From Symmetries to Gravitational Waves

and back

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Monash Centre for Astrophysics

GAMBIT

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some covered papers:

Phys.Rev.D 109 (2024) 6, L061303
Prog.Part.Nucl.Phys. 135 (2024) 104094
Nucl.Phys.B 1002 (2024) 116533
Eur.Phys.J.C 84 (2024) 1, 66
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graphics: Wang, Tian, Balazs 2409.06599

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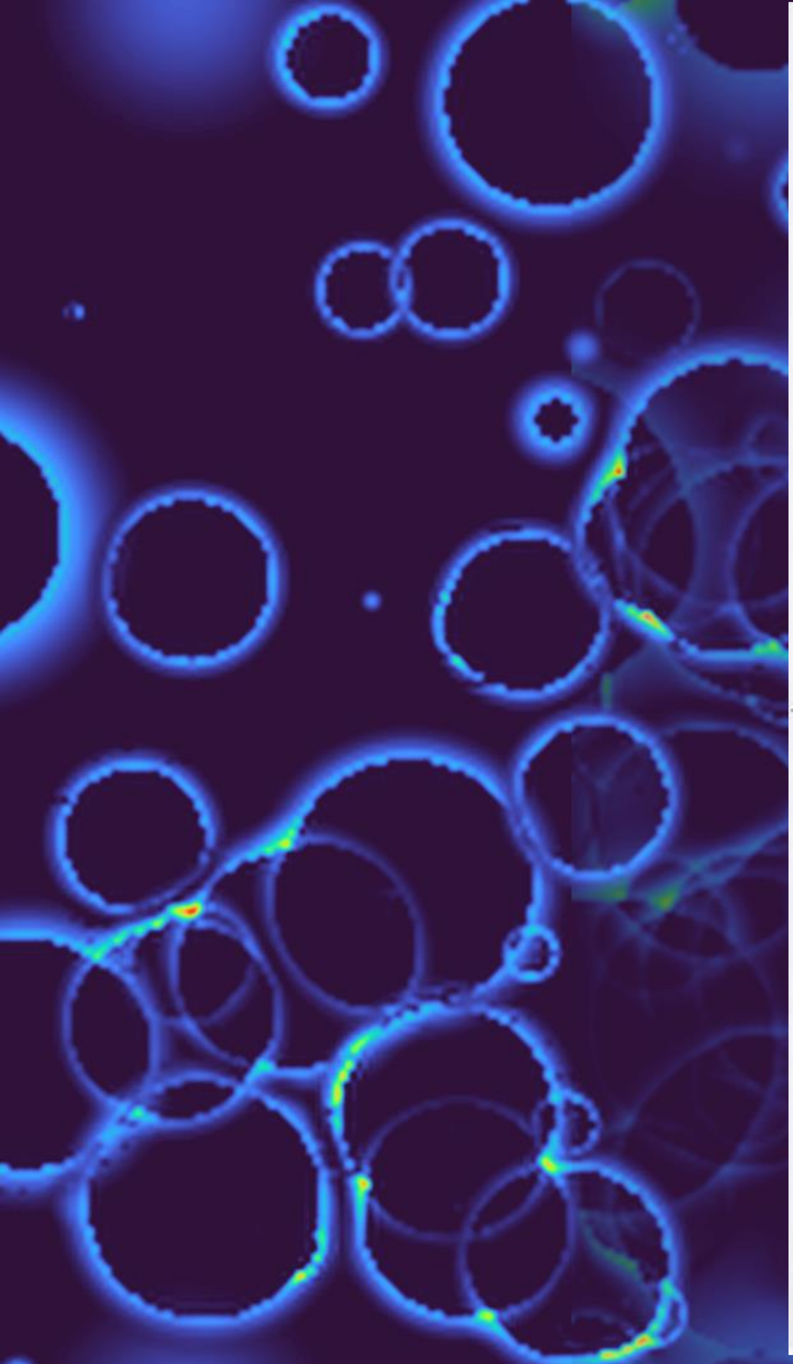
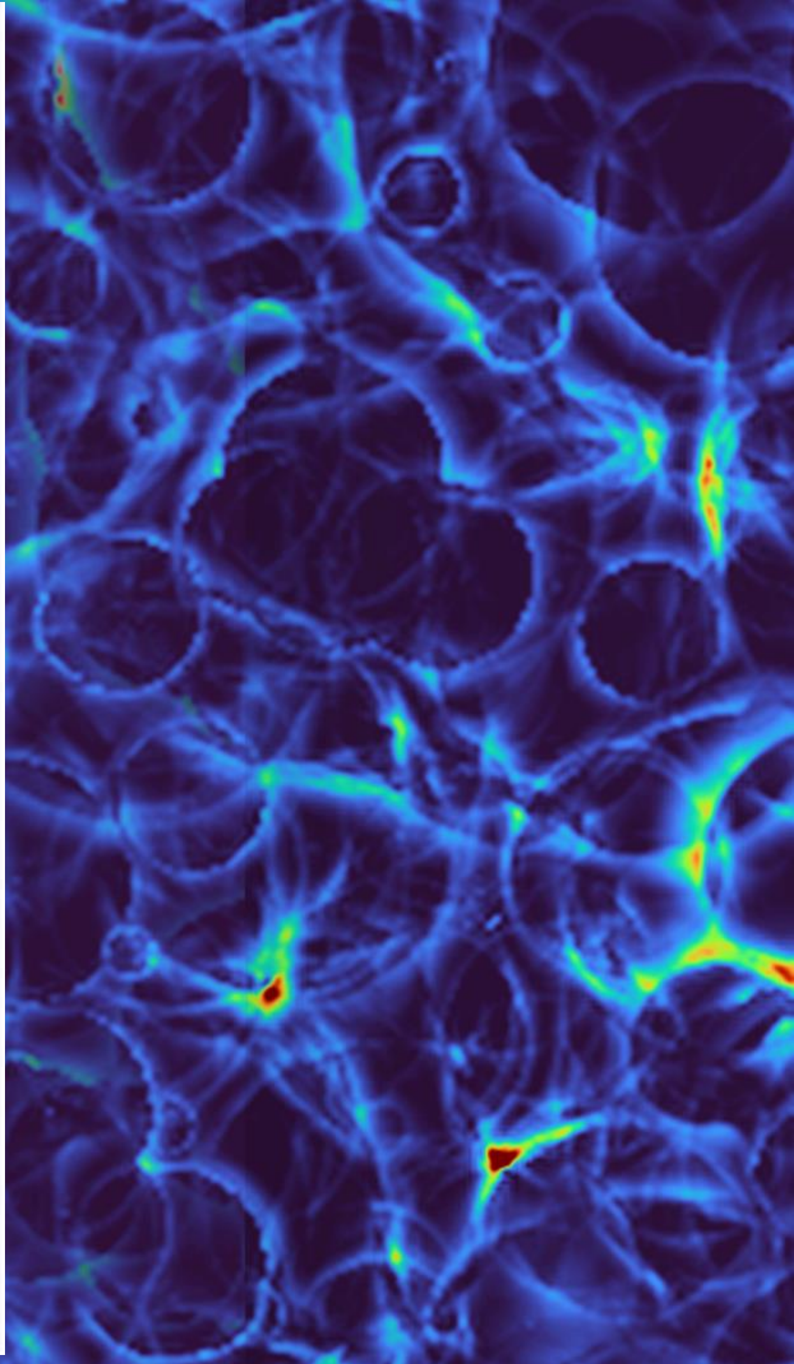
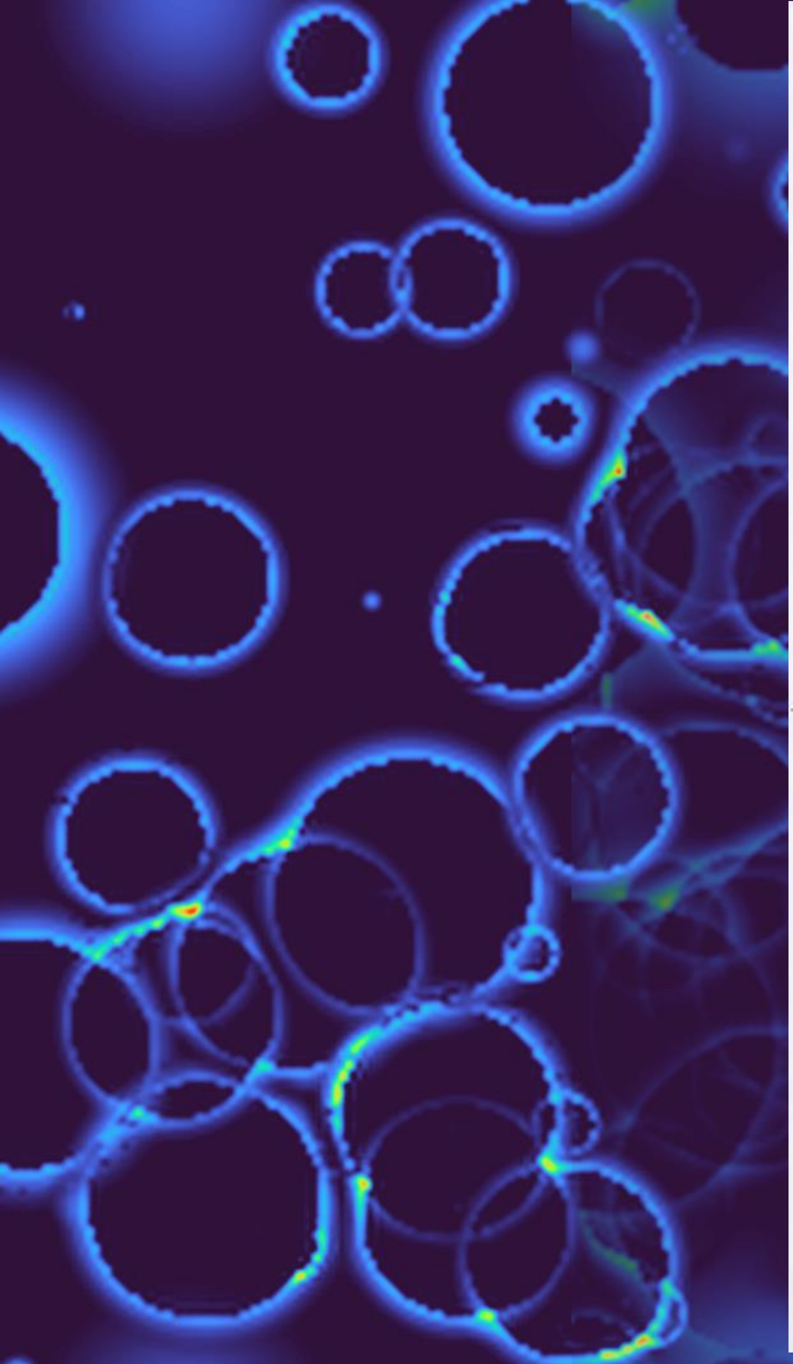


SO, SINCE THE EARLY 1980's, SUSY-GUT's HAVE BEEN THE ATTRACTIVE FORM OF GUT's



slide from Ed Witten's SUSY2002 talk



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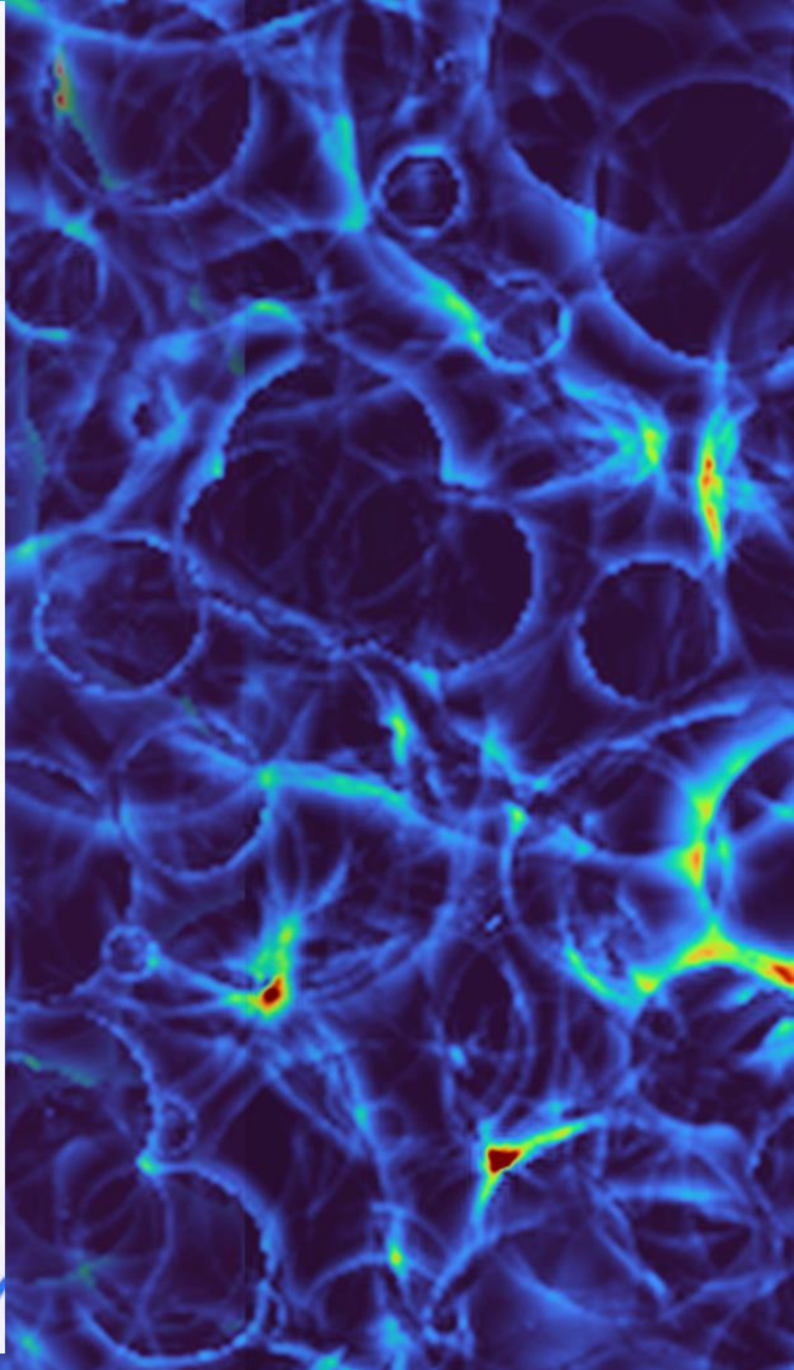
AND THERE ARE FIVE EXCEPTIONS

G_2, F_4, E_6, E_7, E_8

NATURE IS EXCEPTIONAL, SO WHY NOT DESCRIBE IT USING AN EXCEPTIONAL LIE GROUP?

THERE IS ONE THAT BEAUTIFULLY WORKS – THE E_6 MODEL

IT CAPTURES THE SUCCESSES OF THE SO(10) MODEL "EXCEPTIONALLY



slide from Ed Witten's SUSY2002 talk

(28)

SO ONE IS FORCED TO CONTINUE THE GUT CHAIN TO THE END

$$SU(5) \to SO(10) \to E_6 \to E_7 \to E_8$$

AND TO UNIFY THE THREE FERMION GENERATIONS ...

TWO THINGS THAT DIDN'T WORK IN FOUR-DIMENSIONAL GUTs.

slide from Ed Witten's SUSY2002 talk

from symmetries to gravitational waves and back

Nature described by an exceptional group?

Beautiful mathematical dream or physics?

part of
the answer might be encoded in the gravitational wave background

particle physicist: symmetry breaking = cosmologist: phase transition

first order cosmological phase transitions generate gravitational waves

from symmetries to gravitational waves overview

symmetry e.g. $E_8 \times E_8$, …

↓ particle physics, thermodynamics, gravity, …

↓ many assumptions, even more approximations

↓ treacherous analytical and numerical calculations

gravitational wave spectrum $\Omega_{GW}(f_{GW})$

↕ sophisticated statistical inference

experimental observation e.g. NanoGrav, …

↓ trip to Stockholm

new physics

from symmetries to gravitational waves summary

Lagrangian $\mathcal{L}(\phi_i, \psi_j, A_k^\mu, \dots)$

↓ thermal field theory — numerical codes

effective potential $V_{eff}(\phi_i, T)$

↓ semi-classical methods — numerical codes

bounce action & thermal parameters $S(T_*), \alpha, \beta, \gamma, \dots$

↓ --- there's a disconnect here --- — numerical codes

fitting formulae from gravito-hydrodynamical lattice simulations

↓

gravitational wave spectrum $\Omega_{GW}(f_{GW})$

from symmetries to effective potential

symmetries + breaking — e.g. $\mathrm{SU}(2) \times \mathrm{U}(1) \rightarrow \mathrm{U}(1)$

↓ group theory — (GroupMath)

Lagrangian (ia eigenstate para) — $\mathcal{L}(\phi_i, \psi_j, A_k^\mu, M_k, G_i, \Lambda_i, \dots)$

↓ quantum corrections — (DRAlgo)

corrected mass matrix, couplings — $M_i + \Pi_i, \dots$

↓ simultaneous diagonalisation — VefFermi

physical masses, couplings — $m_i, g_i, \lambda_i, \dots$

↓ generic V_{eff} template — VefFermi

effective potential — $V_{eff}(\phi_i, T, m_i, g_k, \lambda_i, \dots)$

from effective potential to critical temp. and beyond

effective potential	$V_{eff}(\phi_i, T, m_i, g_k, \lambda_i, \ldots)$	
↓ minimum finder		PhaseTracer1
thermal phases	$\partial_\phi V_{eff}(\phi_i, \ldots) = 0$	
↓ minimum tracer		PhaseTracer1
critical temperatures	$V_{eff}(\phi_{i,min}, T_C, \ldots)$	
↓ nucleation heuristic		PhaseTracer2
nucleation T (approximate)	T_n	
↓ bounce solver		PhaseTracer2
action, thermal parameters	$S, \alpha, \beta, \gamma, \ldots$	
↓ analytical fitting formulas		PhaseTracer2
GW spectrum	$\Omega_{GW}(f_{GW})$	

from equation of motion to transition

effective potential	$V_{eff}(\phi_i, T)$
↓ analytical methods	
equation of motion	$\ddot{\phi}_i + \frac{n}{\varrho}\dot{\phi}_i = V'_{eff}(\phi_i)$
↓ path finder	BubbleProfiler OptiBounce
Euclidean action	$S_E(T)$
↓ nucleation heuristics	BubbleProfiler OptiBounce
nucleation T (approximate)	T_n
↓ numerical optimiser	BubbleProfiler OptiBounce
bounce solution/action	S_{min}

from action to thermal history (coming soon)

bounce action		S_{min}	
↓	semi-classical appox		TranSolver
nucleation rate		$\Gamma(\mathrm{T}) \sim e^{-S(T)}$	
↓	analytical methods		TranSolver
false vacuum fraction		$P_f(T) \sim \int dT' \Gamma(\mathrm{T}') f(T')$	
↓	efficient integrator		TranSolver
nucleation, percolation, finishing		T_n, T_p, T_f	
↓	transition analyser		TranSolver
“thermal history”		$P_f(T_f)$ for all transitions	

weakest link in traditional calculation

…

1. thermal parameters $S(T_*), \alpha, \beta, \gamma, \ldots$

↓ --- there's a disconnect here ---

2. fitting formulae from gravito-hydrodynamical lattice simulations

- 1. and 2. require eq. of motion, eq. of state, fluid dynamics
- EoM, EoS, FD are particle model dependent, derived from $\mathcal{L}$!
- lattice simulations are independent of particle physics $\mathcal{L}$!
  - generic assumptions are made for EoM, EoS, FD
- it is inconsistent to use fitting formulae to predict gravitational waves

from symmetries to gravitational waves

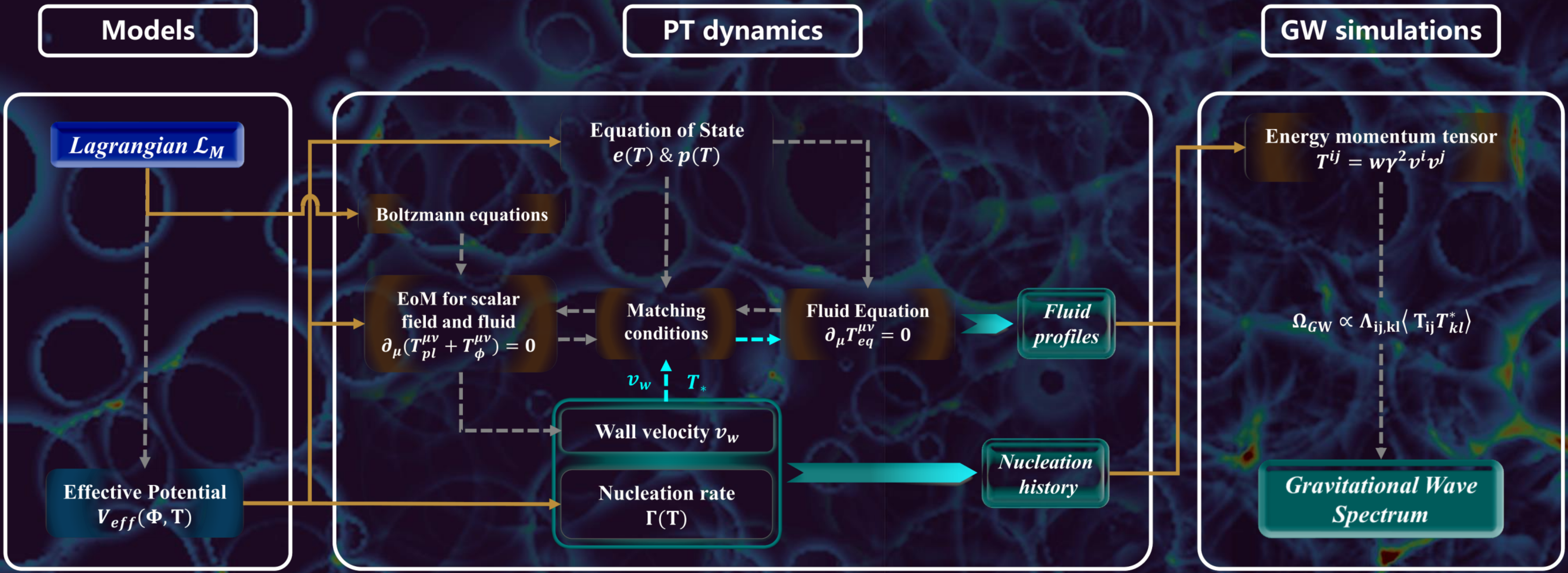


figure from Wang, Tian, Balázs to appear soon

from effective potential to hydrodynamics

effective potential	$V_{eff}(\phi_i, T)$	
↓ thermal field theory		GWCalc
equations of motion & state, $\Gamma(\mathrm{T})$	$p(T) = w\, \varrho(T)$, …	
↓ numerical methods		GWCalc
hydrodynamics	detonation modes	
↓ fast Boltzmann solver		GWCalc
fluid profiles	ϕ, T, v profiles	
↓ semi-analytical methods		GWCalc
wall velocity	v_W	

from hydrodynamics to gravitational waves

nucleation dynamics		$\Gamma(\mathrm{T}), T_*$	
↓	numerical methods		GWCalc
mean bubble separation		HR_*	
hydrodynamics		detonation modes	
fluid profiles		ϕ, v, T profiles	
↓	numerical methods		GWCalc
fluid dynamics		$\partial_\mu T^{\mu\nu} = 0$	
↓	semi-analytic/numerical methods		GWCalc
energy-momentum tensor		$T^{\mu\nu}$	
↓	hydrodynamical lattice simulation		GWCalc
GW spectrum (at T_*)		$\Omega_{GW}(f_{GW})$	

coming soon

- field dependent/thermal spectrum generator generator
  - input gauge symmetries and particle content in Mathematica
  - autogenerate effective potential
  - auto-write it into C++ function
  - auto-compile and -link it to PhaseTracer2
  - run PhaseTracer2 Mathematica to calculate gravitational wave spectrum
- PhaseTracer2
  - end-to-end, from Lagrangian to gravitational waves, calculation
  - 3d and 4d effective potentials for several models in various conventions
  - multiple bounce solving algorithms: perturbative, path deform, OptiBounce
  - nucleation temperature, thermal parameter calculation
  - gravitational wave calculator (via improved analytical fitting formulas)
  - Mathematica interface for running

coming soon

- OptiBounce
  - following Coleman action calculation turned into optimisation problem
    - CasADI optimisation algorithm
  - capable of finding the bounce solution for 500 fields (!)
  - lightning fast, extremely robust
- hydrodynamic calculator
  - equation of state from V_{eff}
  - explosion mode calculation: detonation, deflagration, hybrid mode
  - fluid profiles calculation
  - wall velocity calculation
- hydrodynamic lattice simulator
  - simplified but robust lattice simulation
  - local spherical symmetry (self-similar bubble profiles) is assumed, but
  - evolution of fluid elements is tracked in radial spatial dimension ("hybrid" scheme)
    - generating an anisotropic stress tensor based on a 'spherically symmetric' simulation

coming later: PhaseTracer3 (or 4… or 5…)

- your dream gravitational wave calculator
- fully controllable from Mathematica
- model input via symmetries and particle content
  - auto-calculation and linking of effective potential
    - auto-switching between calculation methods based on parameter values
- robust phase tracing for any effective potentials in high field dimensions
- bounce solution via OptiBounce
- thermal history via TranSolver
- explosion mode and bubble wall velocity calculation
  - auto-switching between modes based on parameter values
- gravito-hydrodynamical simulator
- the best ‘from symmetry to gravitational spectrum’ calculator in the late universe

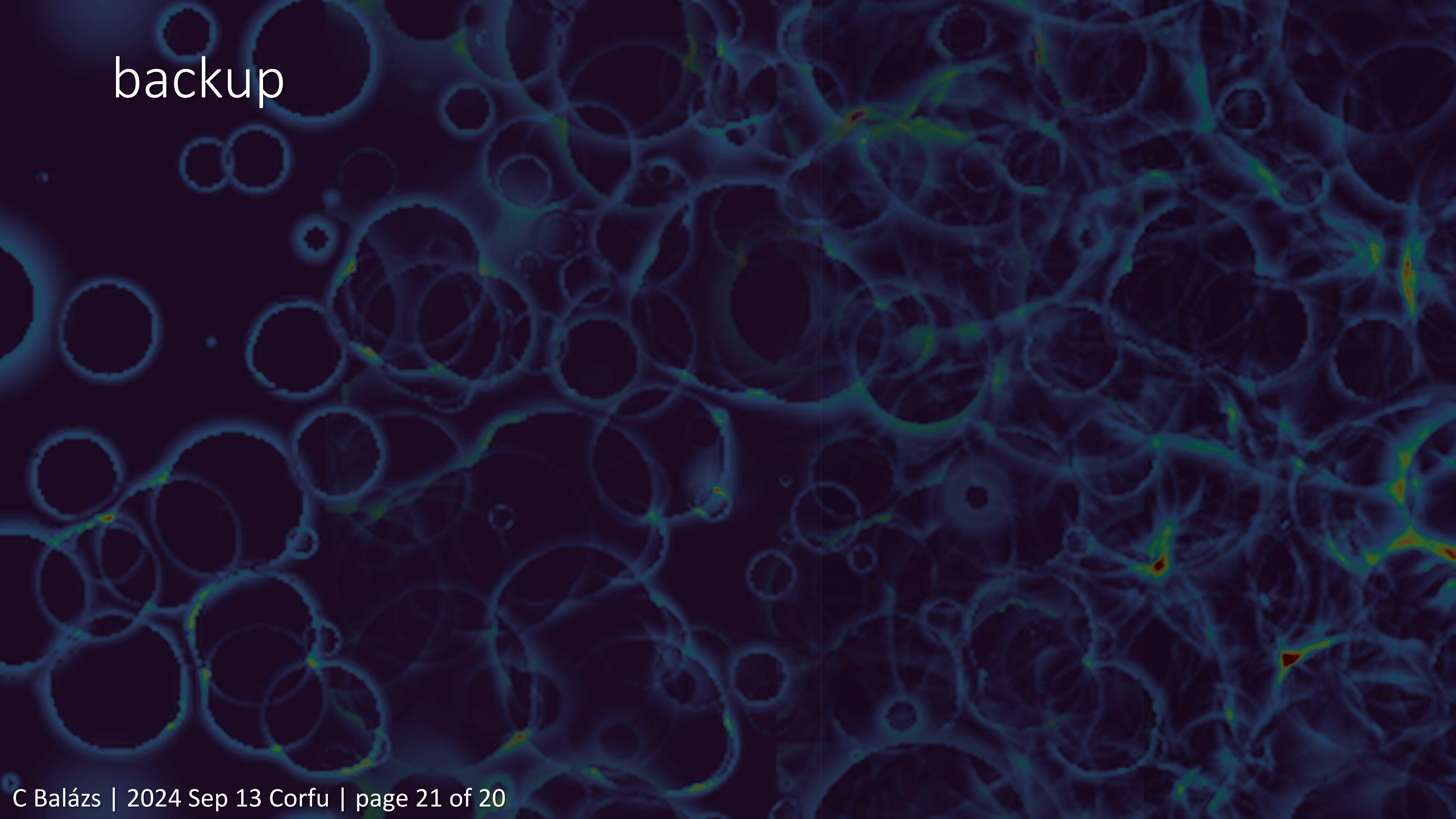
conclusion

Dying to learn if Nature is grand-unified, supersymmetric, or higher dimensional?

Wait for the gravitational background discovery.

Read (part of) the answer off the gravitational wave spectrum.

backup



from symmetry to gravitational wave spectrum and back

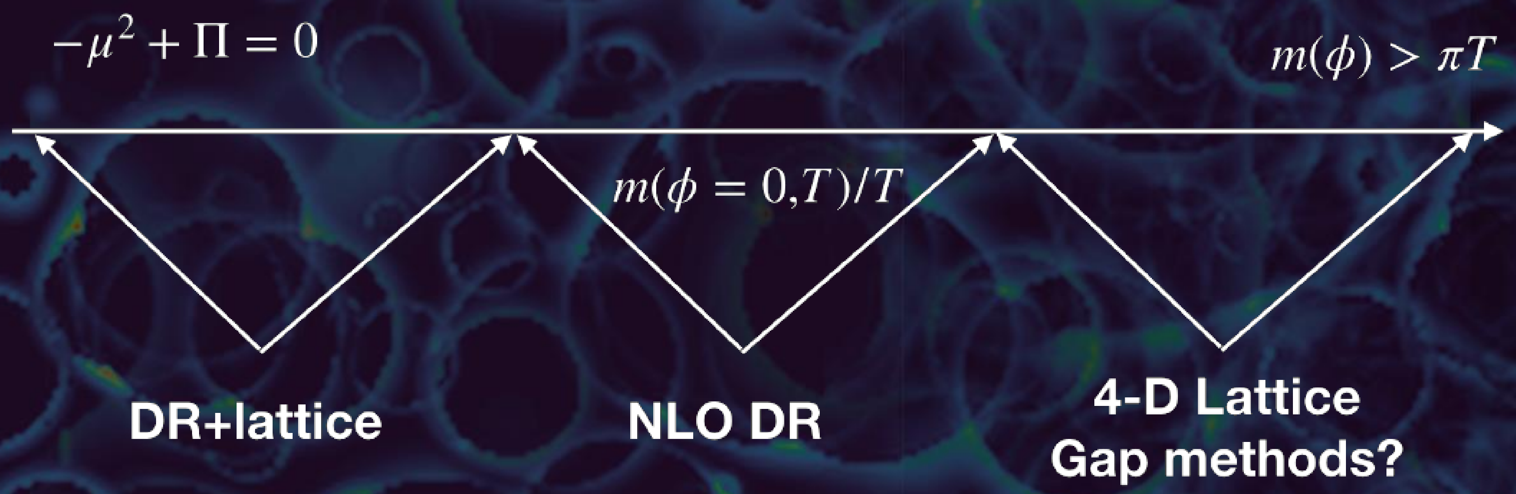


FIG. 9. Perturbation theory is expected to only be valid in a narrow window where the masses are small enough that the high-temperature regime is valid but large enough to avoid infrared divergences near the critical temperature. For lighter masses, dimensional reduction allows the numerical simulation to be more tractable. For larger masses a large 4-dimensional simulation, although perhaps gap equation techniques show some early promise.

figure from Risav Roshan and Graham White arxiv:2401.04388

from symmetry to gravitational wave spectrum and back

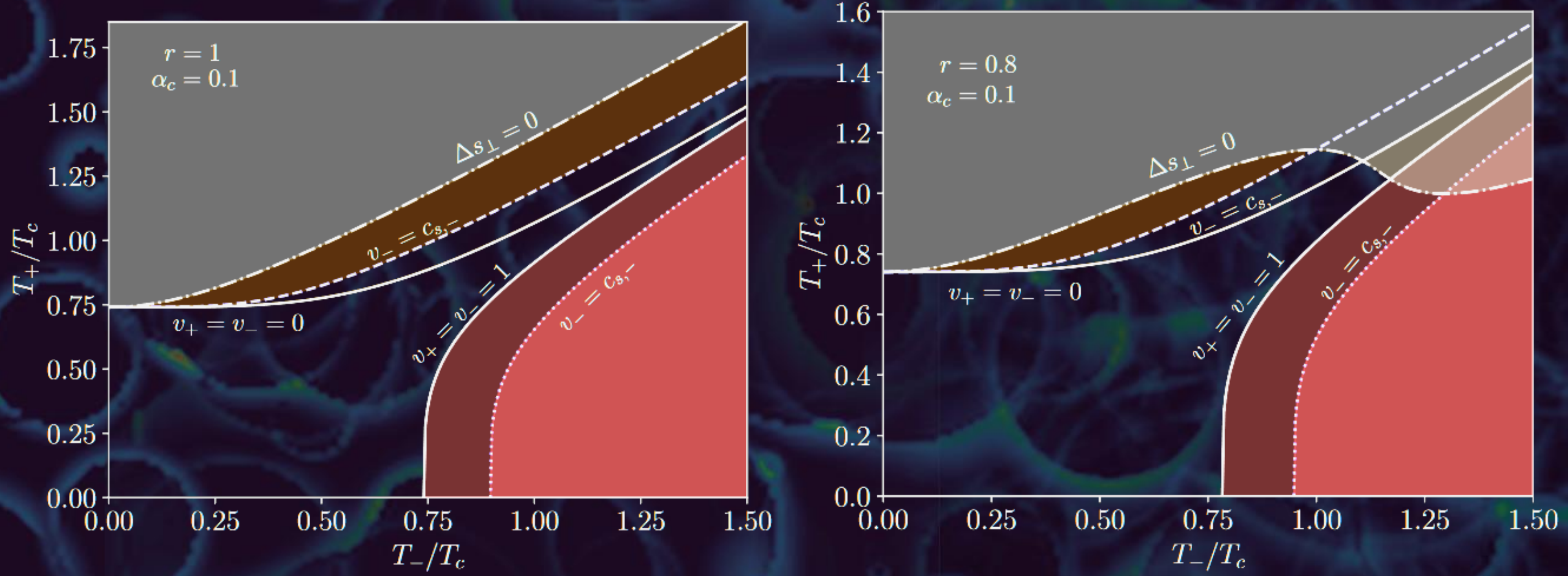


Figure 1. Plots of the curves defined by (4.7) for $\alpha_c = 0.1$ in the T_+-T_- plane of the bag model EoS. The left panel is for $r = 1$ and the right panel for $r = 0.8$. The grey shaded regions are excluded by entropy production. The region lying below the $\Delta s_\perp = 0$ line (black dashed) and above the $v_+ = v_- = 0$ line (black solid) represents deflagrations. The dark orange subregion marks strong deflagrations, and the light orange subregion marks weak deflagrations. The region lying below the $\Delta s_\perp = 0$ line and to the right of the $v_+ = v_- = 1$ line (black solid) covers detonations. The dark red subregion marks strong detonations, and the light red subregion marks weak detonations. The blue dashed and blue dotted lines are the Jouguet lines.

figure from Chi Tian, Xiao Wang, Csaba Balázs to appear soon

from symmetries to gravitational waves

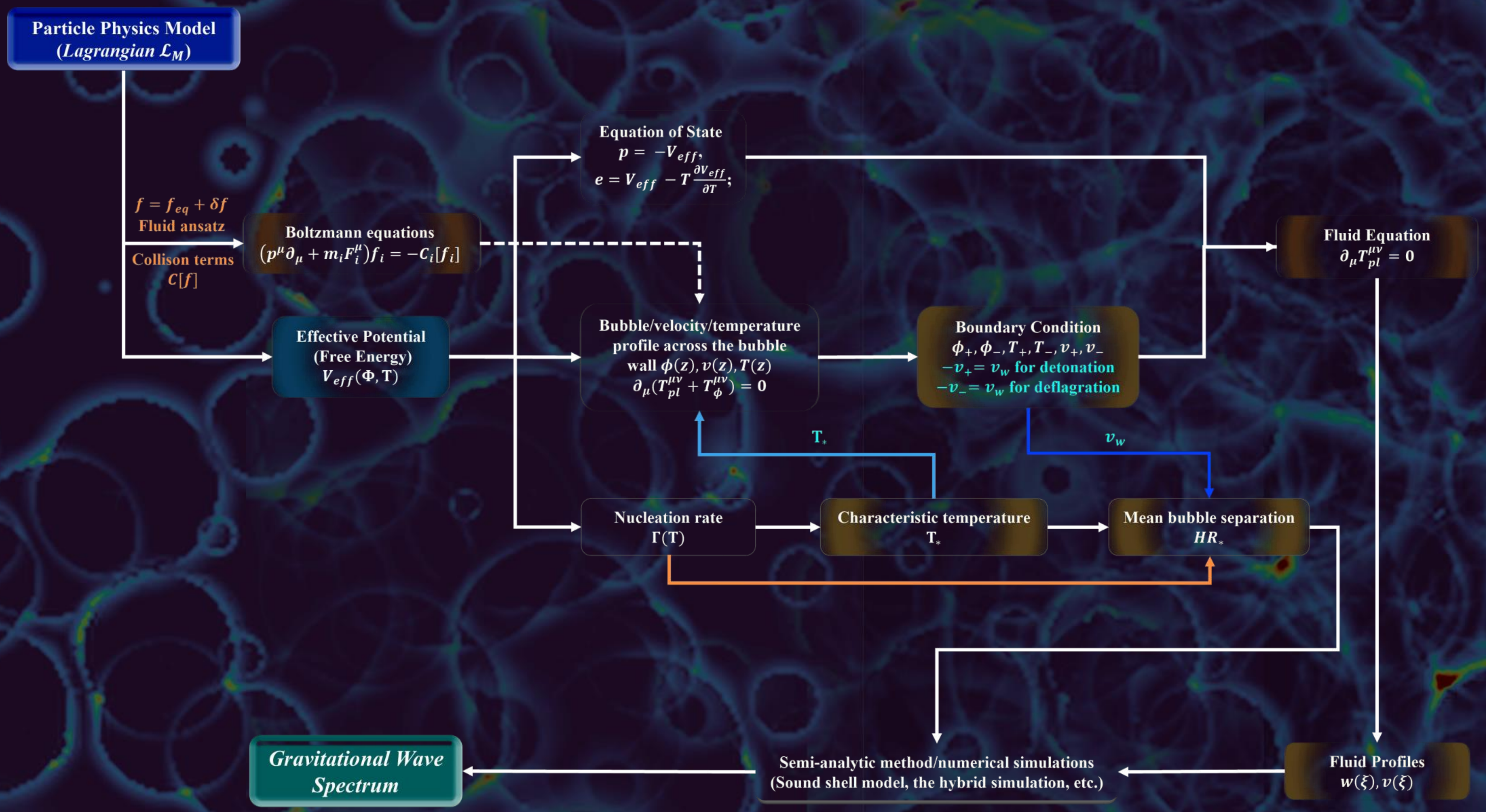


figure from Chi Tian, Xiao Wang, Csaba Balázs to appear soon

lessons learned

- nucleation doesn't imply finishing and… vice versa!
- T_n isn't the best to quantify the temperature of the PT
  - T_n decouples from the progress of the transition for strong supercooling
  - studies that rely on T_n may misclassify the completion of a transition
  - T_p may be a better reference temperature for GW production
- simple nucleation heuristics can be misleading
- from bounce action and bubble wall velocity possible to predict whether a transition completes

supercool ?

- particle physicists like strong signals
  - they tend to cleverly dial their model parameters such that it would lead to the strongest possible phase transition, that is to the highest possible gravitational wave amplitude
- strong transitions are frequently (strongly) supercooled
  - a large portion of GW predictions in the literature are given in the strongly supercooled regime
- in the (strongly) supercooled regime some of the usual assumptions break down/gets modified
  - e.g. the ordering of milestone temperatures can be ‘unusual’

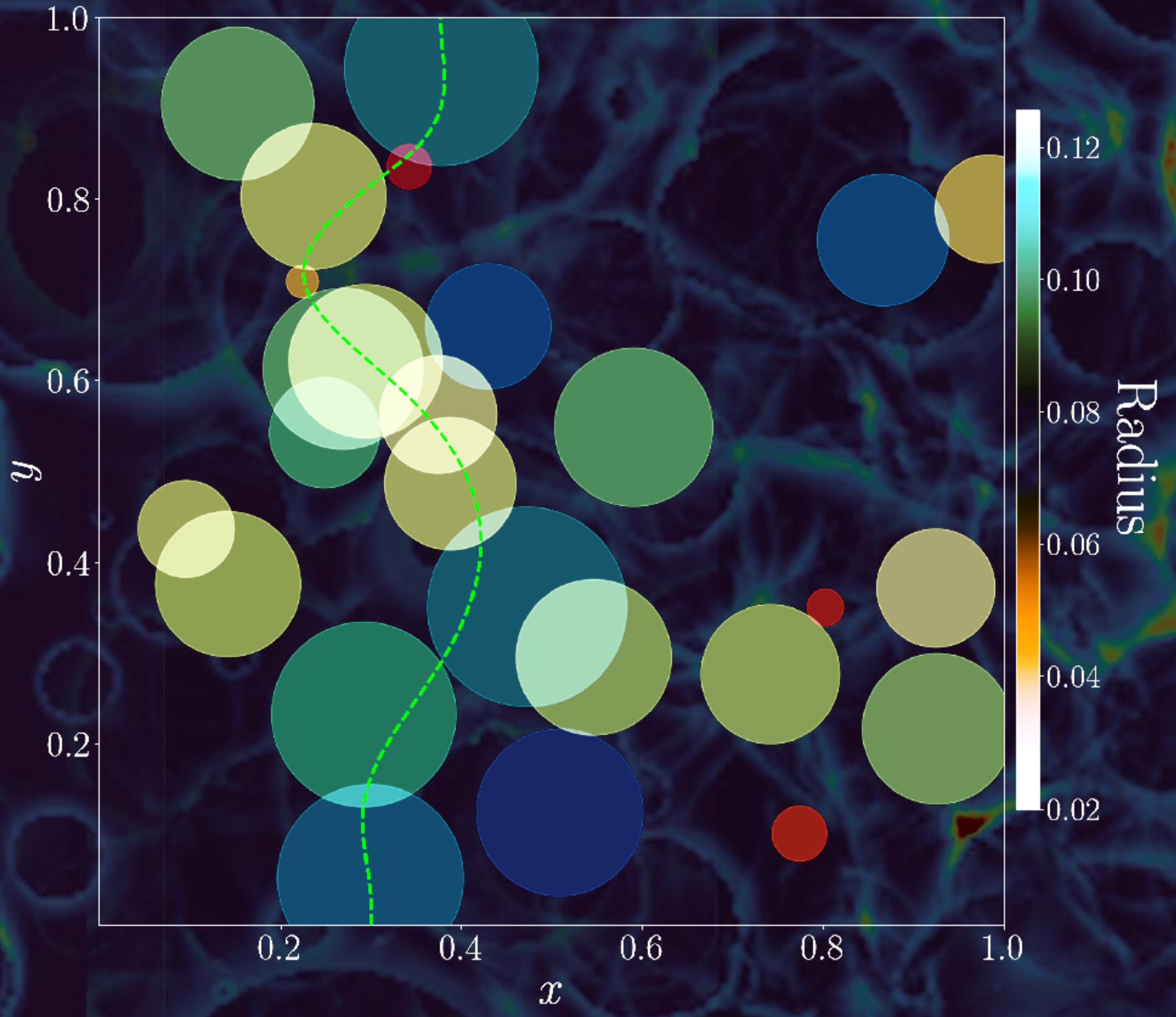
nucleation without percolation

- nucleation condition
  number of nucleated bubbles
  in Hubble volume, N, is 1

- percolation condition
  false vacuum fraction, P_f, is 71%

- nucleation without percolation
  $N(T=0) \geq 1$ & $P_f(T=0) > 0.71$

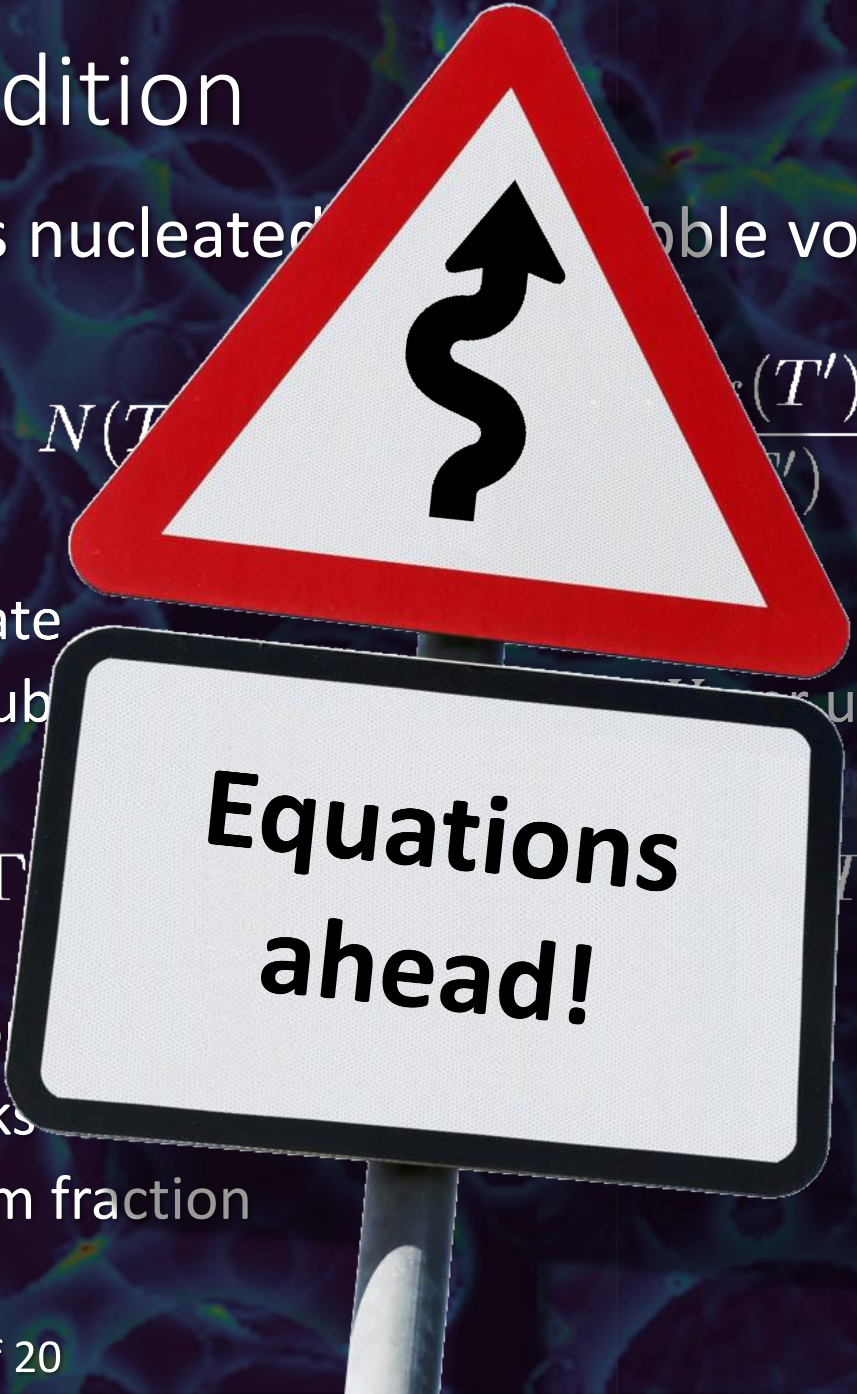
- this happens if (many) bubbles expand relatively slowly



nucleation condition

- number of bubbles nucleated ... bble volume, $V_H \sim H^{-3}$, at T

$$N(T \ldots (T') \ldots$$



- $\Gamma(T)$: nucleation rate
  number of bub ... unit t

$$\Gamma \ldots T))$$

- $S(T)$: bounce actio ...
  strongly peaks ...
- $P_f(T)$: false vacuum fraction

nucleation condition

- number of bubbles nucleated within Hubble volume, $V_H \sim H^{-3}$, at T

$$N(T) = \int_{T_c}^{T} dT' \, \frac{\Gamma(T')P_f(T')}{T' H^4(T')}$$

  - $\Gamma(T)$: nucleation rate

    number of bubbles nucleated per unit V per unit t

$$\Gamma(T) \simeq T^4 \left(\frac{S(T)}{2\pi}\right)^{\frac{3}{2}} \exp(-S(T))$$

  - $S(T)$: bounce action

    strongly peaks around $S_{\min}$
  - $P_f(T)$: false vacuum fraction

false vacuum fraction

$P_f(t) = \ldots$

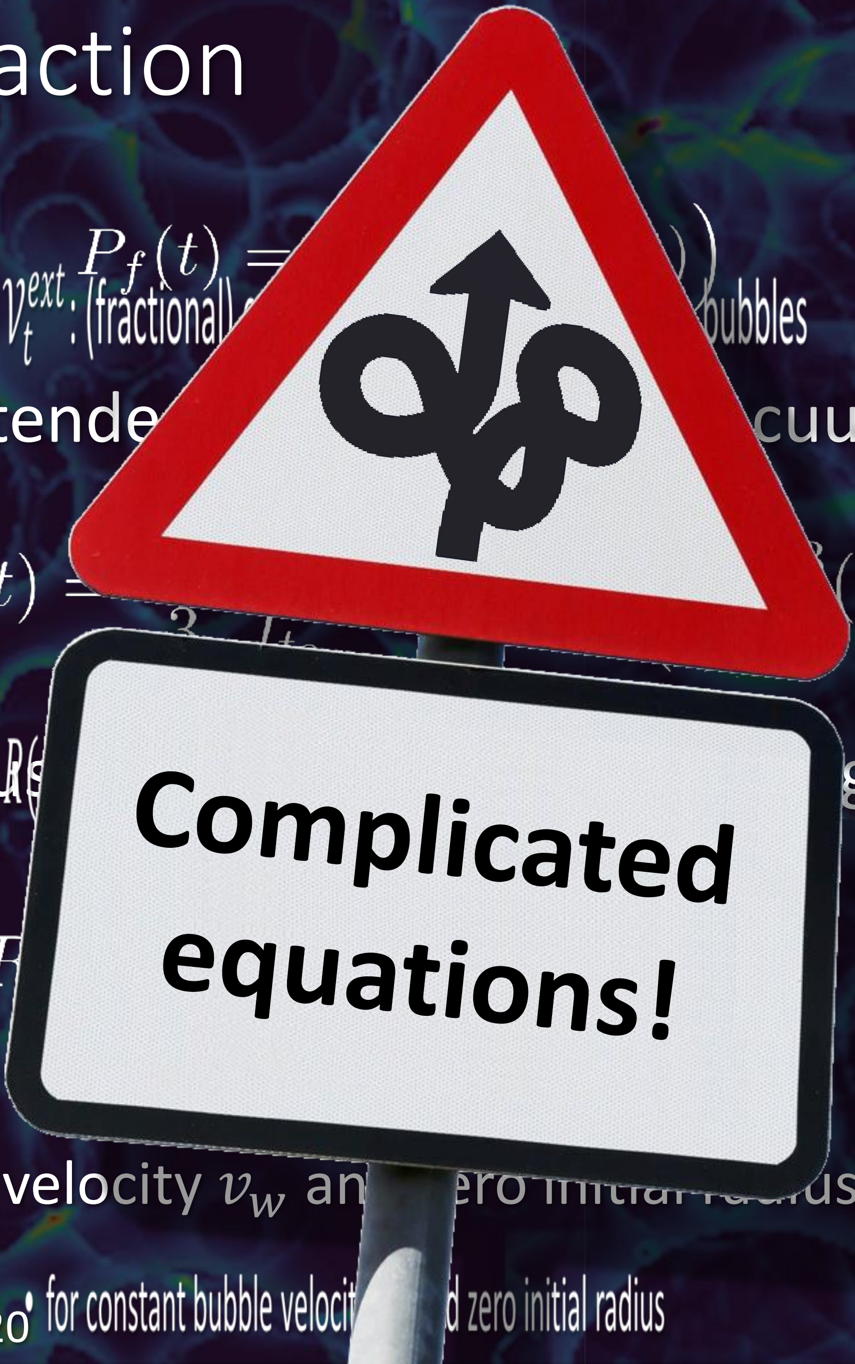
- $\mathcal{V}_t^{ext}$: (fractional) extende… …cuum bubbles

$\mathcal{V}_t^{ext}(t) = \ldots (t', t)$

- $R(t', t)$: bubble radiu… …grown until t)

$R \ldots$

  - for constant bubble velocity v_w an… …ero initial radius



false vacuum fraction

$$P_f(t) = \exp\Big(-\mathcal{V}_t^{\rm ext}(t)\Big)$$

- $\mathcal{V}_t^{\rm ext}$: (fractional) extended volume of t(rue) vacuum bubbles

$$\mathcal{V}_t^{\rm ext}(t) = \frac{4\pi}{3}\int_{t_0}^{t} dt'\,\Gamma(t')\frac{a^3(t')}{a^3(t)}R^3(t',t)$$

- $R(t',t)$: bubble radius (nucleated at time t' and grown until t)

$$R(t',t) = v_w \int_{t'}^{t} dt''\,\frac{a(t)}{a(t'')}$$

assuming constant bubble velocity v_w and zero initial radius

false vacuum fraction

- with the above assumptions

$$P_f(t) = \exp\left(-\frac{4\pi}{3} v_w^3 \int_{t_0}^{t} dt' \, \Gamma(t') \left(\int_{t'}^{t} dt'' \, \frac{a(t')}{a(t'')}\right)^3\right)$$

- assuming adiabatic expansion and converting t to T using

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -TH(T)$$

- the false vacuum fraction is

$$P_f(T) = \exp\left(-\frac{4\pi}{3} v_w^3 \int_{T}^{T_c} dT' \, \frac{\Gamma(T')}{T'^4 H(T')} \left(\int_{T}^{T'} dT'' \frac{1}{H(T'')}\right)^3\right)$$

recap

- nucleation without percolation

$$N(T=0) \geq 1 \;\&\; P_f(T=0) > 0.71$$

  - number of nucleated bubbles in Hubble volume is

$$N(T) = \int_{T_c}^{T} dT' \, \frac{\Gamma(T') P_f(T')}{T' H^4(T')}$$

  - nucleation rate

$$\Gamma(T) \simeq T^4 \left(\frac{S(T)}{2\pi} \right)^{\frac{3}{2}} \exp(-S(T))$$

  - the false vacuum fraction

$$P_f(T) = \exp\left(-\frac{4\pi}{3} v_w^3 \int_T^{T_c} dT' \, \frac{\Gamma(T')}{T'^4 H(T')} \left(\int_T^{T'} dT'' \frac{1}{H(T'')} \right)^3 \right)$$

$N(0)$ and $P_f(0)$ are related

- if no percolation then $0.71 < P_f(T) < 1$, that is $P_f(T) \sim O(1)$ and

$$N(T) \approx \int_T^{T_c} dT' \, \frac{\Gamma(T')}{T' H^4(T')} = \int_T^{T_c} dT' \, X(T')$$

- if $N(0) < 1$ then nucleation happens early, when $P_f(T) \sim 1$, and suppressed late, and the above equations holds again (more in backup)
- incidentally

$$P_f(T) = \exp\left(-\frac{4\pi}{3} \int_T^{T'} dT' \, X(T') Y^3(T', T) \right)$$

with the ratio of comoving radii of the common bubbles to r_{Hubble}

$$Y(T', T) = \frac{v_w}{T'} \int_T^{T'} dT'' \, \frac{H(T')}{H(T'')} = \frac{r(T', T)}{r_H(T')}$$

comparing $N(0)$ to $P_f(0)$

- using $X(T), Y(T', T)$, nucleation without percolation is expressed as

$$\int_0^{T_c} dT\, X(T) Y^3(T,0) < \frac{c_p^3}{N(0)} \int_0^{T_c} dT\, X(T)$$

where $c_p^3 = 3\ln\big(P_f(T_p)\big)/4\pi$

- this can be further simplified to the condition

$$Y(T_\Gamma, 0) < \frac{c_p}{N^{\frac{1}{3}}(0)}$$

where T_Γ is the temperature that maximises the nucleation rate (nearly minimizes the action)

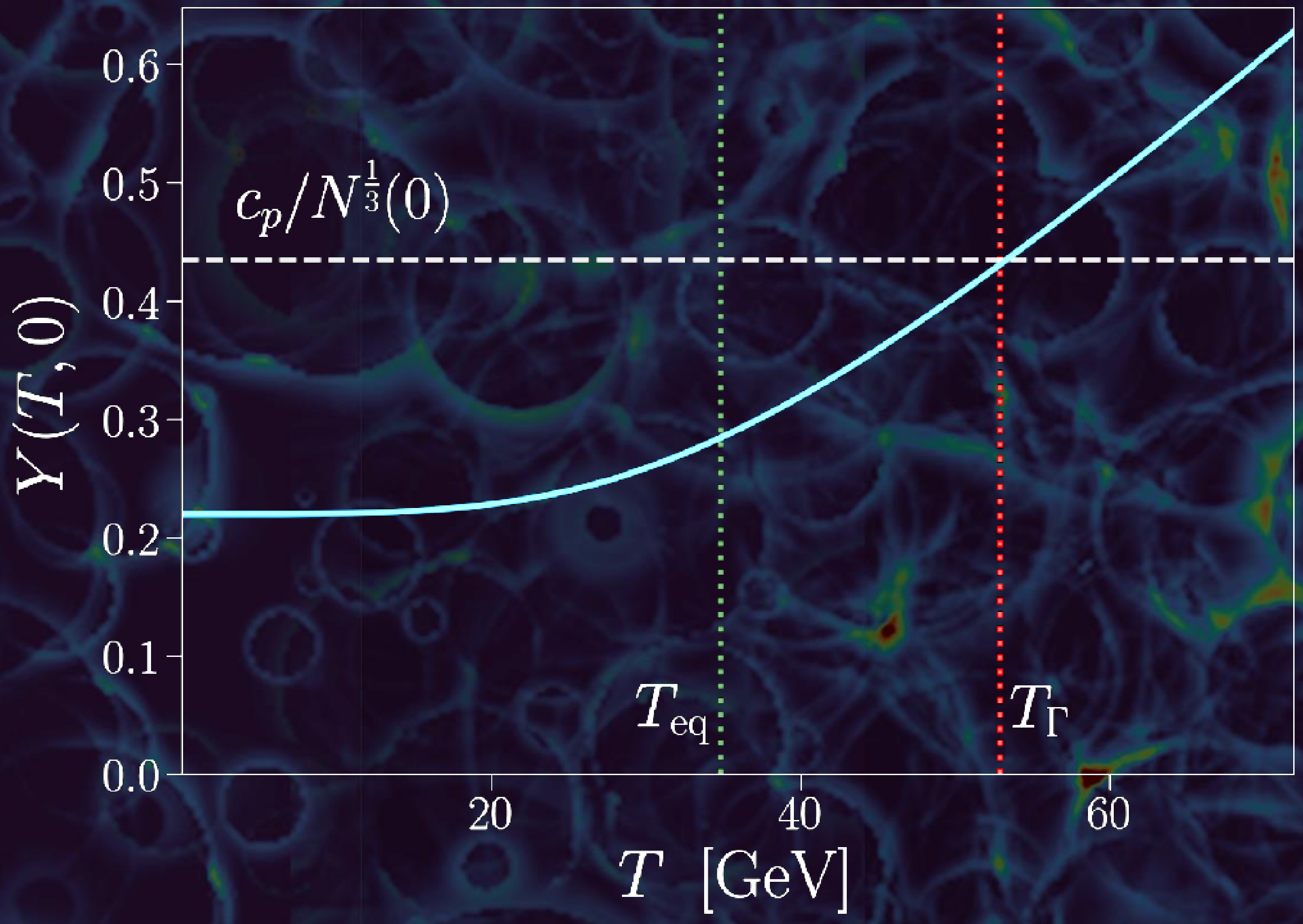
relating $N(0)$ to $P_f(0)$

- for the SM + real singlet we found parameter points where

  $Y^3(T_\Gamma, 0) < c_p^3/N(0)$

- one such benchmark is shown here

- we numerically check that $N(0) < 1$



percolation without (unit) nucleation

- there's percolation without nucleation
- but the percolation condition can be satisfied without satisfying the nucleation condition:

$$N(T = 0) < 1 \ \& \ P_f(T = 0) < 0.71 \ !$$

- this requires a few bubbles nucleating and growing for a long time

percolation without (unit) nucleation

- percolation without (unit) nucleation can be formulated via $Y(T_\Gamma, 0)$

$$Y(T_\Gamma, 0) > \frac{c_p}{N^{\frac{1}{3}}(0)}$$

- using the definition of $Y(T', T)$

$$Y(T', T) = \frac{v_w}{T'} \int_T^{T'} dT'' \, \frac{H(T')}{H(T'')}$$

and assuming a constant vacuum energy density ρ_0

$$H(T) \approx \sqrt{\frac{8\pi G}{3}\rho_0} \sqrt{\left(\frac{T_\Gamma}{T_{\rm eq}}\right)^4 + 1}$$

the T'' integral is doable

percolation without (unit) nucleation

- since $Y(T_\Gamma, 0) \sim v_w$ after doing the T'' integral percolation without (unit) nucleation can be formulated as

$$v_w > \frac{c_p}{N^{\frac{1}{3}}(0)} \left[\sqrt{\left(\frac{T_\Gamma}{T_{\rm eq}}\right)^4 + 1} \; {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\left(\frac{T_\Gamma}{T_{\rm eq}}\right)^4\right) \right]^{-1}$$

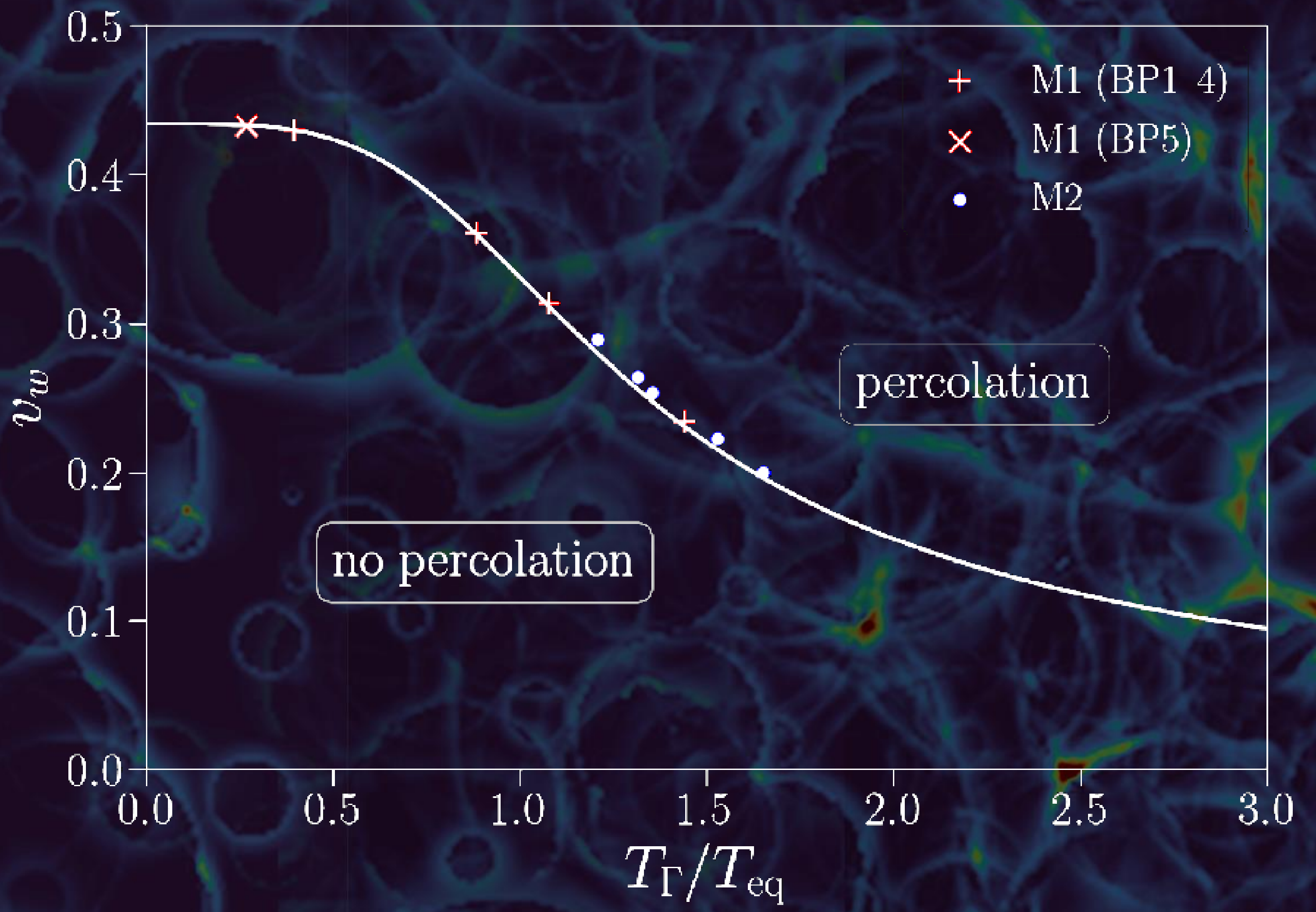
- the above can also be expressed in terms of the average (comoving) bubble radius

$$\frac{r(T_\Gamma, 0)}{r_H(T_\Gamma)} > \frac{c_p}{N^{\frac{1}{3}}(0)}$$

- if bubbles are large and/or a few bubbles expand quickly enough, then percolation without (unit) nucleation can happen

percolation without (unit) nucleation

- assuming $N(T = 0)$ slightly smaller than 1, the condition for percolation is shown by the plot
- there are benchmark points that satisfy the percolation condition for $N(T = 0)$ below 1



completion

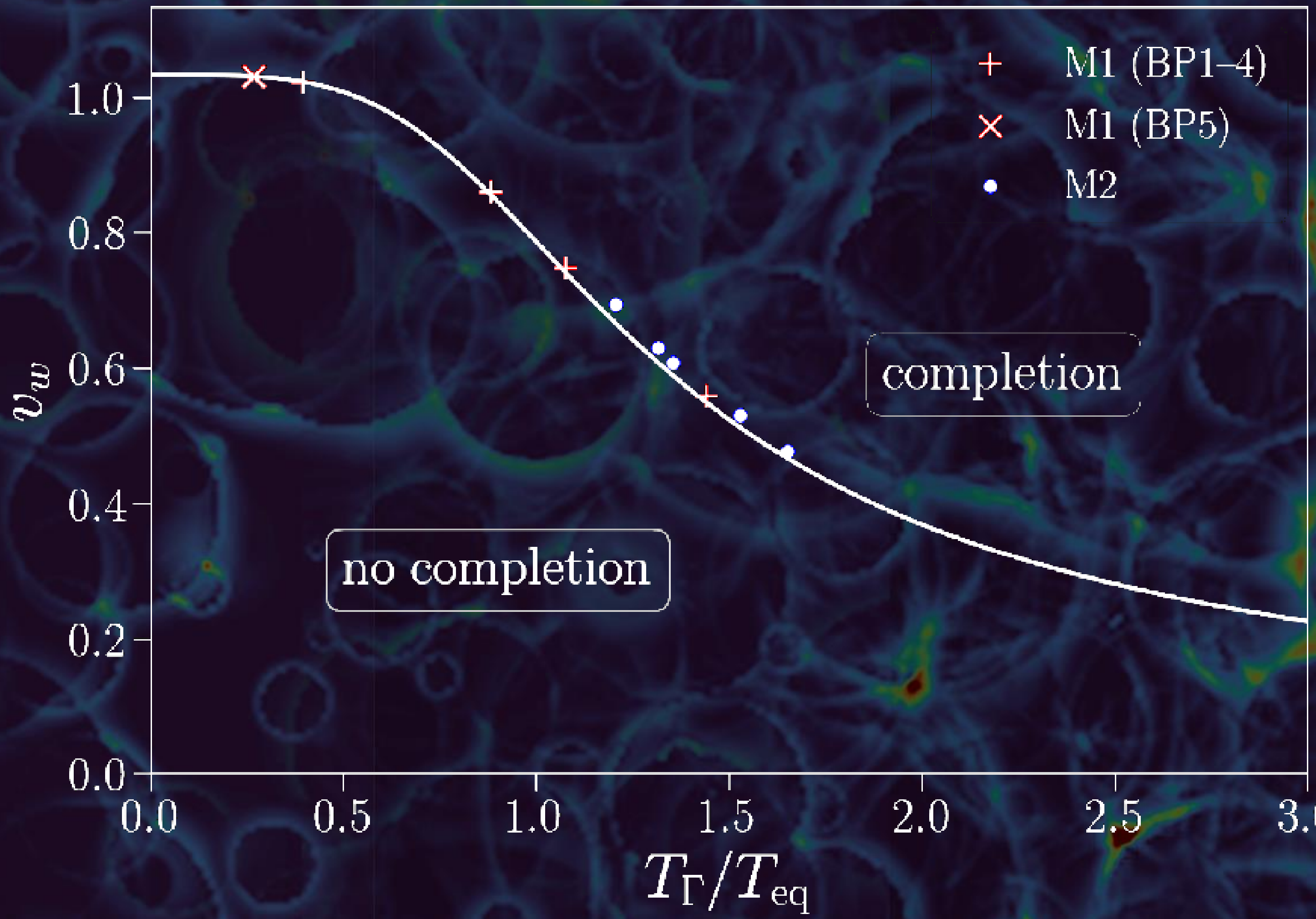
- our analysis can also be applied to completion
- we can replace the percolation condition with a simple completion condition

$$c_p \to c_f = \left(\frac{3 \ln\left(P_f(T_f)\right)}{4\pi} \right)^{1/3}$$

- for the SM + real scalar singlet we can find examples with unit nucleation without completion and completion without unit nucleation

simple completion criterion

- assuming $N(T=0)$ slightly smaller than 1, the condition for completion is shown by the plot
- The previous benchmark points satisfy the finishing condition for $N(T=0)$ below 1



more involved completion

- even if the fraction of space in the false vacuum decreases to some completion threshold

$$P_f\left(T_f\right) = \varepsilon$$

  finite pockets of the false vacuum can persist
- e.g., if the rate of false vacuum conversion does not exceed the rate of expansion of the false vacuum volume
- this raises the need for more involved completion criteria:

more involved completion criteria

when assessing completion these criteria should all be satisfied

- the false vacuum fraction becomes sufficiently small
- the phys. V of the false vacuum, $V_{phs} = a^3 \, P_f$, decreases at $T_{ref.} > T_f$
- V_{phs} decreases on average from percolation to completion
- V_{phs} decreases at T_p and at T_f

in our study we numerically found the bubble wall velocity for which each of these constraints are satisfied

applicability of the nucleation temperature

- in some cases, it's possible to satisfy the traditional percolation and finishing conditions without satisfying the nucleation condition
- this can also be demonstrated in the context of a toy model

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - (ET + A)\phi^3 + \frac{1}{4}\lambda\phi^4 - \frac{\pi^2}{90}g_* T^4$$

- introduce $\mathcal{A} = A/v$

  where v is the field value of the global minimum of V at zero T
- plot benchmark temperatures versus $\mathcal{A}$

applicability of the nucleation temperature

- for extreme values of $\mathcal{A}$ the nucleation temperature 'falls of the scale'
- the nucleation temperature "decouples from the progress of the transition"