Some constraints on interacting dark-fluid models

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Reminders: ΛCDM cosmology

- Modern cosmology based on a maximally symmetric spacetime (FLRW)
  - ✓ Homogeneous: all regions of space look alike, no preferred positions
  - ✓ Isotropic: no preferred directions
  - ✓ Perfect-fluid assumptions
- The isotropy assumption is valid only on very large scales, i.e. on scales bigger than galaxy clusters
- Recent cosmological observations have shown that the universe is currently undergoing accelerated expansion
- Not conclusively known what caused the current accelerated expansion, the prevailing argument being that dark energy caused it, often considered to be sourced by Λ
- Some serious problems (tensions)
  - ✓ Cosmological Constant Problem [1](vacuum catastrophe): measured energy density of the vacuum over 120 orders of magnitude less than the theoretical prediction
    - Worst prediction in the history of physics (and of science in general)
    - Casts doubt on dark energy being a cosmological constant
  - ✓ Cosmic Coincidence Problem [2]: dark matter and dark energy densities have the same order of magnitude at the present moment of cosmic history, while differing with many orders of magnitude in the past and the predicted future
    - The initial conditions of dark matter and dark energy should be fine-tuned to about 95 orders of magnitude to produce a universe where the two densities nearly coincide today, approximately 14 billion years later[3]

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[1] Weinberg, S. Rev. Mod. Phys, 61 (1), 1 (1989)

[2] Velten, H. E. et al., Eur. Phys. J. C, 74 (11), 1 (2014)

[3] Zlatev, I., Wang, L., & Steinhardt, P. J. , Physi. Rev. Lett., 82(5), 896 (1999).

- Latest tensions vis-à-vis precise theoretical predictions and observational measurements:
  - ✓ *The Hubble Tension*: H_0 CMB vs local measurements, more than 5σ discrepancy
    - Planck2018, ΛCDM model
$$H_0 = 67.27 \pm 0.60 \; km/s/Mpc$$
    - Estimate using SNIa measurements (2016)
$$H_0 = 73.24 \pm 1.74 \; km/s/Mpc$$
    - Parallax measurements of Milky Way Cepheids (2018)
$$H_0 = 73.48 \pm 1.66 \; km/s/Mpc$$
  - ✓ *The S_8 discrepancy*: the 3σ level difference between measurements made from the CMB against weak lensing measurements and redshift surveys of the parameter S_8, which quantifies the amplitude of late-time matter fluctuations and structure growth
$$S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$$
    where σ_8 measures the amplitude of the linear power spectrum on the $8h^{-1}$Mpc scale
  - ✓ Ω_K, zero or not zero? ΛCDM assumes flat universe, but Planck temperature and polarisation power spectra give an above 3σ deviation:
$$\Omega_K \approx -0.044^{+0.018}_{-0.015}$$
- Several alternatives proposed, but to mention just a few:
  - ✓ Modified/extended theories of gravity
  - ✓ Interacting vacuum
  - ✓ Evolving fundamental constants
  - ✓ Inhomogeneous/anisotropic models
  - ✓ Interacting dark matter and dark energy $\rightarrow$ non-gravitational interactions

Background thermodynamics

- The standard ΛCDM cosmology is a solution of the Einstein field equations (EFEs) derived from the action

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\,[R + 2\,(L_m - \Lambda)]$$

where R, L_m and L are the Ricci scalar, the matter Lagrangian density and the cosmological constant, respectively. The corresponding EFEs read [4]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

with the first (geometric) term represented by the Einstein tensor, and the RHS of the equation representing the total energy-momentum tensor (EMT) of matter fluid forms.

- Both $G_{\mu\nu}$ and $T_{\mu\nu}$ are covariantly conserved quantities. The EMT for perfect-fluid models is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$$

where ρ and p are the energy density and isotropic pressure of matter, respectively, often related by the barotropic equation of state (EoS) $p = w\rho$ for a constant EoS parameter w. The normalised vector u_α represents the four-velocity of fundamental observers comoving with the fluid

[4] We will set $c = 1$ from here onwards

- The divergence-free EMT leads to the fluid conservation equation

$$T^{\mu\nu}{}_{;\mu} = 0 \implies \dot{\rho} + 3\frac{\dot{a}}{a}(1+w)\rho = 0$$

  where $a(t)$ is the cosmological scale factor whose evolution is given by the Friedmann equation

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$

  where k is the normalised spatial curvature parameter with values $-1\,,0\,,1$ depending on an open, flat or closed spatial geometry.

- In a multi-component fluid system, it is usually assumed that the energy density of each perfect-fluid component is assumed to evolve independently of the other fluids of the system:

$$\dot{\rho}_i + 3H(1+w)\rho_i = 0$$

  where here, we have introduced the Hubble parameter $H \equiv \frac{\dot{a}}{a}$ and in this case the total EMT is the algebraic sum of the EMTs of each fluid, so are the total energy density and total pressure terms the algebraic sums of the individual components

- If we relax this assumption (of conservation) due to the presence of interactions such as diffusion, the individual components do not obey the matter conservation equation, but the total fluid still does.

- In diffusive fluids, the non-conservation equation for the ith component fluid reads:

$$T_i^{\mu\nu}{}_{;\mu} = N_i^{\nu} \tag{1}$$

  where N_i^{ν} corresponds to the current of diffusion term for that fluid

- Assuming that both radiation and baryonic matter are separately conserved, conservation equations for interacting dark energy models:

$$\begin{aligned}
\dot{\rho}_{\rm dm} + 3H\rho_{\rm dm} &= Q \\
\dot{\rho}_{\rm de} + 3H\rho_{\rm de}(1+\omega) &= -Q \\
\dot{\rho}_{\rm bm} + 3H\rho_{\rm bm} &= 0 \\
\dot{\rho}_{\rm r} + 4H\rho_{\rm r} &= 0
\end{aligned} \tag{2}$$

- Q is an arbitrary coupling function whose sign determines how energy (or momentum) is transferred between dark energy and dark matter. If $Q > 0$, then the energy (or momentum) is transferred from dark energy to dark matter and vice versa for $Q < 0$, such that:

$$Q = \begin{cases} > 0 & \text{Dark Energy} \rightarrow \text{Dark Matter (iDEDM regime)} \\ < 0 & \text{Dark Matter} \rightarrow \text{Dark Energy (iDMDE regime)} \\ = 0 & \text{No interaction} \end{cases} \tag{3}$$

- No fundamental theory for the coupling equation Q, freely chosen; we will only consider models where the coupling function Q is either proportional to the dark matter or the dark energy density

- Standard assumptions of background expansion, only the conservation equations get modified:

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_{\rm r} + \rho_{\rm bm} + \rho_{\rm dm} + \rho_{\rm de}\right) - \frac{kc^2}{a^2}$$

$$q = \Omega_{\rm r} + \frac{1}{2}\left(\Omega_{\rm bm} + \Omega_{\rm dm}\right) + \frac{1}{2}\Omega_{\rm de}\left(1 + 3\omega\right)$$

$$\omega^{\rm eff} = \frac{P_{\rm tot}}{\rho_{\rm tot}} = \frac{\frac{1}{3}\Omega_{\rm r} + \omega_{\rm de}\Omega_{\rm de}}{\Omega_{\rm r} + \Omega_{\rm bm} + \Omega_{\rm dm} + \Omega_{\rm de}}$$

- Interaction affects the effective equations of state of both dark matter $\omega_{\rm dm}^{\rm eff}$ and dark energy $\omega_{\rm de}^{\rm eff}$, relative to the uncoupled background equations ($Q = 0$) in:

$$\omega_{\rm dm}^{\rm eff} = -\frac{Q}{3H\rho_{\rm dm}} \qquad \omega_{\rm de}^{\rm eff} = \omega_{\rm de} + \frac{Q}{3H\rho_{\rm de}} \tag{4}$$

- Thus, the effects of an interaction may be understood to imply that if:

$$Q > 0 \text{ (iDEDM)} \begin{cases} \omega_{\rm dm}^{\rm eff} < 0 & \text{Dark matter redshifts \textit{slower} than a}^{-3}\text{ (less DM in past)} \\ \omega_{\rm de}^{\rm eff} > \omega_{\rm de} & \text{Dark energy has \textit{less} accelerating pressure} \end{cases}$$

$$Q < 0 \text{ (iDMDE)} \begin{cases} \omega_{\rm dm}^{\rm eff} > 0 & \text{Dark matter redshifts \textit{faster} than a}^{-3}\text{ (more DM in past)} \\ \omega_{\rm de}^{\rm eff} < \omega_{\rm de} & \text{Dark energy has \textit{more} accelerating pressure} \end{cases}$$

- Even if $\omega_{\rm de} = -1$, when $Q < 0$ or $Q > 0$, then the dark energy may behave like either uncoupled quintessence $\omega_{\rm de}^{\rm eff} > -1$ or uncoupled phantom $\omega_{\rm de}^{\rm eff} < -1$ dark energy respectively

- Consider the ratio r of $\rho_{\rm dm}$ to $\rho_{\rm de}$ for interacting dark energy models:

$$r = \frac{\rho_{\rm dm}}{\rho_{\rm de}} = \frac{\rho_{({\rm dm},0)}\, a^{-3(1+\omega_{\rm dm}^{\rm eff})}}{\rho_{({\rm de},0)}\, a^{-3(1+\omega_{\rm de}^{\rm eff})}} = r_0 a^{-\zeta} \qquad \zeta_{\rm IDE} = 3\left(\omega_{\rm dm}^{\rm eff} - \omega_{\rm de}^{\rm eff}\right) \tag{5}$$

  with ζ indicating the magnitude of the coincidence problem
- The smaller the difference between $\omega_{\rm dm}^{\rm eff}$ and $\omega_{\rm de}^{\rm eff}$ the more the coincidence problem will be alleviated while being solved if $\zeta = 0$, which happens when $\omega_{\rm dm}^{\rm eff} = \omega_{\rm de}^{\rm eff}$
- Achieved if dark matter redshifts slower $\omega_{\rm dm}^{\rm eff} < \omega_{\rm dm}$ and dark energy redshifts faster $\omega_{\rm de}^{\rm eff} > \omega_{\rm de}$, which coincides with the iDEDM ($Q > 0$) scenario
- The opposite holds for the iDMDE ($Q < 0$) scenario, with $\zeta_{\Lambda{\rm CDM}} = 3$:

$$\zeta_{\rm IDE} = 3\left(\omega_{\rm dm}^{\rm eff} - \omega_{\rm de}^{\rm eff}\right) \begin{cases} Q > 0 \text{ (iDEDM):} & \zeta_{\rm IDE} < \zeta_{\Lambda{\rm CDM}} \quad \textit{alleviates} \text{ coincidence} \\ Q < 0 \text{ (iDMDE):} & \zeta_{\rm IDE} > \zeta_{\Lambda{\rm CDM}} \quad \textit{worsens} \text{ coincidence} \end{cases}$$

- $\omega_{\rm dm}^{\rm eff} < 0$ for iDEDM, DM redshifts *slower*, which leads to *less* DM in the past and the radiation-matter equality happening *later*, which in turns causes suppression in the matter power spectrum, *alleviating* the S_8 discrepancy
- $\omega_{\rm de}^{\rm eff} > \omega_{\rm de}$, such that DE redshifts faster, causing *more* DE in the past. Less DM and more DE in the past have the consequence that both the cosmic jerk and the matter-dark energy equality happen *earlier* in cosmic history
- From the Friedmann equation, we can see that this overall suppression of dark matter density causes a lower value of the Hubble parameter at late times. This lower value of H_0 worsens the Hubble tension with regard to late time probes
- The universe should also be *older*
- The opposite holds for the iDMDE scenario

Cosmological implications

Event/Parameter/Problem	$Q > 0$	$Q < 0$
Energy flow	DE $\rightarrow$ DM (iDEDM)	DM $\rightarrow$ DE (iDMDE)
Effective equations of state	$\omega_{\rm dm}^{\rm eff} < \omega_{\rm dm}$; $\omega_{\rm de}^{\rm eff} > \omega_{\rm de}$	$\omega_{\rm dm}^{\rm eff} > \omega_{\rm dm}$; $\omega_{\rm de}^{\rm eff} < \omega_{\rm de}$
Coincidence problem	Alleviates ($\zeta_{\rm IDE} < \zeta_{\Lambda\rm CDM}$)	Worsens ($\zeta_{\rm IDE} > \zeta_{\Lambda\rm CDM}$)
Hubble tension	Worsens	Alleviates
S_8 discrepancy	Alleviates	Worsens
Age of universe	Older	Younger
Radiation-matter equality	Later ($z_{\rm IDE} < z_{\Lambda\rm CDM}$)	Earlier ($z_{\rm IDE} > z_{\Lambda\rm CDM}$)
Cosmic jerk ($q = 0$)	Earlier ($z_{\rm IDE} > z_{\Lambda\rm CDM}$)	Later ($z_{\rm IDE} < z_{\Lambda\rm CDM}$)
Matter-dark energy equality	Earlier ($z_{\rm IDE} > z_{\Lambda\rm CDM}$)	Later ($z_{\rm IDE} < z_{\Lambda\rm CDM}$)

Dynamical evolution

- Consider models which contain radiation $\Omega_{\rm r}$, baryonic matter $\Omega_{\rm bm}$, dark matter $\Omega_{\rm dm}$ and dark energy $\Omega_{\rm de}$
- Defining

$$\Omega_{\rm x} = \frac{8\pi G}{3H^2}\rho_{\rm x} \qquad\qquad x \in (\text{r, bm, dm, de})$$

$$\begin{aligned}
\dot{\Omega}_{\rm de} &= \Omega_{\rm de} H\left[2\Omega_{\rm r} + \Omega_{\rm bm} + \Omega_{\rm dm} + \Omega_{\rm de}\left(1 + 3\omega_{\rm de}\right) - 1 - 3\omega_{\rm de}\right] - \frac{8\pi G}{3H^2} Q \\
\dot{\Omega}_{\rm dm} &= \Omega_{\rm dm} H\left[2\Omega_{\rm r} + \Omega_{\rm bm} + \Omega_{\rm dm} + \Omega_{\rm de}\left(1 + 3\omega_{\rm de}\right) - 1\right] + \frac{8\pi G}{3H^2} Q \\
\dot{\Omega}_{\rm bm} &= \Omega_{\rm bm} H\left[2\Omega_{\rm r} + \Omega_{\rm bm} + \Omega_{\rm dm} + \Omega_{\rm de}\left(1 + 3\omega_{\rm de}\right) - 1\right] \\
\dot{\Omega}_{\rm r} &= \Omega_{\rm r} H\left[2\Omega_{\rm r} + \Omega_{\rm bm} + \Omega_{\rm dm} + \Omega_{\rm de}\left(1 + 3\omega_{\rm de}\right) - 2\right]
\end{aligned} \tag{6}$$

- Interested in the parameter space of how dark matter and dark energy evolve with regard to each other

$$\frac{d\Omega_{\rm de}}{d\Omega_{\rm dm}} = \frac{\Omega_{\rm de} H\left[2\Omega_{\rm r} + \Omega_{\rm bm} + \Omega_{\rm dm} + \Omega_{\rm de}\left(1 + 3\omega_{\rm de}\right) - 1 - 3\omega_{\rm de}\right] - \frac{8\pi G}{3H^2} Q}{\Omega_{\rm dm} H\left[2\Omega_{\rm r} + \Omega_{\rm bm} + \Omega_{\rm dm} + \Omega_{\rm de}\left(1 + 3\omega_{\rm de}\right) - 1\right] + \frac{8\pi G}{3H^2} Q} \tag{7}$$

- Obtain a set of trajectories or flow lines in the $(\Omega_{\rm dm}, \Omega_{\rm de})$-plane, which have stable attractor and unstable repeller points, in turn used to see if the ratio of dark matter to dark energy becomes fixed in the past or present, thus potentially addressing the model's potential to solve the coincidence problem

Two linear interaction models

- Two most commonly used interaction models:

$$Q_1 = \delta H \rho_{\rm de} \qquad\qquad Q_2 = \delta H \rho_{\rm dm} \tag{8}$$

  with δ a dimensionless coupling constant that determines the strength of the interaction [5]
- For these models, we assume that $\delta < -3\omega$ so that the coupling strength $|\delta|$ is not too strong [6]
- This condition implies $(\delta < -3\omega) \rightarrow (\delta + 3\omega < 0)$
- Requiring $H > 0$; $\rho_{\rm dm} > 0$; $\rho_{\rm de} > 0$ the sign of δ will determine the direction of energy flow
- $\delta > 0 \rightarrow Q > 0$ corresponds to the iDEDM regime and $\delta < 0 \rightarrow Q < 0$ to the iDMDE regime
- The greatest qualitative difference between the two coupling functions is that $Q_1 \propto \rho_{\rm de}$ and $Q_2 \propto \rho_{\rm dm}$, which implies that the effect of the coupling will be either most prominent during early dark matter domination, or later dark energy domination respectively

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[5] Denoted by ξ in Eleonora's talk, in Giaré et al arxiv: 2404.02110, etc

[6] J. Vliviita, E. Majerotto, and R. Maartens, JCAP, 07 020 (2008)

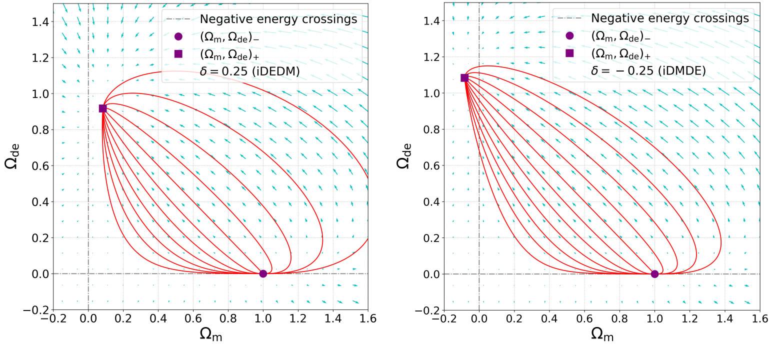
- The model $Q_1 = \delta H \rho_{\rm de}$ is one of the most commonly used interaction functions
- All energy densities remain positive throughout the past universe history, even in the iDMDE ($\delta < 0$) regime, unlike Q_2 which results in $\rho_{\rm de} < 0$ in the iDMDE regime's past evolution
  - ✓ In the iDMDE ($\delta < 0$) regime, these models will always suffer from negative dark matter energy densities ($\rho_{\rm dm} < 0$) during future expansion
    - During future expansion, the dark matter density will decrease, and energy will be transferred away from DM to DE until the DM density eventually reaches zero density
    - No mechanism to stop this energy transfer (since energy transfer is only proportional to dark energy density), the energy transfer will still continue, *inevitably leading to negative dark matter densities ($\rho_{\rm dm} < 0$) in the future*
- Exact conditions needed to ensure positive energy densities throughout the past and future expansion
- For Q_1, negligible radiation, putting $\rm dm$ and $\rm bm$ together, we have:

$$\frac{d\Omega_{\rm de}}{d\Omega_{\rm m}} = \frac{\Omega_{\rm de}\left[\Omega_{\rm m} + \Omega_{\rm de}\left(1 + 3\omega_{\rm de}\right) - 1 - 3\omega_{\rm de} - \delta\right]}{\Omega_{\rm m}\left[\Omega_{\rm m} + \Omega_{\rm de}\left(1 + 3\omega_{\rm de}\right) - 1\right] + \delta\Omega_{\rm de}} \tag{9}$$

- Express evolution of matter and dark energy with a phase portrait in the ($\Omega_{\rm m}$, $\Omega_{\rm de}$)-plane
- Equilibrium points

$$(\Omega_{\rm m}, \Omega_{\rm de})_- = (1, 0) \qquad (\Omega_{\rm m}, \Omega_{\rm de})_+ = \left(-\frac{\delta}{3\omega}, 1 + \frac{\delta}{3\omega}\right)$$

- Each trajectory starts at the unstable repellor point$(1, 0)$ and diverges, and converges again at the attractor stable point $\left(-\frac{\delta}{3\omega}, 1 + \frac{\delta}{3\omega}\right)$



Phase portraits for $\Omega_{\rm dm}$ and $\Omega_{\rm de}$ $(Q = \delta H \rho_{\rm de})$

- Same repeller point as ΛCDM but attractor point shifted by the dark coupling, showing effect of coupling is more dominant during dark energy dominance for $Q \propto \rho_{\rm de}$

- The ratio of the coordinates of the equilibrium points indicates which value r tends to in the past r_- or the future r_+
- The ΛCDM model has $r_- \to \infty$ in the past, whilst approaching $r_+ \to 0$ in the future
- IDE models that can find a constant non-zero or non-infinite value for either r_- or r_+ should solve the coincidence problem in either the past or the future, respectively

$$r_- = \frac{\Omega_{(\mathrm{m},-)}}{\Omega_{(\mathrm{de},-)}} = \frac{1}{0} \to \infty \qquad r_+ = \frac{\Omega_{(\mathrm{dm},+)}}{\Omega_{(\mathrm{de},-)}} \approx \frac{\Omega_{(\mathrm{m},+)}}{\Omega_{(de,+)}} = \frac{-\frac{\delta}{3\omega}}{1+\frac{\delta}{3\omega}} \to -\frac{\delta}{\delta+3\omega}$$

- Model not solving the coincidence problem in the past as r_- but will stabilising r in the future r_+, thereby solving the coincidence for future expansion
- Positive $\delta > 0$ (iDEDM) solves the coincidence problem, but $\delta < 0$ (iDMDE) causes negative energy densities
- $\Omega_{\mathrm{dm},+} \approx \Omega_{m,+} = -\frac{\delta}{3\omega}$, alongside $(\omega < 0) \implies \delta < 0$ (iDMDE) leads to a negative energy attractor solution for Ω_{dm}

- For Q_1, solving the conservation equations yields expressions for $\rho_{\rm dm}$ and $\rho_{\rm de}$:

$$\rho_{\rm dm} = \left(\rho_{({\rm dm},0)} + \rho_{({\rm de},0)} \frac{\delta}{\delta + 3\omega}\left[1 - a^{-(\delta+3\omega)}\right]\right) a^{-3}$$
$$\rho_{\rm de} = \rho_{({\rm de},0)} a^{-(\delta+3\omega+3)}$$

- Corresponding density parameters

$$\Omega_{\rm dm} = \frac{H_0^2}{H^2}\left(\Omega_{({\rm dm},0)} + \Omega_{({\rm de},0)} \frac{\delta}{\delta + 3\omega}\left[1 - (1+z)^{(\delta+3\omega)}\right]\right)(1+z)^3$$
$$\Omega_{\rm de} = \frac{H_0^2}{H^2}\Omega_{({\rm de},0)}(1+z)^{(\delta+3\omega+3)}$$

- The effective equation of states for this model can be obtained by substituting the coupling equation $Q = \delta H \rho_{\rm de}$ into (4). The dark matter effective equation of state is then:

$$\omega_{\rm dm}^{\rm eff} = -\frac{Q}{3H\rho_{\rm de}} = -\frac{\delta H \rho_{\rm de}}{3H\rho_{\rm dm}} = -\frac{\delta}{3}\frac{\rho_{\rm de}}{\rho_{\rm dm}} = -\frac{\delta}{3}\frac{1}{r} \qquad (10)$$

- Similarly, for dark energy, we have the effective equation of state:

$$\omega_{\rm de}^{\rm eff} = \omega + \frac{Q}{3H\rho_{\rm dm}} = \omega + \frac{\delta H \rho_{\rm de}}{3H\rho_{\rm de}} = \omega + \frac{\delta}{3} \qquad (11)$$

- Notice that $\omega_{\rm dm}^{\rm eff}$ is dynamical with a dependence on r, while, in contrast $\omega_{\rm de}^{\rm eff}$ is constant

- For this model, ρ_{de} is always positive for all values of δ, while ρ_{dm} has multiple terms which could become negative
- Four scenarios where the energy density may possibly cross zero and become negative, for $\delta < 0$ and $\delta > 0$ scenarios for either the *past* or the *future*:

$$a^{-(\delta+3\omega)} = 1 + r_0 \left(\frac{\delta + 3\omega}{\delta} \right) \qquad \delta + 3\omega < 0 \tag{12}$$

$$\delta < 0 \Rightarrow \begin{cases} \text{Past} & a < 1 \quad \rightarrow \quad 0 < \text{lhs} < 1 \qquad \text{rhs} > 1 \qquad \textbf{(A)} \\ \text{Future} & a > 1 \quad \rightarrow \quad \text{lhs} > 1 \qquad \text{rhs} > 1 \qquad \textbf{(B)} \end{cases}$$

$$\delta > 0 \Rightarrow \begin{cases} \text{Past} & a < 1 \quad \rightarrow \quad 0 < \text{lhs} < 1 \qquad \text{rhs} < 1 \qquad \textbf{(C)} \\ \text{Future} & a > 1 \quad \rightarrow \quad \text{lhs} > 1 \qquad \text{rhs} < 1 \qquad \textbf{(D)} \end{cases}$$

- $\rho_{\rm dm}$ will always remain positive for scenario's **(A)** (*past* expansion with $\delta < 0$) and **(D)** (*future* expansion with $\delta > 0$)
- In scenario **(B)** the dark energy density will always become negative in the *future* as shown in the attractor point of previous phase portrait

$$1 + r_0 \left(\frac{\delta + 3\omega}{\delta} \right) < 0 \qquad \rightarrow \qquad \delta < -\frac{3\omega}{\left(1 + \frac{1}{r_0}\right)} \tag{13}$$

- Scenario **(C)** (*Past* expansion with $\delta > 0$ will always have positive energy densities
- Since both **(C)** and **(D)** will always have positive energy densities, the positive coupling $\delta > 0$ may be seen as physical
- Since the condition (13) holds, it implies that the condition $\delta < -3\omega$ must necessarily hold as well. Taking the conditions $(\delta > 0)\,;(\delta < -3\omega)$ and $\left(\delta < -\frac{3\omega}{(1+1/r_0)}\right)$ together, a general condition is obtained to ensure positive energy densities for this model:

$$0 < \delta < -\frac{3\omega}{\left(1 + \frac{1}{r_0}\right)} \tag{14}$$

- For the coupling $Q_1 = \delta H \rho_{\rm de}$, only IDE models where energy flows from dark energy to dark matter iDEDM ($\delta > 0$) should be considered seriously as couplings where energy flows from dark matter to dark energy iDMDE ($\delta < 0$) will always lead to either negative energies in the past or the future

Conditions	$\rho_{\rm dm}$ (pst)	$\rho_{\rm dm}$ (fut)	$\rho_{\rm de}$ (pst)	$\rho_{\rm de}$ (fut)	physical?
$0 < \delta < -\frac{3\omega}{\left(1+\frac{1}{r_0}\right)}$	$+$	$+$	$+$	$+$	$\surd$
$\delta > 0$, $\delta > -\frac{3\omega}{\left(1+\frac{1}{r_0}\right)}$	$-$	$+$	$+$	$+$	X
$\delta < 0$	$+$	-	$+$	$+$	X

Conditions for positive energy densities throughout cosmic evolution ($Q_1 = \delta H \rho_{\rm de}$)

Revisiting coincidence

- Let's now try to resolve the CCP using analytic expressions, starting with the ratio:

$$r(z) = \frac{\rho_{\mathrm{dm}}(z)}{\rho_{\mathrm{de}}(z)} = \left(r_0 + \frac{\delta}{\delta + 3\omega}\right)(1+z)^{-(\delta+3\omega)} - \frac{\delta}{\delta + 3\omega} \tag{15}$$

- The r scales with the scale factor (ignoring the constant terms), such that:

$$r \propto a^{(\delta+3\omega)} \qquad \rightarrow \qquad \zeta_{\mathrm{Q}_1} = \zeta_{\mathrm{Q}} = -3\omega - \delta \tag{16}$$

- For the ΛCDM model $\zeta_{\Lambda\mathrm{CDM}} = 3$, thus:

$$\zeta_{\mathrm{Q}} = -3\omega - \delta \rightarrow \begin{cases} \text{if } \delta > 0 \text{ (iDEDM)} & \rightarrow \quad \zeta_{\mathrm{Q}} < \zeta \quad \textit{alleviates} \text{ coincidence problem} \\ \text{if } \delta < 0 \text{ (iDMDE)} & \rightarrow \quad \zeta_{\mathrm{Q}} > \zeta \quad \textit{worsens} \text{ coincidence problem} \end{cases}$$

- The effect becomes more extreme in both the distant past and the distant future:

$$\lim_{(1+z)\to\infty} r_- \to \infty, \qquad ; \qquad \lim_{(1+z)\to 0} r_+ = \to -\frac{\delta}{\delta + 3\omega} \tag{17}$$

- In the distant future, r has the proportionality:

$$\lim_{(1+z)\to 0} r_+ \propto a^0 \quad \rightarrow \quad \zeta_{(\mathrm{Q},+)} = 0 \tag{18}$$

- Since r is constant and $\zeta_{(\mathrm{Q},+)} = 0$, this model solves the coincidence problem for future expansion in the $\delta > 0$ (iDEDM) regime

- $\delta < 0$ (iDMDE) leads to a negative constant r_+ due to $\rho_{\rm dm}$ which becomes negative, which is unphysical
- Thus, for $(1+z) \to 0$ in the future, we have:

$$\lim_{(1+z)\to 0} \zeta_{\rm Q} = 0 \begin{cases} \text{if } \delta > 0 \to r_- = +constant & \textit{solves} \text{ coincidence problem} \\ \text{if } \delta < 0 \to r_- = -constant & \text{negative energy densities (unphysical).} \end{cases}$$

- Recalling that

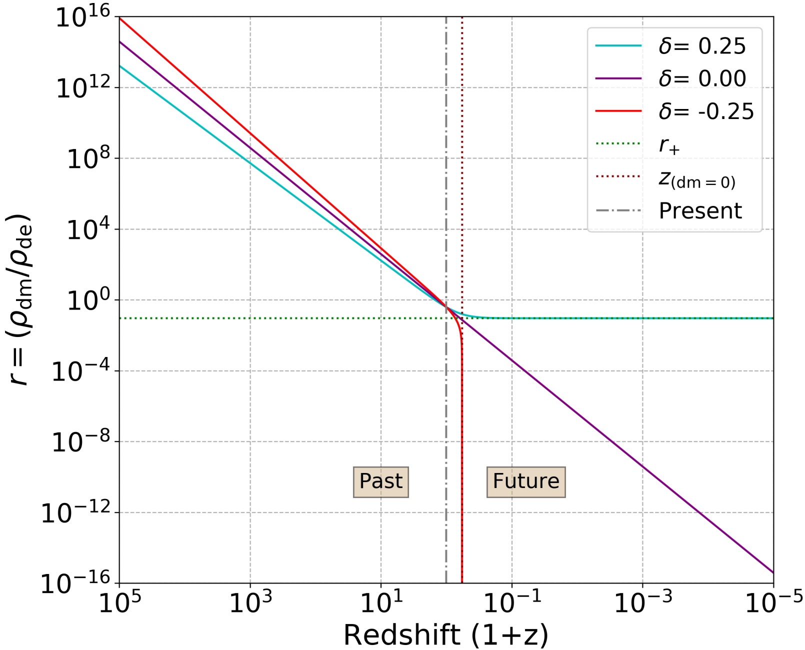
$$\omega_{\rm dm}^{\rm eff} = -\frac{\delta}{3}\frac{1}{r} = -\frac{\delta}{3}\frac{1}{\left(r_0 + \frac{\delta}{\delta+3\omega}\right)(1+z)^{-(\delta+3\omega)} - \frac{\delta}{\delta+3\omega}} \tag{19}$$

we see that in the distant past, $r_- \to \infty$, while in the distant future $r_+ \to -\frac{\delta}{\delta+3\omega}$, as was also shown in the phase portraits

- Noting that $\omega_{\rm de}^{\rm eff} = \omega + \frac{\delta}{3}$ from (11), we can see how the dynamical effective equation of state $\omega_{\rm dm}^{\rm eff}$ behaves in both the distant past and future:

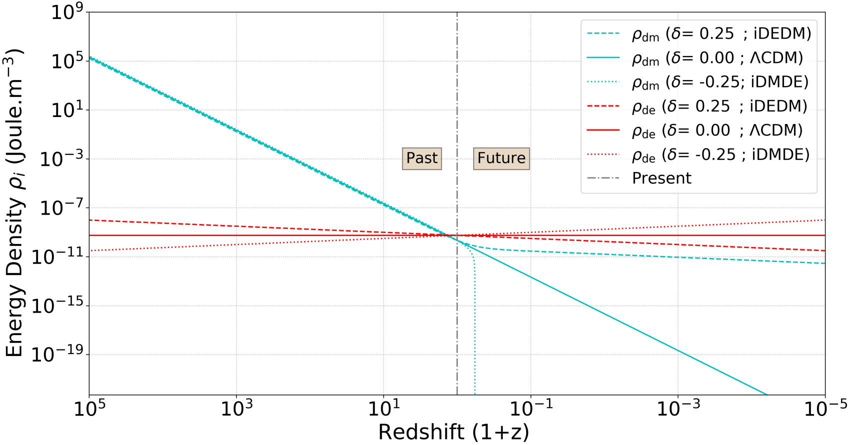
$$\omega_{\rm dm}^{\rm eff} = -\frac{\delta}{3}\frac{1}{r} \begin{cases} \text{Distant past} \quad (r = r_-): & \omega_{\rm dm}^{\rm eff} = -\frac{\delta}{3}\frac{1}{\infty} = 0 = \omega_{\rm dm} \\ \text{Distant future} \ (r = r_+): & \omega_{\rm dm}^{\rm eff} = -\frac{\delta}{3}\left(\frac{\delta}{\delta+3\omega}\right) = \omega + \frac{\delta}{3} = \omega_{\rm de}^{\rm eff} \end{cases} \tag{20}$$

- DM and DE redshift and dilute at the same rate in the future, effectively solving the CCP by keeping the ratio of dark matter to dark energy constant
- In the distant past, $\omega_{\rm dm}^{\rm eff} = \omega_{\rm dm} \implies$ effect of the coupling on dark matter will thus become negligible for past expansion, effectively mimicking the behaviour of uncoupled dark matter



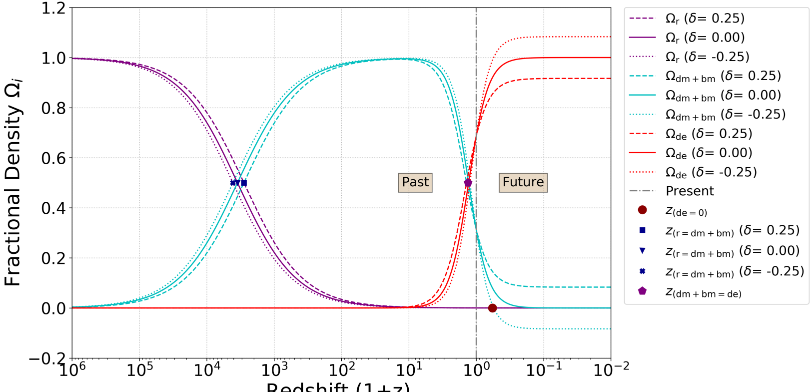
Coincidence problem ($Q_1 = \delta H \rho_{\mathrm{de}}$), $\delta > 0$ (iDEDM): r differs with many orders of magnitude in the past but converges to a constant value in the future $r \to r_+$

- For $\delta > 0$ (iDEDM), dark matter receives energy from dark energy, causing $\rho_{\rm dm}$ to redshift slower $\omega_{\rm dm}^{\rm eff} < \omega_{\rm dm}$ (smaller slope), while $\rho_{\rm de}$ redshifts faster (greater slope), alleviating the coincidence problem in the past
- In the future the slope at which $\rho_{\rm dm}$ and $\rho_{\rm de}$ redshift becomes the same, coinciding with $\omega_{\rm dm}^{\rm eff} = \omega_{\rm de}^{\rm eff}$ and the coincidence problem being solved, while $\rho_{\rm dm}$ dilutes similar to the ΛCDM model in the past where $\omega_{\rm dm}^{\rm eff} = \omega_{\rm dm}$



Energy densities ρ vs redshift - $(Q_1 = \delta H \rho_{\rm de})$

- If $\delta > 0$, $\rho_{\rm de}$ decreases over time; while if $\delta < 0$, $\rho_{\rm de}$ increases over time
- $\implies$ DE effectively behaves like either quintessence or phantom dark energy, respectively, with an equation of state $\omega_{\rm de}^{\rm eff} = \omega + \frac{\delta}{3}$
- Since this continues indefinitely, it may cause a big rip singularity in the future



Density parameters vs redshift - ($Q = \delta H \rho_{\rm de}$)

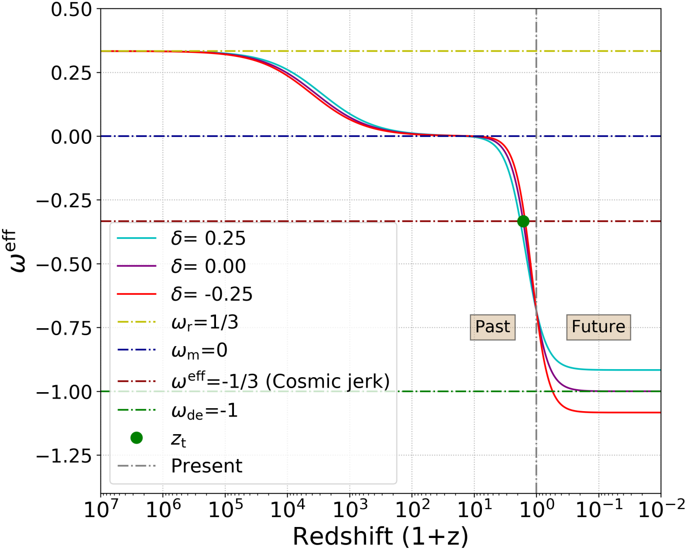
- Note that for $\delta > 0$ the matter-radiation equality happens *later* and the matter-dark energy equality *earlier* in cosmic history, with the opposite holding for $\delta < 0$

- Analytical expressions giving the exact redshift where the radiation-matter $z_{(\mathrm{r=dm+bm})}$ and matter dark energy $z_{(\mathrm{dm+bm=de})}$ equalities occur:

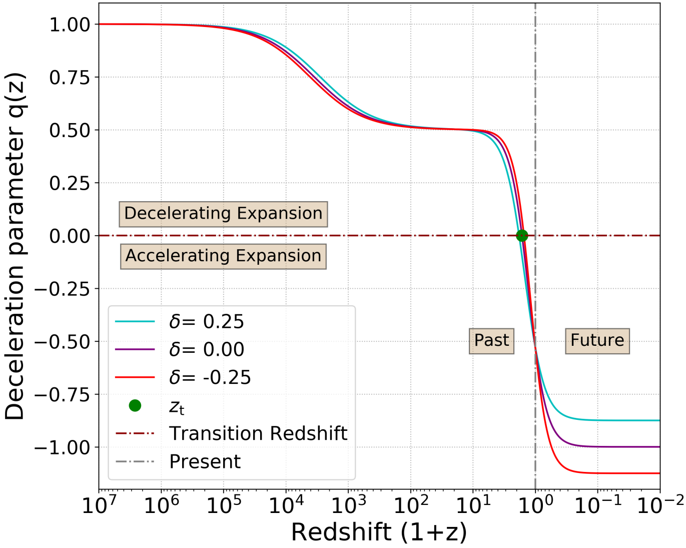
$$z_{(\mathrm{r=dm+bm})} \approx \left(\frac{\Omega_{(\mathrm{bm},0)} + \Omega_{(\mathrm{dm},0)} + \Omega_{(\mathrm{de},0)} \frac{\delta}{\delta+3\omega}}{\Omega_{(\mathrm{r},0)}} \right) - 1 \qquad (21)$$

$$z_{(\mathrm{dm+bm=de})} = \left(\frac{\frac{\Omega_{(\mathrm{bm},0)}+\Omega_{(\mathrm{dm},0)}}{\Omega_{(\mathrm{de},0)}} + \frac{\delta}{\delta+3\omega}}{\left(1+\frac{\delta}{\delta+3\omega}\right)} \right)^{\frac{1}{\delta+3\omega}} - 1 \qquad (22)$$

$\delta > 0$ (iDEDM)	Rad-matt eq:	$z_{\mathrm{IDE}} < z_{\Lambda\mathrm{CDM}}$	happens *later* than ΛCDM
	Matt-DEy eq:	$z_{\mathrm{IDE}} > z_{\Lambda\mathrm{CDM}}$	happens *earlier* than ΛCDM
$\delta < 0$ (iDMDE)	Rad-matt eq:	$z_{\mathrm{IDE}} > z_{\Lambda\mathrm{CDM}}$	happens *earlier* than ΛCDM
	Matt-DE eq:	$z_{\mathrm{IDE}} < z_{\Lambda\mathrm{CDM}}$	happens *later* than ΛCDM



Evolution of effective equation of state ω^{eff} with redshift ($Q_1 = \delta H \rho_{\mathrm{de}}$)



Evolution of deceleration parameter q with redshift $(1+z)$ $(Q_1 = \delta H \rho_{\rm de})$

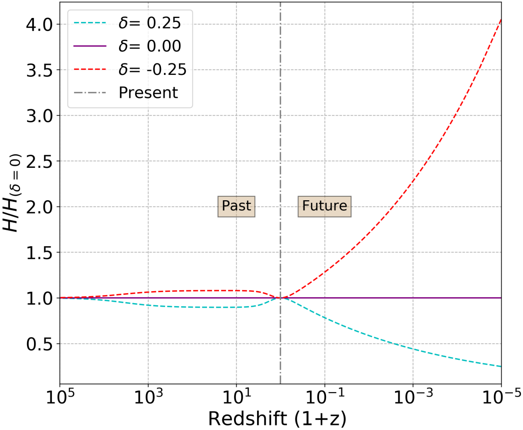
- Past expansion history for the coupled models is almost identical to that of the ΛCDM model, with initial deceleration followed by acceleration from the cosmic jerk onwards
- Model experiences complete radiation-domination $(\Omega_{\rm r}, \Omega_{\rm dm+bm}, \Omega_{\rm de}) \approx (1,0,0) \rightarrow q = 1$; $\omega^{\rm eff} = 1/3$, followed by complete matter-domination $(\Omega_{\rm r}, \Omega_{\rm dm+bm}, \Omega_{\rm de}) \approx (0,1,0) \rightarrow q = 1/2$; $\omega^{\rm eff} = 0$
- No complete dark energy domination: we have $(\Omega_{\rm r}, \Omega_{\rm dm+bm}, \Omega_{\rm de}) \approx \left(0, -\frac{\delta}{3\omega}, 1 + \frac{\delta}{3\omega}\right)$, with deceleration parameter during dark energy domination $q_{+} = \frac{1}{2}\left(1 + 3\omega_{\rm de}^{\rm eff}\right)$
- This cosmic jerk occurs at the transition redshift $z_{\rm t}$, for which an analytical expression can be derived by setting $q = 0$ in eq. (4), giving:

$$z_{\rm t} = \left[- \frac{\frac{\Omega_{\rm (bm,0)} + \Omega_{\rm (dm,0)}}{\Omega_{\rm (de,0)}} + \frac{\delta}{\delta + 3\omega}}{1 + 3\omega + \frac{\delta}{\delta+3\omega}} \right]^{\frac{1}{\delta+3\omega}} - 1 \tag{23}$$

- We can see that:

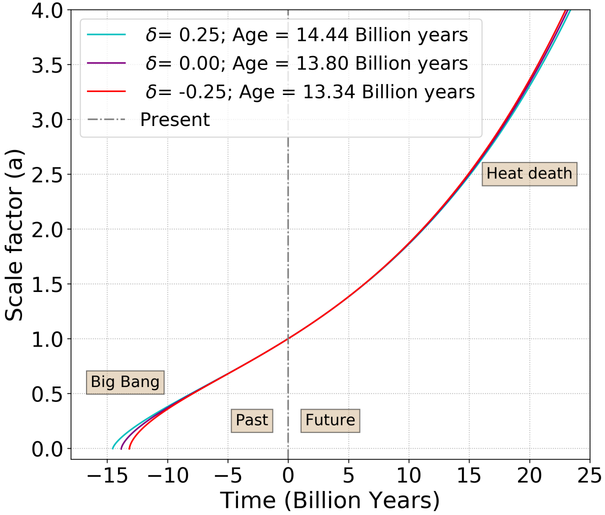
$$\text{Cosmic jerk } (z_{\rm t}) \begin{cases} \delta > 0 \text{ (iDEDM):} & z_{\rm IDE} > z_{\Lambda\rm CDM} \quad \text{happens } \textit{earlier} \text{ than } \Lambda\text{CDM} \\ \delta < 0 \text{ (iDMDE):} & z_{\rm IDE} < z_{\Lambda\rm CDM} \quad \text{happens } \textit{later} \text{ than } \Lambda\text{CDM} \end{cases}$$

- The interaction Q will affect the age of the universe since both the deceleration parameter and the total effective equation of state deviate from ΛCDM expansion, in turn affecting the evolution of H
- $H/H_{\delta=0} < 1$ for $\delta > 0$ (iDEDM) throughout most of the expansion history, indicating a slower expansion rate



Relative Hubble parameter ($H/H_{\delta=0}$) vs redshift ($Q_1 = \delta H \rho_{\rm de}$)

- Due to $\delta > 0$ (iDEDM) having a slower expansion rate, more time is needed for the universe to evolve from a singularity ($a = 0$) to its current size ($a = 1$), causing an older age for the universe
  - ✓ The opposite of this holds for $\delta < 0$ (iDEDM)



Evolution of scale factor with time ($Q_1 = \delta H \rho_{\rm de}$)

Cosmic events comparison: $Q_1 = \delta H \rho_{\mathrm{de}}$

Event: $\delta = 0$	z	t_0 **(Gyr)**	$\rho_{\mathbf{r}}$	$\rho_{\mathbf{m}}$	ρ_{de} (J/m^3)
Big bang singularity	∞	13.80	∞	∞	∞
Radiation-matter equality	3499	13.80	10.9	10.9	5.5e-10
Cosmic jerk	0.63	6.12	5.2e-13	1.2e-9	5.5e-10
Matter-dark energy equality	0.30	3.50	2.1e-13	1.1e-9	5.5e-10
Event: $\delta = 0.25$	z	t_0 **(Gyr)**	$\rho_{\mathbf{r}}$	$\rho_{\mathbf{m}}$	ρ_{de} (J/m^3)
Big bang singularity	∞	14.44	∞	∞	∞
Radiation-matter equality	2807	14.44	4.5	4.5	4.0e-9
Cosmic jerk	0.82	7.23	8.0e-13	1.3e-9	6.4e-10
Matter-dark energy equality	0.39	4.35	2.7e-13	6.0e-10	6.0e-10
Event: $\delta = -0.25$	z	t_0 **(Gyr)**	$\rho_{\mathbf{r}}$	$\rho_{\mathbf{m}}$	ρ_{de} (J/m^3)
Big bang singularity	∞	13.34	∞	∞	∞
Radiation-matter equality	4084	13.34	20.3	20.3	6.6e-11
Cosmic jerk	0.52	5.29	7.8e-13	1.3e-9	6.4e-10
Matter-dark energy equality	0.24	2.93	1.7e-13	5.3e-10	5.3e-10

- The coupling between the dark sectors will influence the evolution of dark matter and dark energy perturbations [7] [8]
- Introduce so-called doom factor: combination of parameters used to avoid instabilities

$$\mathbf{d} = \frac{Q}{3H\rho_{\rm de}(1+\omega)} \tag{24}$$

- May induce non-adiabatic instabilities in the evolution of the dark energy perturbations
- Sign of $\mathbf{d}$ will determine if there is an early time instability. It was shown that if the doom factor is positive and large $\mathbf{d} > 1$; the dark energy perturbations will become dominated by the terms which are dependent on the coupling function Q, leading to a runaway; unstable growth regime
- For model should be free of non-adiabatic instabilities on large scales, $\mathbf{d} < 0$
- For $Q = \delta H\rho_{\rm de}$, we have

$$\mathbf{d} = \frac{Q}{3H\rho_{\rm de}(1+\omega)} = \frac{\delta H\rho_{\rm de}}{3H\rho_{\rm de}(1+\omega)} = \frac{\delta}{3(1+\omega)} \tag{25}$$

[7] M.B. Gavela, D. Hernandez, L. Lopez Honorez, et al., JCAP, 07 034 (2009)

[8] J. Vliviita, E. Majerotto, and R. Maartens, JCAP, 07 020 (2008)

- For this interaction model:

$$\mathbf{d} < 0 \begin{cases} \delta < 0 & \omega > -1 \quad \text{(Quintessence regime)} \\ \delta > 0 & \omega < -1 \quad \text{(Phantom regime)} \end{cases} \rightarrow \text{No instabilities expected}$$

$$\mathbf{d} > 0 \begin{cases} \delta > 0 & \omega > -1 \quad \text{(Quintessence regime)} \\ \delta < 0 & \omega < -1 \quad \text{(Phantom regime)} \end{cases} \rightarrow \text{Instabilities if } \mathbf{d} > 1$$

δ	ω	Dark energy	**d**	Stable	$\rho_{\rm dm} > 0$	$\rho_{\rm de} > 0$	Viable
$+$	< -1	Phantom	-	$\surd$	$\surd$	$\surd$	$\surd$
$+$	> -1	Quintessence	$+$	X	$\surd$	$\surd$	X
-	< -1	Phantom	$+$	X	X	$\surd$	X
-	> -1	Quintessence	-	$\surd$	X	$\surd$	X

Stability and positive energy criteria ($Q = \delta H \rho_{\rm de}$)

- Note that the only scenario that is free from both negative energy densities and instabilities is phantom dark energy $\omega < -1$ in the $\delta > 0$ (iDEDM) regime
- These models will thus violate many of the energy conditions of general relativity

- Since $\omega_{+}^{\rm eff} = \omega_{\rm dm}^{\rm eff} = \omega_{\rm de}^{\rm eff} = \omega + \frac{\delta}{3}$ in the future, the value of δ will determine if the universe model will experience a late time big rip singularity [9]
- For a big rip to occur, we need $\rho_{\rm de} \to \infty$ in a finite time
- Occurs for this model if $\rho_{\rm de}$ increases with scale factor as the universe expands, which only happens if the effective equation of state $\omega_{\rm de}^{\rm eff} = \omega + \frac{\delta}{3} < -1$:

$$\rho_{\rm de} = \rho_{({\rm de},0)} a^{-3(1+\omega+\frac{\delta}{3})}, \qquad -3\left(1+\omega+\frac{\delta}{3}\right) > 0 \quad \text{if} \quad \omega_{\rm de}^{\rm eff} = \omega + \frac{\delta}{3} < -1 \tag{26}$$

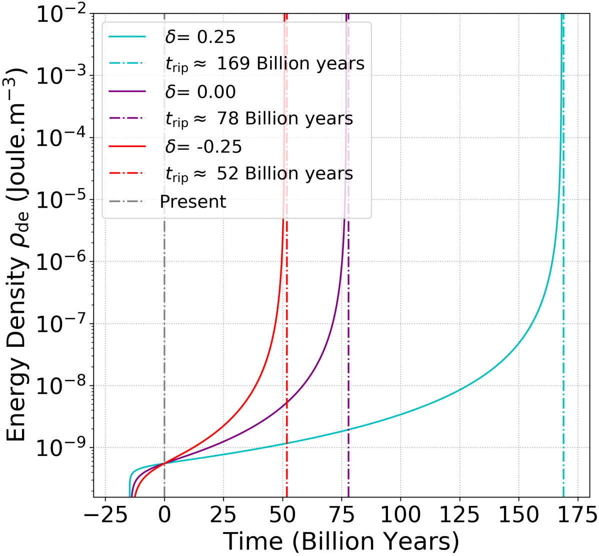
- If this condition is obeyed, the time of the rip $t_{\rm rip}$ can be given by

$$t_{rip} \approx -\frac{2}{3H_0(1+\omega+\frac{\delta}{3})\sqrt{\left(1-\frac{\delta}{\delta+3\omega}\right)\left(1-\Omega_{({\rm dm+bm},0)}\right)}} \tag{27}$$

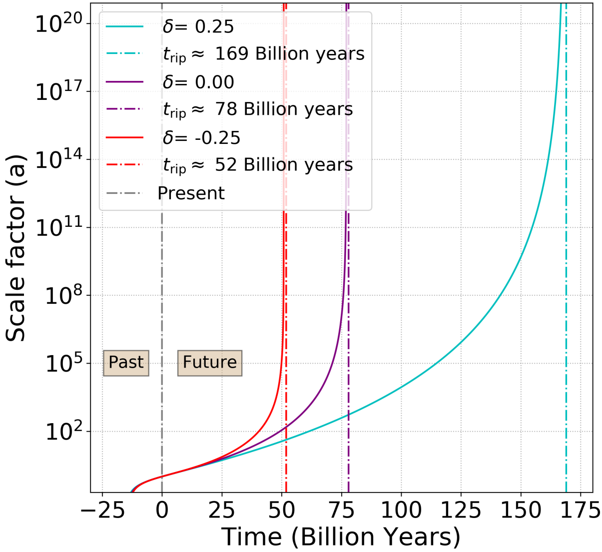
which reduces back to the uncoupled case [10] if $\delta = 0$

[9] S. Pan, J. de Haro, W. Yang, and J. Amors, Phys. Rev. D, 101 12 2470-0029 (2020)

[10] R.R. Caldwell, M. Kamionkowski, and N. N. Weinberg, PRL, 91 (7) (2003)



Evolution of energy density and the big rip for phantom ($\omega = -1.15$) IDE models - ($Q_1 = \delta H \rho_{\rm de}$)



Evolution of factor and the big rip for phantom ($\omega = -1.15$) IDE models - ($Q_1 = \delta H \rho_{\rm de}$)

Similarly for $Q_2 = \delta H \rho_{\rm dm}$

- Analytic solutions of densities

$$\rho_{\rm dm} = \rho_{\rm (dm,0)}\, a^{(\delta-3)},$$
$$\rho_{\rm de} = \left[\rho_{\rm (de,0)} + \rho_{\rm (dm,0)} \frac{\delta}{\delta+3\omega}\left(1 - a^{\delta+3\omega}\right)\right] a^{-3(1+\omega)}$$

- Effective eos parameters

$$\omega_{\rm dm}^{\rm eff} = -\frac{\delta}{3} \qquad \omega_{\rm de}^{\rm eff} = \omega_{\rm de} + \frac{\delta}{3} r$$

- Positive energy densities

$$0 < \delta < -\frac{3\omega}{(1+r_0)}$$

Conditions	$\rho_{\rm dm}$ (pst)	$\rho_{\rm dm}$ (fut)	$\rho_{\rm de}$ (pst)	$\rho_{\rm de}$ (fut)	physical
$0 < \delta < -\frac{3\omega}{(1+r_0)}$	$+$	$+$	$+$	$+$	$\surd$
$\delta > 0$; $\delta > -\frac{3\omega}{(1+r_0)}$	$+$	$+$	$+$	$-$	X
$\delta < 0$	$+$	$+$	$-$	$+$	X

Conditions for positive energy densities throughout cosmic evolution ($Q_2 = \delta H \rho_{\rm dm}$)

$$\delta > 0 \text{ (iDEDM)} \begin{cases} \text{Past expansion:} & \omega_{\rm dm}^{\rm eff} = \omega_{\rm de}^{\rm eff} \; (\zeta_{\rm Q} = 0) \quad \textit{solves} \text{ coincidence problem} \\ \text{Future expansion:} & \omega_{\rm dm}^{\rm eff} < \omega_{\rm dm} \; (\zeta_{\rm Q} < \zeta) \quad \textit{alleviates} \text{ coincidence problem} \end{cases}$$

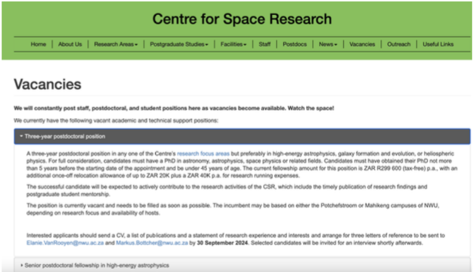
$$\delta < 0 \text{ (iDMDE)} \begin{cases} \text{Past expansion:} & \omega_{\rm dm}^{\rm eff} = \omega_{\rm de}^{\rm eff} \; (\rho_{\rm de} < 0) \quad \text{negative energy densities} \\ \text{Future expansion:} & \omega_{\rm dm}^{\rm eff} > \omega_{\rm dm} \; (\zeta_{\rm Q} > \zeta) \quad \textit{worsens} \text{ coincidence problem} \end{cases}$$

δ	ω	Dark energy	**d**	a priori stable	$\rho_{\rm dm} > 0$	$\rho_{\rm de} > 0$	Viable
$+$	< -1	Phantom	-	$\surd$	$\surd$	$\surd$	$\surd$
$+$	> -1	Quintessence	$+$	X	$\surd$	$\surd$	X
-	< -1	Phantom	$+$	X	$\surd$	X	X
-	> -1	Quintessence	-	$\surd$	$\surd$	X	X

Stability and positive energy criteria ($Q_1 = \delta H \rho_{\rm dm}$)

Summary

- Clarified cosmological consequences of IDE models for any generic interaction Q
- Two case studies of linear dark energy couplings, $Q_1 = \delta H \rho_{\rm de}$ and $Q_2 = \delta H \rho_{\rm dm}$
- Positive energy conditions $0 < \delta < -3\omega/(1 + \frac{1}{r_0})$ and $0 < \delta < -3\omega/\left(1 + r_0\right)$
- The $\delta < 0$ (iDMDE) regime will always lead to $\rho_{\rm dm} < 0$ in the future for $Q = \delta H \rho_{\rm de}$ and $\rho_{\rm de} < 0$ in the past for $Q = \delta H \rho_{\rm dm}$ $\implies$ $\delta < 0$ (iDMDE) regime should not be taken seriously as a potential dark energy candidate for these models
- The $\delta > 0$ (iDEDM) regime:
  - ✓ The model $Q = \delta H \rho_{\rm de}$ could solve the coincidence problem in the future whilst alleviating the problem for the past
  - ✓ The model $Q = \delta H \rho_{\rm dm}$ can solve the coincidence problem in the past and alleviate the problem for the future
  - ✓ Later radiation-matter equality for both models, earlier matter-dark energy equality and cosmic jerk; older age of the universe; opposite holds for $\delta < 0$ (iDMDE)
  - ✓ The only viable regime for both these models, which avoid both negative energy densities and gravitational instabilities, is phantom dark energy $\omega < -1$ in the $\delta > 0$ (iDEDM) regime
  - ✓ This has the consequence that model $Q_2 = \delta H \rho_{\rm dm}$ will always end with a future big rip singularity, while $Q_1 = \delta H \rho_{\rm de}$ may avoid this fate
  - ✓ The model $Q = \delta H \rho_{\rm de}$ will only experience a big rip future singularity if the condition $\omega_{\rm de}^{\rm eff} = \omega + \frac{\delta}{3} < -1$
  - ✓ This big rip may be avoided if the conditions $3(\omega + 1) < \delta < -3\omega \left(1 + \frac{1}{r_0}\right)$ if $\omega_{\rm de} < -1$
- Emphasis on importance of choosing the correct parameter space (iDEDM regime, with phantom dark energy) to avoid both negative energies and instabilities
- Future outlook: nonlinear IDE models more realistic? Observational constraints!



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