Corfu Summer Institute - DSU24

How to define tension in Gravity

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DARK SIDE OF THE UNIVERSE

narrow : dark matter & dark energy

broad : what we suspect but have not (yet) seen

q-gravity, supersymmetry, M theory

This talk is a comment on the broad DSU, triggered by a question:

How to define the **tension** of an extended object **in gravity ?**

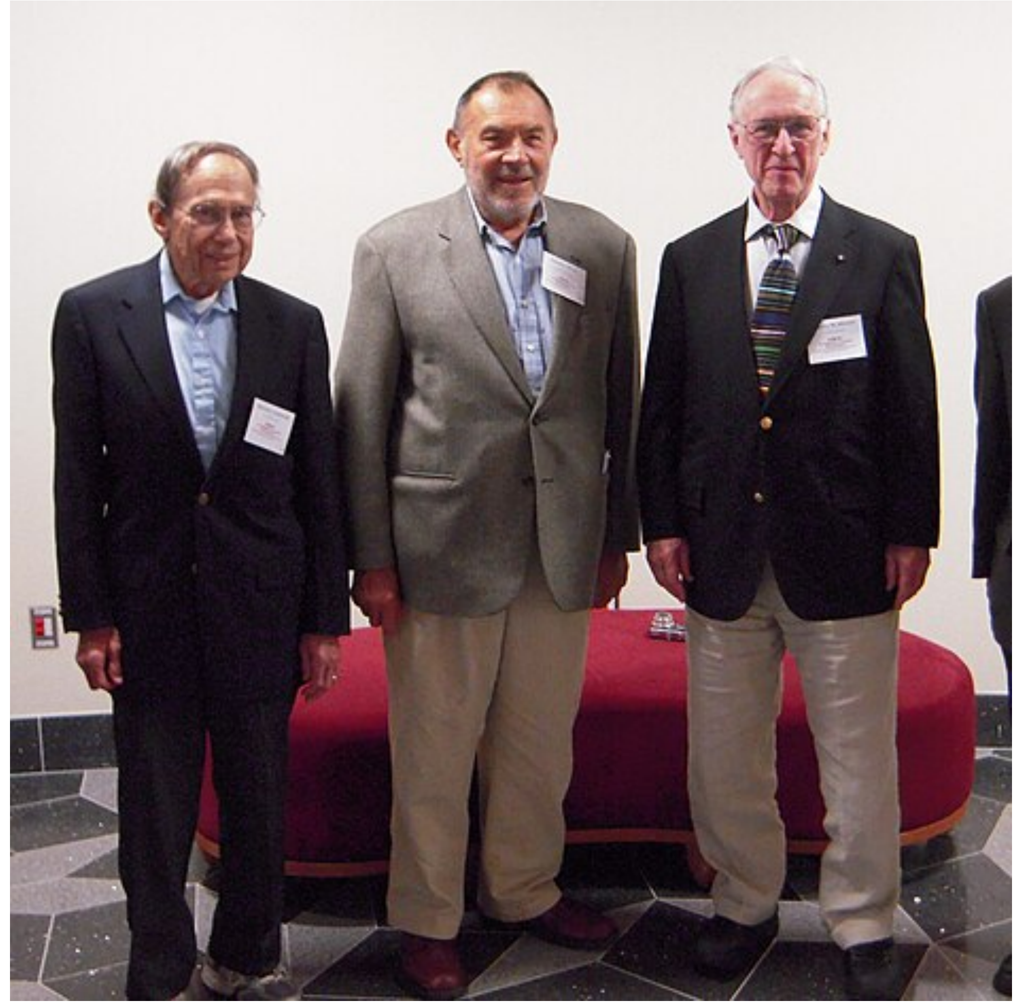
based on work with Zhongwu Chen

2404.14998 [hep-th], and ongoing

Let me first explain the question:

There is no local definition of **mass/energy** in gravity

But in asymptotically-flat spacetime mass can be given an invariant meaning in terms of the fall-off of the metric at infinity



Arnowitt, Deser, Misner '59

The definition extends to asymptotic AdS

Abbott, Deser '82;
Hawking, Horowitz '96

Holography, alias AdS/CFT:

ADM Energy in aAdS $\longleftrightarrow$ dilatation charge of dual CFT operator

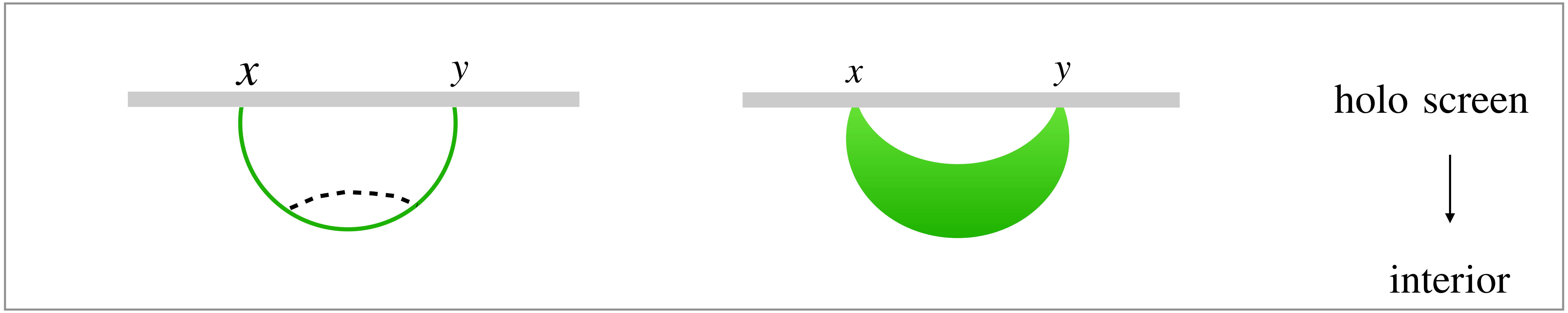
For a free scalar particle in *unit-radius* AdS4:

$$\Delta = \frac{3}{2} + \sqrt{m_0^2 + \frac{9}{4}} = m_0 \left[1 + \frac{3}{2m_0} + \frac{9}{8m_0^2} + \cdots \right] \qquad \lambda_{\text{Compton}} \ll 1$$

Taking into account interactions :

$$\left[\cdots + G_N m_0 + (G_N m_0)^2 + \cdots \right] \qquad r_{\text{Schwarzschild}} \ll 1$$

Can compute Δ from $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \dfrac{1}{|x-y|^{\Delta}}$



probe particle worldline

backreacting "banana geometry"

Given Δ can calculate microscopic entropy $S(\Delta)$ in CFT

$\Longrightarrow$ first glimpse of the UV structure of Black Holes

Strominger, Vafa '96; . . . .

Now Q-gravity is a theory of relativistic *p*-dim **extended objects**

[strings, membranes, . . .]
$p = 1$ $p = 2$

Does their **tension** σ admit a similar invariant definition ?

The bare tension is a parameter in the effective Lagrangian

$$\mathscr{L}_{\text{eff}} = \sigma_0 \, (\text{Area of worldvolume})$$

and we expect $\sigma \simeq \sigma_0$ only for a **classical probe** brane

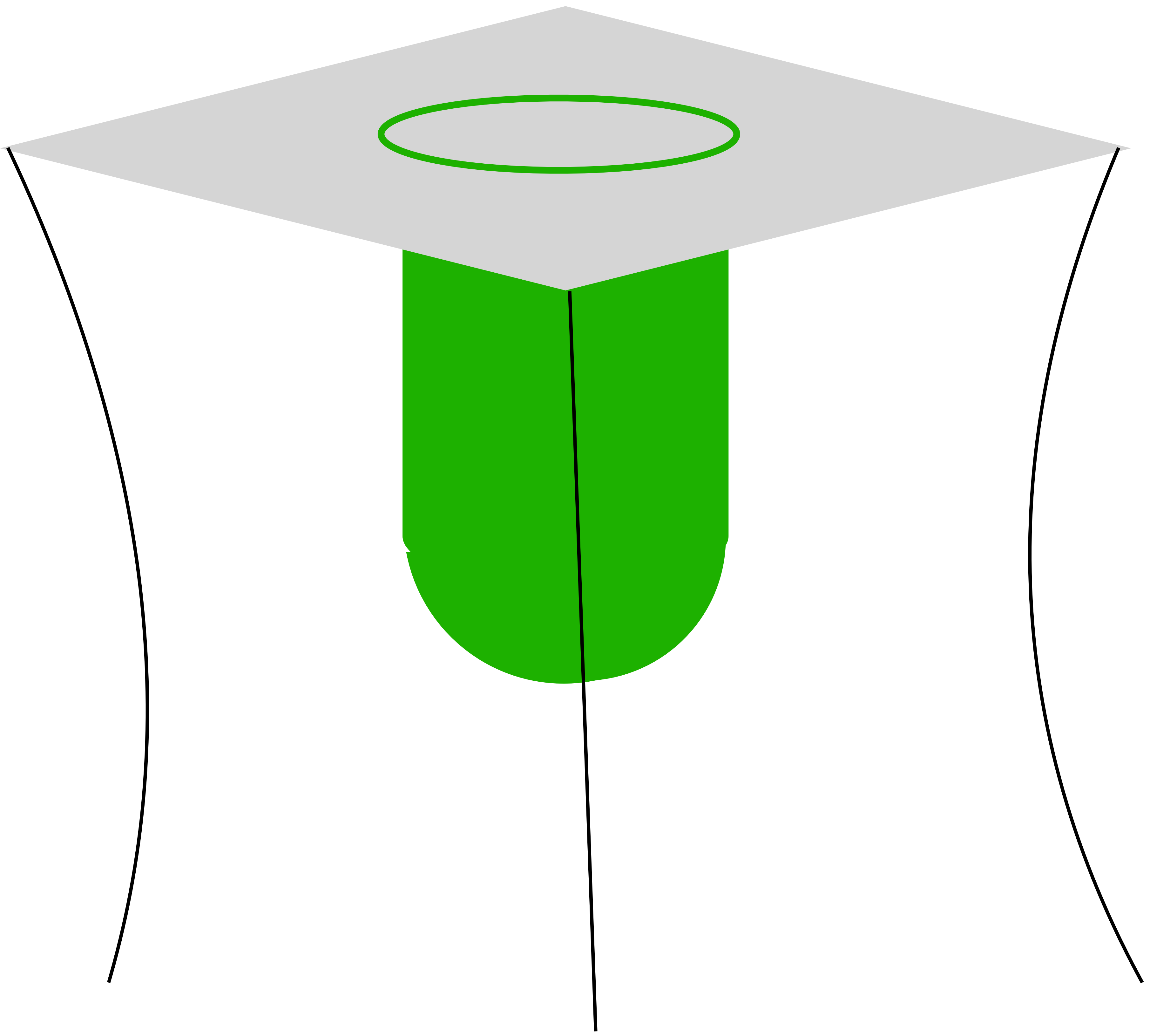
What we are after are quantum and gravitational corrections

Difficulty: for an infinitely-extended object spacetime cannot be asymptotically flat nor asymptotically AdS

Earlier works assumed a "transverse asymptotically flat" or "transverse aAdS" spacetime, and a Killing isometry along the brane. This allows an extension of the definition of ADM mass

Deser, Soldate '89; Myers '99;
Townsend, Zamaklar '01; Harmark, Obers '04;
Traschen et al '04

Holography allows to formulate the question in the modern language of **Defect Conformal Field Theory (DCFT).** This is a proxy for the asymptotic behaviour of gravity fields, but with the option to embedd it in UV complete q-gravity.



ex. of DCFT: a string worldsheet intersecting the holo screen on a defect line

New: contrary to the case of point-particles, there exist two independent definitions of invariant tension :

I. gravitational (ADM like) tension

II. stiffness (`inertial' tension)

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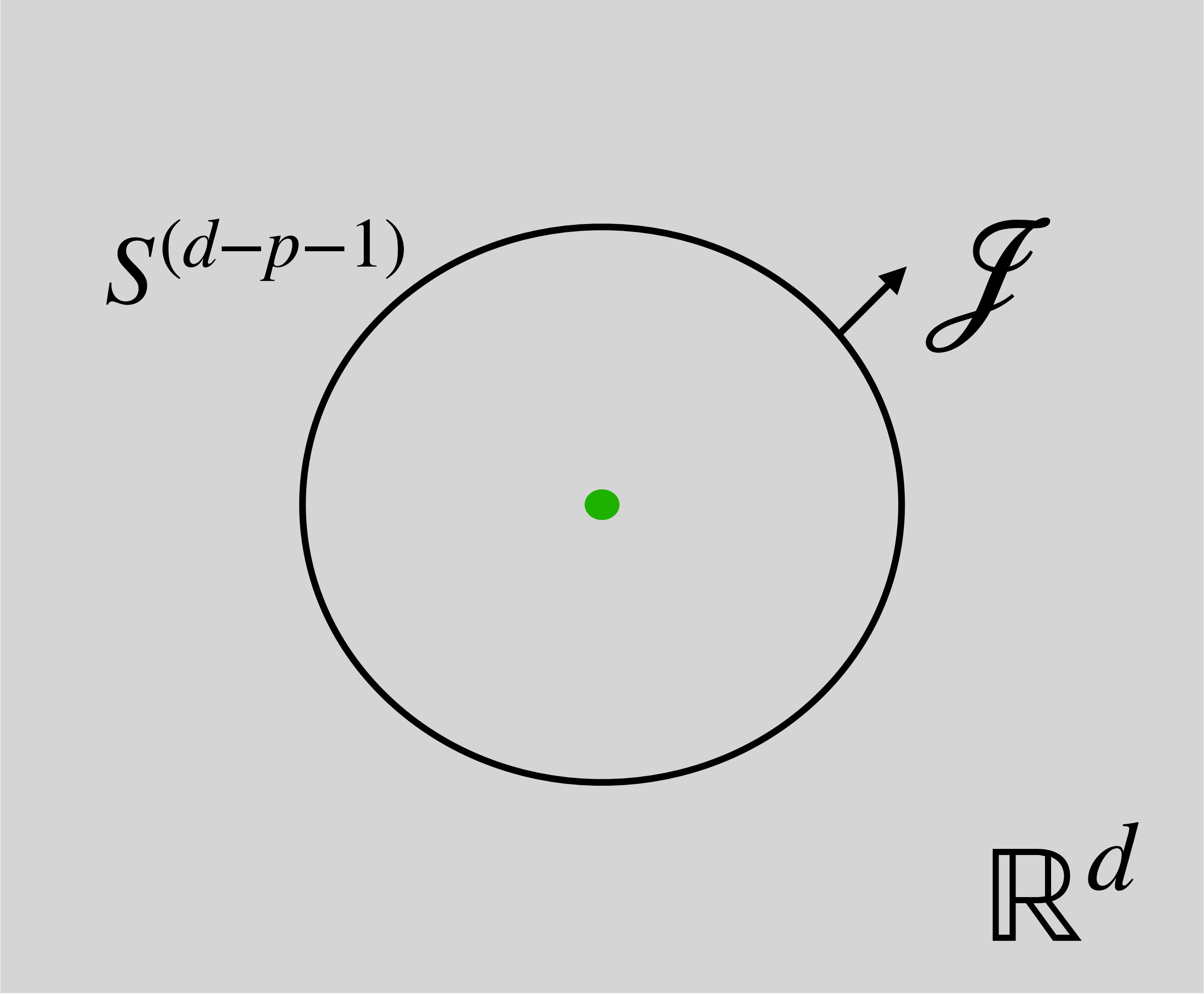
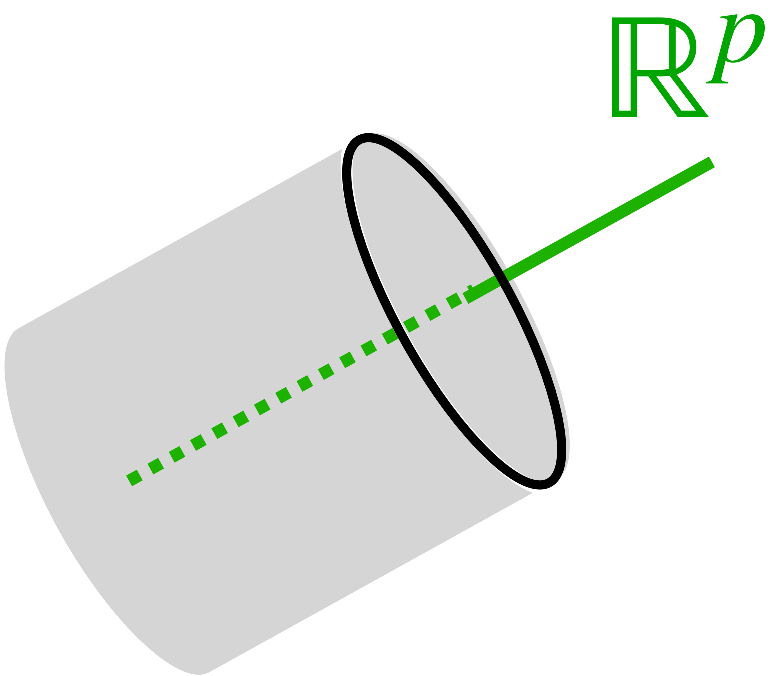
gravitational agrees with ADM mass for $p = 0$



$\partial \mathrm{AdS}_{d+1}$ = holographic screen



dilatation current

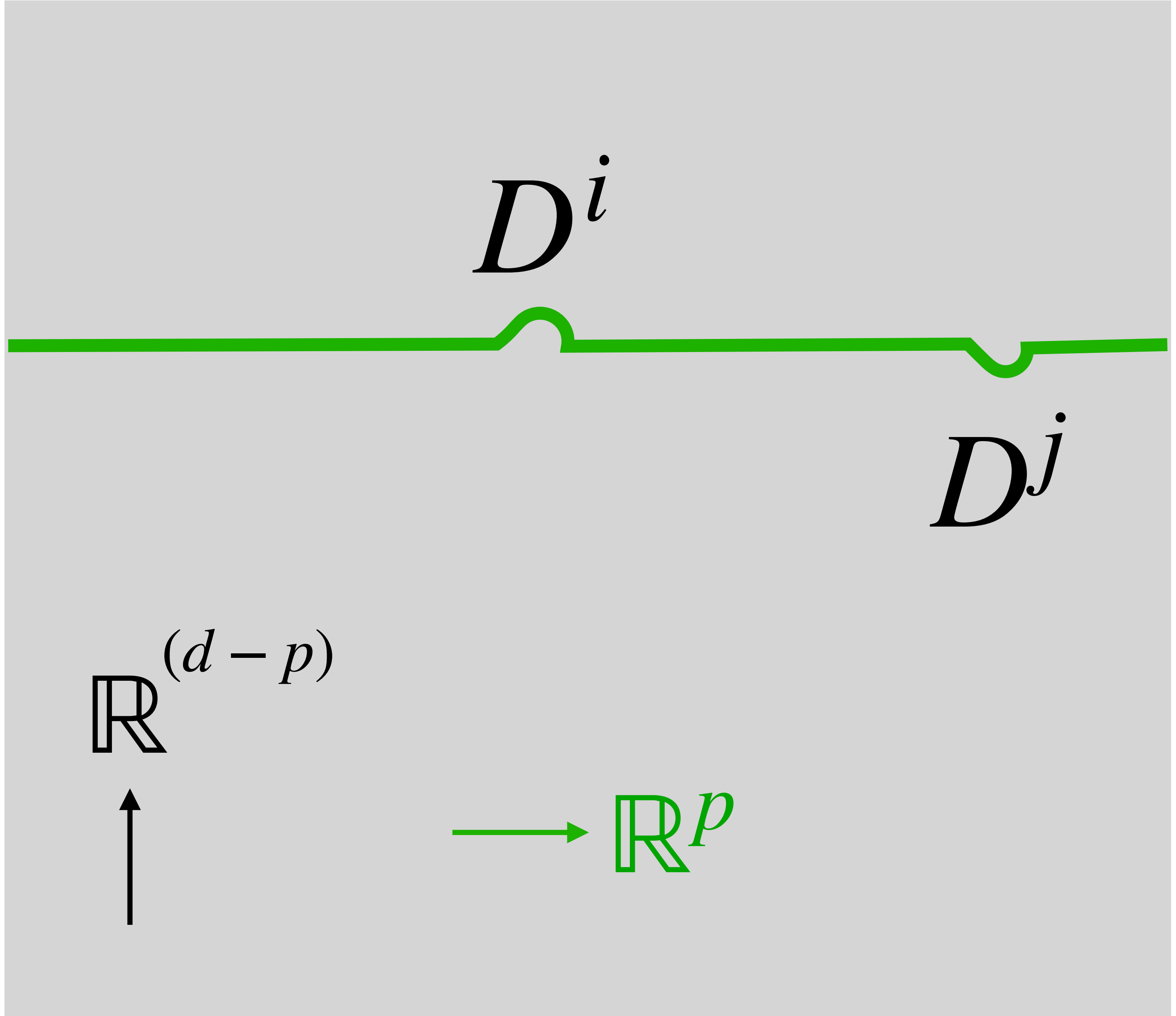
$$\sigma_{(\mathrm{gr})} := \left(\frac{d-1}{d-p-1}\right) \oint ds^j \langle \mathcal{J}_j \rangle$$

$$x^k \langle T_{kj} \rangle = a_T (\text{universal})$$

piece of DCFT data

vev of em tensor

stiffness $(p > 0)$ does not exist for point particle



piece of DCFT data

displacement norm

$$\sigma_{(\text{stiff})} := C_D \frac{\pi^{p/2}\,\Gamma(\frac{p}{2}+1)}{(p+2)\,\Gamma(p+1)}$$

$$\langle D^j(x) D^k(y)\rangle \sim \frac{C_D\,\delta^{jk}}{(x-y)^{p+1}}$$

We have fixed the prefactors in these definitions by computing the relevant Feynman-Witten diagrams in gravity and requesting that $\sigma_{\rm gr} \simeq \sigma_{\rm stiff} \simeq \sigma_0$ in the limit of a classical, elementary, probe brane

Witten '98

What makes the computation non-trivial is the absence of global Fefferman-Graham coordinates where the standard AdS/CFT dictionary is defined. Thus there is no universal AdS cutoff for both bulk and brane fields. We side-stepped this difficulty by imposing non-trivial DCFT identities at leading non-trivial order.

$$\langle TD \rangle \sim \langle T \rangle + \langle DD \rangle$$

Billo, Goncalves, Lauria, Meineri '15

Checks of the normalization:

1. Maldacena-Wilson line in d=4 SYM

$$C_D = -18a_T = 12B = \frac{6}{\pi^2}\lambda\partial_\lambda \log\langle W_\odot\rangle$$

known $\forall \quad \lambda, N$

in the limit $\lambda, N \to \infty$

$$\sigma_{\rm gr} = \sigma_{\rm stiff} \simeq \frac{\sqrt{\lambda}}{2\pi}$$

$\longleftarrow$ F-string tension ✓

2. Interfaces in d=2

$$C_D = \frac{6\sigma_0/\pi}{1+4\pi G_N\sigma_0}$$

CB, Chapman, Ge, Policastro '20

in the limit $G_N \to 0$

$$\sigma_{\rm stiff} \simeq \sigma_0$$ ✓

3. Graham-Witten anomalies for *p*=2,4 in d=4,6 ✓

All this is a little abstract, so let's focus on the first example

Note that the holographic correspondence adds/removes one dimension

gravity in AdS_{d+1}	CFT_d
particle	local operator
string	(Wilson) line
membrane	"surface operator"
etc	

Heavy external quarks, in particular, are dual to strings in AdS

Recall also that $\sigma_{\rm stiff} \propto C_D$ and $\sigma_{\rm gr} \propto a_T$

These parameters have a simple physical meaning in the gauge theory in terms of the energy radiated by an accelerating heavy quark:

$$\mathcal{E}_{\rm rad} = \frac{\pi}{6} C_D \int dt\, a^2$$

'Bremstrahlung function'

Correa, Henn, Maldacena, Sever '12;
Fiol, Garolera, Lewkowycz '12; . . .

Furthermore the energy deposited at infinity by the accelerating quark is

$$\mathcal{E}_{\infty} \propto \langle T_{\mu\nu} \rangle \propto a_T$$

$\mathscr{E}_\infty = \mathscr{E}_{\rm rad}$ requires a linear relation $C_D = -\,18\,a_T$

This is valid for supersymmetric quarks, but fails for non-susy ones

Maldacena, Lewkowycz '13

L.Bianchi, Lemos, Meineri '18

What goes wrong in the absence of susy is not totally clear, but there seems to be a problem with extracting divergent self-energy corrections from $\mathscr{E}_\infty$

Does this mean that non-susy "elementary quarks" in a theory with an exact (UV complete) gravity dual are not consistent ?

No known conundrum for other values of p, d but from known examples it was conjectured that susy Ward identities imply the linear relation

$$C_D = -a_{\rm T} \frac{2(d-1)(p+2)\Gamma(p+1)}{d\,\pi^{p-d/2}\,\Gamma(\frac{p}{2}+1)\Gamma(\frac{d-p}{2})}$$

L.Bianchi, Lemos '19

Inserting in our formulae this gives

$$\sigma_{\rm gr} = \sigma_{\rm stiff}$$

i.e. supersymmetry implies the equality of gravitational tension and stiffness

This is a peculiar BPS protection (usually $\sigma = \# \, q$)

It has the flavour of the principle of equivalence $m_{\rm gr} = m_{\rm in}$

Curiosity, or a profound fact about susy in the UV ?

Take away messages:

There exist two invariant definitions of the tension of extended objects in AdS gravity

Supersymmetry equates them, and may be needed in the deep UV to avoid a contradiction

Thank you for your attention