On the origin of species thermodynamics and the black hole-tower correspondence

Alvaro Herraez

Based on [arXiv:2406.17851] with D. Lüst, J. Masías, M.Scalisi

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The species Scale

- Maximum UV cut-off in QG in the presence of N light species [Dvali '07] [Dvali, Reedi '08] [Dvali, Lüst '10] [Dvali, Gómez '10]

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Species Scale of the d-dim EFT $\begin{cases} M_{\mathrm{Pl},\,d+p} & \text{(decompactification of } p \text{ dimensions)} \\ M_{\mathrm{str}} & \text{(weakly coupled string limits)} \end{cases}$

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- Radius of the smallest BH *in the EFT*

$$S = \frac{A}{4G_{N,d}} \sim \left(\frac{M_{{\rm Pl},d}}{\Lambda_{\rm sp}}\right)^{d-2} \simeq N_{\rm sp}$$

Species Scale of the d-dim EFT
- $M_{{\rm Pl},\, d+p}$ (decompactification of p dimensions)
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Gravitational Collapse and the Covariant Entropy Bound

- Configuration of energy E in a box of size L can collapse gravitationally unless

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[Bousso´99]

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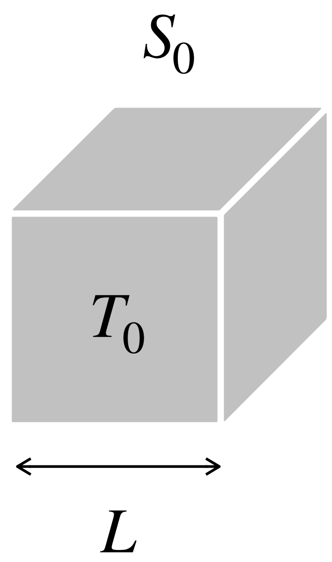

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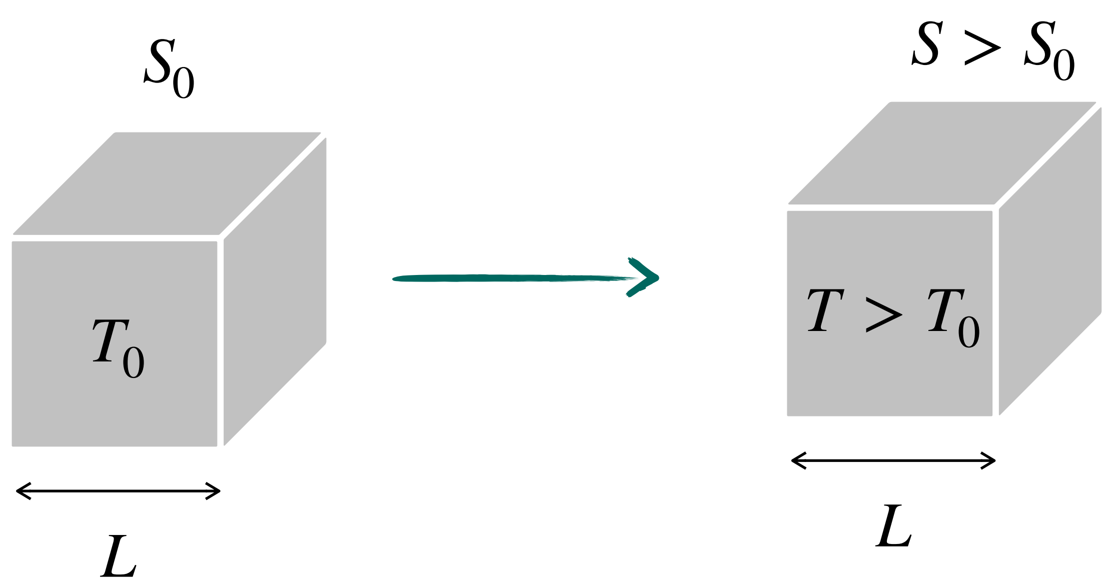

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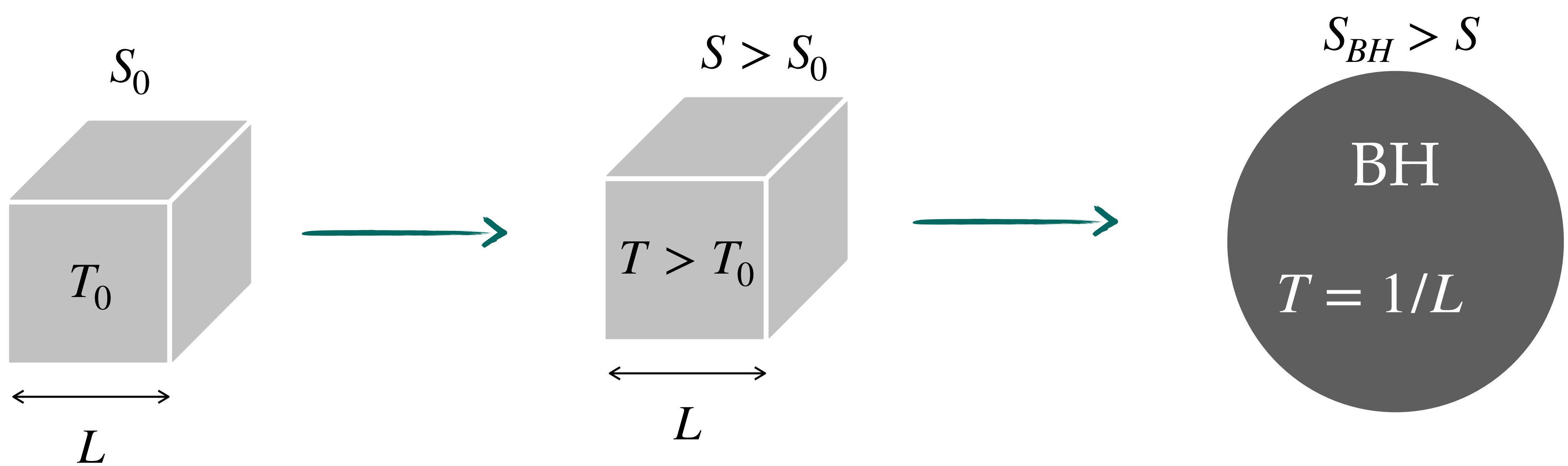

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- Coincide for $T \simeq 1/L$

[Castellano, AH, Ibáñez ´21] [AH, Lüst, Masías, Scalisi ´24]

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1. Motivation: Species Scale, Covariant Entropy Bound and Gravitational Collapse
2. Field Theory Entropy vs. Species Entropy
3. Black Hole-Tower Correspondence

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Field Theory Entropy

[AH, Lüst, Masias, Scalisi '24]

- Configuration of particles in a box of size L with a spectrum of species $m_n = n^{1/p} m_t$ all at a common T (neglect energy in the interactions $M_{\mathrm{Pl},d} \gg \Lambda_{\mathrm{sp}} \geq T$)

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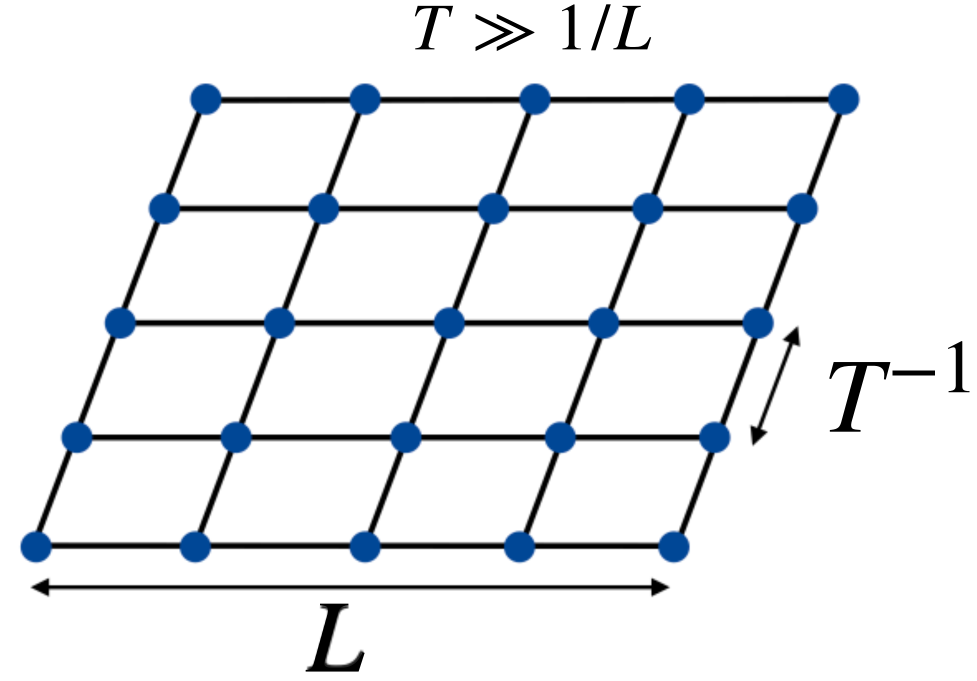

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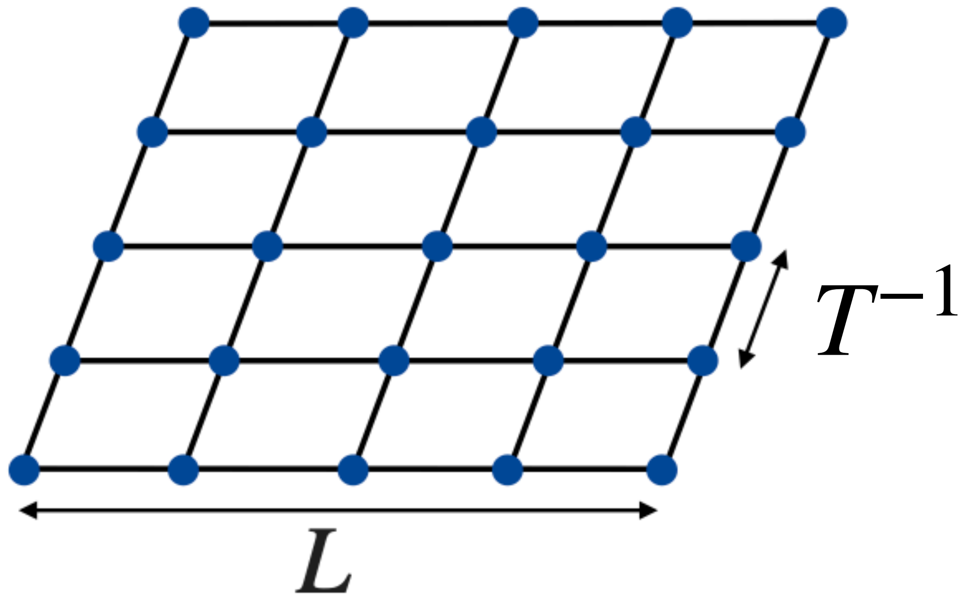

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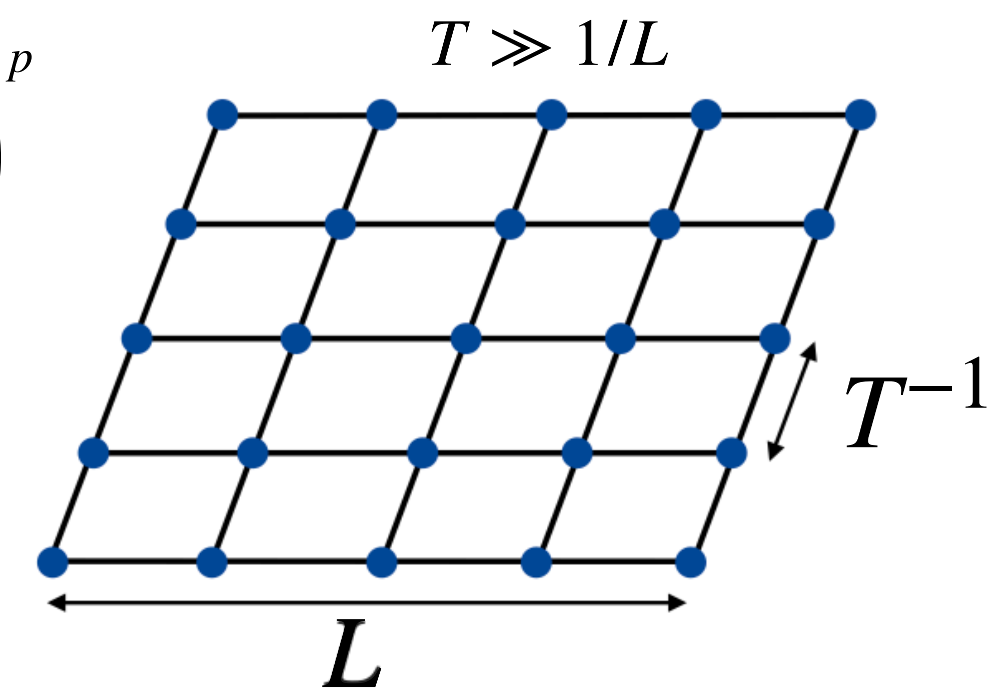

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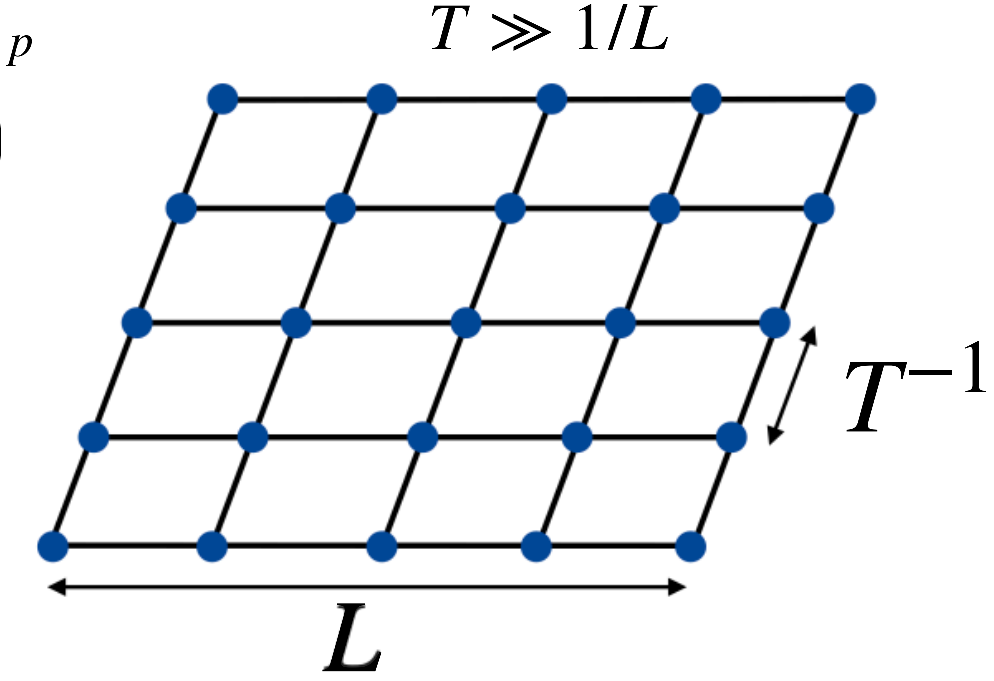

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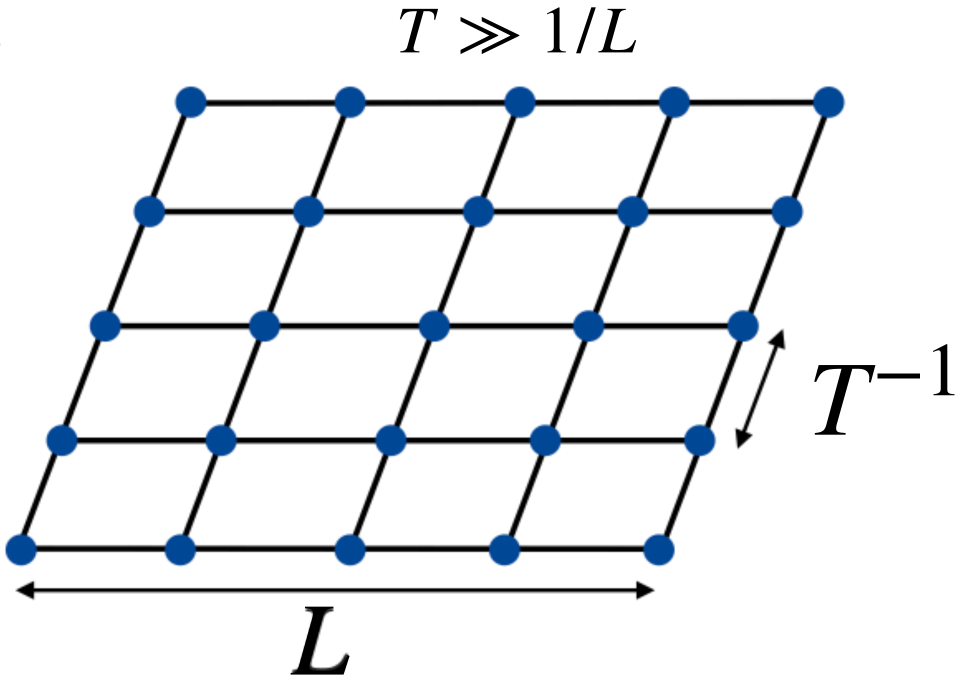

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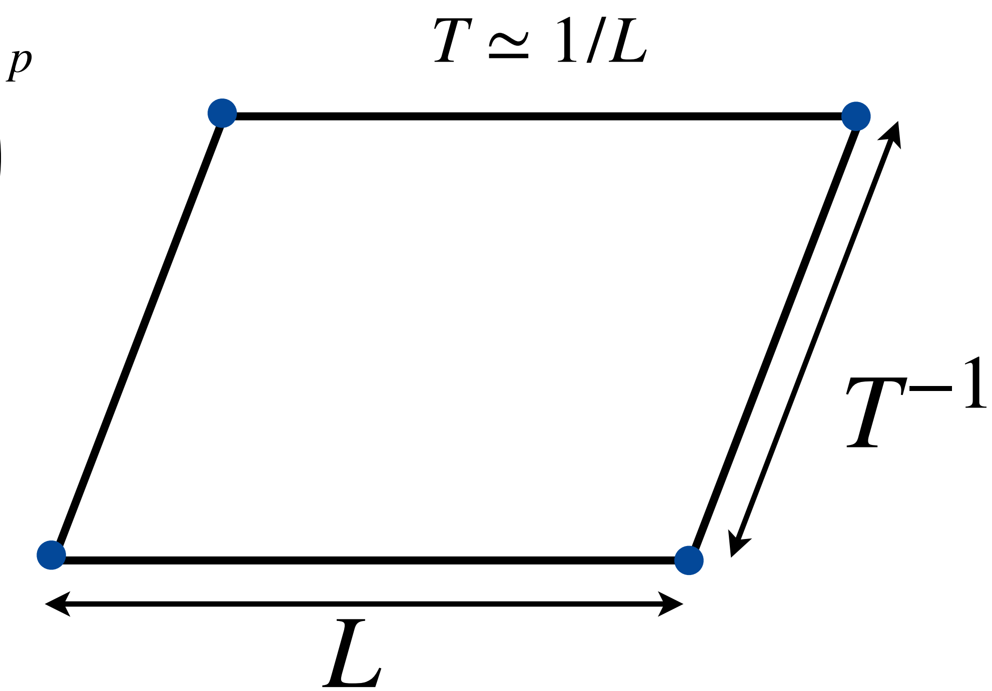

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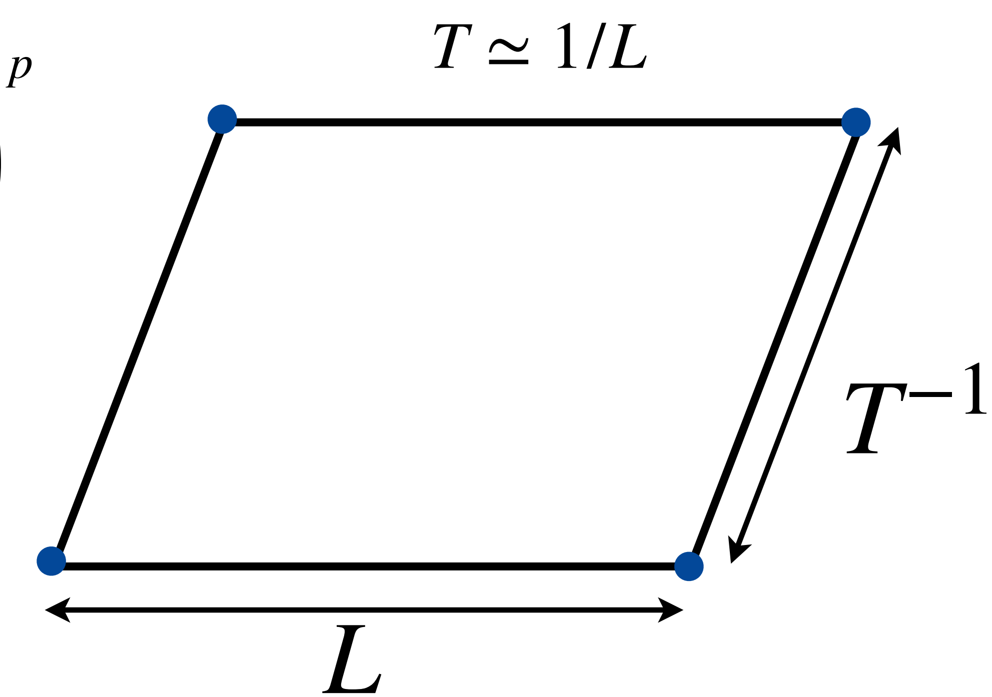

Species Entropy from EFT thermodynamics in the limit

- CEB and Gravitational Collapse Bound coincide for $T \simeq 1/L$
  [AH, Lüst, Masías, Scalisi ´24]

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$$E \simeq \Lambda_{\text{sp}} N_{\text{sp}} = \sum_i m_i \qquad S \simeq N_{\text{sp}}$$

Species Thermodynamics

[Cribiori, Lüst, Montella, ´23]

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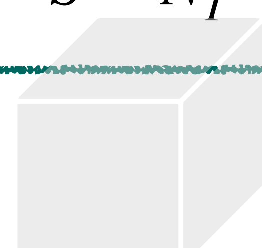
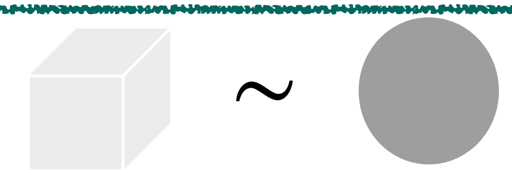
Volume Scaling — **Species/BH Scaling**

$\Lambda_{\mathrm{sp}} \gg T \gg 1/L$	$\Lambda_{\mathrm{sp}} \gg T \simeq 1/L$	$T \simeq 1/L \to \Lambda_{\mathrm{sp}}$
$S \sim N_T\, (TL)^{d-1}$	$S \sim N_T$	$S \sim N_{\mathrm{sp}}$





On the origin of species thermodynamics and the black hole-tower correspondence

1. Motivation: Species Scale, Covariant Entropy Bound and Gravitational Collapse
2. Field Theory Entropy vs. Species Entropy
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Black Hole-String Correspondence

[Susskind ´93]
[Horowitz, Polchinski ’96 ’97]

[Chen, Maldacena, Witten ’21]
[Susskind ’21]
[Ceplack, Emparan, Puhm, Tomasevic ‘22]
[Bedroya, Vafa, Wu ‘23]

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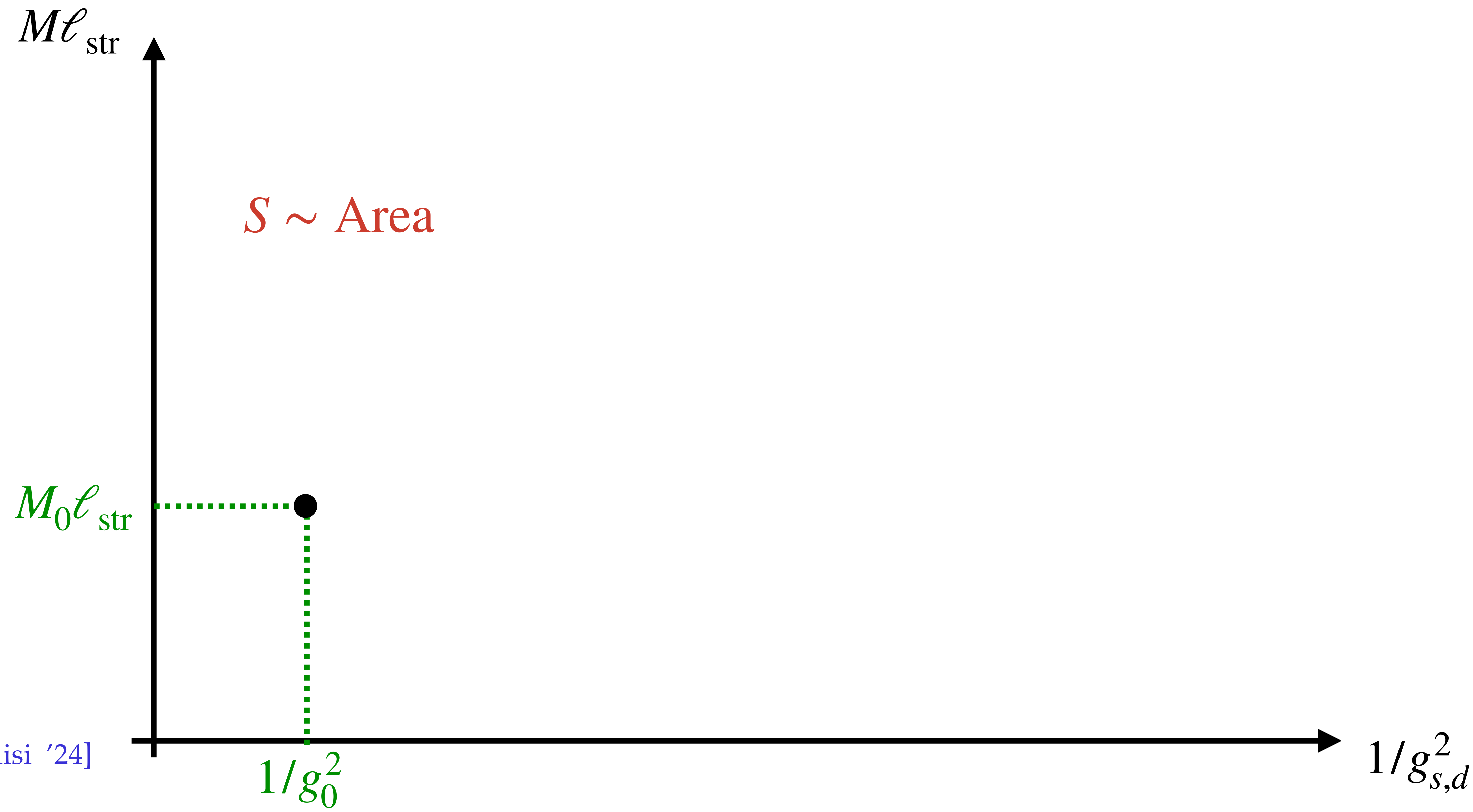

[AH, Lüst, Masias, Scalisi ’24]

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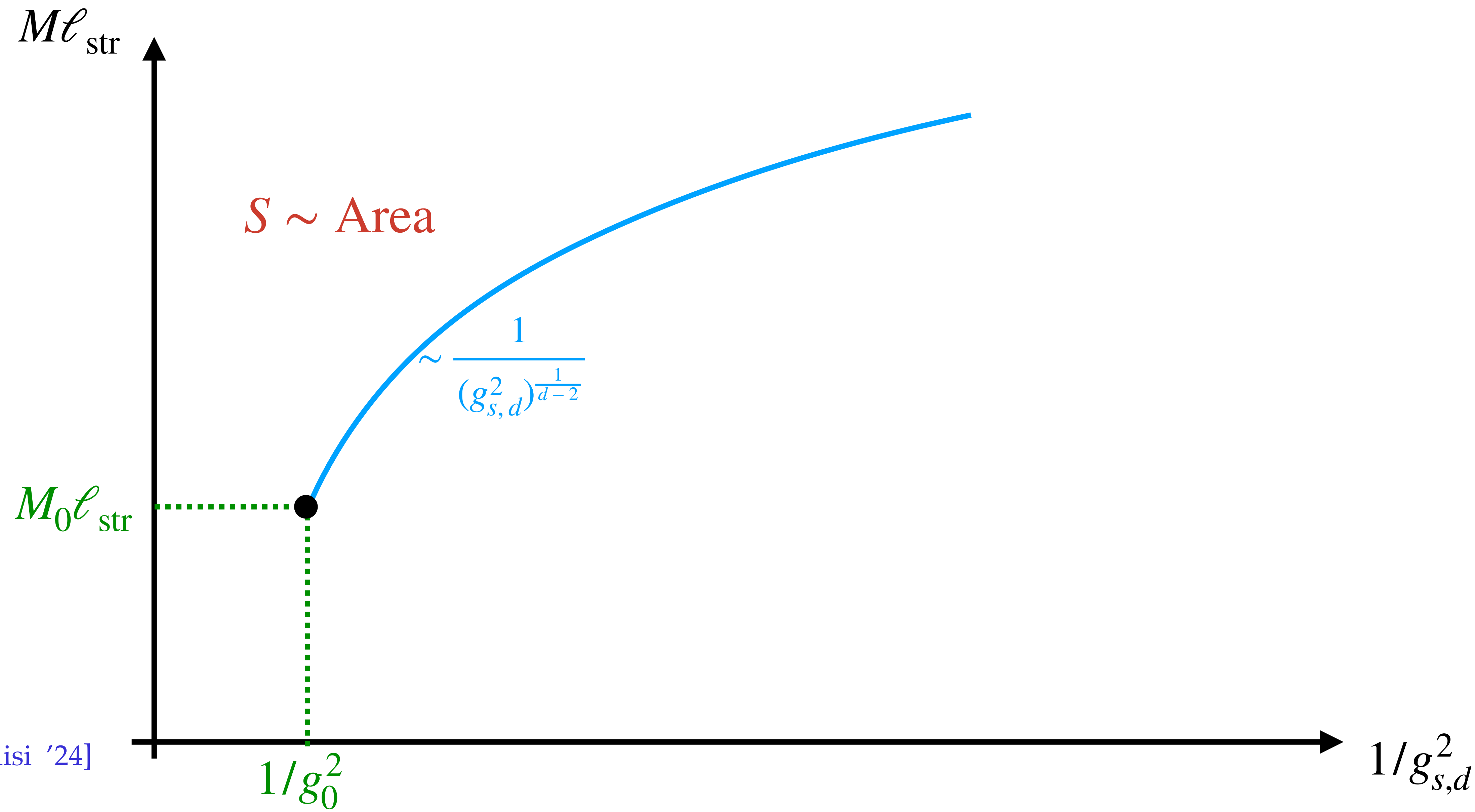

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(Free) String

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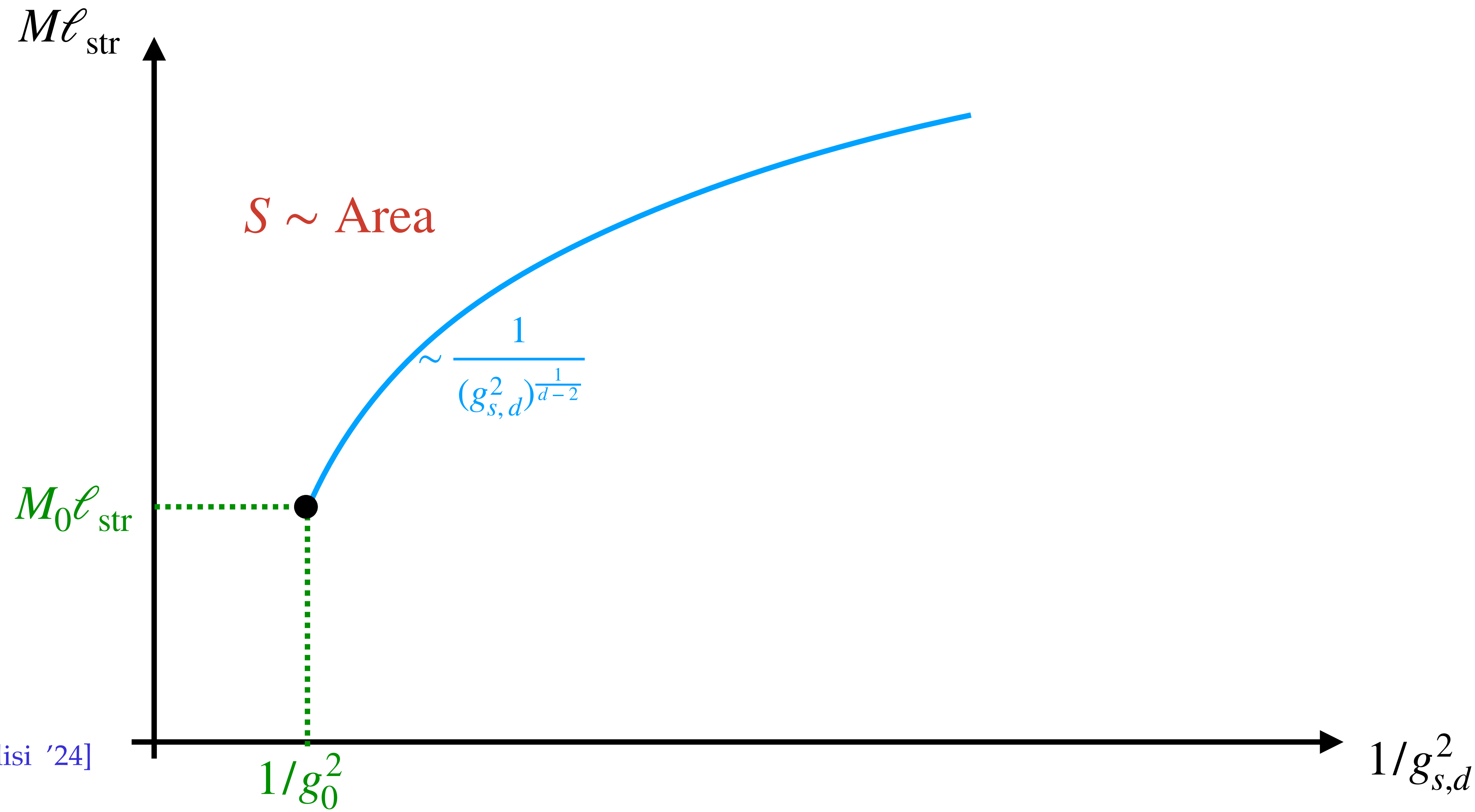

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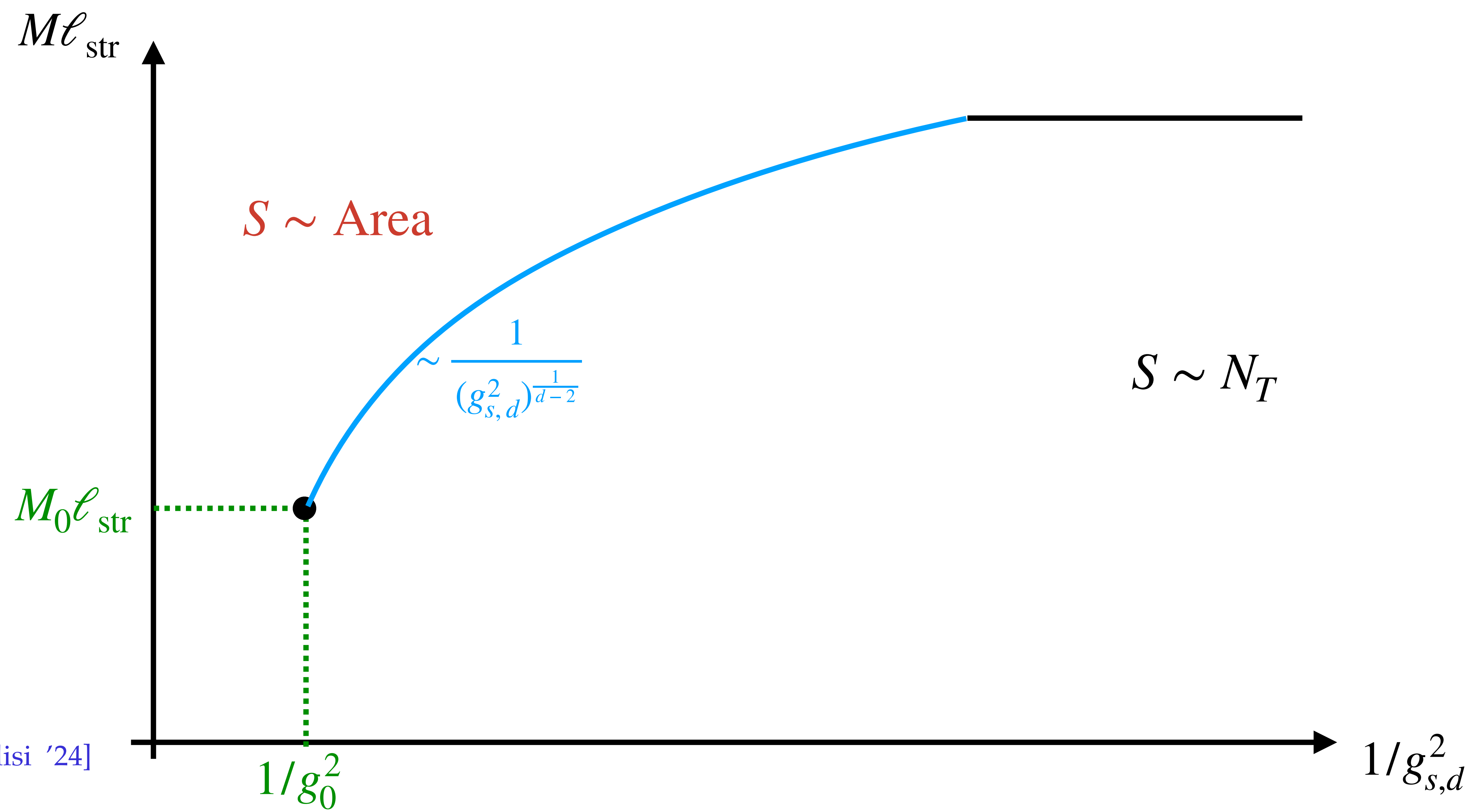

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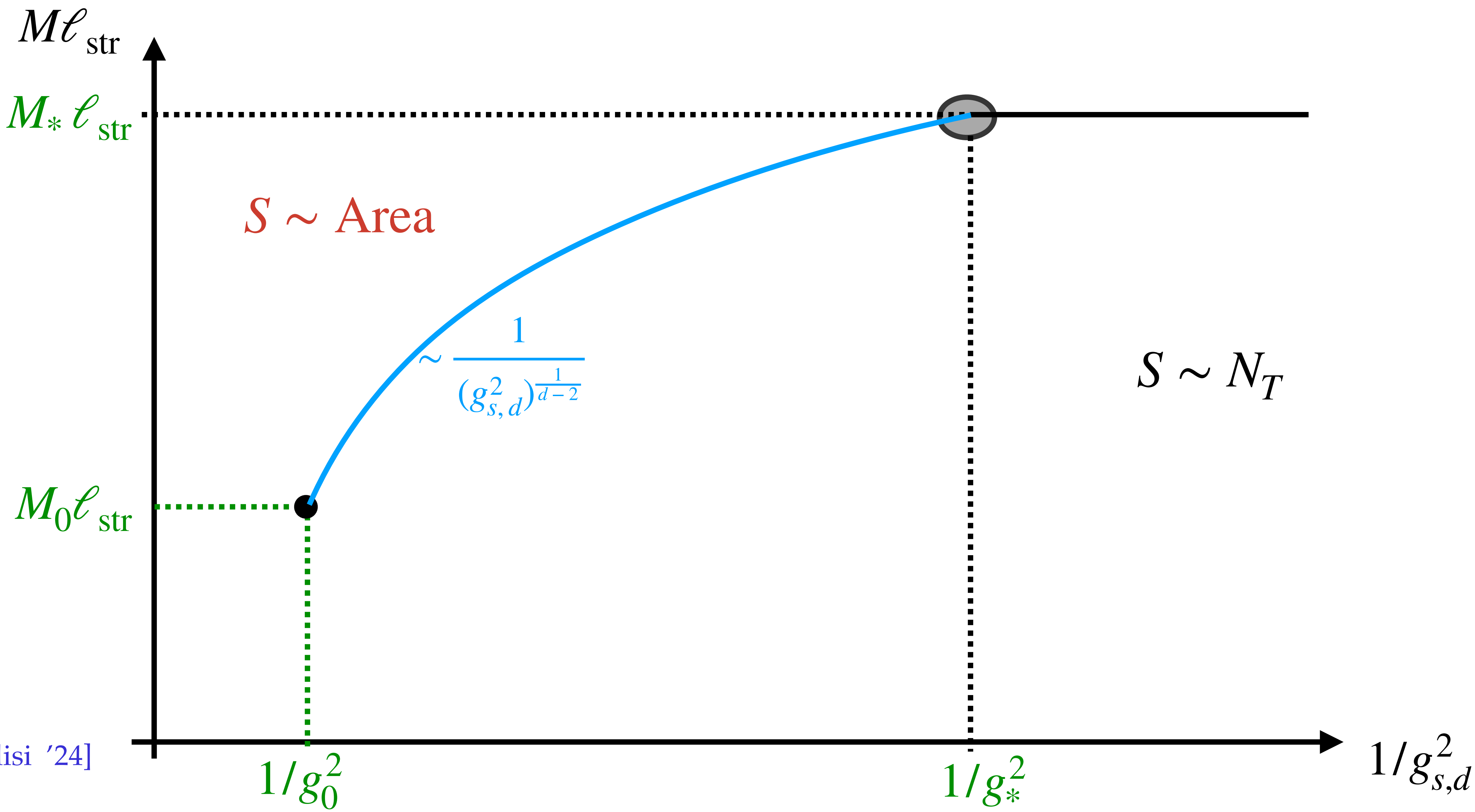
Correspondence point

$$R_{\mathrm{BH}} \sim \ell_{\mathrm{str}}$$

$$S = \frac{1}{g_{s,*}^2} = N_{\mathrm{sp}}$$



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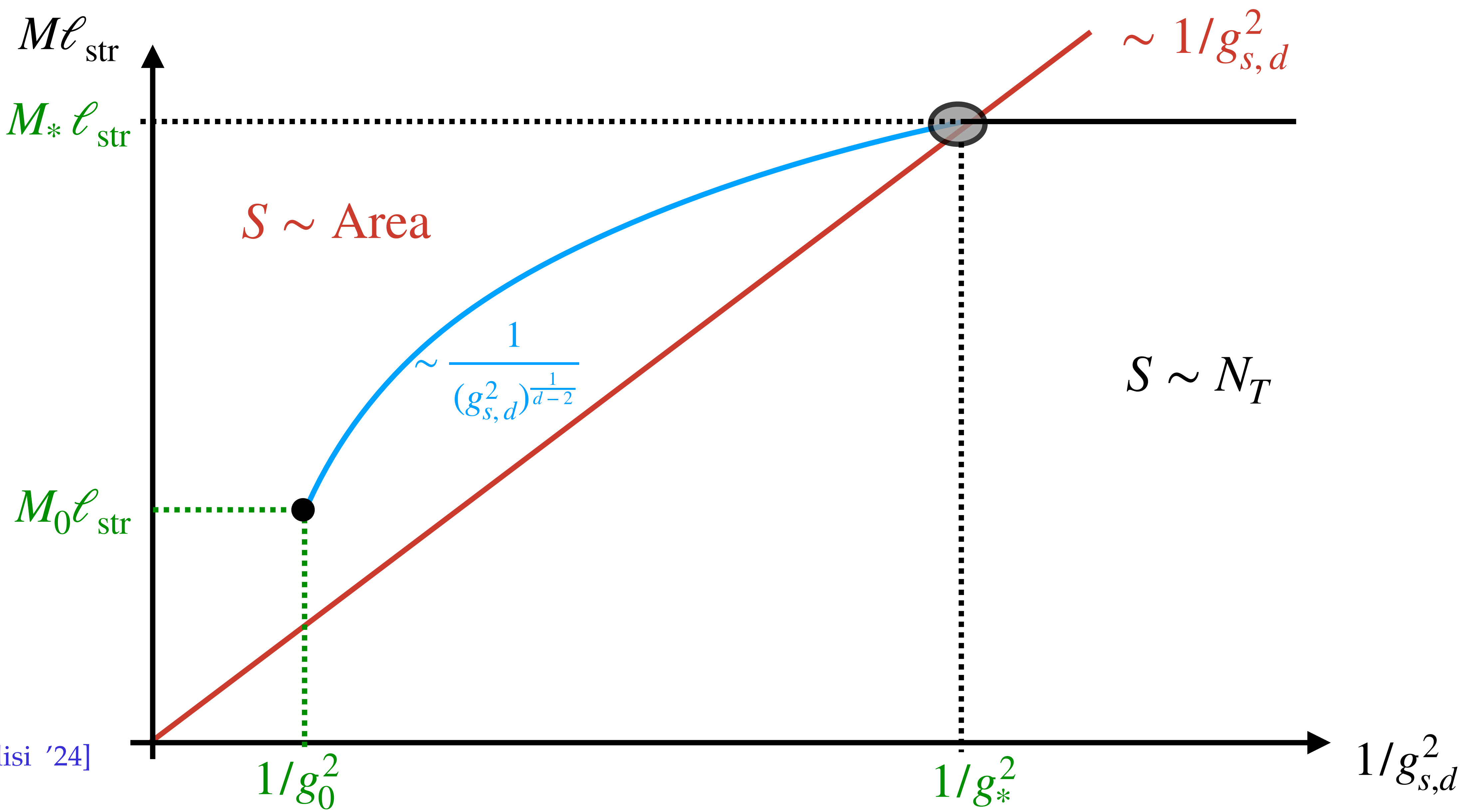

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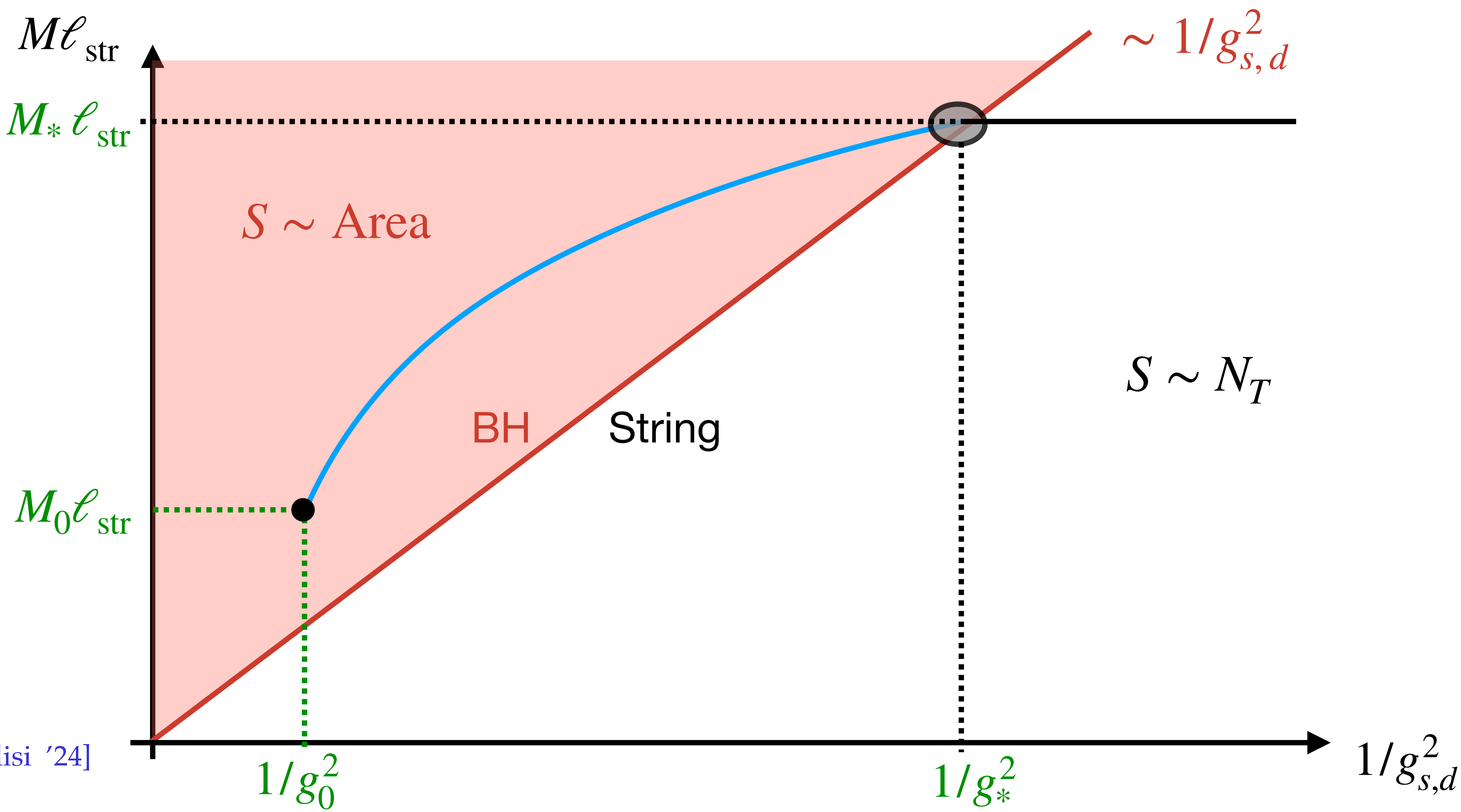


[AH, Lüst, Masias, Scalisi ’24]

Black Hole-Tower Correspondence

$$\ell_{\mathrm{Pl,d}}^{d-2} = \frac{\ell_{\mathrm{sp}}^{d-2}}{\mathscr{V}}$$

$$\ell_{\mathrm{str}} \to \ell_{sp} = \Lambda_{\mathrm{sp}}^{-1}$$
$$g_s^{-2} \to \mathscr{V}$$

Black Hole

$$M_{\mathrm{BH}} \sim \frac{R_{\mathrm{BH}}^{d-3}}{\ell_{\mathrm{Pl,d}}^{d-2}} \sim \frac{R_{\mathrm{BH}}^{d-3}\,\mathscr{V}}{\ell_{\mathrm{sp}}^{d-2}}$$

$$S_{\mathrm{BH}} \sim \left(R_{\mathrm{BH}}/\ell_{\mathrm{Pl,d}}\right)^{d-2} \sim \left(\frac{M_{\mathrm{BH}}^{d-2}\ell_{\mathrm{sp}}^{d-2}}{\mathscr{V}}\right)^{\frac{1}{d-3}}$$

Box of species

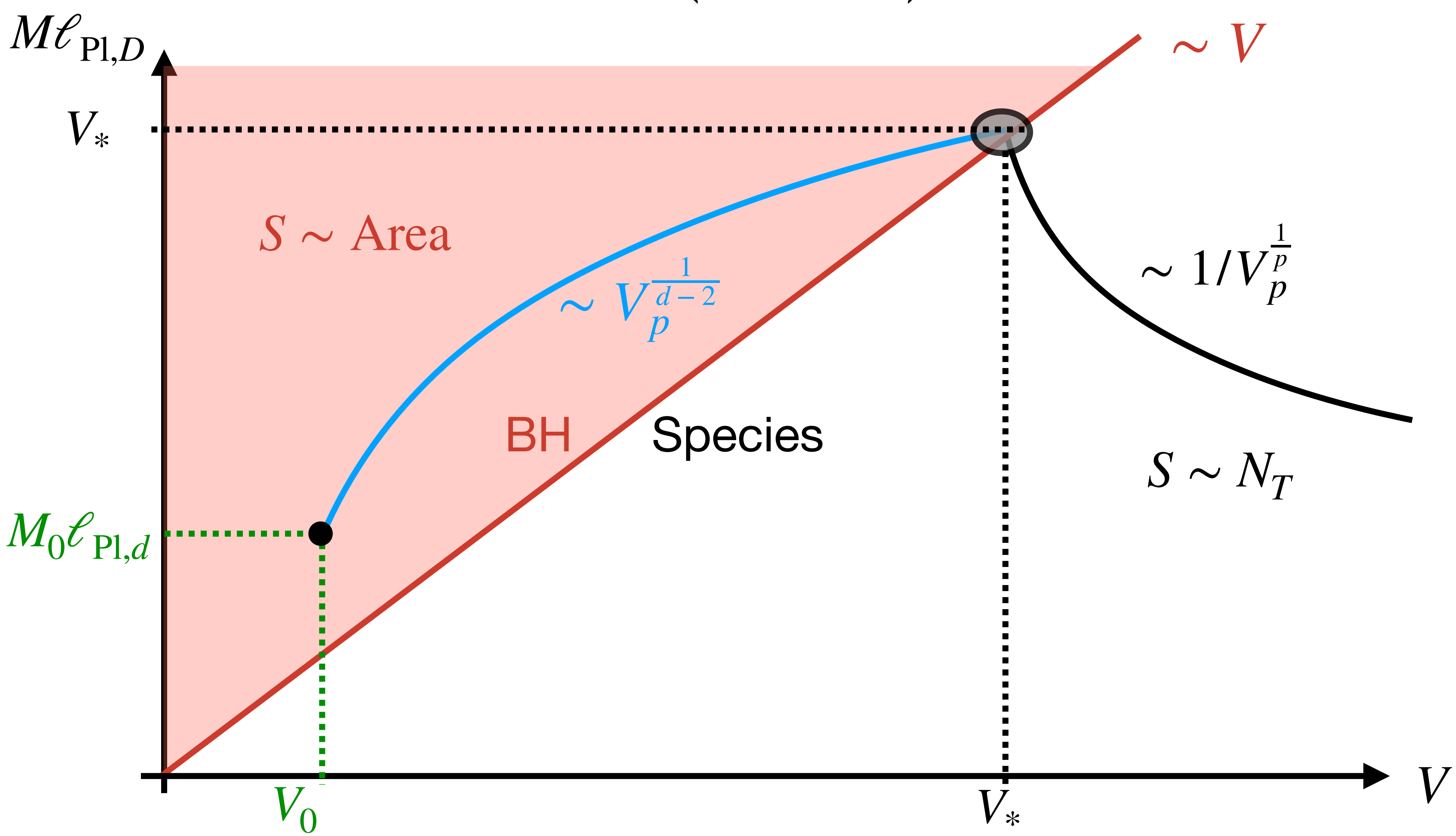
$$L^{-1} = T \leq \Lambda_{\mathrm{sp}}$$

$$S \sim N_T (TL)^{d-1}$$

Correspondence point (line)

$$R_{\mathrm{BH}} \sim \ell_{\mathrm{sp}}$$

$$S = \mathscr{V}_* = N_{\mathrm{sp},*}$$



[AH, Lüst, Masias, Scalisi '24]

Towers in the swampland

- What about *other kinds of towers*? ⟶ Emergent String Conjecture
  [Basile, (Cribiori), Montella, Lüst ´23 ´24] [Bedroya, Mishra, Wiesner ’24] [Lee, Lerche, Weigand ´19]

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  ✳ Superexponential ⟶ Divergent Canonical Partition Function ✗

  ✳ Subpolynomial: $p \to 0$ ⟶ $\dfrac{S}{N_T} \simeq \dfrac{p+1}{p} \to \infty$ ✗

On the origin of species thermodynamics and the black hole-tower correspondence

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- Study configurations that do not collapse gravitationally and approach maximum possible entropy ($T \simeq 1/L$)

- Recover volume scaling and species thermodynamics scalings in the right limits $S \sim (TL)^{d-1}$ for $T \gg 1/L$ and $S \simeq N_T \to N_{\rm sp}$ for $T \simeq 1/L \to \Lambda_{\rm sp}$

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- These towers can give rise to a tower-black hole correspondence and the scaling of the entropy of black holes can be accounted by the entropy of the free system

- Polynomial and exponentially degenerate towers have the right scaling of entropy as $T \to \Lambda_{sp}$ (i.e. recover area law for small black holes) $\longrightarrow$ Emergent String Conjecture

Open Questions — Food for Thought

- $\Lambda_{\rm sp} \simeq M_s$ vs. $\Lambda_{\rm sp} \simeq M_s \log(M_{\rm pl}/M_s)$ $\longrightarrow$ Interpreted as maximum temperature

  $T_{\rm max} = \Lambda_{\rm sp} \simeq M_s$

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- Details on the transition between black holes/branes and towers of species (dynamics)

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- Details on the transition between black holes/branes and towers of species (dynamics)

- Relation to the Emergence Proposal?
  - $T \simeq 1/L \to \Lambda_{\rm sp}$ (no kinetic energy for species at maximum T)
  - Entropy of strongly coupled gravitational BH from towers of states in the decoupling limit

Thank you!

Gravitational Collapse and the Covariant Entropy Bound

- Configuration of energy E in a box of size L can collapse gravitationally unless

$$L \geq R_{\rm BH}(E) = \left(\frac{E}{M_{{\rm Pl},d}}\right)^{\frac{1}{d-3}} M_{{\rm Pl},d}^{-1} \longrightarrow \quad E \lesssim L^{d-3} \qquad S \lesssim \frac{L^{d-3}}{T}$$

- Covariant Entropy Bound

[Bousso´99]

$$S \leq \frac{A}{4G_{N,d}} \sim (LM_{{\rm Pl},d})^{d-2} \xrightarrow{S \sim \frac{E}{T}} \quad E \lesssim TL^{d-2} \qquad S \lesssim L^{d-2}$$

- Coincide for $T \simeq 1/L$ $\longrightarrow$ $E_{\rm max}(T) \simeq \frac{1}{T^{d-3}} \geq \frac{1}{\Lambda_{\rm sp}^{d-3}} \simeq \frac{N_{\rm sp}}{\Lambda_{\rm sp}}$

[Castellano, AH, Ibáñez ´21] [AH, Lüst, Masías, Scalisi ´24]

All one-particle states in the box with non-zero momentum have a large Boltzmann suppression $\sim e^{-E_m/T}$

$E_m = \sqrt{m_m + p_m}$ $\qquad p = n^2/L^2$

$$S_{\rm max}(T) \simeq \frac{1}{T^{d-2}} \geq \frac{1}{\Lambda_{\rm sp}^{d-2}} \simeq N_{\rm sp}$$