Gravitational Production of Non-Minimal Vector Dark Matter

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based on:

- A. Ahmed, BG, A. Socha, JHEP 02 (2023) 196, e-Print: 2207.11218,
- A. Ahmed, BG, A. Socha, Phys.Lett.B 831 (2022) 137201, e-Print: 2111.06065,
- A. Ahmed, BG, A. Socha, JHEP 08 (2020) 059, e-Print: 2005.01766,
- BG, A. Socha, work in progress.

"The Dark Side of the Universe",
September 12th, 2024, Corfu, Greece

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Background dynamics

The FLRW metric:

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2 = a^2(\tau)\left[d\tau^2 - d\vec{x}^2\right]$$

The action:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_\phi + \mathcal{L}_{\rm VB}\right]$$

The α-attractor T-model

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$$

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{\sqrt{6\alpha}M_{\rm Pl}}\right) \simeq \begin{cases} \Lambda^4 & |\phi| \gg M_{\rm Pl} \\ \boxed{\Lambda^4\left|\dfrac{\phi}{M_{\rm Pl}}\right|^{2n}} & |\phi| \ll M_{\rm Pl} \end{cases},$$

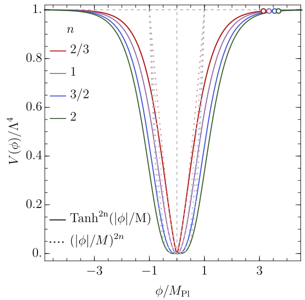
where $n > 0, 6\alpha = 1, \Lambda = 3.0 \times 10^{-3}\, M_{\rm Pl}$

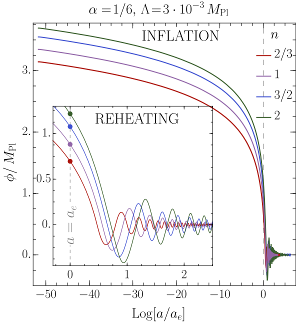
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Classical background equations of motion:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \quad \text{with} \quad H \equiv \frac{\dot{a}}{a}$$





Assumption: $\rho_X \ll \rho_\phi$

Vector boson spectator (dark matter)

$$S_{VB} = \int d^4x\sqrt{-g}\left\{-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{m_X^2}{2}g^{\mu\nu}X_\mu X_\nu + -\frac{\xi_1}{2}g^{\mu\nu}RX_\mu X_\nu + \frac{\xi_2}{2}R^{\mu\nu}X_\mu X_\nu\right\},$$

where $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$ with $Z_2 : X_\mu \to -X_\mu$.

O. Özsoy and G. Tasinato, "Vector dark matter, inflation and non-minimal couplings with gravity", 2310.03862,
C. Capanelli, L. Jenks, E.W. Kolb, E. McDonough, "Runaway Gravitational Production of Dark Photons", 2403.15536,
BG., A. Socha, "Purely gravitational production of dark vectors non-minimally coupled to gravity", in progress,
A. Ahmed, BG, A. Socha, "Gravitational production of vector dark matter", JHEP 08 (2020) 059, 2005.01766.

Gravitational production of DM

$$X_\mu(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} X_\mu(\tau,\vec{k}) e^{i\vec{k}\cdot\vec{x}}, \qquad \vec{X}(\tau,\vec{k}) = \sum_{\lambda=\pm,L} \vec{\epsilon}_\lambda(\vec{k}) X_\lambda(\tau,\vec{k}),$$

$$S_\mathrm{T} = \sum_{T=\pm} \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2}|X_\mathrm{T}'(\tau,\vec{k})|^2 - \frac{1}{2}[k^2 + a^2 m^2_{\mathrm{eff,x}}(a)]|X_\mathrm{T}(\tau,\vec{k})|^2 \right\},$$

$$S_\mathrm{L} = \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2} \frac{1}{A_\mathrm{L}^2(a,k)} |X_\mathrm{L}'(\tau,\vec{k})|^2 - \frac{1}{2} a^2 m^2_{\mathrm{eff,x}}(a) |X_\mathrm{L}(\tau,\vec{k})|^2 \right\},$$

where $k^2 \equiv |\vec{k}|^2$, and

$$A_\mathrm{L}^2(a,k) \equiv \frac{k^2 + a^2 m^2_{\mathrm{eff,t}}(a)}{a^2 m^2_{\mathrm{eff,t}}(a)},$$

$$m^2_{\mathrm{eff,t}}(a) \equiv m_X^2 - \xi_1 R(a) + \frac{1}{2}\xi_2 R(a) + 3\xi_2 H^2(a),$$

$$m^2_{\mathrm{eff,x}}(a) \equiv m_X^2 - \xi_1 R(a) + \frac{1}{6}\xi_2 R(a) - \xi_2 H^2(a).$$

$$m^2_{\text{eff,t}}(a) = m^2_X - 3\left[\left(\xi_1 - \frac{1}{2}\xi_2\right)(3w(a)-1) - \xi_2\right] H^2(a),$$

$$m^2_{\text{eff,x}}(a) = m^2_X - \left[3\left(\xi_1 - \frac{1}{6}\xi_2\right)(3w(a)-1) + \xi_2\right] H^2(a).$$

where

$$w(a) \equiv \frac{p(a)}{\rho(a)} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}, \qquad w(a) \in [-1, 1]$$

- For minimal couplings, i.e. $\xi_1 = \xi_2 = 0$: $m^2_{\text{eff,x}}(a) = m^2_{\text{eff,t}}(a) = m^2_X$.
- During inflation (dS)

$$m^2_{\text{eff,x}}(a) = m^2_{\text{eff,t}}(a) = m^2_X + 3(4\xi_1 - \xi_2)H^2(a) \simeq \text{const.}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

$$H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

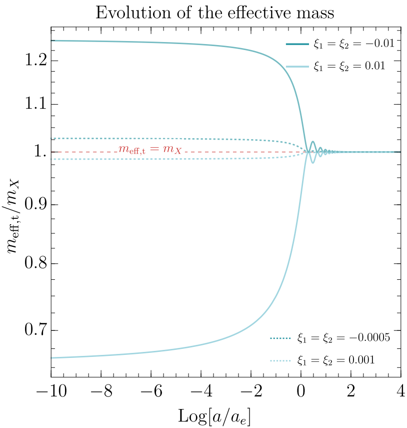


Figure 1:

$$s(a) \equiv \text{sign}\left\{\frac{k^2 + a^2 m^2_{\text{eff,t}}(a)}{a^2 m^2_{\text{eff,t}}(a)}\right\}$$

To avoid ghost instability $s(a) > 0$ for any a: $\rightsquigarrow m^2_{\text{eff,t}}(a) > 0$

$$\Downarrow$$

$$f(w(a), \xi_1, \xi_2) \le \left(\frac{m_X}{H_e}\right)^2 \equiv \eta_e^{-1}$$

with

$$f(w(a), \xi_1, \xi_2) \equiv 3\left[\left(\xi_1 - \frac{1}{2}\xi_2\right)(3w(a) - 1) - \xi_2\right] \text{ for } w(a) \in [-1, 1]$$

For

$$\xi_1 = \frac{1}{2}\xi_2.$$

The Lagrangian density reads

$$\sqrt{-g}\mathcal{L}_X^{\rm NM} = \sqrt{-g}\left[-\frac{\xi_1}{2}Rg_{\mu\nu}X^\mu X^\nu + \frac{\xi_2}{2}R_{\mu\nu}X^\mu X^\nu\right] = \sqrt{-g}\frac{1}{2}\xi_2 G_{\mu\nu}X^\mu X^\nu,$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$.

$$m^2_{\rm eff,x}(a)\Big|_{\xi_1=\xi_2/2} = m_X^2 - w(a)3\xi_2 H^2(a)$$

$$m^2_{\rm eff,t}(a)\Big|_{\xi_1=\xi_2/2} = m_X^2 + 3\xi_2 H^2(a)$$

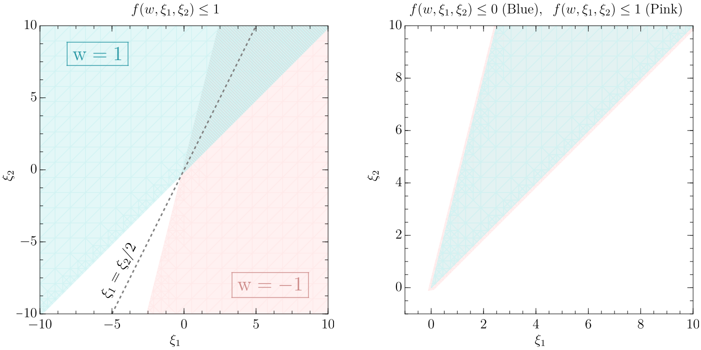


Figure 2: Left: Region in the $\xi_1 - \xi_2$ parameter space satisfying $f(w(a), \xi_1, \xi_2) \lesssim 1$, i.e. for $\eta_e = 1$, with two limiting choices of the equation-of-state parameter $w = -1$ (light pink region) and $w = 1$ (light cyan region). Right: Values of $\xi_1 - \xi_2$ ensuring the positivity of $m^2_{\mathrm{eff,t}}(a)$ for two values of $\eta_e^{-1} \in \{0, 1\}$.

$$S_{\rm L} = \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2}\frac{1}{A_{\rm L}^2(a,k)} |X_{\rm L}'(\tau,\vec{k})|^2 - \frac{1}{2} a^2 m_{\rm eff,X}^2(a) |X_{\rm L}(\tau,\vec{k})|^2 \right\}$$

$$X_L(\tau,\vec{k}) = A_{\rm L}(a,k)\mathcal{X}_L(\tau,\vec{k})$$

Integrating by parts and dropping the boundary term

$$S_{\rm L} = \frac{1}{2}\int d\tau \int \frac{d^3k}{(2\pi)^3} \times$$
$$\times \left\{ |\mathcal{X}_{\rm L}'(\tau,\vec{k})|^2 - \left[a^2 m_{\rm eff,X}^2(a) A_{\rm L}^2(a,k) + \frac{A_{\rm L}''(a,k)}{A(a,k)} - 2\left(\frac{A_{\rm L}'(a,k)}{A_{\rm L}(a,k)}\right)^2 \right] |\mathcal{X}_{\rm L}(\tau,\vec{k})|^2 \right\}$$

$$X_{\rm T}''(\tau,\vec{k}) + \omega_{\rm T}^2(\tau,k) X_{\rm T}(\tau,\vec{k}) = 0,$$
$$\mathcal{X}_{\rm L}''(\tau,\vec{k}) + \omega_{\rm L}^2(\tau,k) \mathcal{X}_{\rm L}(\tau,\vec{k}) = 0,$$

where the time-dependent frequencies are defined as

$$\omega_{\rm T}^2(\tau,k) \equiv k^2 + a^2 m_{\rm eff,X}^2(a),$$

$$\omega_{\rm L}^2(\tau,k) \equiv a^2 m_{\rm eff,X}^2(a) A_{\rm L}^2(a,k) + \frac{A_{\rm L}''(a,k)}{A_{\rm L}(a,k)} - 2\left(\frac{A_{\rm L}'(a,k)}{A_{\rm L}(a,k)}\right)^2$$

$$\omega_{\mathrm{T}}^2(a,k) = k^2 + a^2 m_X^2 - a^2 H^2(a)\left[3(3w(a)-1)\left(\xi_1 - \frac{1}{6}\xi_2\right) + \xi_2\right]$$

$$\omega_{\mathrm{L}}^2(a,k) = k^2 \frac{m_{\mathrm{eff,x}}^2}{m_{\mathrm{eff,t}}^2} + a^2 m_{\mathrm{eff,x}}^2(a) - \frac{k^2}{k^2 + a^2 m_{\mathrm{eff,t}}^2(a)} \times$$

$$\times \left[\frac{a''}{a} + \frac{m''_{\mathrm{eff,t}}}{m_{\mathrm{eff,t}}} + 2\frac{a'}{a}\frac{m'_{\mathrm{eff,t}}}{m_{\mathrm{eff,t}}} - 3\frac{(a' m_{\mathrm{eff,t}} + m'_{\mathrm{eff,t}} a)^2}{k^2 + a^2 m_{\mathrm{eff,t}}^2(a)}\right]$$

For $\xi_1 = \xi_2 = 0$, the frequency recovers the standard formula

$$\omega_{\mathrm{L}}^2(\tau,k) = \omega_{\mathrm{L}}^2(\tau,k)\,\big|_{\xi_1=\xi_2=0} = k^2 + a^2 m_X^2 - \frac{k^2}{k^2 + a^2 m_X^2}\left[\frac{a''}{a} - 3\frac{a'^2 m_X^2}{k^2 + a^2 m_X^2}\right]$$

see also

- A. Ahmed, B.G. and A. Socha, “Gravitational production of vector dark matter,” JHEP **08** (2020), 059
- E. W. Kolb and A. J. Long, “Completely dark photons from gravitational particle production during the inflationary era,” JHEP **03** (2021), 283

UV behaviour, i.e. $k^2 \to \infty$:

$$\omega_T^2(a,k) \to k^2, \qquad \omega_L^2(a,k) \to k^2 \frac{m^2_{\text{eff,x}}(a)}{m^2_{\text{eff,t}}(a)}$$

$$\frac{m^2_{\text{eff,x}}(a)}{m^2_{\text{eff,t}}(a)} \lesseqgtr 0$$

$$\frac{m^2_{\text{eff,x}}(a)}{m^2_{\text{eff,t}}(a)} < 0 \Rightarrow \text{massive creation of short-wavelength modes}$$

Remark:

- during dS inflation $m^2_{\text{eff,x}}(a) = m^2_{\text{eff,t}}(a)$, therefore for $k^2 \to \infty$ $\omega_T^2(a,k) = \omega_L^2(a,k) = k^2$, i.e. no massive production of short-wavelength modes, i.e. no "runaway production"

Credibility might be restored:

- One could impose the positivity condition on $m^2_{\mathrm{eff},X}(a)$ analogously to $m^2_{\mathrm{eff},t}(a)$:

$$\tilde{f}(w, \xi_1, \xi_2) \lesssim \left(\frac{m_X}{H(a_e)}\right)^2 = \eta_e^{-1},$$

with

$$\tilde{f}(w, \xi_1, \xi_2) \equiv 3\left[3w(a) - 1\right]\left(\xi_1 - \frac{1}{6}\xi_2\right) + \xi_2.$$

- For $m_X \to 0$ and $\xi_1, \xi_2 \neq 0$ there is no region such that $m^2_{\mathrm{eff},X}(a) > 0$ and $m^2_{\mathrm{eff},t}(a) > 0$ for arbitrary $w \in [-1, 1]$.
- If $m^2_{\mathrm{eff},X}(a) > 0$ for any a, then $\omega_T^2(\tau, k) \equiv k^2 + a^2 m^2_{\mathrm{eff},X}(a) > 0$, so no tachyonic production of X_T.

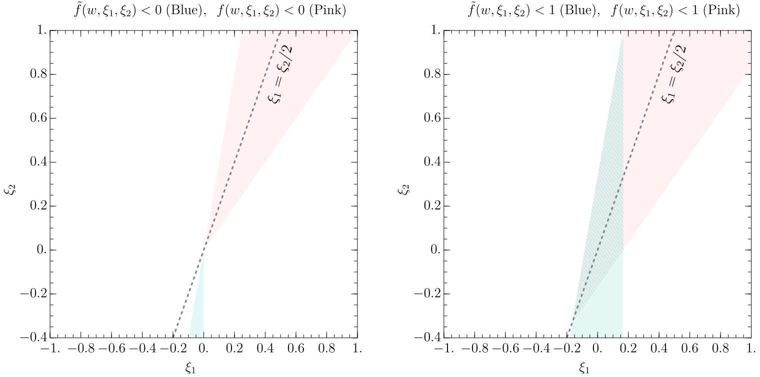


Figure 3: Left: Region in the $\xi_1 - \xi_2$ parameter space satisfying $f(w(a), \xi_1, \xi_2) \lesssim \eta_e^{-1}$ and $\tilde{f}(w(a), \xi_1, \xi_2) \lesssim \eta_e^{-1}$, for $\eta_e^{-1} = 0$ (left panel) and $\eta_e^{-1} = 1$ (right panel).

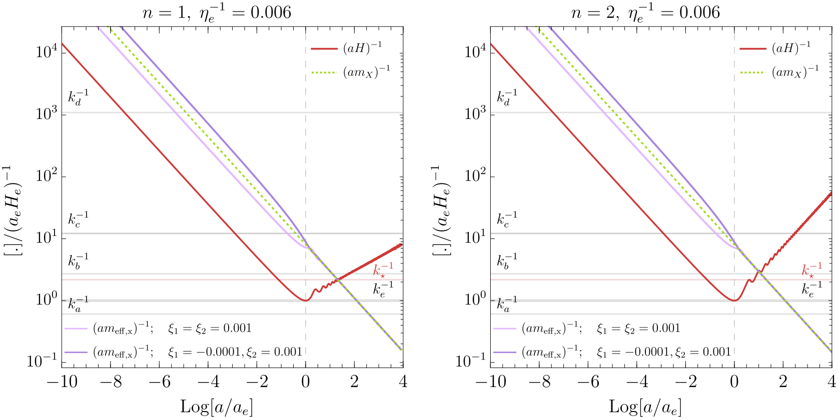


Figure 4: Evolution of various length scales, $\eta_e \equiv \left(\frac{H_e}{m_X}\right)^2$.

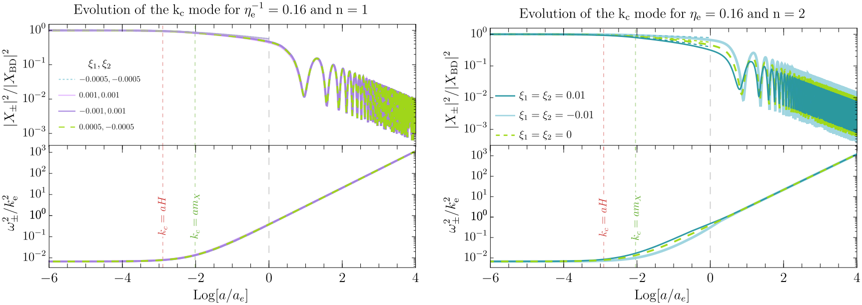


Figure 5: Evolution of the k_c momentum mode of the transversely polarized vectors with mass $m_X = 5 \cdot 10^{12}\mathrm{GeV}$ for different non-minimal couplings for $n = 1$ (left) and $n = 2$ (right). Lower panels: Evolution of the transverse frequency $\omega_\pm^2/k_e^2$, $\eta_e \equiv \left(\frac{H_e}{m_X}\right)^2$.

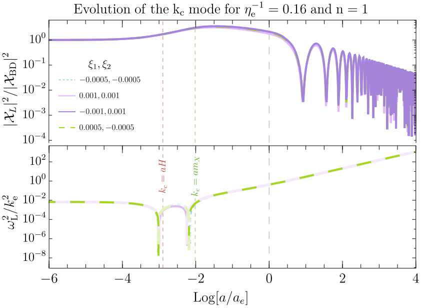


Figure 6: Evolution of the k_c momentum mods of the redefined longitudinal polarization with mass $m_X = 5 \cdot 10^{12}\mathrm{GeV}$ (left) and $m_X = 10^{12}\mathrm{GeV}$ (right) for different non-minimal couplings assuming quadratic inflaton potential during reheating, i.e., $n = 1$. Lower panels: Evolution of longitudinal frequency $\omega_\mathrm{L}^2/k_e^2$, $\eta_e \equiv \left(\frac{H_e}{m_X}\right)^2$.

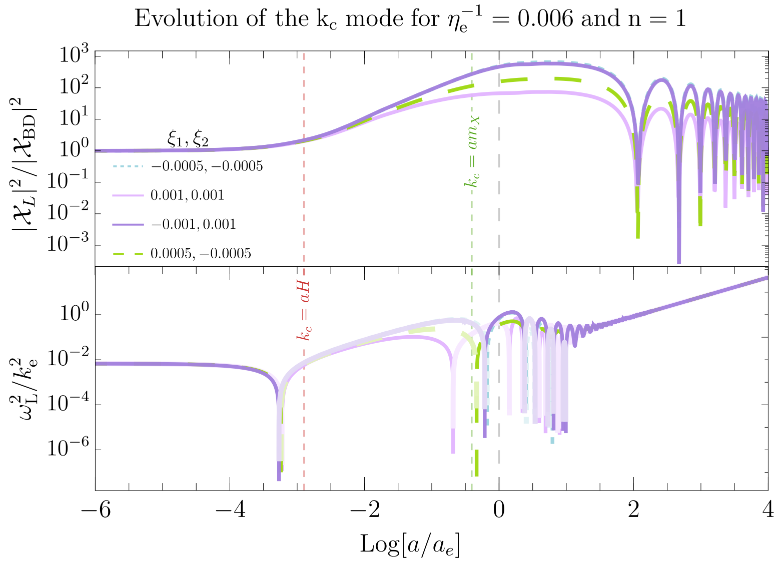


Figure 7: Evolution of the k_c momentum mod of the redefined longitudinal polarization with mass $m_X = 5 \cdot 10^{12}\,\mathrm{GeV}$ (left) and $m_X = 10^{12}\,\mathrm{GeV}$ (right) for different non-minimal couplings assuming quadratic inflaton potential during reheating, i.e., $n = 1$. Lower panel: Evolution of longitudinal frequency ω_L^2/k_e^2, $\eta_e \equiv \left(\frac{H_e}{m_X}\right)^2$.

Energy density

$$T_{\mu\nu} := \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{M}}}{\delta g^{\mu\nu}}, \qquad T^{\mathrm{X}}_{\mu\nu} = T^{\mathrm{M}}_{\mu\nu} + T^{\xi_1}_{\mu\nu} + T^{\xi_2}_{\mu\nu}$$

$$T^{\mathrm{M}}_{\mu\nu} = g_{\mu\nu}\left(\frac{1}{4} g^{\rho\sigma} g^{\alpha\beta} X_{\rho\alpha} X_{\sigma\beta} - \frac{m_X^2}{2} g^{\alpha\beta} X_\alpha X_\beta\right) - g^{\alpha\beta} X_{\mu\alpha} X_{\nu\beta} + m_X^2 X_\mu X_\nu$$

$$T^{\xi_1}_{\mu\nu} = \xi_1 \Big[- R X_\mu X_\nu - G_{\mu\nu} g^{\rho\sigma} X_\rho X_\sigma +$$

$$- g_{\mu\nu} g^{\rho\sigma} g^{\alpha\beta} \nabla_\sigma \nabla_\rho (X_\alpha X_\beta) + g^{\rho\sigma} \nabla_\mu \nabla_\nu (X_\rho X_\sigma) \Big]$$

$$T^{\xi_2}_{\mu\nu} = \frac{\xi_2}{2} \Big[- g_{\mu\nu} g^{\alpha\rho} g^{\beta\sigma} R_{\rho\sigma} X_\alpha X_\beta + 2 g^{\rho\sigma} R_{\nu\sigma} X_\mu X_\rho + 2 g^{\rho\sigma} R_{\mu\sigma} X_\nu X_\rho +$$

$$+ g^{\rho\sigma} \nabla_\rho \nabla_\sigma (X_\mu X_\nu) + g_{\mu\nu} g^{\lambda\rho} g^{\kappa\sigma} \nabla_\lambda \nabla_\kappa (X_\rho X_\sigma) - g^{\lambda\sigma} \nabla_\mu \nabla_\sigma (X_\lambda X_\nu) +$$

$$- g^{\lambda\sigma} \nabla_\nu \nabla_\sigma (X_\lambda X_\mu) \Big]$$

$$\hat{\vec{X}}(\tau, \vec{x}) = \sum_{\lambda=\pm,\mathrm{L}} \int \frac{d^3 k}{(2\pi)^3} \vec{\epsilon}_\lambda(\vec{k}) e^{i\vec{k}\cdot\vec{x}} \hat{\mathcal{X}}_\lambda(\tau, k),$$

$$\hat{\mathcal{X}}_\lambda(\tau, k) \equiv \hat{a}_\lambda(\vec{k}) \mathcal{X}_\lambda(\tau, k) + \hat{a}^\dagger_\lambda(-\vec{k}) \mathcal{X}^\star_\lambda(\tau, k)$$

The power spectra:

$$\langle 0|\hat{\mathcal{X}}_\lambda(\tau, k) \cdot \hat{\mathcal{X}}_{\lambda'}(\tau, q)|0\rangle = \delta_{\lambda\lambda'} (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{q}) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{X}_\lambda}(\tau, k)$$

$$\langle 0|\hat{\mathcal{X}}'_\lambda(\tau, k) \cdot \hat{\mathcal{X}}'_{\lambda'}(\tau, q)|0\rangle = \delta_{\lambda\lambda'} (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{q}) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{X}'_\lambda}(\tau, k)$$

$$\langle 0|\hat{\mathcal{X}}_\lambda(\tau, k) \cdot \hat{\mathcal{X}}'_{\lambda'}(\tau, q)|0\rangle + \langle 0|\hat{\mathcal{X}}'_\lambda(\tau, k) \cdot \hat{\mathcal{X}}_{\lambda'}(\tau, q)|0\rangle = \delta_{\lambda\lambda'} (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{q}) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{X}_\lambda \mathcal{X}'_\lambda}(\tau,$$

where $\lambda, \lambda' = \pm, \mathrm{L}$

$$\langle 0|\hat{\rho}_X|0\rangle = \langle 0|\hat{\rho}_\mathrm{L}|0\rangle + \langle 0|\hat{\rho}_\pm|0\rangle,$$

where

$$\langle 0|\hat{\rho}_\mathrm{L}|0\rangle = \langle 0|\hat{\rho}^\mathrm{M}_\mathrm{L}|0\rangle + \langle 0|\hat{\rho}^{\xi_1}_\mathrm{L}|0\rangle + \langle 0|\hat{\rho}^{\xi_2}_\mathrm{L}|0\rangle,$$

$$\langle 0|\hat{\rho}_\pm|0\rangle = \langle 0|\hat{\rho}^\mathrm{M}_\pm|0\rangle + \langle 0|\hat{\rho}^{\xi_1}_\pm|0\rangle + \langle 0|\hat{\rho}^{\xi_2}_\pm|0\rangle$$

$$\langle 0|\hat{\rho}^{\mathrm{M}}_{\pm}|0\rangle = \frac{1}{2a^4}\int \frac{d^3k}{(2\pi)^3}\frac{2\pi^2}{k^3}\left\{\mathcal{P}_{\mathcal{X}'_{\pm}}(\tau,k) + (k^2 + a^2 m_X^2)\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k)\right\}$$

$$\langle 0|\hat{\rho}^{\xi_1}_{\pm}|0\rangle = \frac{\xi_1}{a^4}\int \frac{d^3k}{(2\pi)^3}\frac{2\pi^2}{k^3}\left\{-3a^2H^2\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k) + 3aH\mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}}\right\}$$

$$\langle 0|\hat{\rho}^{\xi_2}_{\pm}|0\rangle = \frac{\xi_2}{a^4}\int \frac{d^3k}{(2\pi)^3}\frac{2\pi^2}{k^3}\left\{2a^2H^2\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k) - 3aH\mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}}\right\}$$

Spectral energy densities

$$\frac{d\langle 0|\hat{\rho}^{\mathrm{M}}_{\pm}|0\rangle}{d\ln k} \propto \frac{1}{2a^4}\left\{\mathcal{P}_{\mathcal{X}'_{\pm}}(\tau,k) + (k^2 + a^2 m_X^2)\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k)\right\}$$

$$\frac{d\langle 0|\hat{\rho}^{\xi_1}_{\pm}|0\rangle}{d\ln k} \propto \frac{\xi_1}{a^4}\left\{-3a^2H^2\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k) + 3aH\mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}}\right\}$$

$$\frac{d\langle\hat{\rho}^{\xi_2}_{\pm}|0\rangle}{d\ln k} \propto \frac{\xi_2}{a^4}\left\{2a^2H^2\mathcal{P}_{\mathcal{X}_{\pm}}(\tau,k) - 3aH\mathcal{P}_{\mathcal{X}_{\pm}\mathcal{X}'_{\pm}}\right\}$$

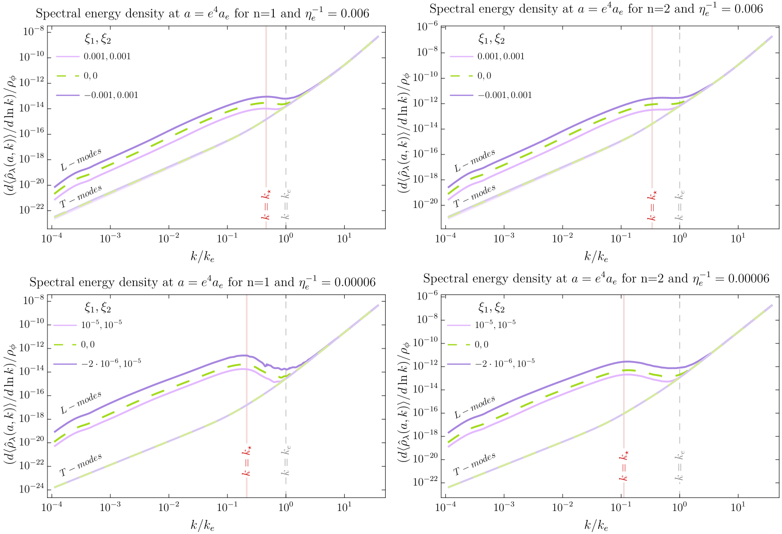


Figure 8: Spectral energy density of the longitudinal (L) and transverse (T) components of the minimally (dashed green) and non-minimally (violet curves) coupled vector field for the quadratic (left panel) and quartic (right panel) reheating model.

Normal ordering

The physical expectation value of the energy density is calculated with respect to the initial vacuum state, e.g., Bunch-Davies vacuum, whereas the normal ordering is performed with respect to the late-time ladder operators. At late time, i.e., when the evolution of the modes becomes adiabatic, the total energy density can be approximated by the following formula:

$$\langle 0^{IN} | : \hat{\rho}_{\rm L} : | 0^{IN} \rangle \simeq \lim_{\tau \to \infty} \langle \hat{\rho}_{\rm L} \rangle \approx \frac{1}{a^4} \int \frac{d^3 k}{(2\pi)^3} \omega_{\rm L} |\beta_k^{\rm L}|^2 ,$$

$$\lim_{\tau \to \infty} |\beta_k^{\rm L}|^2 = \frac{1}{2\omega_{\rm L}} |\mathcal{X}_{\rm L}'|^2 + \frac{\omega_{\rm L}}{2} |\mathcal{X}_{\rm L}|^2 - \frac{1}{2} ,$$

and similarly for $\langle 0^{IN} | : \hat{\rho}_{\rm T} : | 0^{IN} \rangle$.

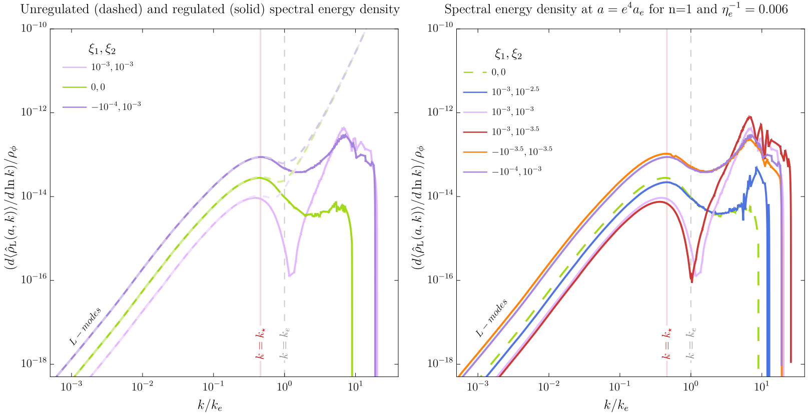


Figure 9: Normally ordered spectral energy density of longitudinal modes. Left: Comparison of the unregulated (solid curves) and regularized (dashed curves) spectral energy density. The results perfectly overlap in the low-k regime.

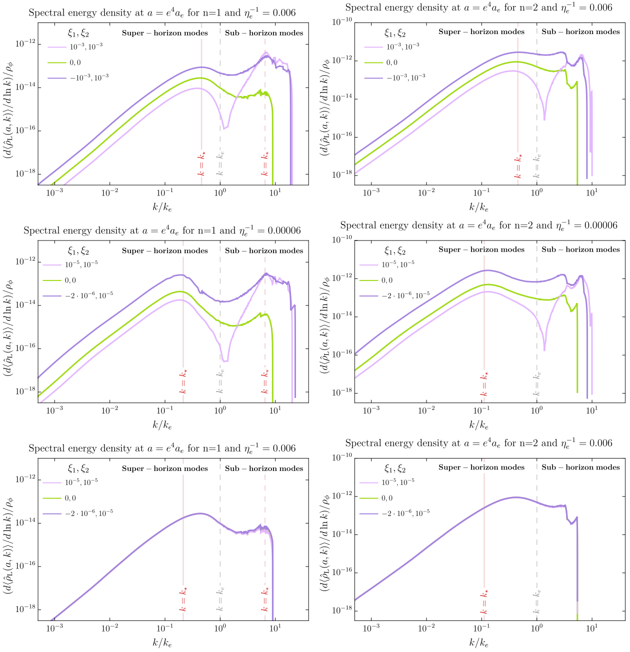


Figure 10: Normally ordered spectral energy density of longitudinal (L) and transverse (T) components of the minimally (dashed green) and non-minimally (violet curves) coupled vector field for the quadratic (left panel) and quartic (right panel) reheating potential.

Summary

- Gravitational production of Abelian massive gauge fields, candidates for dark matter, that are coupled non-minimally to gravity has been discussed.
- The α-attractor T-model potential for the inflaton field has been adopted:

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{\sqrt{6\alpha}M_{\rm Pl}}\right) \begin{cases} \Lambda^4 & |\phi| \gg M_{\rm Pl} \\ \boxed{\Lambda^4 \left|\frac{\phi}{M_{\rm Pl}}\right|^{2n}} & |\phi| \ll M_{\rm Pl} \end{cases}$$

- Spectator vector X_μ: $\rho_X \ll \rho_\phi$
- Energy density corresponding to various polarization components of the vector field have been calculated.

- It has been shown that the presence of the non-minimal couplings may imply a massive, tachyonic production of high-momentum modes of the gauge field.
- For $m_X \to 0$ and $\xi_1, \xi_2 \neq 0$ there is no region such that $m^2_{\text{eff,x}}(a) > 0$ and $m^2_{\text{eff,t}}(a) > 0$ for arbitrary $w \in [-1, 1]$.
- If $m^2_{\text{eff,x}}(a) > 0$ for any a, then $\omega^2_{\text{T}}(\tau, k) \equiv k^2 + a^2 m^2_{\text{eff,x}}(a) > 0$, so no tachyonic production of X_T.
- During dS inflation $m^2_{\text{eff,x}}(a) = m^2_{\text{eff,t}}(a)$, therefore for $k^2 \to \infty$ $\omega^2_{\text{T}}(a, k) = \omega^2_{\text{L}}(a, k) = k^2$, i.e. no massive production of short-wavelength modes, i.e. no "runaway production".
- Appearance of a second maximum in normally ordered spectral energy density of longitudinal modes has been noticed.

Backup slides

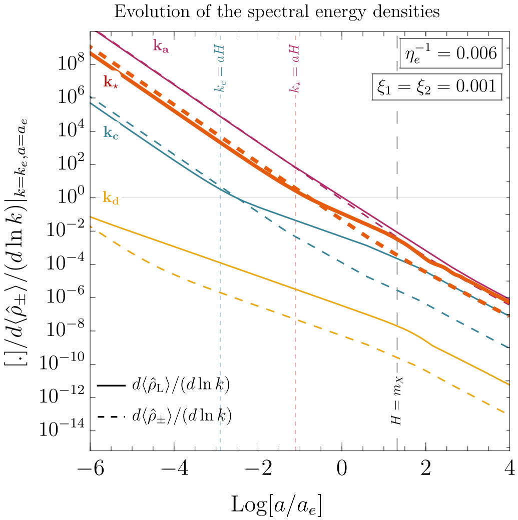


Figure 11:

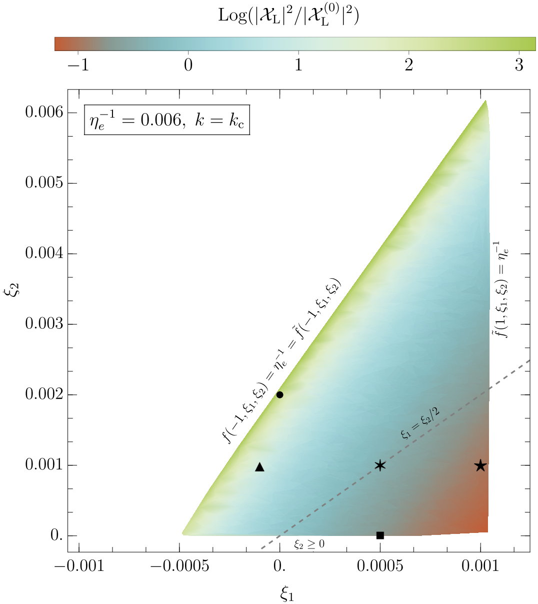


Figure 12: The amplitude squared of the longitudinal polarization normalized to the Bunch-Davies value for different choices of the non-minimal coupling ξ_1, ξ_1 satisfying constraints $f(w(a), \xi_1, \xi_2) \gtrsim \eta_e^{-1}$, and $\tilde{f}(w(a), \xi_1, \xi_2) \gtrsim \eta_e^{-1}$.