# Entanglement harvesting in superposed Minkowski spacetime[1]

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# Understanding Quantum Gravity using operational approach

An operational approach is one that grounds some phenomena in measurements of physical observable using tools such as detectors, rods, and clocks → "Bottom up" approach

This work aims to use such approach by studying quantum information structure of space-time specifically encoded by a superposed Minkowski spacetime.

Specifically, study of entanglement generation in superposed Minkowski spacetime

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# Entanglement harvesting

- Take two **uncorrelated** particle detectors and allow them to *locally interact* with a free quantum field.

- After some time, these two detectors will *become entangled*, even if they remain space-like separated.

- This can be attributed to the entanglement existing in the quantum vacuum. Entanglement vanishes if the separation between the detectors and energy gap of the detectors is very large.

- How does global structure of spacetime[2] affect entanglement harvesting phenomena?

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- Gravity is inherently a theory of Geometry

- One uniqueness of QM is superposition principle

- Geometry + Superposition

$\rightarrow$ Superposition of "semi-classical" space-time states - respective amplitudes not related by global coordinate transformation.

**Disclaimer!!** Not a full theory of QG! But assume that this is a valid solution within an anticipated theory .

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# Reference


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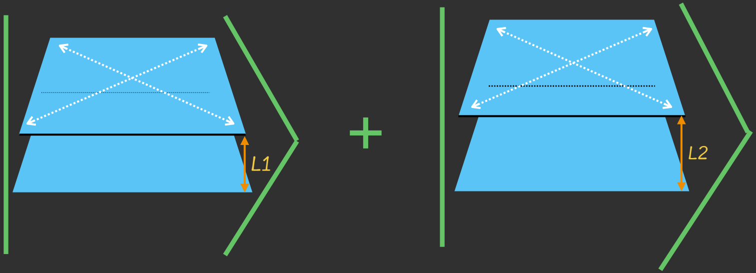
# Plan of Talk

# Setting the background geometry

# Quotient Minkowski spacetime $\mathcal{M}_0 \sim \mathcal{M}/J_0$

$$\mathcal{M}_0 \simeq \mathcal{M}/J_0$$
$$J_0^{L_1} : (t,x,y,z) \to (t,x,y,z+L_1)$$
$$J_0^{L_2} : (t,x,y,z) \to (t,x,y,z+L_2)$$

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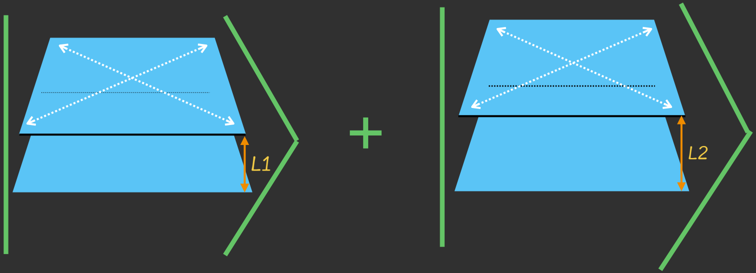


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# Quantum fields on $\mathcal{M}_0$

$$\hat{\Phi}(x) = \frac{1}{\sqrt{\mathcal{N}}} \sum_n \gamma^n \hat{\phi}(J_0^n x)$$

$\mathcal{N} = \sum_n \gamma^{2n}$, $\quad [\hat{\Phi}(x), \dot{\hat{\Phi}}(x')] = \delta(x - x') +$ image terms

$\gamma = \pm 1$ denoting untwisted and twisted field.

Vacuum [3]: Same as Minkowski vacuum $|0\rangle_F$

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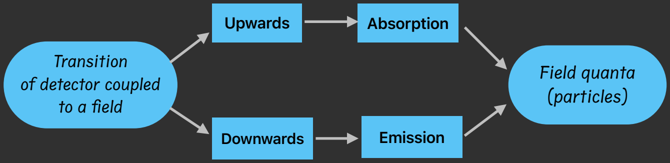
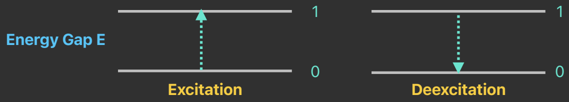
# Quantum Detector

- Model systems that couples to quantum fields
- Operational approach to probe quantum fields
- Example: Atom-Field interaction




Unruh deWitt Detector



# Some Calculational Details

# Calculation

Hilbert space: $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_\Phi \otimes \mathcal{H}_S$

$$\Downarrow$$

Initial state: $|\psi_i\rangle = |0,0\rangle \otimes |0\rangle_F \otimes |+\rangle$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|L_1\rangle \pm |L_2\rangle)$$

Interaction Hamiltonian:

$$H_D^{int}(\tau_D) = \lambda\chi_D(\tau_D) \underbrace{\left(\sigma_+(\tau_D) + \sigma_-(\tau_D)\right)}_{\text{SU(2) ladder operator}} \otimes \underbrace{\sum_{i=1,2} \hat{\Phi}[x_D^i(\tau_D)]}_{\text{Quantum Field}} \otimes \underbrace{|L_i\rangle\langle L_i|}_{\text{Spacetime}}$$

where $\sigma_+(\tau_D) = e^{i\,\Omega_D \tau_D}\,|1\rangle\langle 0|, \quad \sigma_-(\tau_D) = e^{-i\,\Omega_D \tau_D}\,|0\rangle\langle 1|,$
$\Omega_D$=Energy gap of detector

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Unitary evolution:

$$U = \hat{\mathcal{T}}\left[\exp\left\{-\mathrm{i}\int dt\,\left(\frac{d\tau_A}{dt}\right) H_A(\tau_A(t))\otimes\mathbf{I} + \mathbf{I}\otimes\left(\frac{d\tau_B}{dt}\right) H_B(\tau_B(t))\right\}\right]$$

Final state:

$$|\psi_f\rangle = \sum_n \lambda^n |\psi_f^{(n)}\rangle = \sum_n U_n|\psi_i\rangle$$

Joint density operator of the detectors:

$$\rho_{AB}^{\pm} = \mathrm{Tr}_{\Phi}[\langle\pm|\psi_f\rangle\langle\psi_f|\pm\rangle]$$

$$= \begin{pmatrix} 1 - P_{\pm}^{E,A} - P_{\pm}^{E,B} + E_{\pm} & 0 & 0 & X_{\pm} \\ 0 & P_{\pm}^{E,B} - E_{\pm} & C_{\pm} & 0 \\ 0 & C_{\pm}^{*} & P_{\pm}^{E,A} - E_{\pm} & 0 \\ X_{\pm}^{*} & 0 & 0 & E_{\pm} \end{pmatrix} \sim \text{‘X’ state}$$

$$E_{\pm} = P_{\pm}^{E,A} P_{\pm}^{E,B} + |C_{\pm}|^2 + |X_{\pm}|^2 \sim \vartheta(\lambda^4)$$

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# Transition Probability

$$P_{\pm}^{E,D} = \frac{1}{4}[P_D^{L_1} + P_D^{L_2} \pm 2P_D^{L_1 L_2}]$$

$$P_D^{L_i} = \int dt\, dt' \eta_D(t)\eta_D(t')\, \mathrm{e}^{-\mathrm{i}\,\Omega_D(\tau_D(t)-\tau_D(t'))}\; W^{L_i}\Big(x_D(t), x_D(t')\Big)$$

where, $\eta_D(t) = \mathrm{e}^{-\frac{t^2}{2\sigma^2}}$

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$$W^L(x(t), x(t')) = \Big\langle 0 \Big| \hat{\Phi}^L(x)\, \hat{\Phi}^L(x') \Big| 0 \Big\rangle$$

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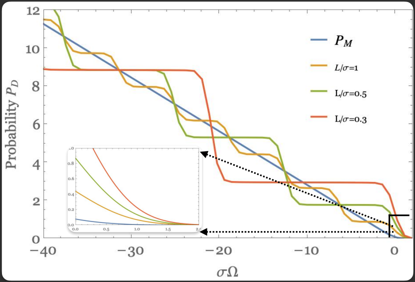
$$P_D^{L_1 L_2} = \frac{1}{\mathcal{N}} \sum_{L_1 n = L_2 m} P_D^M +$$

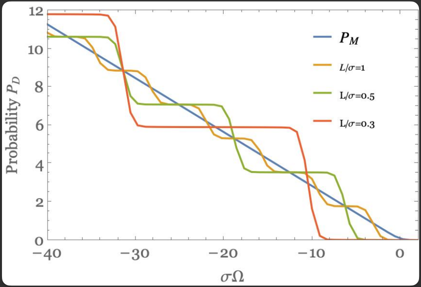
$$\frac{\sigma}{2\sqrt{\pi}\mathcal{N}} \sum_{L_1 n \neq L_2 m} \frac{\mathrm{e}^{-(L_1 n - L_2 m)^2/4\sigma^2}}{(L_1 n - L_2 m)} \left[ \mathrm{Im}\left( \mathrm{e}^{\mathrm{i}(L_1 n - L_2 m)\Omega_D} \,\mathrm{erf}\left( \frac{\mathrm{i}(L_1 n - L_2 m)}{2\sigma} + \sigma\Omega_D \right) \right) \right.$$

$$\left. - \sin(\Omega_D (L_1 n - L_2 m)) \right]$$

# Results

# Comparison of transition probabilities for different characteristic lengths

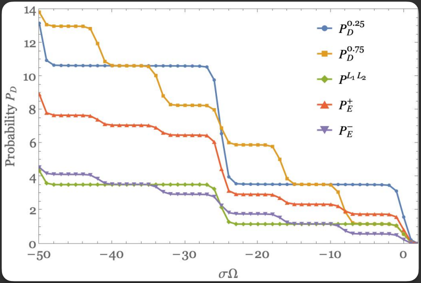


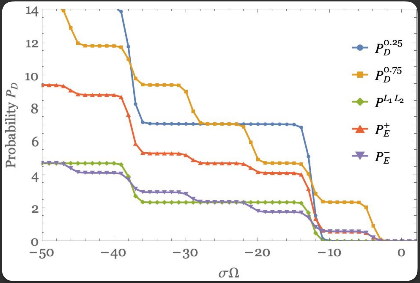

Untwisted Field

Twisted Field

# Comparison of transition probabilities for single and superposed cylindrical space




Untwisted Field




Twisted Field

# Measure of entanglement

Peres-Horodecki criterion: *Necessary condition, for the joint density matrix of two quantum mechanical systems to be separable*

$$\mathbf{N} = \frac{\|\rho^{\Gamma_A}\| - 1}{2} = \sum |\text{Negative eigenvalues of } \rho^{\Gamma_A}|$$

Conditions to produce entanglement:

$$|X| > P^{D,E} \quad , \qquad |C| > \sqrt{E}$$

$E = P_A P_B + |C|^2 + |X|^2 \rightarrow$ Second condition is never satisfied

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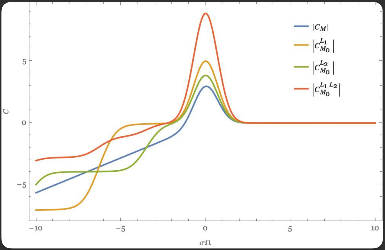
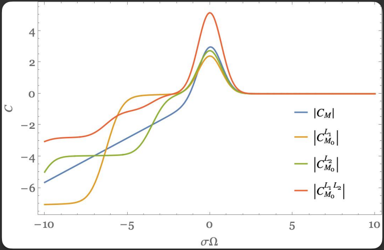
# Comparison of Concurrence $\mathcal{C} = 2\mathbf{N}$ for $\frac{a}{\sigma} = 0.09$, $\frac{L_1}{\sigma} = 0.5$, $\frac{L_2}{\sigma} = 0.9$




**Untwisted Field**

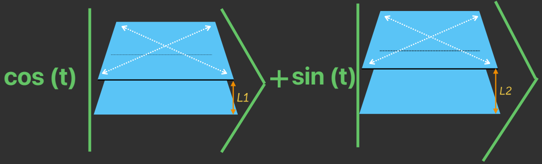



**Twisted Field**

$a = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2}$, For simplicity we consider $a = z_A - z_B$.

# Other ongoing works

- Arbitrary superposition of two spacetime states and measure of joint density matrix in arbitrary control state




- Consider trajectory of the detectors to be also in superposed state

- Consider field vacuum is also quantum controlled $\leftrightarrow$ Gravitational and matter degrees of freedom are coupled.

- Future direction : Connection to Noncommutative geometry?

**Acknowledgement:** *'The Quantum Information Structure of Spacetime' Project (QISS), by grant 63132 from the John Templeton Foundation*

*Thanks For Your Attention!*

*Question ?*