θ-vacua, non-perturbative condensates in the Standard Model and (super)gravity

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Takeaway 1

- The non-conservation of a charge due to the quantum anomaly is necessarily accompanied by the spontaneous breaking of the associated symmetry. The vacuum condensates triggering the breaking have intrinsically topological origin and are accompanied by (pseudo)Goldstone states.

Takeaway 2

- Within the Standard Model we predict a new (pseudo)Goldstone particle state, η_w , associated with the spontaneous breaking of anomalous $U(1)_{B+L}$ symmetry.

Takeaway 3

- S-matrix formulation of gravity mandates its supersymmetric extension. Furthermore, the supergravity vacuum is inherently asymmetric and thus accompanied by emergent fields that form the Goldstone supermultiplet (dilaton, R-axion, goldstino).

Topology, θ-vacua, anomalies and condensates: general considerations

- Consider SU(2) pure Yang-Mills theory (prototype for QCD and the EW sectors of the standard model or even GR)

$$\mathcal{L} = -\frac{1}{2g^2}\mathrm{Tr} F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{16\pi^2}\mathrm{Tr} F_{\mu}\tilde{F}^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]\,,\ A_\mu = A^a_\mu \sigma^a/2,\ \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}$$

- θ-term is topological:

$$\mathrm{Tr} F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv \partial_\mu K^\mu,\ K^\mu = \frac{1}{6}\epsilon^{\mu\nu\rho\sigma}C_{\nu\rho\sigma}$$

$$C_{\nu\rho\sigma} = \mathrm{Tr}\left(A_{[\nu}\partial_\rho A_{\sigma]} - \frac{2i}{3}A_{[\nu}A_\rho A_{\sigma]}\right)$$ - Chern-Simons 3-form

Topology, θ-vacua, anomalies and condensates: general considerations

- θ-term is mandatory (!) to preserve causality (cluster property) and results from topological properties of the ground (vacuum) state

- Topology of vacuum states :

$$A_0 = 0,\ A_i = g\partial_i g^{-1},\ g(\vec{x}) = \mathrm{e}^{i\alpha^a(\vec{x})\sigma^a/2} \in SU(2)$$

$$g(\vec{x}):\ S^3 \to SU(2),\ \pi_3(SU(2)) = \mathbb{Z}$$

$$n = \frac{1}{16\pi^2}\int d^4x \mathrm{Tr} F_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{g^3}{96\pi^2}\int d^3\vec{x}\ \epsilon_{0ijk}\epsilon^{abc}A_i^a A_j^b A_k^c$$

Topology, θ-vacua, anomalies and condensates: general considerations

- The ground state

$$|n\rangle = \int \mathcal{D}\alpha^a(\vec{x}) \; |\mathrm{e}^{i\alpha^a(\vec{x})\sigma^a/2} A_i^a(n)\rangle$$

  - Invariant under “small” gauge transformations [$\alpha^a(\vec{x})$ is smooth on S^3]
  - “Large” gauge transformations: $|n\rangle \rightarrow |n+\nu\rangle$ [ν winding #]

Fully gauge invariant ground state:

$$|\theta\rangle = \sum_n \mathrm{e}^{i\theta n}|n\rangle, \quad H_\theta|\theta\rangle = E_{\theta,\min}|\theta\rangle$$

Topology, θ-vacua, anomalies and condensates: general considerations

- Gauge invariance implies that θ = const., and different θ-vacua are orthogonal, $\langle\, \theta' | \theta \rangle = 0 \; [\theta' \neq \theta + 2\pi k]$

- Fock space of states $$\mathcal{F} = \coprod_{\theta \in \mathrm{R}} \mathcal{F}_\theta$$

- Generating functional

$$Z[J] = \langle \theta | \theta \rangle_J = \int \mathcal{D} A_\mu^{(\nu)} \exp\left(-S[A] - \int d^4x J_\mu A^\mu + \int d^4x \frac{i\theta}{16\pi^2} \mathrm{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

- Quantum transition between different $|n\rangle$ vacua are 'mediated' by instantons Belavin, Polyakov, Schwarz and Tyupkin '75

Topology, θ-vacua, anomalies and condensates: general considerations

- Add (spin-1/2) fermions:

$$Z = \int \mathcal{D}A_\mu^{(\nu)} \mathrm{Det}\left(\not{D}^{(\nu)} + M\right) \exp\left(-S[A] + \int d^4x \frac{i\theta}{16\pi^2} \mathrm{Tr} F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

- Vacuum energy:

$$E(\theta) = -\frac{1}{V}\ln Z \geq 0 \;\; \left[\mathrm{Det}\left(\not{D} + M\right) \geq 0,\;\; S[A] \geq 0\right]$$

$$E(\theta = 0) = 0.$$

Vafa and Witten ‘84

- $\theta \neq 0$ is incompatible with S-matrix formulation of gravity

Dvali and Gomez ‘16

Topology, θ-vacua, anomalies and condensates: general considerations

- If the Dirac operator admits zero-mode solutions

$$\left(\not{D}^{(\nu)} + M\right)\Psi_0^{(\nu)} = 0$$

the vacuum transitions with $\nu \neq 0$ are halted and θ becomes unobservable

- Fermion zero-modes are mandated by the topological index theorem

Atiyah and Singer ‘63

$$\partial_\mu J_5^\mu = \frac{2N}{16\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$Q_5(t=+\infty) - Q_5(t=-\infty) \equiv n_+ - n_- = 2N\nu$$

Topology, θ-vacua, anomalies and condensates: general considerations

- Anomalous U(1) symmetry must be spontaneously broken

$$\mathrm{e}^{i\alpha Q_5/2N} \in U(1), \quad \left[Q_5 = \int d^3\vec{x} J_5^0\right]$$

- Action on the vacuum state:

$$\mathrm{e}^{i\alpha Q_5/2N}|\theta\rangle = |\theta + \alpha\rangle$$

$$\langle\theta|\theta + \alpha\rangle = 0 \Longrightarrow Q_5|\theta\rangle \neq 0$$

Pseudo-Goldstone particle in the spectrum

3-form gauge theory formulation of θ-vacua

$$\theta \mathrm{Tr} F\tilde{F} \to \theta \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}$$

$$F_{\mu\nu\rho\sigma} = \partial_{[\mu} C_{\nu\rho\sigma]}, \quad C'_{\nu\rho\sigma} = C_{\mu\nu\rho} + \partial_\nu \omega_{\rho\sigma} + \partial_\sigma \omega_{\nu\rho} + \partial_\rho \omega_{\sigma\nu}$$

Gauge redundancy -> no propagating dof

- Topological susceptibility of vacuum

$$\chi = \mathrm{F.T.} \langle \theta | \mathrm{Tr} F\tilde{F}(x),\ \mathrm{Tr} F\tilde{F}(0) | \theta \rangle_{p\to 0} = \begin{cases} \text{const.}, & \theta \text{ observable} \\ 0, & \theta \text{ unobservable} \end{cases}$$

$$\mathrm{F.T.} \langle \theta | C_{\mu\nu\rho}(x),\ C_{\alpha\beta\gamma}(0) | \theta \rangle_{p\to 0} \propto \begin{cases} \frac{\rho(0)}{p^2} + \ldots, & \theta \text{ observable (Coulomb phase, Luscher pole)} \\ \frac{\rho(0)}{p^2 - m_\eta^2} + \ldots, & \theta \text{ unobservable (Higgs phase, propagating dof!)} \end{cases}$$

Luscher ‘78, Veneziano ‘79

Dvali ‘15

θ-vacua in General Relativity

- Energy/action is not positive definite
- Eigenvalues of the Dirac operator and topological properties of vacua can be affected by the topology of curved manifolds and their causal structure

- Positive energy/action theorem: The manifolds that are asymptotically Ricci flat ($R_{\mu\nu} = 0$) and admit spinors have positive energy/action and well-defined S-matrix:

$$S_{GR} \geq 0$$

Schoen and Yau ’79
Witten ‘81

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θ-vacua in General Relativity

$$S_{GR} = 0 \Longrightarrow \begin{cases} \eta_{\mu\nu}, & \text{flat spacetime with trivial topology} \\ R_{\mu\nu\rho\sigma} = \pm\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}, & \text{ALE gravitational instantons} \end{cases}$$

Eguchi and Hanson '78
Gibbons and Hawking '78

- Topological invariants

$$\tau = \frac{1}{24\pi^2}\int_M \mathrm{Tr}R\tilde{R} + \text{b.terms} = -1 \text{ (EH instanton)}$$

$$\chi = \frac{1}{64\pi^2}\int_M \epsilon_{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}R^{\mu\nu\alpha\beta}R^{\rho\sigma\beta\gamma} + \text{b.terms} = 2 \text{ (EH instanton)}$$

- Effective GR action: $\int d^4x \frac{\sqrt{-g}M_P^2}{2}R + \theta_g\tau + c\chi$

θ-vacua in General Relativity

- Index of the Dirac operator $I_{1/2} = 0$ for all ALE instantons – no spin 1/2 zero modes!

- Spin 3/2 fermions, $I_{3/2} \neq 0$ $(= 2 \text{ for EH instanton})$

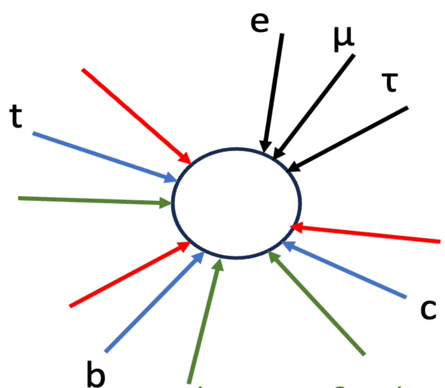
- Hence to nullify θ_g – vacua we must incorporate spin 3/2 fermion => promote GR to Supergravity(!)

The electroweak η_w meson of the Standard Model

- In the Standard Model $U(1)_{B+L}$ is an exact symmetry in the classical approximation and explicitly broken by quantum anomaly

$$\partial_\mu J^\mu_{\mathrm{B+L}} = -\frac{3}{16\pi^2} W^a_{\mu\nu} \tilde{W}^{a\mu\nu} + (\text{hypercharge})$$

$$\Delta Q_{B+l} = 3\nu$$



't Hooft, '76

- $U(1)_{B+L}$ must be broken also spontaneously, the order parameter being 't Hooft's local composite operator comprising of 12 fermionic (quark and lepton) operators. The phase field of the order-parameter is an emergent dof, η_w.

The electroweak η_w meson of the Standard Model

- Toy model: $q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$, $\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$, $u_R,\ d_R,\ e_R, \nu_R + \text{Higgs doublet}$

$$\psi = q_L + \ell_R^c \ , \quad \eta = \begin{pmatrix} u_R \\ d_R \end{pmatrix} + \begin{pmatrix} e_L^c \\ -\nu_L^c \end{pmatrix} \ .$$

$$\Psi = (\psi, \eta)^{\mathrm{T}}$$

$$\mathcal{L}_F = \Psi^\dagger \hat{\mathcal{D}} \Psi \ ,$$

$$\hat{\mathcal{D}} = \begin{pmatrix} -i\bar{\not{D}} \ , & -iM_q P_R + i\epsilon M_\ell^* \epsilon P_L \\ -iM_q^\dagger P_L + i\epsilon M_\ell^{\mathrm{T}} \epsilon P_R \ , & -i\not{\partial} \end{pmatrix}$$

$$M_q = \begin{pmatrix} y_u \phi^{0*} \ , & y_d \phi^+ \\ -y_u \phi^{+*} \ , & y_d \phi^0 \end{pmatrix} \ , \quad M_\ell = \begin{pmatrix} y_\nu \phi^{0*} \ , & y_e \phi^+ \\ -y_\nu \phi^{+*} \ , & y_e \phi^0 \end{pmatrix}$$

The electroweak η_w meson of the Standard Model

- (B+L) symmetry $\Psi \to \mathrm{e}^{i\alpha\Gamma_5/2}\Psi\ , \quad \Psi^\dagger \to \Psi^\dagger \mathrm{e}^{i\alpha\Gamma_5/2}\ , \quad \Gamma_5 = \begin{pmatrix} \gamma_5\ , & 0 \\ 0\ , & -\gamma_5 \end{pmatrix}$

- Despite the fermions are massive, the theory exhibits fermion zero-modes in full agreement with the index theorem

Krasnikov, Rubakov and Tokarev, ’79
Anselm and Johansen, ‘93

- Propagator in the instanton vacuum:

$$\frac{1}{\hat{\mathcal{D}} + i\mu} = \frac{P_0}{i\mu} + \Delta - i\mu\Delta^2 + \mathcal{O}(\mu^2)$$

$$\langle x|P_0|x\rangle = \Psi_0^\dagger(x-z)\Psi_0(x-z)$$

The electroweak η_w meson of the Standard Model

- The instanton gas is an excellent approximation to the non-perturbative vacuum because of the Higgs vev provides a natural infrared cutoff:

$$\langle \Psi^\dagger(x)\Psi(x)\rangle = \lim_{\mu\to 0}\frac{1}{i\mu}\int \frac{d^4z d\rho}{\rho^5}\; D(\rho)\langle x|P_0|x\rangle$$

$$\simeq -iv^3\left(\frac{2\pi}{\alpha_2}\right)^4 \mathrm{e}^{-\frac{2\pi}{\alpha_2}}$$

$$D(\rho) = \left(\frac{2\pi}{\alpha_2(\rho)}\right)^4 \mathrm{e}^{-\frac{2\pi}{\alpha_2(\rho)}-2\pi^2 v^2\rho^2} \quad \mu\rho$$

The electroweak η_w meson of the Standard Model

- Apply the Goldstone theorem with the anomaly contribution:

$$\delta(\Psi^+\Gamma_5\Psi) = 2i\Psi^+\Psi$$

$$\int d^4x \left\langle \left(i\mu\Psi^+\Gamma_5\Psi - \frac{\alpha}{4\pi} W\tilde{W} \right)(x)\ ,\ \Psi^+\Gamma_5\Psi(0) \right\rangle = i\langle\Psi^+\Psi\rangle \neq 0.$$

$$\int d^4x \langle W\tilde{W}(x)\ ,\ \Psi^+\Gamma_5\Psi\rangle_{p=0} \propto \langle\Psi^+\Psi\rangle \Longrightarrow \langle vac|W\tilde{W}|\eta\rangle = B(p) \neq 0$$

$$\langle\eta|\Psi^+\Gamma_5\Psi|vac\rangle = C(p) \neq 0$$

- Electroweak 3-form is Higgsed (hence θ_{ew} is unobservable):

$$FT\langle C^{(\mathrm{CS})}, C^{(\mathrm{CS})}\rangle = \frac{|B(0)|^2}{p^2 - m_\eta^2} + \ldots$$

Supergravity breaking via gravitino condensate

- N=1 pure Supergravity – $g_{\mu\nu},\ \psi_\mu\ + (auxiliary\ fields)$
- ALE gravitational instantons do not respect supersymmetry:

$$\psi_i = \psi_i^t + D_i\lambda\ (gauge\ \psi_0,\ \gamma^\mu\psi_\mu = 0)$$

$$\gamma^i D_i\lambda = 0$$

- The necessary condition for supercharges

$$Q_\alpha = -\oint dS_i D^i\lambda_\alpha\ ,\quad \bar{Q}^{\dot{\alpha}} = -\oint dS_i D^i\bar{\lambda}^{\dot{\alpha}}$$

is the existence of normalisable solutions to

$$D_i\lambda = D_i\bar{\lambda} = 0$$

- No normalisable spin ½ zero-mode solutions in ALE gravitational instantons -> supersymmetry must be broken!

Supergravity breaking via gravitino condensate

- Global U(1) R-symmetry

$$\psi_\mu \to \mathrm{e}^{i\alpha}\psi_\mu$$

is classically exact, broken by gravitational anomaly

$$\partial_\mu J^\mu_R \propto R\tilde{R}$$

- Gravitino condensate $\langle \psi_\mu \sigma^{\mu\nu} \psi_\nu \rangle \neq 0.$
- Emergent Goldstone supermultiplet (dilaton, R-axion, goldstino)

$$\phi,\ \eta_R,\ \lambda$$

Summary

- Non-conservation of an anomalous charge is always accompanied by the spontaneous breaking of the corresponding symmetry. The order parameter of such breaking is inherently related to the topological properties of vacuum states

- Within the Standard Model, we predict a new, yet to be discovered particle state, η_W – the (pseudo)Goldstone boson of spontaneously broken $U(1)_{B+L}$ invariance

- Within S-matrix formalism, the consistency of θ-vacua, mandates the extension of General Relativity to supergravity. The supersymmetry is broken by gravitino condensate, resulting in (pseudo)Goldstone supermultiplet of dilaton, R-axion and goldstino