# Aspects of massive gauge fields

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- The principle of continuity

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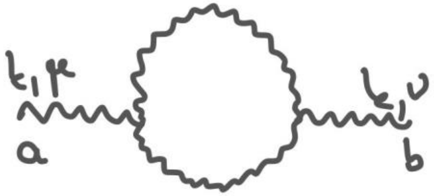


D1


D2

$$\mathrm{Im}\,(D2) = 2\ \mathrm{Im}\,(D1)|_{m\to 0}$$

# The puzzle of masless limit

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$$r_V = \left(\frac{GM}{m_g^4}\right)^{1/5}$$

- The Vainshtein-Kriplovich conjecture

*"...it appears highly probable that outside perturbation theory, a continuous zero-mass limit exists, and the theory is renormalizable."*

# THE GOAL

# THE PLAN

- **THE LANGUAGE**

# The Plan

- The Language
- Massive Yang-Mills Theory

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- The language
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- Other gauge fields
- Non-minimal coupling with the Proca fields

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# THE BASICS

WHY?

Lorentz-like gauges do not fix the gauge uniquely
and thus lead to the fictitious modes.

$$W_\mu \to \tilde{W}_\mu = W_\mu + \partial_\mu \lambda \qquad \phi \to \tilde{\phi} = \phi - \lambda.$$

Coulomb gauge

$$\partial_i \tilde{W}_i = 0 \quad \to \quad \lambda = -\partial_i W_i$$

$$W_\mu = \phi = 0 \quad \to \quad \lambda = 0$$

Lorentz gauge

$$\partial_\mu \tilde{W}^\mu = 0 \quad \to \quad \partial^2 \lambda = -\partial_\mu W^\mu$$

$$W_\mu = \phi = 0 \quad \to \quad \partial^2 \lambda = 0$$

$$\lambda_k = C_1 e^{ikt} + C_2 e^{-ikt}$$

# The curiosity

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## MANIFESTLY COVARIANT APPROACH

$$h_{\mu\nu} = l^T_{\mu\nu} + \partial_\mu A^T_\nu + \partial_\nu A^T_\mu + \left(\partial_\mu \partial_\nu - \frac{1}{4}\partial^2 \eta_{\mu\nu}\right)\mu + \frac{1}{4}\lambda\eta_{\mu\nu}$$

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## MANIFESTLY NON-COVARIANT APPROACH

$$h_{00} = 2\phi$$
$$h_{0i} = B_{,i} + S_i$$
$$h_{ij} = 2\psi\delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + h^T_{ij}$$
$$F_{i,i} = 0, \quad S_{i,i} = 0 \quad h^T_{ij,i} = 0, \quad h^T_{ii} = 0,$$

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## LOOK INTO

A. Hell, D. Lüst & G. Zoupanos,
On the degrees of freedom of R2 gravity in flat spacetime
JHEP 02 (2024) 039

# The first case

Massive Yang-Mills theory

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + m^2\,\mathrm{Tr}(A_\mu A^\mu)$$

BASED ON

A. Hell, The strong couplings of massive Yang-Mills Theory JHEP 03 (2022) 167

# The main ingredients

## Field decomposition

$$(A_0, A_i)$$

$$A_i = \zeta A_i^T \zeta^\dagger + \frac{i}{g}\zeta_{,i}\zeta^\dagger$$

$$A^T_{i,i} = 0$$

$$\zeta = e^{-ig\chi}$$

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$$A_i = A_i^a T^a$$

$$\begin{aligned} A_i^a =& A_i^{Ta} + \chi_{,i}^a \\ & - g\varepsilon^{abc}(A_i^{Tb}\chi^c + \frac{1}{2}\chi_{,i}^b\chi^c) \\ & - \frac{g^2}{2}\varepsilon^{fab}\varepsilon^{fcd}(\chi^b A_i^{Tc}\chi^d + \frac{1}{3}\chi^b\chi_{,i}^c\chi^d). \end{aligned}$$

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$$S = \frac{1}{2}\int d^4x\,(\phi_{,\mu}\phi^{,\mu} - m^2\phi^2) \qquad \omega_k^2 = k^2 + m^2$$

$$\delta\phi_L^2 = \langle 0|\,\phi(\vec{x},t)\,\phi(\vec{y},t)\,|0\rangle \sim \left.\frac{k^3}{\omega_k}\right|_{k\sim\frac{1}{L}} \qquad L = |\vec{x} - \vec{y}|$$

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## Quantum fluctuations

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$$\delta\phi_L\big|_{k^2 \gg m^2} \sim \frac{1}{L}$$

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## The constraint

$$\left(-\Delta + m^2\right) A_0 = -\,\dot{A}_{i,i} + ig[\dot{A}_i, A_i]$$

$$+\, ig\left(2\,[A_i, A_{0,i}] + [A_{i,i}, A_0]\right)$$

$$+\, g^2\,[A_i, [A_0, A_i]]$$

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$$\mathcal{L}_0 = \mathrm{Tr}\left[-\chi\left(\partial^2 + m^2\right)\frac{-\Delta m^2}{-\Delta + m^2}\chi - A_i^T\left(\partial^2 + m^2\right)A_i^T\right]$$

$$\mathcal{L}_{int} \sim \mathrm{Tr}\left\{-2igm^2 A_i^T \chi\chi_{,i} + \frac{m^2 g^2}{6}\left(\chi_{,\mu}\chi\chi^{,\mu}\chi - \chi_{,\mu}\chi^{,\mu}\chi^2\right)\right\}$$

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$$\delta\chi_L \sim \frac{1}{mL} \qquad \delta A_L^T \sim \frac{1}{L}$$

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$$\sim \frac{g}{L^4}$$

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# THE MASSIVE YANG-MILLS THEORY

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + m^2\,\mathrm{Tr}(A_\mu A^\mu)$$

## ☼ THE CONSTRAINT

$$\left(-\Delta + m^2\right) A_0 = -\,\dot{A}_{i,i} + ig[\dot{A}_i, A_i]$$

$$+\, ig\left(2\left[A_i, A_{0,i}\right] + \left[A_{i,i}, A_0\right]\right)$$

$$+\, g^2\left[A_i, \left[A_0, A_i\right]\right]$$

$$\mathcal{L}_0 = \mathrm{Tr}\left[-\chi\left(\partial^2 + m^2\right)\frac{-\Delta m^2}{-\Delta + m^2}\chi - A_i^T\left(\partial^2 + m^2\right)A_i^T\right]$$

$$\mathcal{L}_{int} \sim \mathrm{Tr}\left\{-2igm^2 A_i^T \chi\chi_{,i} + \frac{m^2 g^2}{6}\left(\chi_{,\mu}\chi\chi^{,\mu}\chi - \chi_{,\mu}\chi^{,\mu}\chi^2\right)\right\}$$

$$\sim \frac{g}{L^4} \qquad\qquad \sim \frac{g^2}{(mL)^2\,L^4}$$

## ☼ QUANTUM FLUCTUATIONS

$$\delta\chi_L \sim \frac{1}{mL} \qquad\qquad \delta A_L^T \sim \frac{1}{L}$$

# The Massive Yang-Mills Theory

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$$(\partial^2 + m^2)\chi \sim \frac{g^2\chi^3}{L^2}$$

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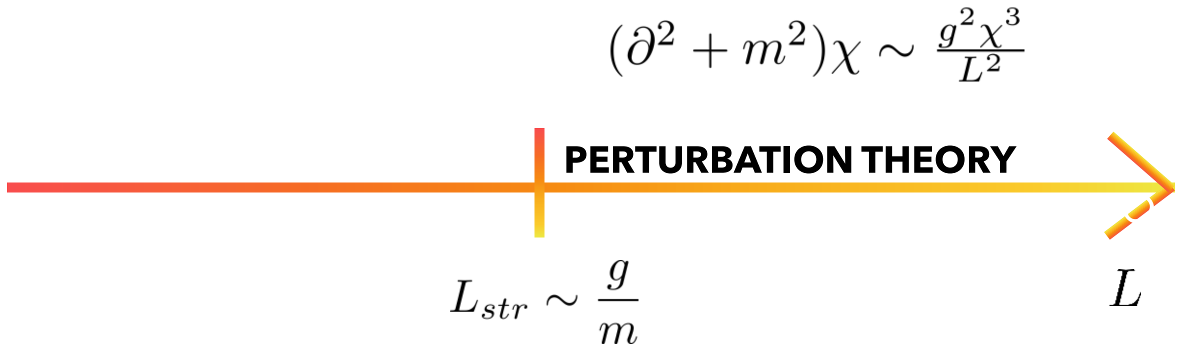

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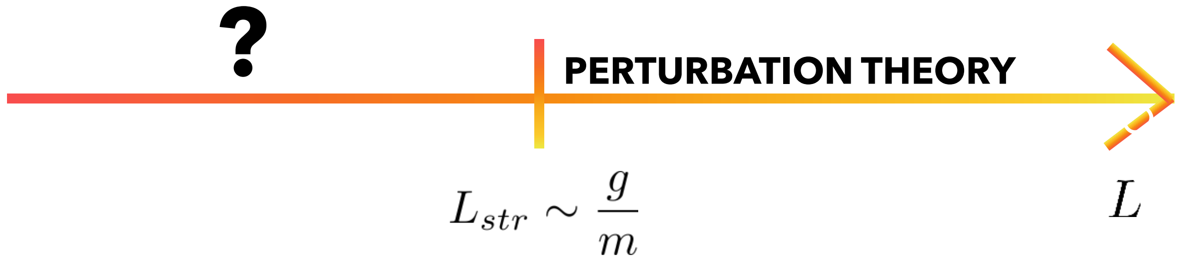

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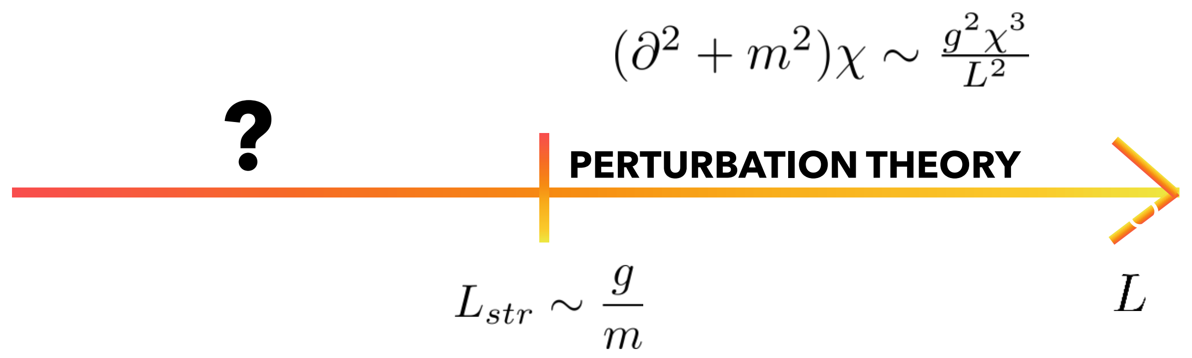



## ☼ Beyond the Strong Coupling Scale

$$\mathcal{L} \sim \mathrm{Tr}\left[\frac{m^2}{g^2}\zeta^\dagger_{,\mu}\zeta^{,\mu} - A_i^T\left(\partial^2 + m^2\right)A_i^T\right] - \frac{2im^2}{g}\mathrm{Tr}\left(A_i^T\zeta^\dagger\zeta_{,i}\right)$$

$$A_i^{T(1)} \sim -i\frac{m^2}{g}\zeta^\dagger\zeta_{,i} \sim \frac{g}{L^3}\frac{L}{L_{str}}$$

What about other gauge fields?

BASED ON

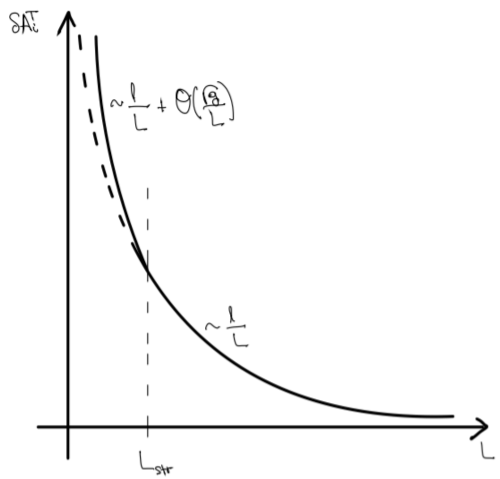
A. Hell, On the duality of massive Kalb-Ramond and Proca fields, JCAP 01 (2022) 01, 056

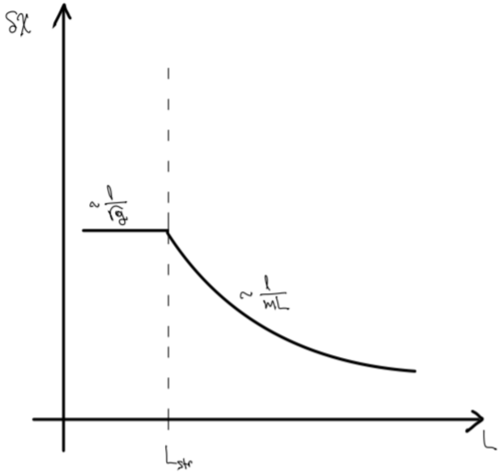
# PROCA THEORY

$$\mathcal{L}_P = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu + \frac{g^2}{4}\left(A_\mu A^\mu\right)^2$$

$$\mathcal{L}_0 = -\frac{1}{2}\chi(\partial^2 + m^2)\frac{m^2(-\Delta)}{-\Delta + m^2}\chi - \frac{1}{2}A_i^T(\partial^2 + m^2)A_i^T$$

$$\mathcal{L}_{int} \sim \frac{g^2}{4}\left(\chi_{,\mu}\chi^{,\mu}\right)^2 - g^2\chi_{,\mu}\chi^{,\mu}\chi_{,i}A_i^T$$



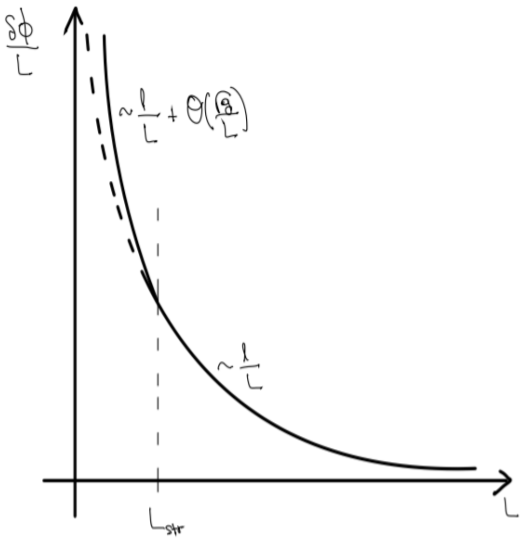



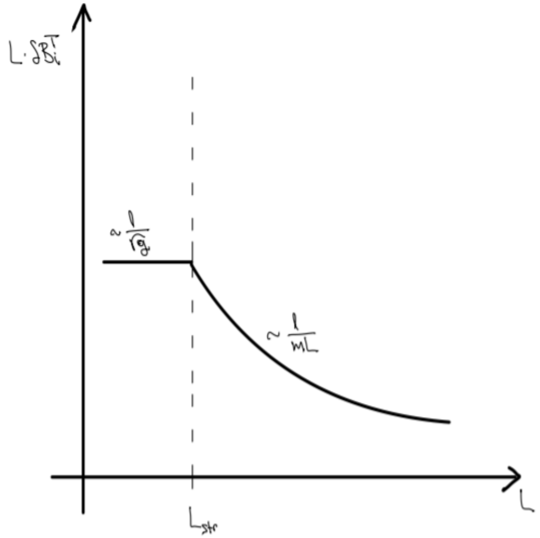

# KALB – RAMOND THEORY

$$\mathcal{L}_{KB} = \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{m^2}{4}B_{\mu\nu}B^{\mu\nu} + \frac{g^2}{16}\left(B_{\mu\nu}B^{\mu\nu}\right)^2$$

$$\mathcal{L}_0 = -\frac{1}{2}B_i^T(\partial^2 + m^2)\frac{m^2}{-\Delta + m^2}B_i^T - \frac{1}{2}\phi_n(\partial^2 + m^2)\phi_n$$

$$\mathcal{L}_{int} \sim g^2\left(B^T\right)^4 + g^2\left(B^T\right)^3\phi_n$$



# Dual Theories

- F. Quevedo & C. A. Trugenberger (1997)
- H. Kawai (1981)
- M. Shifman and A. Yung (2018)
- R. D'Auria and S. Ferrara (2005)
- L. Heisenberg and G. Trenkler (2020)
- A. Smailagic and E. Spallucci (2001)
- C. Markou, F. J. Rudolph and A. Schmidt-May (2019)
- A. Aurilia, P. Gaete, J. A. Helayel-Neto and E. Spallucci (2017)
- I. L. Buchbinder, E. N. Kirillova, and N. G. Pletnev (2008)
- H. Casini, R. Montemayor and L. F. Urrutia (2002)
- D. Dalmazi and R. C. Santos (2011)
- Y. M. Zinoviev (2005)
- S. M. Kuzenko and K. Turner (2021)
- J. Louis and A. Micu (2002)
- M. Gunaydin, S. McReynolds and M. Zagermann (2006)
- G. B. De Gracia (2017)

• • •

# In principle...

Degrees of freedom that appear upon modifying
the theory become strongly coupled when the parameter
characterizing the modification is very small (tends to 0).

# In principle...

BUT

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BUT

How strongly coupled the theory is?

# Non-minimal couplings

$$S = S_{EH} + S_P + S_{nmin},$$

$$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g} R$$

$$S_P = \int d^4x \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} A_\mu A^\mu \right)$$

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## UPSHOT

$$\partial^2 h_{ij}^T \sim -\frac{\beta}{M_{pl}^2} P_{ijkl}^T \partial^2 \left( \chi_{,k} \chi_{,l} \right)$$

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# Non-minimal couplings

## DISFORMAL TRANSFORMATION

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} - \frac{\beta}{M_{pl}^2} A_\mu A_\nu$$

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$$\begin{aligned} A_i^a =& A_i^{Ta} + \chi_{,i}^a \\ & - g\varepsilon^{abc}(A_i^{Tb}\chi^c + \frac{1}{2}\chi_{,i}^b\chi^c) \\ & - \frac{g^2}{2}\varepsilon^{fab}\varepsilon^{fcd}(\chi^b A_i^{Tc}\chi^d + \frac{1}{3}\chi^b\chi_{,i}^c\chi^d). \end{aligned}$$

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$$g_{\mu\nu} = \tilde{g}_{\mu\nu} - \frac{\beta}{M_{pl}^2} A_\mu A_\nu, \qquad \tilde{g}_{\mu\nu} = a^2\eta_{\mu\nu}$$

# Non-minimal couplings

## THE RUNAWAY

2024 Capanelli et al.

$$ds^2 = a^2(\eta)\eta_{\mu\nu}dx^\mu dx^\nu$$

$$S_\chi \sim \int d\eta d^3k \frac{1}{2}\left(\chi'_{n(-k)}\chi'_{nk} - \frac{M_S^2 k^2}{M_T^2}\chi_{nk}\chi_{n(-k)}\right)$$

$$M_T^2 = a^2\left[m^2 + \alpha R - \beta\left(3H^2 - \frac{1}{2}R\right)\right]$$

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## LOOK INTO

A. Hell, Unveiling the inconsistency of Proca theory with non-minimal coupling to gravity, arXiv: 2404.02972

# To take home

THEORY

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THEORY

MODIFIED THEORY

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THEORY

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⟶ NEW DOF

# To take home

THEORY

MODIFIED THEORY

→ NEW DOF

STRONG COUPLING

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**Non-minimal Ricci tensor**

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**Thank You !**

# C: Coupling with the Ricci scalar

- Most important interaction

$$\mathcal{L}_{\alpha int} = -\alpha \left(\Delta\phi + 3\ddot{\psi} - 2\Delta\psi\right) A_\mu A^\mu$$

$$\rightarrow \qquad -m^2(-\partial^2 + m^2)\chi \sim \frac{3\alpha^2}{M_{pl}^2}\partial_\mu \left[\chi^{,\mu}\partial^2(\chi_{,\alpha}\chi^{,\alpha})\right]$$

$$\rightarrow \qquad L_\alpha \sim \left(\frac{\alpha}{M_{pl}m^2}\right)^{1/3}$$

# C: RESOLUTION VIA THE DISFORMAL COUPLING

☼ Tensor modes:

$$M_{pl}^2 \partial^2 h_{ij}^T \sim -2(A_{k,i}^T - A_{i,k}^T)(A_{k,j}^T - A_{j,k}^T) + 2\dot{A}_i^T \dot{A}_j^T + \mathcal{O}\left( (h_{ij}^T)^3 \frac{M_{pl}^2}{L^2} \right)$$

☼ ITS OK!

☼ Longitudinal modes:

$$m^2(-\partial^2 + m^2)\chi^{(2)} \sim \frac{\beta^2}{M_{pl}^2} \mathcal{O}\left(\Delta\chi \dot{\chi}_{,i} \dot{\chi}_{,i}\right) \quad \rightarrow \quad \chi^{(2)} \sim \frac{\beta^2}{M_{pl}^2 m^5 L^7}$$

$$\rightarrow \quad L_{\beta str} \sim \left( \frac{\beta}{M_{pl} m^2} \right)^{1/3}$$

# C: Coupling with the Ricci tensor

- Most important interaction

$$\mathcal{L}_{\beta int} = \frac{\beta}{4}\partial_i\chi\partial_j\chi\partial^2 h^T_{ij}$$

$$(-\partial^2 + m^2)m^2\chi \sim -\frac{\beta}{2}\partial_i\left[\chi_{,j}\partial^2 h^T_{ij}\right] - m^2\partial_i\left(h^T_{ij}\chi_{,j}\right)$$

$$\rightarrow$$

$$\partial^2 h^T_{ij} \sim -\frac{\beta}{M^2_{pl}}P^T_{ijkl}\partial^2\left(\chi_{,k}\chi_{,l}\right)$$

$$\rightarrow \quad h^{(1)}_{ij} \sim \frac{\beta}{M^2_{pl}m^2L^4} \quad \rightarrow$$

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# C: RESOLUTION VIA THE DISFORMAL COUPLING

$$S_{Pnmin} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} - \frac{m^2}{2}g^{\mu\nu}A_\mu A_\nu - \frac{1}{2}\alpha R g^{\mu\nu}A_\mu A_\nu - \frac{1}{2}\beta R^{\mu\nu}A_\mu A_\nu\right)$$

$$ds^2 = a^2(\eta)\eta_{\mu\nu}dx^\mu dx^\nu$$

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# A: mYM

$$\left(-\Delta + m^2\right) A_0 = - \dot{A}_{i,i} + ig[\dot{A}_i, A_i] + ig\left(2\left[A_i, A_{0,i}\right] + \left[A_{i,i}, A_0\right]\right) + g^2\left[A_i, \left[A_0, A_i\right]\right]$$

$$A_0 = \zeta \frac{1}{D}\left(-\frac{i}{g} m^2 \zeta^\dagger \dot{\zeta} + ig\left[\dot{A}_i^T, A_i^T\right]\right)\zeta^\dagger + \frac{i}{g}\dot{\zeta}\zeta^\dagger$$

$$\frac{1}{D} = \frac{1}{-\Delta + m^2 - 2ig[A_i^T, \partial_i \bullet\ ] + g^2[A_i^T, [A_i^T, \bullet\ ]]}$$

$$\mathcal{L}_0^T = tr\left(\dot{A}_i^T \dot{A}_i^T - A_{i,j}^T A_{i,j}^T - m^2 A_i^T A_i^T\right)$$

$$\mathcal{L}_0^\chi = -\frac{m^2}{g^2} tr\left[\zeta^\dagger \dot{\zeta} \frac{-\Delta}{-\Delta + m^2}\left(\zeta^\dagger \dot{\zeta}\right) - \zeta^\dagger \zeta_{,i} \zeta^\dagger \zeta_{,i}\right]$$

$$\mathcal{L}_{int}^{T\chi} = \frac{2im^2}{g} tr\left\{-A_i^T \zeta^\dagger \zeta_{,i} + m^2 \zeta^\dagger \dot{\zeta} \frac{1}{D}\left[A_i^T, \frac{1}{-\Delta + m^2}\partial_i(\zeta^\dagger \dot{\zeta})\right]\right\}$$

$$-m^2 tr\left\{\zeta^\dagger \dot{\zeta}\frac{1}{D}[\dot{A}_i^T, A_i^T] + [\dot{A}_i^T, A_i^T]\frac{1}{D}(\zeta^\dagger \dot{\zeta}) + m^2 \zeta^\dagger \dot{\zeta}\frac{1}{D}\left[A_i^T, \left[A_i^T, \frac{1}{-\Delta + m^2}(\zeta^\dagger \dot{\zeta})\right]\right]\right\}$$

$$\mathcal{L}_{int}^T = tr\left\{-2igA_i^T A_j^T\left(A_{j,i}^T - A_{i,j}^T\right) + g^2\left[\dot{A}_i^T, A_i^T\right]\frac{1}{D}\left[\dot{A}_j^T, A_j^T\right] + g^2\left(A_i^T A_j^T A_i^T A_j^T - A_i^T A_i^T A_j^T A_j^T\right)\right\}$$

# A: mYM

$$\mathcal{L}_0^\chi \sim \frac{m^2}{g^2} tr\left(\zeta^\dagger_{,\mu}\zeta^{,\mu}\right).$$

$$\zeta = e^{-ig\chi}.$$

$$\zeta = \begin{pmatrix} \zeta_2^* & \zeta_1 \\ -\zeta_1^* & \zeta_2 \end{pmatrix}.$$

$$|\zeta_1|^2 + |\zeta_2|^2 = 1.$$

$$\zeta_1 = \cos(g\sigma)e^{ig\theta_1}$$

$$\zeta_2 = \sin(g\sigma)e^{ig\theta_2}.$$

$$\mathcal{L}_0^\chi = \frac{1}{2}\left[4m^2\partial_\mu\sigma\partial^\mu\sigma + f^2(\sigma)\partial_\mu\theta_1\partial^\mu\theta_1 + p^2(\sigma)\partial_\mu\theta_2\partial^\mu\theta_2\right]$$

$$f^2(\sigma) = 4m^2\cos^2(g\sigma) \qquad p^2(\sigma) = 4m^2\sin^2(g\sigma).$$

$$\sigma_n = 2m\sigma \qquad \partial_\mu\theta_{1n} = f(\sigma)\partial_\mu\theta_1 \qquad \partial_\mu\theta_{2n} = p(\sigma)\partial_\mu\theta_2.$$

$$\mathcal{L}_0^\chi = \frac{1}{2}\left[\partial_\mu\sigma_n\partial^\mu\sigma_n + \partial_\mu\theta_{1n}\partial^\mu\theta_{1n} + \partial_\mu\theta_{2n}\partial^\mu\theta_{2n}\right].$$

$$\delta\sigma_L \sim \frac{1}{2g}\frac{k}{k_{str}} \qquad \delta\theta_{1L} \sim \frac{1}{2g}\frac{k}{k_{str}}, \qquad \delta\theta_{2L} \sim \frac{1}{2g}\frac{k}{k_{str}}$$

# A: mYM

$$\Omega_0 = \zeta^\dagger \dot{\zeta}, \qquad \Omega_0 \sim \frac{g}{mL^2}.$$

$$\partial_\mu \Omega_0 \sim \frac{g^2}{(mL)^2 L^2}$$

$$-\frac{2im^2}{g} tr\left(A_i^T \zeta^\dagger \zeta_{,i}\right) \sim g \frac{L}{L_{str}} \frac{1}{L^4}$$

$$tr\left\{\frac{2im^4}{g} \Omega_0 \frac{1}{\Delta}\left[\dot{A}_i^T, \frac{1}{\Delta}\left(\Omega_{0,i}\right)\right]\right\} \sim g^3 \left(\frac{L}{L_{str}}\right)^5$$

$$tr\left\{2m^2 \Omega_0 \frac{1}{\Delta}\left[\dot{A}_i^T, A_i^T\right]\right\} \sim g^2 \left(\frac{L}{L_{str}}\right)^2$$

$$A_i^{T(1)} \sim -i \frac{m^2}{g} \zeta^\dagger \zeta_{,i} \sim \frac{g}{L^3} \frac{L}{L_{str}}$$

# The comparison

## Proca theory

$A_0$ $\qquad A_i = A_i^T + \chi_{,i}, \qquad A_{i,i}^T = 0$

## Kalb – Ramond theory

$B_{0i} = C_i^T + \mu_{,i}, \qquad C_{i,i}^T = 0$

$B_{ij} = \varepsilon_{ijk} B_k, \qquad B_i = B_i^T + \phi_{,i}, \qquad B_{i,i}^T = 0$

# The comparison

## Proca theory

$$\mathcal{L}_0 = -\frac{1}{2}\chi(\partial^2 + m^2)\frac{m^2(-\Delta)}{-\Delta + m^2}\chi - \frac{1}{2}A_i^T(\partial^2 + m^2)A_i^T$$

$$\mathcal{L}_{int} \sim \frac{g^2}{4}(\chi_{,\mu}\chi^{,\mu})^2 - g^2\chi_{,\mu}\chi^{,\mu}\chi_{,i}A_i^T$$

$$\sim \frac{g^2}{(mL)^4L^4} \qquad \sim \frac{g^2}{(mL)^3L^4}$$

☼ Quantum fluctuations

$$\delta\chi_L \sim \frac{1}{mL} \qquad \delta A_L^T \sim \frac{1}{L}$$

$$L_{str} \sim \frac{\sqrt{g}}{m}$$

## Kalb – Ramond theory

$$\mathcal{L}_0 = -\frac{1}{2}B_i^T(\partial^2 + m^2)\frac{m^2}{-\Delta + m^2}B_i^T - \frac{1}{2}\phi_n(\partial^2 + m^2)\phi_n$$

$$\mathcal{L}_{int} \sim g^2\left(B^T\right)^4 + g^2\left(B^T\right)^3\phi_n$$

$$\sim \frac{g^2}{(mL)^4L^4} \qquad \sim \frac{g^2}{(mL)^3L^4}$$

☼ Quantum fluctuations

$$\delta\phi_{nL} \sim \frac{1}{L} \qquad B_L^T \sim \frac{1}{mL^2}$$

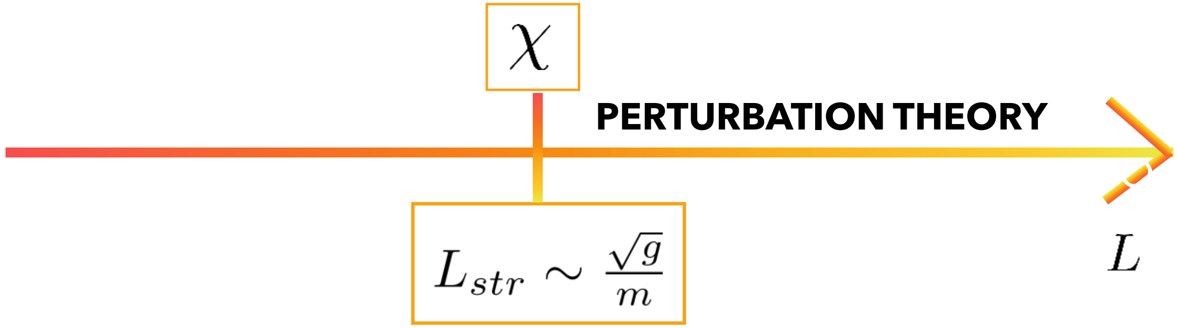
$$\phi_n = \sqrt{-\Delta}\phi$$

# THE COMPARISON

## PROCA THEORY

$$\mathcal{L}_0 = -\frac{1}{2}\chi(\partial^2 + m^2)\frac{m^2(-\Delta)}{-\Delta + m^2}\chi - \frac{1}{2}A_i^T(\partial^2 + m^2)A_i^T$$

$$\mathcal{L}_{int} \sim \frac{g^2}{4}\left(\chi_{,\mu}\chi^{,\mu}\right)^2 - g^2\chi_{,\mu}\chi^{,\mu}\chi_{,i}A_i^T$$




## KALB – RAMOND THEORY

$$\mathcal{L}_0 = -\frac{1}{2}B_i^T(\partial^2 + m^2)\frac{m^2}{-\Delta + m^2}B_i^T - \frac{1}{2}\phi_n(\partial^2 + m^2)\phi_n$$

$$\mathcal{L}_{int} \sim g^2\left(B^T\right)^4 + g^2\left(B^T\right)^3\phi_n$$