Ising Cosmology

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Quantum Gravity, Strings and the Swampland

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Based on:
1. N. Irges, A. Kalogirou, and F. Koutroulis. “Ising Cosmology”. Eur.Phys.J.C 83 (2023), 5,431. arXiv: 2209.09939 [hep-th].
2. N. Irges, A. Kalogirou, and F. Koutroulis. “Thermal effects in Ising Cosmology”. Universe 9 (2023), 10, 434. arXiv: 2209.09938 [hep-th].

Abstract

- We consider the rapidly expanding phase of the universe and model it by de Sitter (dS) space and a real scalar field.
- Using arguments from holography we propose that the deviation of the cosmological spectral index n_S of scalar fluctuations from unity may be controlled almost entirely by the critical exponent η of the $d=3$ Ising model.
- We compute the thermal propagators of the scalar field in dS background and propose that non-trivial thermal effects as seen by an 'out' observer can be encoded by η which fixes completely a number of cosmological observables.

Background & Motivation

- The Universe is believed to have gone through a phase of inflation due to the existence of the Horizon problem.

  The Cosmic Microwave Background (CMB) radiation is strangely isotropic i.e. the temperatures in two present spacelike points are identical, which suggests that at some point in their past, these points where timelike.

- The CMB radiation can be described by the power spectrum at super-Hubble scales for a quantum scalar field $\phi_{|\boldsymbol{k}|}$:

$$\mathcal{P}_S = \left(\frac{|\mathbf{k}|^3}{2\pi^2}\right)\left|\phi^2_{|\mathbf{k}|}\right|$$

  while its scale invariance depends on the value of the scalar spectral index:

$$n_S = 1 + \frac{\partial \ln \mathcal{P}_S}{\partial \ln |\mathbf{k}|}.$$

- When $n_S = 1$, the spectrum is scale invariant, while experiments [Planck Collaboration (2018)] measure that its real value to be approximately $n_S \simeq 0.964$.

Outline

- Part1: The spectral Index
  - The calculation of n_S from dS/CFT
  - Tensor fluctuations
- Part2: The Thermal Effects
  - Propagators and temperature
  - Line of constant physics and other observables
- Part3: What's next?

Part1: The spectral Index, The calculation of n_S from dS/CFT

- The CMB is nearly scale invariant since:

$$\delta n_S = 1 - n_S^{\text{exp}} = 0.036$$

  while various endeavors have been made to describe the above deviations.

- We study the d=4 Poincare patch of dS spacetime which spans from $\tau \in (-\infty, 0]$.

$$ds^2 = dt^2 - a^2(t)dx^2 = a^2(\tau)(d\tau^2 - dx^2), \qquad a^2(\tau) = \frac{1}{H^2\tau^2} \qquad H = \frac{\dot{a}(t)}{a(t)}$$

- The dual theory to a scalar field in dS, on the boudnary is known to be a large N non-unitary CFT

- We consider an observer that starts his journey at the start of inflation $(\tau_{\text{in}} \to -\infty)$ and flows towards a time close to the Horizon $(\tau_{\text{out}} \to 0)$.

  - On the Boundary, this can be seen as a RG flow from a UV to the IR Fixed Point (FP) where the τ_{in} (τ_{out}) is known to correspond to a UV(IR) FP on the dS Boundary. [Antoniadis, Mazur, Mottola (2012)]

Part1: The spectral Index, The calculation of n_S from dS/CFT

- On the Boundary, we suggest that the Ising model is dual to the analytic continuation of the Bulk dS spacetime. The Lagrangian of the model on the IR FP is:

$$\mathcal{L} = \frac{1}{2}\left(\partial_i \sigma\right)^2 - \lambda \sigma^4 .$$

- One of the critical exponents of the above model is $\eta = 2\gamma_\sigma$, with γ_σ being the anomalous dimension of the σ field. Numeric computations show that $\eta \simeq 0.036$! ~[Campostrini et al. (2002)]

- Recall that the anomalous dimensions arise when one lets a theory flow away from one of its fixed points, essentially breaking its scale invariance. This naturally fits with our description!

- We want to check whether there is connection between the deviation of n_S and η i.e. if they satisfy a equation of the form:

$$\delta n_S = \eta \Rightarrow n_S = 1 - \eta$$

or if their similar value is just a coincidence!

Part1: The spectral Index, The calculation of n_S from dS/CFT

- In the dS/CFT correspondence, an operator $\mathcal{O}_{|\mathbf{k}|}$ on the boundary of dimension $\Delta_{\mathcal{O}} = 3$, will be dual to the the scalar curvature perturbations $\zeta_{|k|} = z(\tau)\,\phi_{|k|}$ of dimension $\Delta_\zeta = 0$ if:

$$< \mathcal{O}_{|\mathbf{k}|}\mathcal{O}_{-|\mathbf{k}|} > \sim \frac{1}{<\phi_{|\mathbf{k}|}\phi_{-|\mathbf{k}|}>} \sim \frac{1}{<\zeta_{|\mathbf{k}|}\zeta_{-|\mathbf{k}|}>}$$

~[Maldacena (2003)]

- where:
  - $\phi_{|k|}$ is the bulk scalar field in momentum space,
  - $z(\tau)$ is a factor determined by the (conformal time τ) time-dependent classical background,
  - $<\phi_{|k|}\ \phi_{-|k|}> \simeq <\zeta_{|k|}\ \zeta_{-|k|}>$ is a gauge-invariant result

- Then, just outside the IR FP, the deviation from unity of the scalar spectral index is:

$$n_S = 1 - 2\Gamma_{\mathcal{O}} - \beta_\lambda \frac{\partial}{\partial\lambda} \ln < \mathcal{O}_{|\mathbf{k}|}\mathcal{O}_{-|\mathbf{k}|} >$$

with $\Gamma_{\mathcal{O}} = \Delta_{\mathcal{O}} - [\Delta_{\mathcal{O}}]$ the anomalous dimension of the operator:

$$\Gamma_{\mathcal{O}} = -\mu \frac{\partial}{\partial\mu}\left(Z_\sigma^{-1} z_{\mathcal{O}}\right)$$

Part1: The spectral Index, The calculation of n_S from dS/CFT

- The "total" anomalous dimension of an operator $\mathcal{O}$ is defined as:

$$\gamma_{\mathcal{O}} = \mu \frac{\partial}{\partial \mu} z_{\mathcal{O}}$$

where $z_{\mathcal{O}}$ is the wave renormalization of $\mathcal{O}$.

- These definitions imply that for an operator containing two σ fields:

$$\Gamma_{\mathcal{O}} = -\gamma_{\mathcal{O}} + 2\gamma_{\sigma}$$

so that for any $\Delta_{\mathcal{O}} = 3$ operator with $\Gamma_{\mathcal{O}} = 0$ (f.e. $\mathcal{O} = \Theta$): $\gamma_{\mathcal{O}} = 2\gamma_{\sigma}$ and:

$$\mathrm{n}_S = 1 - \beta_{\lambda} \frac{\partial}{\partial \lambda} \ln < \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} > .$$

$\Theta = \delta^{ij} T_{ij}$
The trace of the stress energy tensor (SET)

- Note that expressing the tilt of the CMB spectrum as an anomalous dimension has been attempted before. However, the slow-roll approximation was used while parameter-fixing was needed ~[Larsen, van der Schaar et al. (2002)]

Part1: The spectral Index, The calculation of n_S from dS/CFT

- Now, if our hypothesis is correct and η truly fixes the deviation of n_S:

$$\beta_\lambda \frac{\partial}{\partial \lambda} < \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} >= 2\gamma_\sigma < \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} >$$

$\mathcal{O} = \Theta$ (or an improvement) should satisfy the above eigenvalue eq.!

- Assuming that in the vicinity of the IR FP ($\lambda \simeq \lambda^*$):

$$< \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} >= c_\mathcal{O} / |x|^{2d}$$

the coupling $c_\mathcal{O}$ satisfies $\beta_\lambda \partial_\lambda c_\mathcal{O} = 2\gamma_\sigma c_\mathcal{O}$ whose leading order solution is:

$$c_\mathcal{O} \simeq \left(\frac{16\pi^2 - 3\lambda}{\lambda} \right)^\eta , \qquad \eta = 2\gamma_\sigma$$

which vanishes at the FP, as it should.

Part1: The spectral Index, Tensor fluctuations

- The dual of dS is a non-unitary CFT with negative central charge.

- Additionally, the 2-point function of the gravitational waves $\gamma_{\mu\nu}$ is inversely proportional to the central charge of the CFT: $\mathcal{P}_T \sim \langle \gamma\gamma \rangle \sim \frac{1}{c_T^*}, \qquad c_T^* \sim -R_{\mathrm{dS}}^2.$

  Here, $\mathcal{P}_T$ is the tensor power spectrum.

- On the other hand, the interesting physics happens just outside the FP, where the ratio:

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_S} > 0$$

  should be positive and small.

- In order to deal with this seemingly inconsistency, we will show that the effective coupling that determines the tensor spectrum is:
  - ➔ positive away from the FP
  - ➔ somewhere before it reaches the FP, becomes negative.

Part1: The spectral Index, Tensor fluctuations

- Let us call such a coupling, the “C-function” $C(e)$, $e = |x_1 - x_2|$.
- The 2-point function of the boundary SET $T_{\mu\nu}$ couples to $<\gamma\gamma>$ (inversely). ~[Maldacena (2003)]
- The C-function is defined as:

$$< \mathbf{T}_{\mu\nu} \mathbf{T}_{\rho\sigma} >= \frac{A_{\mu\nu\rho\sigma}}{e^{2d}} \sim \frac{C(e)}{e^{2d}} F_{\mu\nu\rho\sigma}$$

with $F_{\mu\nu\rho\sigma}$ a tensor structure independent of e.

- We then proceed to decompose the SET into a traceless $T_{\mu\nu}$ and trace Θ part:

$$\mathbf{T}_{\mu\nu} = T_{\mu\nu} + \Theta \frac{\delta_{\mu\nu}}{d}$$

and take advantage of the conservation equation:

$$\partial_\mu < \mathbf{T}^{\mu\nu} \mathbf{T}^{\rho\sigma} >= 0$$

- But unlike the $d=2$ case (which leads to Zamolodchikov’s c-theorem), there is no obvious preference for a particular C-function.

Part1: The spectral Index, Tensor fluctuations

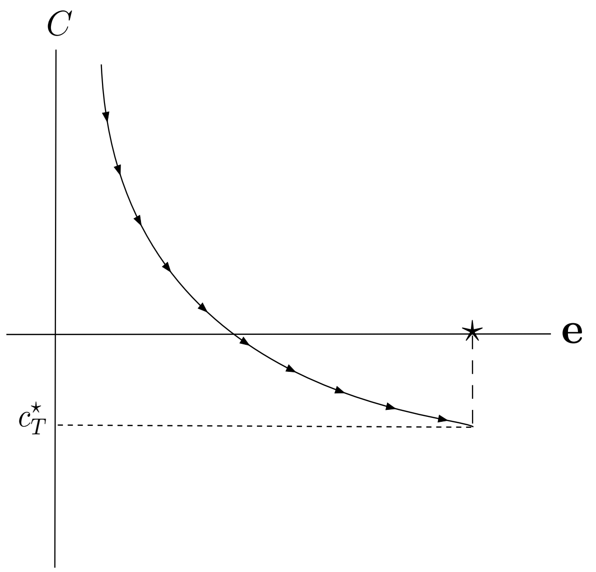
- In our case however, we have the extra constraint:

$$\dot{c}_{\mathcal{O}} = -\eta c_{\mathcal{O}}, \qquad \dot{} = \frac{d}{de}$$

which enables us to fix the most general form of the C-function:

$$C = \left(a \left[\frac{e}{e_*} \right]^{-\eta} + b \right) c_T^*$$

where a,b arbitrary coefficients and c_T^* the tensor coupling.



- The conditions needed give:

$$C^* = c_T^* \Rightarrow a + b = 1$$

$$\dot{C} < 0 \Rightarrow a < 0$$

$$C > 0 \Rightarrow a < a_c \equiv \frac{1}{1 - \left[\frac{e}{e_*}\right]^{-\eta}}$$

As we approach the IR FP $(e \to e_*)$, $a_c \to -\infty$ meaning that at some point C becomes negative!

Part1: The spectral Index, Tensor fluctuations

- To compute the actual from of the C-function, one needs to tackle the problem simultaneously from both the bulk and the boundary.
  - On the boundary, the finite parts of the renormalized correlator $<T_{\mu\nu}T_{\rho\sigma}>$ need to be calculated via the ϵ-expansion.
  - In the bulk, in order to compute both $\mathcal{P}_S$ and $\mathcal{P}_T$ a further understanding of the time-dependent background is needed.
- Finally, the dual of dS is ought to be a non-unitary large N theory as well.
- Unfortunately, the Ising model is unitary and of $N=1$. Is this a problem?
  - First of all, $\eta = 0.036$ is pretty much N-independent. ~[Reviewed by Henriksson (2022)]
  - Secondly, the analytic continuation of the dS-scalar system is an AdS system which is dual to a unitary CFT. The critical exponent η and its effect on n_S survives this analytic continuation.

Part2: The Thermal Effects, Propagators and temperature

- The rapidly expanding phase of the universe can be modeled by $d{+}1\,dS$ space and a scalar field.
  - The action: $\mathcal{S} = \int d^{d+1}x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(m^2+\xi\mathcal{R})\phi^2\right]$
  - The metric: $ds^2 = a^2(d\tau^2 - dx^2)$

- The corresponding equation of motion (eom) is:

$$\Phi''_{|\mathbf{k}|} + \omega^2_{|\mathbf{k}|}\Phi_{|\mathbf{k}|} = 0, \qquad \phi_{|\mathbf{k}|} = \frac{\Phi_{|\mathbf{k}|}}{a}$$

with

$$\omega^2_{|\mathbf{k}|} = |\mathbf{k}|^2 + m^2_{\rm dS}, \qquad m^2_{\rm dS} = \frac{1}{\tau^2}\left(\frac{m^2}{H^2} + 12\xi - \frac{d^2-1}{4}\right), \qquad \mathcal{R} = 12H^2$$

and its general solution is a linear combination of

$$H^{\pm}_{\nu_{\rm cl}}(\tau,|\mathbf{k}|) = J_{\nu_{\rm cl}}(\tau,|\mathbf{k}|) \pm iY_{\nu_{\rm cl}}(\tau,|\mathbf{k}|), \qquad \nu_{\rm cl} = \frac{d}{2}\sqrt{1 - \frac{4m^2}{d^2H^2} - \frac{48\xi}{d^2}}$$

Part2: The Thermal Effects, Propagators and temperature

- We already know that dS is characterized by a temperature, the dS temperature:

$$T_{dS} = \frac{H}{2\pi}$$

hence, we suspect that the corresponding thermal effects have left an imprint on the CMB.

- In order to regulate these effects, we will proceed to calculate the thermal propagators of our theory.

- There are three known ways to incorporate temperature in a system:

  i. The imaginary time (Matsubara) formalism

  ii. The Schwinger-Keldysh (SK) formalim

  iii. Thermo-field Dynamics (TFD)

The SK and TFD formalisms are both real-time formalisms and will be our main focus.

We will show that their results are equivalent.

Part2: The Thermal Effects, Propagators and temperature

- In either the SK or TFD, we are forced to double the degrees of freedom by a defining a copy of our Hilbert space.

- In the context of the SK path integral, the doubled Hilbert space is related to a forward (+) and a backward (-) branch in conformal time evolution.

- The field propagator D is the 2×2 matrix:

$$\mathrm{D}(\tau_1-\tau_2)\equiv\begin{pmatrix}D_{++} & D_{+-}\\ D_{-+} & D_{--}\end{pmatrix}=\begin{pmatrix}\langle 0|\mathcal{T}\Phi(\tau_1)\Phi(\tau_2)|0\rangle & \langle 0|\tilde{\Phi}(\tau_1)\Phi(\tau_2)|0\rangle\\ \langle 0|\Phi(\tau_1)\tilde{\Phi}(\tau_2)|0\rangle & \langle 0|\tilde{\mathcal{T}}\tilde{\Phi}(\tau_1)\tilde{\Phi}(\tau_2)|0\rangle\end{pmatrix}$$

  where $\mathcal{T}(\tilde{\mathcal{T}})$ is the time (anti-time) order product.

- Note that the Kubo-Martin-Schwinger (KMS) condition:

$$\langle 0|\Phi(\tau_1)\Phi(\tau_2)|0\rangle=\langle 0|\Phi(\tau_2)\Phi(\tau_1-i\tfrac{\beta}{2})|0\rangle\,,\qquad \beta=\frac{1}{T}$$

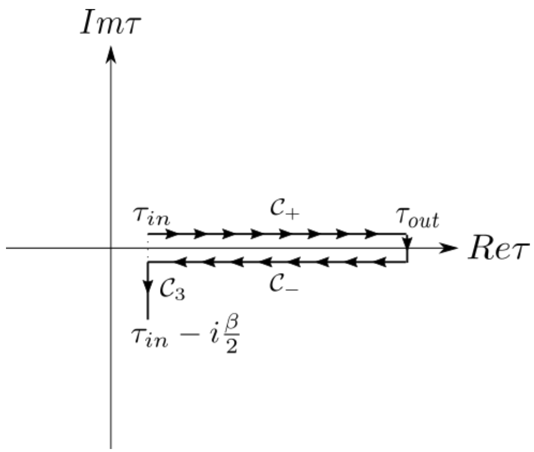
  is the most basic condition a thermal propagator needs to satisfy.

Part2: The Thermal Effects, Propagators and temperature

- In order to calculate the thermal propagator in a dS background, we will introduce the SK contour

which decomposes into three individual parts:

- the forward branch ($\mathcal{C}_+$)
- the backward branch ($\mathcal{C}_-$)
- The thermal branch ($\mathcal{C}_3$)



We introduce the propagators:

$$D_{33} = \langle 0|\mathcal{T}\{\Phi^3(\tau_1)\Phi^3(\tau_2)\}|0\rangle$$

$$D_{3+} = \langle 0|\Phi(\tau_1)\Phi^3(\tau_2)|0\rangle$$

$$D_{3-} = \langle 0|\tilde{\Phi}(\tau_1)\Phi^3(\tau_2)|0\rangle$$

- Then, there are three junction points $(\tau_{\rm out}, \tau_{\rm in}, \tau_{\rm in} - i\,\beta/2)$ which form a closed system of equation whose solution is the thermal propagator!

- Since the chosen contour allows for imaginary time flow, we assume that there is no inflation in that direction.

Part2: The Thermal Effects, Propagators and temperature

- Assuming the BD vacuum at $\tau = \tau_{\rm in} \to -\infty$, the mode-functions $\Phi_{|\boldsymbol{k}|}$ are expressed in terms of Hankel functions of $\nu_{\rm cl} = 3/2$ order.

- Then, the in-in SK thermal propagator is:

$$\mathrm{D}_{\beta/2} = D + n_B(\beta/2)\left(D_{++} + D^*_{++}\right)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} D_{++} & D_{+-} \\ D_{-+} & D_{--} \end{pmatrix}$$

with

$$n_B(\beta) = \frac{e^{-\beta\omega_{|\mathbf{k}|}}}{1 - e^{-\beta\omega_{|\mathbf{k}|}}}$$

the Bose-Einstein number density.

- For future reference, we define the parametrization:

$$s(\beta) \equiv \sinh\theta_{|\mathbf{k}|}(\beta) = \sqrt{n_B(\beta)}, \qquad c(\beta) \equiv \cosh\theta_{|\mathbf{k}|}(\beta)$$

so that:

$$c^2(\beta) - s^2(\beta) = 1$$

Part2: The Thermal Effects, Propagators and temperature

- In the TFD approach, the doubled Hilbert space is seen as the tensor product of the Hilbert spaces of positive and negative momenta.

- The corresponding fields living on these spaces can be Fourier-expanded:

$$\Phi(\tau,\mathbf{x}) = \int d^3\mathbf{k}\left[\alpha^-_{\mathbf{k}} u^*_{|\mathbf{k}|}(\tau) + \alpha^+_{\mathbf{k}} u_{|\mathbf{k}|}(\tau)\right] e^{i\mathbf{kx}}$$

$$\tilde{\Phi}(\tau,\mathbf{x}) = \int d^3\mathbf{k}\left[\tilde{\alpha}^+_{\mathbf{k}} u^*_{|\mathbf{k}|}(\tau) + \tilde{\alpha}^-_{\mathbf{k}} u_{|\mathbf{k}|}(\tau)\right] e^{i\mathbf{kx}}$$

  while the ladder operators can be rotated into their thermal analogues:

$$\begin{pmatrix} \alpha^-_{\mathbf{k}}(\beta) \\ \tilde{\alpha}^+_{\mathbf{k}}(\beta) \end{pmatrix} = \mathrm{U}(\beta;\mathbf{k}) \begin{pmatrix} \alpha^-_{\mathbf{k}} \\ \tilde{\alpha}^+_{\mathbf{k}} \end{pmatrix}$$

  where U can be thought of as a BT!

- The thermal ladder operators define the thermal vacuum:

$$\alpha^-_{\mathbf{k}}(\beta)\,|0;\beta\rangle = 0$$

Part2: The Thermal Effects, Propagators and temperature

- The thermal propagator in the TFD formalism is equal to:

$$D_{\beta'} = U_{\beta'}^{-1} D U_{\beta'}^{-1\,T}$$

with

$$\mathrm{U}_{\beta'}^{-1} \equiv \begin{pmatrix} c(\beta') & s(\beta') \\ s(\beta') & c(\beta') \end{pmatrix}$$

$$\mathrm{D} = \begin{pmatrix} \langle \Phi\Phi \rangle & \langle \Phi\tilde{\Phi} \rangle \\ \langle \tilde{\Phi}\Phi \rangle & \langle \tilde{\Phi}\tilde{\Phi} \rangle \end{pmatrix}$$

- The rotation results into:

$$\mathrm{D}_{\beta'} = D + \left(s^2(\beta') + s(\beta')c(\beta')\right)\left(D_{++} + D^*_{++}\right)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

- The following identity holds:

$$\frac{e^{-\beta\omega_{|\mathbf{k}|}}}{1-e^{-\beta\omega_{|\mathbf{k}|}}} + \frac{e^{-\frac{\beta}{2}\omega_{|\mathbf{k}|}}}{1-e^{-\beta\omega_{|\mathbf{k}|}}} = \frac{e^{-\frac{\beta}{2}\omega_{|\mathbf{k}|}}}{1-e^{-\frac{\beta}{2}\omega_{|\mathbf{k}|}}}$$

$$s(\beta) = \sqrt{n_B(\beta)}$$

$$c(\beta) = \sqrt{1 - n_B}$$

Thus. for $\beta' = 2\beta$ the SK and TFD propagators match!

Part2: The Thermal Effects, Line of constant physics and other observables

- The thermal scalar spectrum is:

$$\mathcal{P}_{S,\beta}\mathbf{1} = D_\beta \mathbf{1}\Big|_{\tau_1=\tau_2}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

- We define the parameter:

$$\kappa \equiv \omega_{|\mathbf{k}|}|\tau|\Big|_{|\mathbf{k}\tau|=1} = \sqrt{\frac{5-d^2}{4} + \frac{m^2}{H^2} + 12\xi}$$

  - For $d=3$ and $m^2/H^2 + 12\xi = 0 \Rightarrow \kappa = i$, which corresponds to a scale invariant CMB

or

$$\nu_{\rm cl} = 3/2$$

- Moving away from $\tau_{\rm in}$ will introduce a time-dependent BT of the frequency:

$$\Omega_{|\mathbf{k}|} = \omega_{|\mathbf{k}|}\left(|c|^2 + |s|^2\right)$$

$$\kappa \to \Lambda = \kappa\left(1 + 2\frac{e^{-2\kappa x}}{1-e^{-2\kappa x}}\right) = \kappa \coth(x\kappa), \qquad x = \frac{\pi H}{2\pi T}$$

The transformation of $\omega_{|k|}$ corresponds to the appearance of a thermal mass.

Part2: The Thermal Effects, Line of constant physics and other observables

- The thermal mass shifts the weight away from $3/2$:

$$\nu_{\rm cl} \to \nu = \nu_{\rm cl} + \nu$$

which in return shifts the scaling dimension of the scalar field:

$$\Delta_- = \frac{d}{2} - \nu = \Delta_{{\rm cl},-} - \nu_q$$

The dual partner will have:

$$\Delta_+ = \frac{d}{2} + \nu$$

We are interested in $(\Delta_-, \Delta_+)_{\rm cl} = (0,3)$

- The appearance of the thermal mass can be also seen from the eom:

$$\phi'' + 2aH\phi' + (\frac{m^2}{H^2} + \xi\frac{\mathcal{R}}{H^2})a^2H^2\phi = 0, \qquad H' = -\frac{1}{2a}\phi'^2$$

$$' = \frac{\partial}{\partial\tau}$$

since there seems to be a deformation of H at late times.

Part2: The Thermal Effects, Line of constant physics and other observables

- The thermal scalar spectral index will be equal to:

$$n_{S,\beta} = 1 + \frac{d\ln\left(|\mathbf{k}|^3 \mathcal{P}_{S,\beta}\right)}{d\ln|\mathbf{k}|}$$

so that its thermal deviation is:

$$\delta n_S = n_{S,\beta} - 1 = -\frac{2x}{\Lambda}\left[\frac{e^{-x\Lambda}}{1-e^{-2x\Lambda}}\right]$$

- Two temperatures are interesting for $\kappa = i$:

  - zero-temperature $(T = 0 \Rightarrow x \to \infty)$
  - dS-temperature $(T = T_{dS} \Rightarrow x = \pi)$

$$\delta n_S = 0$$

Scale invariance is restored in zero and dS temperatures!

- This agrees with our assumption that on the boundary we flow from an UV towards an IR FP!

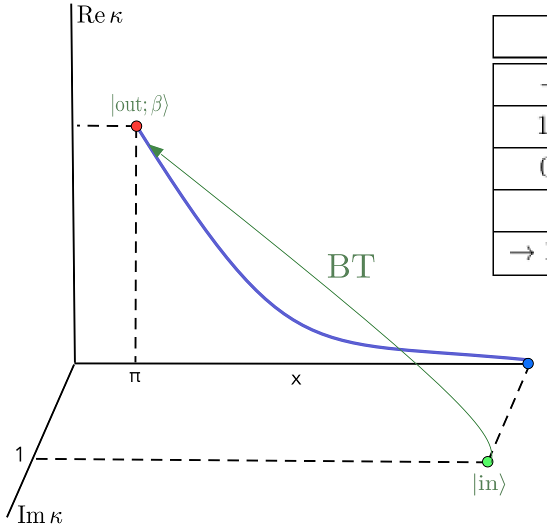
Part2: The Thermal Effects, Line of constant physics and other observables

- For the intermediate temperatures $(x \geq \pi)$: $\delta n_S \neq 0$ and the scale invariance breaks-down
- We can now fix the value of $n_{S,\beta} = 0.964$.

$$n_{S,\mathrm{exp}} = 0.9649 \pm 0.0042$$

~[Planck Collaboration, 2018]

- We observe a line of constant physics (LCP) which heats up our system up to the dS temperature!



Λ	x
$\to 0$	$\to \infty$
10^{-6}	$3.5 \cdot 10^{7}$
0.01	1600
0.5	14.8
$\to 1.5117$	$\to \pi$

Our system:

- Starts at $\tau = \tau_{\mathrm{in}}$, $\kappa = i$,
- Flows towards $\tau = \tau_{\mathrm{out}}$ and heats up $(T < T_{dS})$,
- There, for different values x, different values of Λ satisfy $\delta n_{S,\beta} =$-0.036,
- Reaches $\tau = \tau_{\mathrm{out}}$ and $T = T_{dS}$ and the CMB becomes scale invariant.

Part2: The Thermal Effects, Line of constant physics and other observables

- Our system has no free parameters, thus we can directly compute several other observables.

- The running of the spectral index:

$$n^{(1)}_{S,\beta} = \frac{dn_{S,\beta}}{d\ln|\mathbf{k}|} \quad \longrightarrow \quad n^{(1)}_{S,\beta} = \delta n_S\left[2 - \frac{1}{\Lambda^2} - \frac{x}{\Lambda}\left(1 + \frac{2e^{-2x\Lambda}}{1-e^{-2x\lambda}}\right)\right]$$

which close to dS temperature $(x \simeq \pi, \Lambda \simeq 1.5117)$ gives:

$$n^{(1)}_{S,\beta} = 0.0186 \qquad n^{(1)}_{S,exp} = 0.013 \pm 0.012$$

~[Planck Collaboration (2018)]

- The non-Gaussianity parameter:

$$f_{NL} = \frac{5}{6}\frac{N_{\rho\rho}}{N_\rho^2}, \qquad N = \int_{t_i}^{t_f} dtH, \qquad N_\rho = \frac{\partial N}{\partial \mathcal{P}_{S,\beta}}$$

~[Maldacena (2003)]

$$f_{NL} = -\frac{5\left[x(-1+\Lambda^2)^2\left(1+x\Lambda\cot(\frac{x\Lambda}{2})\right)+2\Lambda^3\sinh(x\Lambda)\right]}{6\Lambda^2\left[x(-1+\Lambda^2)+\Lambda\sinh(x\Lambda)\right]} \xrightarrow[\Lambda \simeq 1.5117]{x \simeq \pi} f_{NL} = -1.7138$$

$$f_{NL,exp} = -1.7 \pm 5.2$$

~[Planck Collaboration (2018)]

What's next?

- The thermal corrections should trigger a backreaction described by:

$$\mathrm{G}_{\mu\nu} = 8\pi G \left\langle T_{\mu\nu} \right\rangle_{\beta}, \;\; \left\langle T_{\mu\nu} \right\rangle_{\beta} = \langle 0;\beta | T_{\mu\nu} | 0;\beta \rangle$$

  where $\mathrm{T}_{\mu\nu}$ would be the zero-temp SET.

- One then needs to:
  - expand $\mathrm{T}_{\mu\nu}$ in terms of the zero-temp ladder-operators,
  - deal with the divergences arising from the k-integral
  - solve the Einstein equations for the new metric $\tilde{g}_{\mu\nu}$.

- Furthermore, there should be a frame of reference for which the thermal effects correspond to a thermal effective action (with extra interactive terms) which can result to the thermal propagator D_{β}.

$$\left\langle T_{\mu\nu} \right\rangle_{\beta} = \frac{2}{\sqrt{-g}} \frac{\delta\Gamma}{\delta g^{\mu\nu}}$$

Conclusion

- We considered a thermal scalar in dS background in the BD vacuum. Time-evolution placed us in the interior of the finite temperature phase diagram.

- Through holography, the above can be understood as an RG flow of the boundary theory.

  - ➔ We presented indications that the boundary theory is in the universality class of the 3d Ising model

  - ➔ The η critical exponent of the boundary theory could be the parameter that characterized the breaking of scale invariance.

- Using Thermal field theory, we managed to describe how thermal effects appear in the propagator.

- We calculated the thermal power spectrum and the thermal deviation of n_S which gave us clues for the existence of a LCP.

- We calculated additional cosmological observables $(n^{(1)}_{S,\beta}, f_{NL})$ which where well within current experimental bonds.

Thank you!