SO(10) : a Necessary Step for Physics Beyond the Standard Model

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Summary

1) THE PROGRESS OF PHYSICS HAS BEEN ACHIEVED IN THE PAST BY EXPLAINING DIFFERENT PHENOMENA WITHIN A SET OF LAWS WITH A MATHEMATICAL ELEGANCE ,

2) THE GRAVITATIONAL THEORY BY NEWTON, WHICH ACCOUNTS FOR GRAVITY AND KEPLERO LAWS, MAXWELL EQUATIONS FOR THE ELECTROMAGNETIC INTERACTIONS, SPECIAL AND GENERAL RELATIVITY, QUANTUM MECHANICS STANDARD MODEL FOR QUANTUM ELECTRODYNAMICS, WEAK AND STRONG INTERACTIONS .

3) THE QUANTUM NUMBERS OF A FAMILY OF FERMIONS ARE A STRONG HINT FOR UNIFICATION .

4) SO(10) PROVIDES THE SCENARIO FOR NEUTRINO OSCILLATIONS AND LEPTOGENESIS .

5) CONCLUSION

Gravity and Keplero's Laws

The experimental discovery by Galileo of gravity and the laws by Keplero, which describe the motion of the planets around the son, are accounted by the Newton theory of gravitation based on the formula, which gives the actraction of tho masses :

$$F = \frac{G m_1 m_2}{r^2}$$

Newton, who proposed the second law of the dynamics,

$$F = ma$$

invented the differential calculus to deduce the consequences of that equation .

The Maxwell Equations

The electromagnetic phenomena are described by the very elegant Maxwell equations :

$$div(E) = \frac{\rho}{\epsilon_0}$$

$$div(H) = 0$$

$$rot(E) = -\frac{1}{c}\frac{\delta H}{\delta t}$$

$$rot(H) = \frac{1}{c}\frac{\delta E}{\delta t} + \mu_0 \rho v$$

which allowed him to understand that the light is an electromagnetic wave, which in the vacuum propagates with velocity c.

From Lorentz Transformation to Special Relativity

The invariance of Maxwell equations with respect to Lorentz transformations, for which the combination

$$x^2 + y^2 + z^2 - c^2t^2$$

does not change for systems of reference with a relative constant velocity, lead Einstein to propose the special relativity, where the time is not absolute .

By taking advantage of the studies of Levi-Civita and Ricci Curbastro , Einstein invented a relativistic invariant theory of gravitation, experimentally confirmed by the precession of the perihelium of Mercury and by the deviation of the direction of the light passing in the proximity of the son.

Quantum Mechanics

To explain the Planck formula for the black body, the photoelectric effect and the Bohr description of the discrete energies of the hidrogen atom, quantum mechanics has been formulated with two different formulations, the mechanics of the matrices by Heisenberg with the famous indetermination principle and the Schrodinger equation : in both cases the Planck constant h plays fundamental role . It appears in the commutator :

$$[q, p] = \frac{h}{2\pi}$$

and in

$$E = i\frac{h}{2\pi}\frac{\delta}{\delta t}$$

and in

$$p = -i\frac{h}{2\pi}\Delta$$

Quantum Mechanics

With the duality wave-corpuscole at difference from classical mechanics where particles are corpuscoles and the electromagnetic field is a wave, quantum mechanics implies that both these physical objects have both properties, which show up in different experiments : photons describe the particle behaviour of the electromagnetic field, while particles show a wave behaviour in the diffusion on a diffractive lattice, where the distances are of the order of magnitude of the wavelenght associated to the particle related to its momentum by the relation

$$p\lambda = h$$

proposed by the philosophe De Broglie.

Quantum Electrodynamics

In the quantum theory of the electromagnetic interactions of the electrons and the photons the quadrivector A_μ plays an important role, while in the classical theory is an auxiliary field to descrive the electric and magnetic fields :

$$F_{\mu\nu} = \frac{\delta}{\delta\mu} A_\nu - \frac{\delta}{\delta\nu} A_\mu$$

Quantum electrodynamics allows to compute with a great precision the magnetic moments of the electron and of the muon in terms of

$$\alpha = \frac{e^2}{4\pi}$$

.

From QED to QCD

The theory of the strong, electromagnetic and weak interaction has a formal description similar to QED, but with important differences, which give rise to different phenomenological properties . While QED is a gauge theory with only one conserved charge, the charges of the strong interactions have the same commutators of the 3×3 traceless hermitian matrices, which imply the opposite dependance on the scale with respect to QED, giving rise to the confinement of the quarks and to the asymptotic freedhom, which accounts for the fact that quarks may not be isolated and for the scale invariance in deep inelastic scattering .

The Unified Electroweak Theory

The assumption by Glashow that the quantum field associated to the photons A^μ is a combination of W_3^μ completing the weak isospin triplet with the charged components, which mediate the weak interactions, and of another field, X^μ :

$$A^\mu(x) = \sin\theta_G W_3^\mu(x) + \cos\theta_G X^\mu(x)$$

implies the existence of the field :

$$Z^\mu(x) = \cos\theta_G W_3^\mu(x) - \sin\theta_G X^\mu(x)$$

coupled to the combination of the charges :

$$T_{3W} - (\sin\theta_G)^2 Q$$

Spontaneous Symmetry Breaking to Obtain the Masses of the Weak Bosons

Experiment implies tha the weak bosons mediating the charged and the neutral weak currents have large masses around $100 GeV$. To get them in a renormalizable theory one needs the mechanism of the spontaneous symmetry breaking, which in gauge theories implies that the bosons associated to the symmetries spontaneously broken, since the vacuum is not invariant under the symmetry transformatios, acquire a mass .. Therefore also the weak theory as the electromagnetic and the strong ones is a gauge theory related to QED, since the gauge group $SU(2)_W \times U(1)_Y$ is spontaneously broken to $U(1)_Q$ with the electric charge Q the linear combination

$$Q = T_{3W} + Y$$

.

THE QUANTUM NUMBERS OF A FAMILY OF FERMIONS PROVIDE A STRONG MOTIVATION TO SU(5) UNIFICATION

In the gauge theories the interaction with the gauge fields depends on the transformation properties with respect to the gauge group . In the standard model there are three families of fundamental left-handed fermions transforming as

$$(3,2,+\frac{1}{6})+(\bar{3},1,-\frac{2}{3})+(1,1,+1)+(\bar{3},+\frac{1}{3})+(1,2,-\frac{1}{2})$$

. The trace of Y both for the first three representations and for the second two vanishes, which is a property of the representations of a simple group . Also the ratio of the traces of the SU(3) and SU(2) Casimir and Y^2 of the first three representations and the second ones is the same, 3, which is also a property of the simple groups .

CONSEQUENCES OF THE SU(5) UNIFICATION

The first three representation may be classified in the 10 representation of SU(5), while the second ones in the $\bar{5}$ The additional twelve gauge bosons implied by SU(5) unification should give rise to proton decay, while the lower limit for its lifetime is very high,

$$10^{34}$$

years . Also the unification implies the same value for the g_3 and g_2 coupling constants and a slghtly smaller value for g_1 . Indeed the three coupling constants at the scale of the neutral weak boson, Z_0 have the property :

g_3 larger than g_2 larger than g_1

THE RENORMALIZATION GROUP EQUATIONS IMPLY THAT THE THREE GAUGE CONSTANTS APPROACH EACH OTHER AT A HIGHER SCALE

More precisely g_1 crosses first g_2 and then g_3, which cross at a scale sufficiently high to be consistent with the lower limit on proton lifetime . Supersymmetry may give rise to unification consistent with experiment, but up to now there is no evidence for the supersymmetric partners of the existing particles . The two SU(5) multiplets may merge in the spinorial representation of SO(10), the 16, together with a SU(5) singlet, which may be identified as a left-handed antineutrino .

SO(10) IS A PROPER SCENARIO FOR NEUTRINO OSCILLATIONS

Neutrino oscillations, advocated by Pontecorvo several years before the proposal of the standard model, are up to now the only experimental phenomenon not explained within the standard model . The study of the neutrino oscillations allowed to determine the three mixing angles and the CP violating phase in the mixing matrix of left-handed neutrinos, as well the two square mass differences, which account for the solar and atmospheric neutrino oscillations . The sum of the masses of the three left-handed neutrinos has a very lower astrophysics limit of the order of 0.1 eV .

SPONTANEOUS SYMMETRY BREAKING FROM SO(10) TO $SU(3) \times SU(2) \times SU(1)$

The spontaneous symmetry breaking from SO(10) to $SU(3) \times SU(2) \times SU(1)$ may be realized with the $SU(4) \times SU(2) \times SU(2)$ singlet of the 210 representation of SO(10), T_{78910}, and by the SU(5) singlet of the bispinorial, 126 . If the vacuum expectation value of the 210 is larger than the one of the 126 one has the intermediate symmetry $SU(4) \times SU(2) \times SU(2)$ at the scale of about 10^{11} GeV and a sufficiently high scale of SO(10) symmetry breaking consistent with the lower limit for the lifetime of the proton .

THE BARYON ASYMMETRY OF THE UNIVERSE

In the universe there is a baryon asymmetry, which is fixed in the standard model of cosmology by the ratio

$$\frac{\Delta B}{N_\gamma} = 8.61510^{-11}$$

. At the weak scale sphalerons violate baryon and lepton numbers, conserving their difference

$$B - L$$

. The baruon asymmetry may arise by a lepton asymmetry at a high scale, which at the weak scale gives rise to a baryon asymmetry . A sufficient lepton asymmetry may be generated in the decay of the right-handed neutrinos predicted by SO(10) if their masses are not yerarchical, but of the same order, and there is a CP violation in their decays .

THE SEA-SAW MODEL FOR THE MASSES OF THE LEFT-HANDED NEUTRINOS

To explain why the square mass differences of the neutrinos are many order of magnitude smaller than the square of the masses of the charged fermions it has been proposed that the Majorana masses of the right-handed neutrinos are very large compared to the masses of the chqrged fermions with the consequence of the sea-saw formula for the masses of the left handed neutrinos :

$$m_{_L} = m^d d M^{-1} m^d_T$$

In the SO(10) gayge theory, we have described, the matrix elements of M are of order

$$10^{11}$$

GeV, the right order of magnitude to get masses of the neutrinos consistent with the low values of the difference of their squares .

DIRAC NEUTRINO MASSES ARE EXPECTED TO BE GERARCHICAL

A success of SU(5) unification has been the prediction that assuming that the isospin doublet responsible for the spontaneous symmetry breaking for the unified electro-weak model transforms as the fundamental representation, the 5 of SU(5), at the unification scale the mass of the b quark and of the τ lepton are equal . Keeping into account the variation of the masses up to the scale given by the mass of the neutral weak boson, the Z^0, the equality is realized around

$$10^{11}$$

GeV, just the scale of the breaking of the intermediate Pati-Salam symmetry

$$SU(4) \times SU(2) \times SU(2)$$

. In SO(10) a similar property hplds for the masses of the t quark and the Dirac mass of the neutrino of the third generation .

CONSTRAINTS ON THE MASS OF THE LEFT-HANDED NEUTRINOS TO GET A NON YERARCHICAL SPECTRUM FOR THE RIGHT-HANDED NEUTRINOS

By inverting the see-saw formula one gets for the Mayorana mass matrix of the right-handed neutrinos :

$$M = m_{TD} m^{-1} m_D$$

to avoid contributions proportional to

$$(m_{D_\tau}^2)$$

and

$$m_{D_\tau})(m_{D_\mu}$$

one has two constrints for the inverse of the Majorana masses of the left-handed neutrinos .

By neglecting in the CKM matrix terms proportional to

$$(\sin\theta_C)^2$$

the first constraint is :

$$\frac{(\sin\theta_s)^2}{m_1} + \frac{(\cos\theta_s)^2}{m_2} + \frac{1}{m_3} = 0$$

where m_i are the Majorana masses of the left-handed neutrinos with their phase . There is a lower limit for the modulus of the lightest neutrino $|m_1|$.

REQUIRING A COMPACT SPECTRUM FOR RIGHT.HANDED NEUTINOS AND THE BARYON ASYMMETRY MAY COMPLETELY FIX THE MASS MATRIX OF THE LEFT-HANDED NEUTRINOS

By assuming that the leptogenesis scenario to get the baryon asymmetry

$$\Delta B = 8.16510^{-11}$$

and a compact spectrum for the right-handed neutrinos one gets three constraints, which allow to completely determine the left-handed neutrino mass matrix, since the oscillations fixed the three mixing angles, the CP violating phase and the two mass square differences . Small values are predicted for the m_{uu} matrix element appearing in the neutrinoless beta decay and the effective neutrino mass in tritium decay , For the lowest neutrino mass one gets a value in the range $(0.02, 0.04)$ eV .

THE SCALAR REPRESENTATIONS APPEARING IN THE SPONTANEOUS SYMMETRY BREAKING OF SO(10)

Attempts to get the scalar necessary for the spontaneous symmetry breaking of the electroweak theory through condensates of a pair of fermions have been tried . It is interesting that the Higgs necessary for the spontaneous symmetry breaking of SO(10) belong to the representation contained in the products :

$$16 \times 16 = 10 + 120 + 126$$

$$16 \times \bar{1}6 = 1 + 45 + 210$$

THE SCALAR REPRESENTATIONS APPEARING IN THE SPONTANEOUS SYMMETRY BREAKING OF SO(10)

The

$$SU(4) \times SU(2) \times SU(2)$$

singlet of the 210 and the $SU(5)$ singlet of the 126 are on a critic orbit, which makes easily to build definite positive invariants, which vanish in the directions able to realize the spontaneous symmetry breaking pattern :

$$SO(1O \rightarrow SU(4) \times SU(2) \times SU(2) \rightarrow SU(3) \times SU(2) \times U(1)$$

Conclusion

1) The examples of the gravitational theory by Newton, the Maxwell equations, the special and general relativity, quantum mechanics and the standard model of strong, weak and electromagnetic interactions show that progress in physics has been obtained by a unified descriptions of different phenomena with an elegant mathematic formalism .

2) Between the proposal of going beyond the standard model SO(10) gauge theory has both the property of physical elegance and of being the framework of the only phenomenon beyond the standard model : neutrino oscillation .

3) It also provides the framework for baryogenesis from leptogenesis .

4) It is reasonable to assume that it is an intermediate step in the research beyond the standard model .