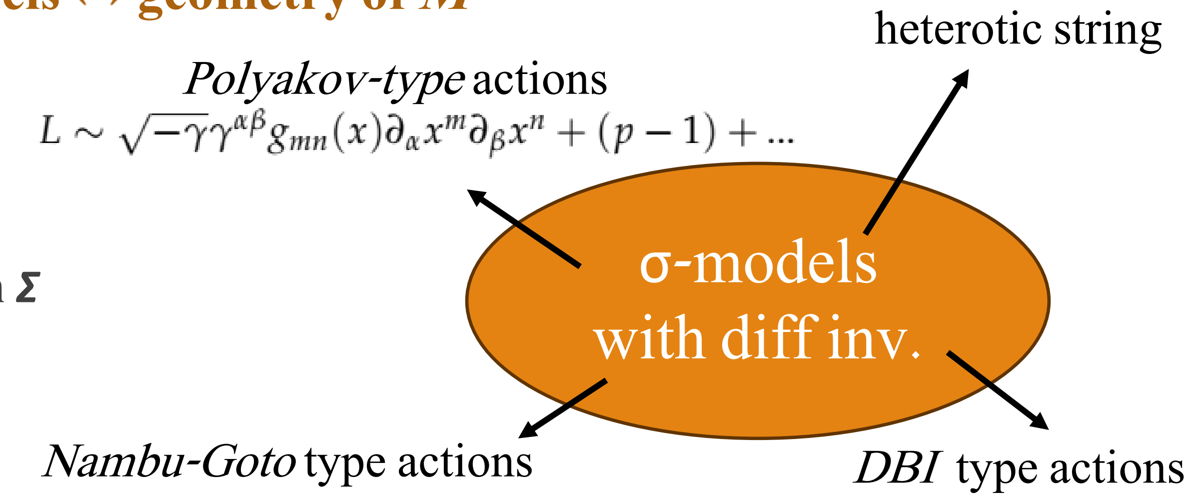

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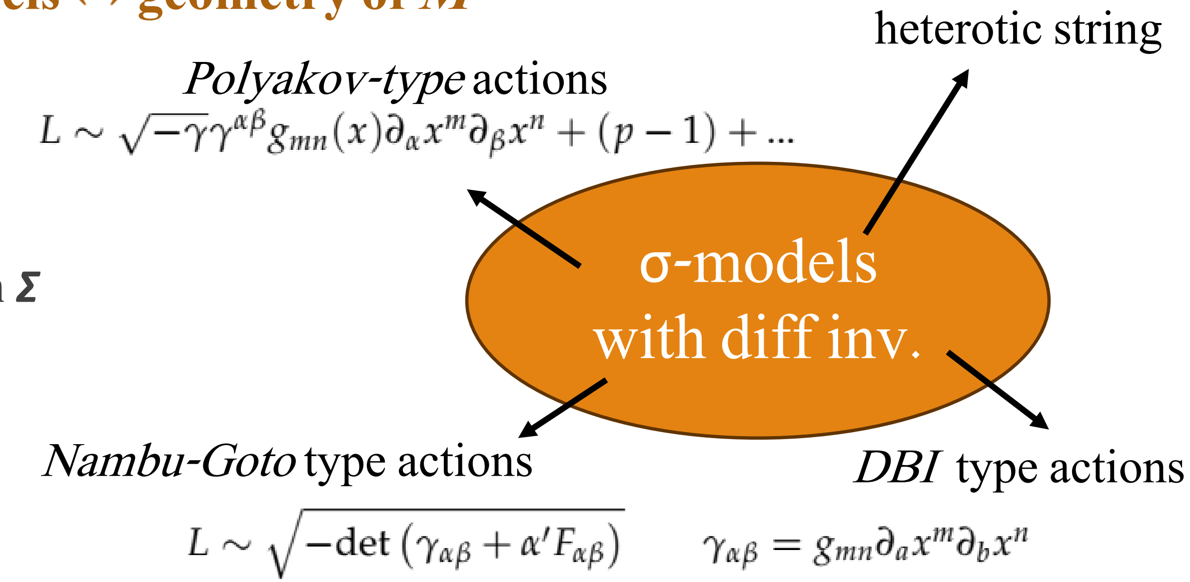

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1. **Generalised geometry unifying language of brane σ-models** *[Duff et al 90-, Siegel et al 92-, Tseytlin 92, Hull 06-, Alekseev/Strobl 04, Hatsuda et al 12-, Sakatani/Uehara 16-, Arvanitakis/Blair 17- ,DO 19-]*

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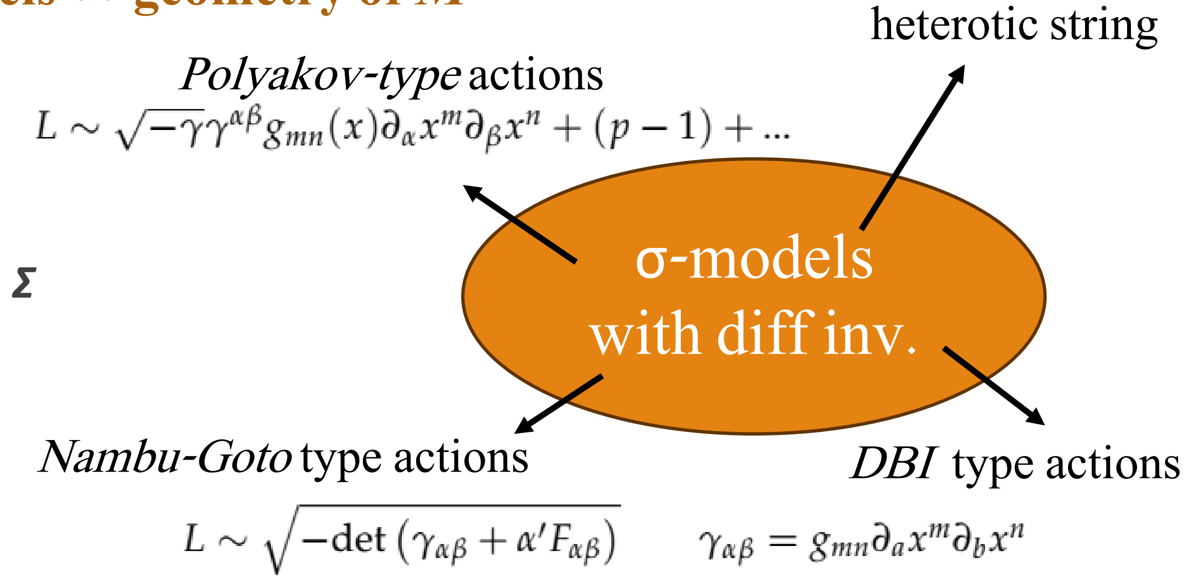

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2. **Covariance under exceptional generalised geometry restricts to ½-BPS branes (string, M-branes, D-branes, non-perturbative/exotic branes, …) in supergravity**
   [DO 21,23,24, building on Arvanitakis/Blair 17-22]
   - only considering bosonic part *(i.e. no supersymmetry necessary)*

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Hamiltonian formulation of brane σ-models

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- ex. 1: **2dim σ-models and O(d,d) generalised geometry** $S \sim \int \left(g_{mn} \mathrm{d}x^m \wedge \star \mathrm{d}x^n + B_{mn} \mathrm{d}x^m \wedge \mathrm{d}x^n \right)$

[Siegel 92, Tseytlin 92]

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  - Currents $t_M(\sigma) = (p_m(\sigma), \partial x^m(\sigma))$
  - Poisson brackets $\{t_M(\sigma), t_N(\sigma')\} = \eta_{MN} \delta'(\sigma - \sigma')$

[Siegel 92, Tseytlin 92]

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[Hatsuda/Kamimura 12, DO 21]

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Generalised geometry (overview)

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physical objects	metric g	metric g , $(p+1)$-form gauge fields C, … , dilaton
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Generalised geometry (overview)

	Riemannian geometry	**Generalised Geometry**
physical objects	metric g	metric g, ($p+1$)-form gauge fields C, … , dilaton
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- example – membrane currents:

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only ½-BPS branes (string, membrane, D-branes, …) are allowed.

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$$\boxed{t^{(p+1)}_{K_{p+1}} \wedge \mathrm{d}X^{L_1}\partial_{L_1} = D_{K_{p+1}}{}^{M_p,L_1} t^{(p)}_{M_p} \partial_{L_1}}$$

- **Lessons/Results** for $G = E_{d(d)}$: *[DO, 21,23,24]*
  - Constraints are very restrictive:

    only ½-BPS branes (string, membrane, D-branes, …) are allowed.
  - remarkable, as only bosonic part of σ-models
  - Still: solutions break $E_{d(d)} \longrightarrow GL(d)$

Generalised Lie bracket and brane charge constraint

- **All** diff-invariant σ-models can be realised like that (Claim 1).
  It turns that it is not a very strong statement, concrete choices of $t^{(q)}(\sigma)$ can break G (duality) symmetry completely.

- Stronger statement: **G-covariance on phase space**
  For this, the ***generalised Lie bracket*** should be realised on phase space:

$$V = \int V^{M_1}(x) t^{(1)}_{M_1} \in \mathcal{R}_1, \quad \phi = \int \phi^{M_p}(x) t^{(p)}_{M_p} \in \mathcal{R}_p, \qquad \boxed{\{\phi, V\} \overset{!}{=} \int (\mathcal{L}_V \phi)^{M_p} t^{(p)}_{M_p}}$$

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summary & outlook

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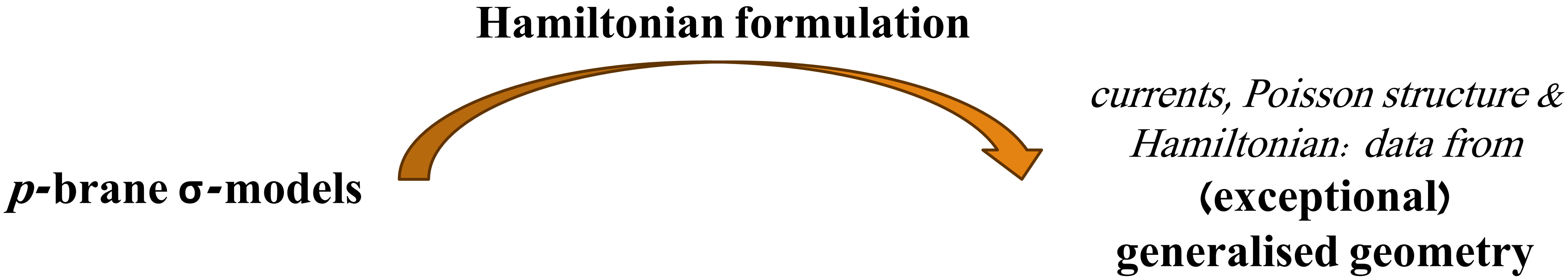
***p*-brane σ-models**

summary & outlook



Hamiltonian formulation

***p*-brane σ-models**

currents, Poisson structure & Hamiltonian: data from

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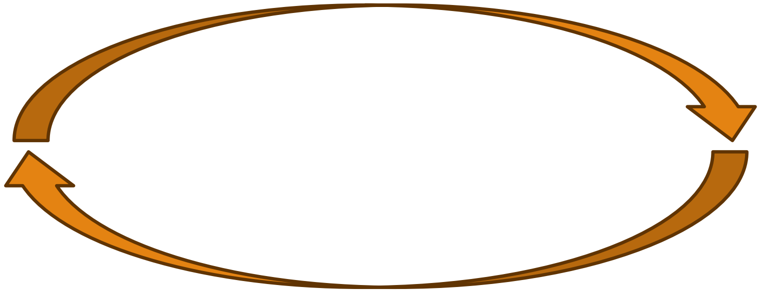
summary & outlook



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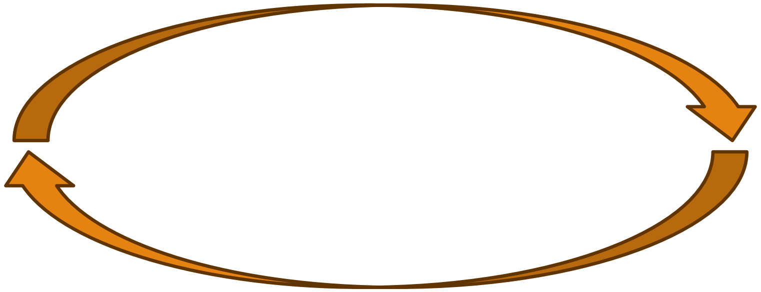

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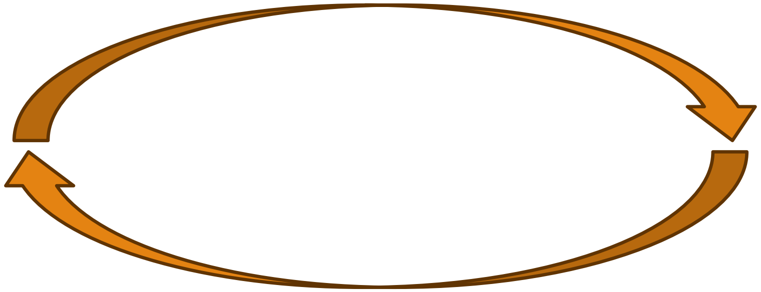

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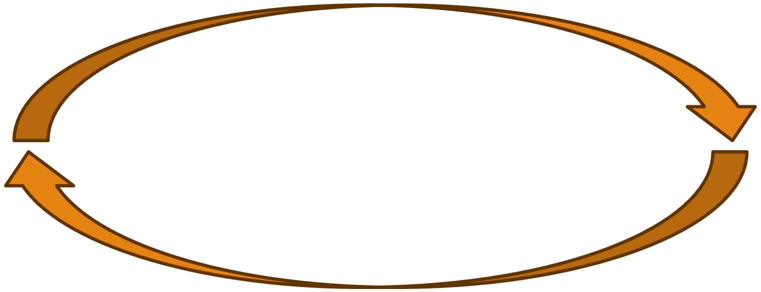

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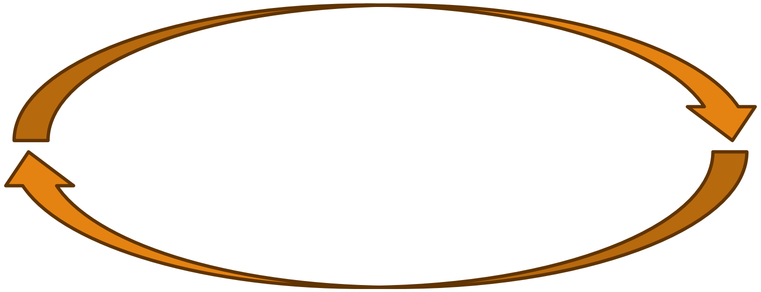

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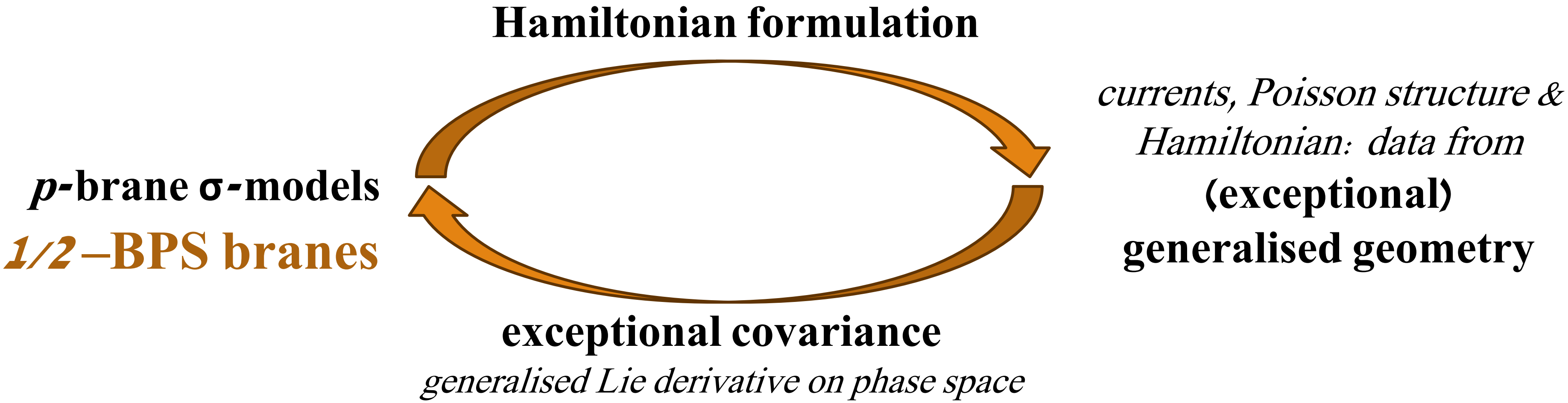

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- A-theory *[Hatsuda, Hulik, Linch, Siegel, Wang, Wang 23]* : non-conventional brane theories, without requiring exceptional covariance (brane charge constraints)

Thank you for your attention!