Large extra dimensions from higher-dimensional inflation

I. Antoniadis

LPTHE, CNRS/Sorbonne University, Paris

and

HEP Research Unit, Faculty of Science, Chulalongkorn University

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Large hierarchies in particle physics and cosmology

Particle physics: why gravity appears so weak compared to other forces?

$$M_p/M_w \sim 10^{16}$$

Cosmology: why the Universe is so large compared to our causal horizon?

at least 10^{26} larger

Possible connection: through large extra dimensions

their existence is required in string theory

Large size extra dimensions $\Rightarrow$ low scale quantum gravity

$M_p = M_*(2\pi RM_*)^{d/2}$: $RM_* >> 1 \Rightarrow M_* << M_p$

Horizon problem can be explained by a period of inflation

expansion rate faster than speed of light

Extra dimensions may obtain large size by higher-dim inflation

Anchordoqui-IA-Lust '22

Compact dimensions and inflation

If 4d inflation occurs for fixed size extra dimensions $\Rightarrow$

$$H \lesssim 1/R \text{ (Higuchi bound)} \Rightarrow R < 10^{-16} \text{ cm for } H \gtrsim \text{TeV}$$

For larger sizes there are 2 possibilities:

$$\Rightarrow \begin{cases} \text{- } R \text{ gets a large value by a potential after the end of inflation} \\ \text{- extra dimensions expand with time} \\ \quad \text{from } R_0 \sim M_*^{-1} \text{ to } \sim R_0 \left(\frac{M_p}{M_*}\right)^{2/d} \text{ to explain the mass hierarchy} \end{cases}$$

Question: can uniform $(4+d)$ inflation relate the 2 hierarchies?

size of the observable universe to the observed weakness of gravity

compared to the fundamental (gravity/string) scale M_*

Anchordoqui-IA '93

4D decomposition of higher-dim metric

Start with $(4+d)$-dim gravity with d compact dimensions of size R:

$$S_{4+d} = \int [d^4x]\,[d^dy]\left(\frac{1}{2}M_*^{2+d}\mathcal{R}^{(4+d)} - \Lambda_{4+d}\right)$$

4D decomposition in the Einstein frame:

$$ds^2_{4+d} = \left(\frac{r}{R}\right)^d ds^2_4 + \left(\frac{R}{R_0}\right)^2 ds^2_d \qquad r \equiv \langle R\rangle_{\text{final}} \qquad \Rightarrow$$

internal volume normalised to $(2\pi R_0)^d$

$$S_4 = \int [d^4x]\left(\frac{1}{2}M_p^2\mathcal{R}^{(4)} - \frac{d(d+2)}{4}M_p^2\left(\frac{\partial R}{R}\right)^2 - (2\pi r)^{2d}\frac{\Lambda_{4+d}}{(2\pi R)^d}\right)$$

$M_p^2 = M_*^{2+d}(2\pi r)^d$; scalar potential: $V = \frac{M_p^2}{M_*^{2+d}}\frac{\Lambda_{4+d}}{(R/r)^d}$ [14]

maximal symmetric solution: $(4+d)$-dim de Sitter

$$ds^2_{4+d} = \hat{a}^2_{4+d}(\tau)(-d\tau^2 + d\vec{x}^2 + dy^2)$$

$$\hat{a}_{4+d}(\tau) = \frac{1}{H\tau} \qquad H^2 = \frac{2\Lambda_{4+d}}{(3+d)(2+d)M_*^{2+d}}$$

$$= \left(\frac{R_0}{R}\right)^d ds_4^2 + \left(\frac{R}{R_0}\right)^2 dy^2 \quad ; \quad ds_4^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$$

$$\nearrow$$

$$a\left(\tau_0 = H^{-1}\right) = 1\,; \quad a\left(\tau_{\mathrm{end}}\right) = (r/R_0)^{1+\frac{d}{2}} \quad \Rightarrow$$

$$a(\tau) = \left(\frac{R(\tau)}{R_0}\right)^{1+\frac{d}{2}} = \hat{a}^{1+\frac{d}{2}}(\tau) \quad ; \quad \frac{R(\tau)}{R_0} = \hat{a}(\tau) = a^{\frac{2}{2+d}}(\tau)$$

$\hat{N}$ **e-folds in** $(4+d)$**-dims** $\Rightarrow N = \left(1+\frac{d}{2}\right)\hat{N}$ **e-folds in 4D**

Large extra dimensions from higher-dim inflation

Solution to the Horizon problem: $N \gtrsim 30 - 60 \quad (N_{\rm min} \sim \ln \frac{M_I}{\rm eV})$

$$M_p^2 = (2\pi r)^d M_*^{2+d} \quad ; \quad r = R_0\, a^{\frac{2}{2+d}} = R_0\, e^{\frac{2N}{2+d}} \quad \Rightarrow$$

$$M_* = M_p e^{-\frac{dN}{2+d}} \lesssim 10^{13}\,\text{GeV}$$

Impose $M_* = M_p\, e^{-dN/(2+d)} \gtrsim 10$ TeV

$\gtrsim 10^8$ GeV for $d = 1 \quad (r \lesssim 30\,\mu\text{m})$

$\gtrsim 10^6$ GeV for $d = 2 \quad (r^{-1} \gtrsim 10$ keV$)$

$\Rightarrow$ **the horizon problem is solved for any** d

$$14\,(\ln 10) \geq \frac{d}{d+2} N \quad \Rightarrow \quad N \lesssim 32\left(1 + \frac{2}{d}\right)$$

$4D$ decomposition of $(4+d)$ dimensional de Sitter

Higher-dim proper time $\hat{t} = -H^{-1}\ln(H\tau)$

$$\hat{a}(\hat{t}) = e^{H\hat{t}} \quad \Rightarrow \quad a(\hat{t}) = e^{(1+d/2)H\hat{t}} \quad ; \quad R(\hat{t}) = R_0 e^{H\hat{t}}$$

However 4D proper time $t \neq \hat{t}$ since $a(\tau) = (H\tau)^{-(1+d/2)}$:

exponential expansion in higher-dims $\Rightarrow$ power low inflation in 4D

$$Ht = \frac{2}{d}(H\tau)^{-\frac{d}{2}} \quad \Rightarrow \quad a(t) = \left(\frac{d}{2}Ht\right)^{1+\frac{2}{d}} \; ; \; R(t) = R_0\left(\frac{d}{2}Ht\right)^{\frac{2}{d}}$$

$$d = 1 \quad : \quad a(t) \sim (Ht)^3 \quad ; \quad R(t) \sim R_0 (Ht)^2$$
$$d = 2 \quad : \quad a(t) = (Ht)^2 \quad ; \quad R(t) = R_0 Ht$$

Precision of CMB power spectrum measurement

Physical distances change from higher to 4 dims

equal time distance between two points in 3-space

$$d^{\tau}_{\rm phys}(x,x') = \underset{\text{co-moving distance}}{d(x,x')}\, a(\tau) = d(x,x')\,\hat{a}(\tau)\left(\frac{R}{R_0}\right)^{d/2} = \hat{d}^{\tau}_{\rm phys}(x,x')\frac{M_p(\tau)}{M_*}$$

precision of CMB data: angles $\lesssim 10$ degrees, distances $\lesssim$ Mpc (Gpc today)

Mpc $\to$ Mkm at $M_I \sim$ TeV with radiation dominated expansion

$$\left.\begin{array}{l} \times\text{TeV}/M_I \text{ at a higher inflation scale } M_I \sim M_* \\ \times M_*/M_P \text{ conversion to higher-dim distances} \end{array}\right\} \times \text{TeV}/M_p$$

$\simeq$ micron scale $\Rightarrow d = 1$ is singled out! with $M_* \sim 10^9$ GeV

$d > 1$: needs a period of 4D inflation for generating scale invariant density perturbations

Density perturbations from 5D inflation

inflaton (during inflation) $\simeq$ massless minimally coupled scalar in dS space

$\Rightarrow$ logarithmic growth at large distances (compared to the horizon H^{-1})

scale invariant (flat) power spectrum at low momenta

Equal time 2-point function in momentum space at late cosmic time

$$\langle \Phi^2(\hat{k},\tau)\rangle = \frac{\pi\tau}{4\,\hat{a}^3}\left[J_\nu^2(\hat{k}\tau)+Y_\nu^2(\hat{k}\tau)\right] \quad ; \quad \nu=\frac{D-1}{2}=2$$

$$\tau\to 0: \quad \simeq \frac{4}{\pi}\frac{H^3}{(\hat{k}^2)^2} \quad ; \quad \hat{k}^2=k^2+n^2/R_0^2$$

2-point function on the Standard Model brane (located at $y=0$):

$$\sum_n \langle \Phi^2(\hat{k},\tau)\rangle_{\tau\to 0} \simeq \frac{2R_0H^3}{k^2}\left(\frac{1}{k}\coth(\pi kR_0)+\frac{\pi R_0}{\sinh^2(\pi kR_0)}\right)$$

CMB power spectrum from 5D inflation

physical wavelength:

$$\lambda = 2\pi \frac{a}{k} = \left(\frac{R}{R_0}\right)^{1/2} \hat{\lambda} \quad ; \quad \hat{\lambda} = 2\pi \frac{\hat{a}}{k} \text{ in 5D}$$

$\Rightarrow \quad \pi k R_0 = 2\pi^2 R/\hat{\lambda} > 1$ for $\hat{\lambda} \lesssim$ micron $\quad (\lambda \lesssim$ km$)$

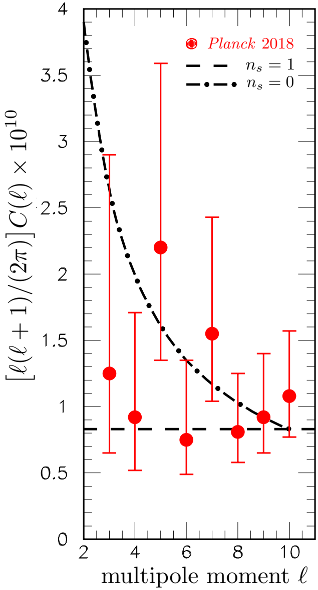
Amplitude of the power spectrum: $\mathcal{A} = \frac{k^3}{2\pi^2} \langle \Phi^2(k,\tau) \rangle_{y=0}$ [12]

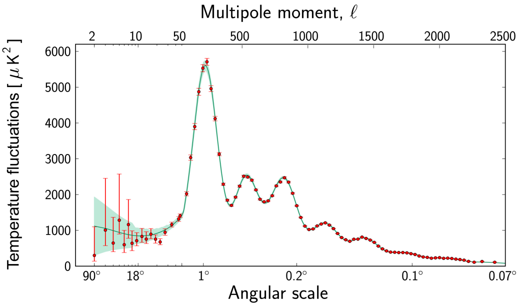
- $\pi k R_0 > 1$ ('small' wave lengths) $\Rightarrow \mathcal{A} \sim \frac{H^2}{\pi^2} R_0 H \qquad n_s \simeq 1$
- $\pi k R_0 < 1$ ('large' wave lengths) $\Rightarrow \mathcal{A} \simeq \frac{2H^3}{\pi^3 k} \qquad n_s \simeq 0$

summation over n is crucial for scale invariance

it amounts a 'tower' of 4D inflatons

Large-angle CMB power spectrum





Detailed computation of primordial perturbations:

IA-Cunat-Guillen '23

5D: inflaton + metric (5 gauge invariant modes) $\Rrightarrow$

4D: 2 scalar modes (inflaton + radion), 2 tensor modes, 2 vector modes [10]

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{1}{3\varepsilon}\mathcal{A}\left[\left(\frac{k}{\hat{a}H}\right)^{2\delta-5\varepsilon} + \varepsilon\left(\frac{k}{\hat{a}H}\right)^{-3\varepsilon} \times \begin{cases} \frac{5}{24} & R_0 k >> 1 \\ \frac{1}{3} & R_0 k << 1 \end{cases}\right]$$

$$\mathcal{P}_{\mathcal{T}} \simeq \frac{4H^2}{\pi^2}\left(\frac{k}{\hat{a}H}\right)^{-3\varepsilon} \times \begin{cases} R_0 H & R_0 k >> 1 \\ \frac{2H}{\pi k} & R_0 k << 1 \end{cases} \qquad r = 24\varepsilon$$

$$\mathcal{P}_{\mathcal{V}} \simeq \frac{4R_0 H^3}{\pi^2}\left(\frac{k}{\hat{a}H}\right)^{-3\varepsilon} \times \begin{cases} 1 & R_0 k >> 1 \\ \frac{\pi^3}{45}(R_0 k)^3 & R_0 k << 1 \quad S^1/Z_2\,(n \neq 0) \end{cases}$$

$$\mathcal{P}_{\mathcal{S}} \simeq \frac{9\varepsilon^2}{16}\mathcal{P}_{\mathcal{R}} \quad \text{entropy} \Rrightarrow \beta_{\text{isocurvature}} = \frac{\mathcal{P}_{\mathcal{S}}}{\mathcal{P}_{\mathcal{R}} + \mathcal{P}_{\mathcal{S}}} \simeq \frac{9\varepsilon^2}{16} < 0.038\,\text{exp}$$

slow-roll parameters: $\varepsilon = -\frac{\dot{H}}{H^2}$; $\delta = \varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon} \simeq \eta - \varepsilon$

End of inflation

5D inflation can be implemented in the framework of Dark Dimension

Montero-Vafa-Valenzuela '22

5D inflaton should couple to the brane for SM particle production

e.g. via a 'Yukawa' coupling suppressed by the bulk volume $\sim y/(RM_*)^{d/2}$

$$\left(\Gamma^{\Phi}_{\rm SM} \sim y^2 \frac{m_\Phi}{(RM_*)^d}\right) \quad > \quad \left(\Gamma^{\varphi}_{\rm grav} \sim \frac{m^4_\Phi}{M^3_*}\right)$$

$$\Rightarrow \qquad m_\Phi < M_* \left(\frac{M_*}{M_p}\right)^{2/(2+d)} \simeq 1\,\mathrm{TeV} \quad (d=1)$$

Also: specific realisation of the Dynamical Dark Matter framework

internal graviton decays for small violation of KK-momentum conservation

Gonzalo-Montero-Obied-Vafa '22

Radion potential

5D cosmological constant at the minimum of the inflaton potential

$\Rightarrow$ runaway radion potential: [21]

$$V_0 = 2\pi r^2 \frac{\Lambda_5^{\rm min}}{R}\,; \qquad (\Lambda_5^{\rm min})^{1/5} \lesssim 100\,{\rm GeV} \quad \text{(Higuchi bound)}$$

canonically normalised radion: $\phi = \sqrt{3/2}\ln(R/r)$ [4]

$\Rightarrow$ exponential quintessence-like form $V_0 \sim e^{-\alpha\phi}$ with $\alpha \simeq 0.8$

just at the allowed upper bound: Barreiro-Copeland-Nunes '00

Alternatively, radion could be stabilised

During inflation: $R(t) \sim t^2 \Rightarrow$ radion $\phi \sim \ln t$ and $\dot{\phi} \sim 1/t$

it is therefore expected to oscillate around the minimum if it exists

Radion stabilisation at the end of 5D inflation

Anchordoqui-IA '23

Potential contributions stabilising the radion:

$$V = \left(\frac{r}{R}\right)^2 \hat{V} + V_C \quad ; \quad \hat{V} = 2\pi R \Lambda_5^{\min} + T_4 + 2\pi \frac{K}{R}$$

T_4: 3-branes tension, K: kinetic gradients, V_C: Casimir energy

$\uparrow$ Arkani-Hamed, Hall, Tucker-Smith, Weiner '99

Radion mass m_R: $\sim$ eV (m_{KK}) to 10^{-30} eV (m_{KK}^2/M_p) depending on K

- $K \sim M_*$, all 3 terms of $\hat{V}$ of the same order, V_C negligible [21]

  tune $\Lambda_4 \sim 0_+ \Rightarrow m_R \lesssim m_{KK} \sim$ eV

- K negligible, all 3 remaining terms of the same order [22]

  $\Rightarrow$ minimum is driven by a +ve $V_C = \dfrac{2\pi r^2}{32\pi^7 R^6}(N_F - N_B)$

  Arkani-Hamed, Dubovsky, Nicolis, Villadoro '07

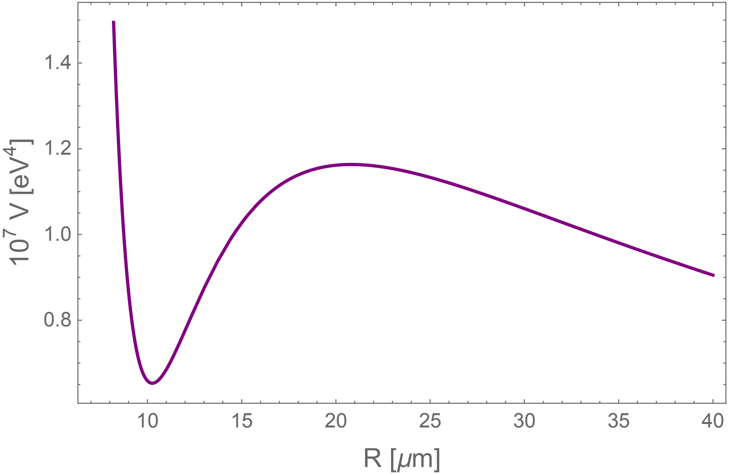
  no tuning of Λ_4 but $\Lambda_5^{\min}$ should be order $(\text{subeV})^5$ [20]

Casimir potential

$$V_C = 2\pi R \left(\frac{r}{R}\right)^2 \mathrm{Tr}(-)^F \rho(R,m) \quad m : 5D \text{ mass}$$

$$\rho(R,m) = -\sum_{n=1}^{\infty} \frac{2m^5}{(2\pi)^{5/2}} \frac{K_{5/2}(2\pi Rmn)}{(2\pi Rmn)^{5/2}} \begin{cases} mR \to \infty & \text{exp suppressed} \\ \\ mR \to 0 & 1/R^5 \end{cases}$$

Example of Radion stabilisation potential



$(\Lambda_5^{\min})^{1/5} = 25\ \mathrm{meV},\ |T_4|^{1/4} = 27\ \mathrm{meV},\ N_F - N_B = 7$

$N_F = 12$ (3 bulk R-neutrinos) $N_B = 5$ (5D graviton)

Cosmic discrepancies and Hubble tension

Anchordoqui-I.A.-Lust '23, AAL-Noble-Soriano '24

5σ tension between global and local measurements

$H_0 = 67.4 \pm 0.5$ km/s/Mpc   Planck data

$H_0 = 73.04 \pm 1.04$ km/s/Mpc SH0ES supernova

This tension can be resolved if Λ changes sign around redshift $z \simeq 2$

Akarsu-Barrow-Escamilla-Vasquez '20, AV-Di Valentino-Kumar-Nunez-Vazquez '23

AdS$\rightarrow$dS transition is hard to implement due to a swampland conjecture:

non-SUSY AdS vacua are at infinite distance in moduli space

However it could happen due to quantum tunnelling effects

AdS→dS transition due to false vacuum decay in 5D

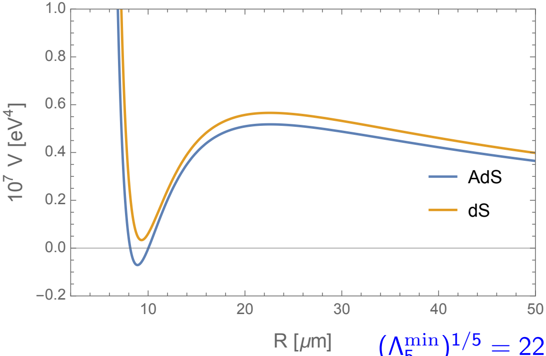
5D scalar at a false vacuum with light mass (lighter than $R_{\rm max}^{-1}$)

$N_F - N_B = 6 \Rrightarrow$ AdS vacuum

decay to a (almost degenerate $\delta\epsilon < \Lambda$) true vacuum with heavy mass

$N_F - N_B = 7 \Rrightarrow$ dS vacuum

slow transition at $z \simeq 2$



$(\Lambda_5^{\rm min})^{1/5} = 22.6$ meV, $|T_4|^{1/4} = 24.2$ meV

Conclusions

Large extra dimensions from higher dim inflation

- connect the weakness of gravity to the size of the observable universe
- scale invariant density fluctuations from 5D inflation
- radion stabilization

smallness of some physical parameters might signal

a large distance corner in the string landscape of vacua

such parameters can be the scales of dark energy and SUSY breaking

mesoscopic dark dimension proposal: interesting phenomenology

neutrino masses, dark matter, cosmology, SUSY breaking

Stabilisation neglecting V_C

minimum with $\hat{V}' = \hat{V} = 0 \Rrightarrow$

$$r = \left(\frac{K}{\Lambda_5^{\min}}\right)^{1/2} \;; \quad T_4 = -4\pi(K\Lambda_5^{\min})^{1/2}\,; \quad V'' = \hat{V}''\,|_{R=r} = 4\pi\frac{K}{r^3}$$

all terms of $V_{\min}$ of order $|T_4|$ with $T_4 < 0$ and $m_\phi^2 = \frac{4}{9}\frac{|T_4|}{M_p^2}$

maximum: $R_{\max} = 3r \quad ; \quad V_{\max} = \frac{2}{27}|T_4|$

$V_{\min} < V_{\max}$ satisfying the Higuchi bound $\Rrightarrow m_\phi \leq 3/r$

$$\left.\begin{array}{l}\text{experimental bounds on new forces implying } m_\phi \gtrsim 0.1 \text{ eV} \\ \text{Higuchi bound on } \Lambda_5 \text{ [14]}\end{array}\right\} \Rrightarrow$$

$K^{1/3} \sim M_*\,; \quad (\Lambda_5^{\min})^{1/5} \sim 100$ GeV$\,; \quad |T_4|^{1/4} \sim 1$ TeV [15]

Stabilisation with $K = 0$

$R_{\max} = -\frac{T_4}{\pi \Lambda_5^{\min}} \quad \Rightarrow \quad T_4 < 0$

A minimum can be generated at lower R from V_C if $N_F - N_B > 0$ [15]

$N_B = 5$ (5D graviton) $\Rightarrow$ need at least 2 5D-fermions with masses $\lesssim R_{\max}^{-1}$

for instance 3 R-handed neutrinos: $N_F = 12$

minimum at $V \simeq 0_+$ and $< R > \sim$ micron implies as before

all 3 terms of $V_{\min}$ same order: $\left(\Lambda_5^{\min}\right)^{1/5} \sim |T_4|^{1/4} \sim V_C^{1/4} \sim$ subeV

Bulk masses $\mu_i \gtrsim \Delta m_{32} > \Delta m_{21}$ avoid strict bounds from ν-oscillations

$(\mu_i = 0 \Rightarrow r \lesssim 0.2\,\mu\mathrm{m})$ Anchordoqui-IA-Cunat '23

$T_4 = 0$: $R_{\max}^{-1} \sim (\Lambda_5^{\min})^{1/5}$ from $V_0 + V_C\,|_{\rm bos} \rightarrow$ min from $V_C\,|_{\rm fer}$

$N_B = 5 \Rightarrow$ maximum at ~ 10 microns