Radiation exchange in primordial gravitational waves

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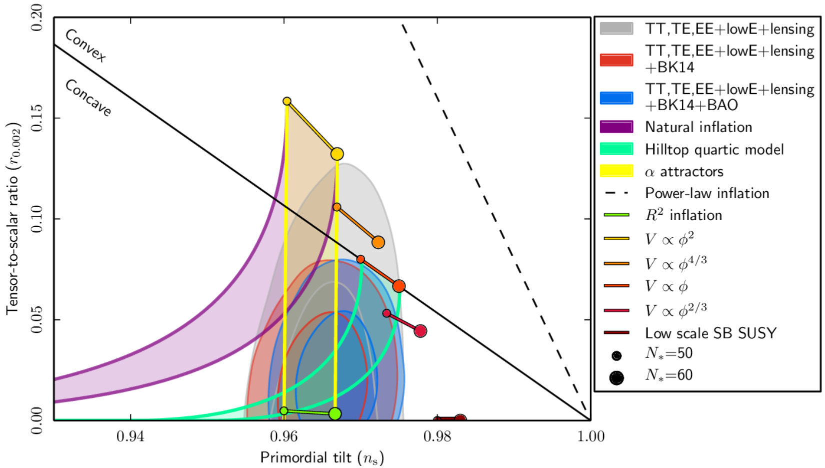
What we did

We computed thermal corrections to primordial gravitational wave (GW) spectrum during radiation dominant universe. The super-horizon primordial GWs are enhanced.

*Questions

1. Superhorizon conservation of GWs?
2. What is the observational implications?

Cosmological perturbations in the sky



GWs in Radiation dominated universe

*Linearized Einstein equation with hydrodynamical approximation:

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \partial_k^2 h_{ij} = 0.$$

1. Massless.
2. Constant superhorizon modes.
3. Radiation drives the background dynamics only.
4. h_{ij} is a free field in the FLRW background.

*RD universe realized by massless scalar field χ with a local thermal state ϱ_χ.

$$S_\chi = -\frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$$
$$= \frac{1}{2}\int d^4x\ a^2(\chi'^2 - \delta^{ij}\partial_i\chi\partial_j\chi) + \cdots ,$$

1. Energy momentum tensor of χ.

$$T_{\chi,\mu\nu} = \partial_\mu\chi\partial_\nu\chi - \frac{g_{\mu\nu}}{2}g^{\rho\sigma}\partial_\rho\chi\partial_\sigma\chi$$

2. Matching condition:

$$T_{\gamma,\mu\nu} = \mathrm{Tr}[\hat{\varrho}_\chi\hat{T}_{\chi,\mu\nu}] \to P = \frac{\rho}{3}$$

Coupling between h_{ij} and χ

*Individual χ feels gravity and vice versa:

$$
\begin{aligned}
S_\chi &= -\frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \\
&= \cdots + \frac{1}{2}\int d^4x\ a^2 h^i{}_k\delta^{kj}\partial_j\chi\partial_i\chi \\
&- \frac{1}{4}\int d^4x\ a^2 h^i{}_k h^k{}_l\delta^{lj}\partial_j\chi\partial_i\chi\cdots .
\end{aligned}
$$

*Radiation χ and h_{ij} are coupled.

Inconsistency?

Hydrodynamical approx.	(A simple) UV theory
h_{ij} is free	h_{ij} couples to χ.

Something might be missing in the standard cosmological perturbation theory based on hydrodynamical approximation.

Short comings of hydrodynamical approach

*Hydrodynamical approximation:

$$\mathrm{EoM}(\hat{h}, \hat{\chi}) = 0 \rightarrow \mathrm{Tr}\left[\hat{\varrho}\mathrm{EoM}(\hat{h}, \hat{\chi})\right]_{\chi} = 0.$$

*$\hat{h}$ is regarded as external field when averaging χ.
→ Mean field approximation for χ.

Mean field approximation?

*Consider effective mass of ϕ in

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - g\phi^2\chi,$$

*Wrong answer:

$$m^2_{\text{eff.},\phi} = 2g\langle\chi\rangle?$$

*Correct answer: find the pole of the 1-loop propagator.

The in-in formalism

*Hydrodynamics v.s. PT in interaction picture.

*Interaction Hamiltonian:

$$H_{\text{int}} = \epsilon H_{\hat{h}\hat{\chi}\hat{\chi}} + \epsilon^2 H_{\hat{h}\hat{h}\hat{\chi}\hat{\chi}} + \mathcal{O}(\epsilon^3)$$

*Operator evolution[Weinberg 2008]:

$$\begin{aligned} h =& \hat{h} + \lambda \int_{\tau_{\text{R}}}^{\tau} d\tau_1 [H_{\text{int},1}, \hat{h}] \\ &+ \lambda^2 \int_{\tau_{\text{R}}}^{\tau} d\tau_1 \int_{\tau_{\text{R}}}^{\tau_1} d\tau_2 [H_{\text{int},2}, [H_{\text{int},1}, \hat{h}]] + \cdots \end{aligned}$$

*Inflationary adiabatic vacuum & local equilibrium density operators:

$$\varrho^{\text{tot}} \sim \varrho_{h,\text{adi.}} \otimes \varrho_{\chi,\beta}$$

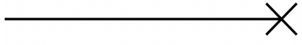
*Power spectrum:

$$\text{Tr}[\varrho^{\text{tot}} h_{\mathbf{q}} h_{\mathbf{q}'}] = P_h(q)(2\pi)^3 \delta(\mathbf{q} + \mathbf{q}')$$

*Feynman graphs in the in-in formalism:

$$\Theta(\tau)[\partial\hat{\chi}, \partial\hat{\chi}] =$$

$$\Theta(\tau)[\hat{h}, \hat{h}] =$$

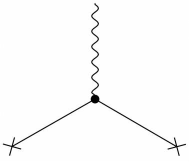

$$\partial\hat{\chi} =$$


$$\hat{h} =$$


$$\int d^4x = \bullet$$

*Single scattering with 3-pt interaction:

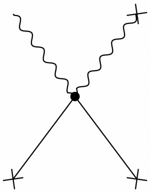
$$\delta\hat{h}_{ij}^{(1,2)}(x) = i\int^{x^0} dy^0[\hat{H}_{h\chi\chi}(y^0), \hat{h}_{ij}(x)] =$$



*Gravitational waves are produced by (random motion of) radiation.

*Single scattering with 4-pt interaction:

$$\delta\hat{h}_{ij}^{(1,3)}(x) = i\int^{x^0} dy^0 [\hat{H}_{hh\chi\chi}(y^0), \hat{h}_{ij}(x)] =$$

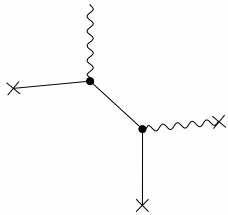


*The effective mass.

Evolution of h_{ij}

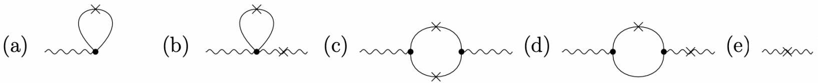
*Double scattering of 3-pt interaction:

$$\delta\hat{h}_{ij}^{(2,3)}(x) = i\int^{x^0} dy^0 i\int^{y^0} dz^0 \left[\hat{H}_{h\chi\chi}(z^0), \left[\hat{H}_{h\chi\chi}(y^0), \hat{h}_{ij}(x)\right]\right]$$

$=$ 

*Not included in hydrodynamics (beyond MFA).
*I named this radiation exchange.

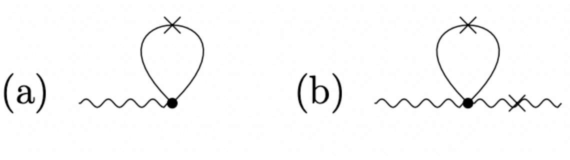
GW spectrum up-to 1-loop order



(a) Tadpole
(b) Effective mass
(c) Induced GWs
(d) Radiation exchange
(e) Tree level spectrum

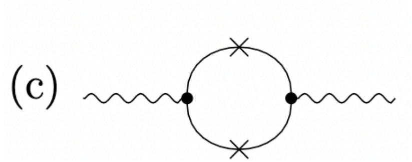
Cancellation of effective mass

*General covariance prohibits the mass of graviton:



- The effective mass from (b): $m_{\text{eff}}^2 = 2H^2$.
- (a) is perturbed by the local thermal state (extra h_{ij} from coordinate transf.).
- The perturbed tadpole cancels the effective mass.

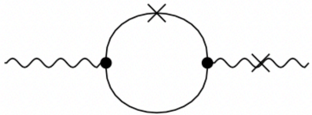
*Interactions of plasma generates GWs from nothing.



- Causal production happens only in the subhorizon scale.
- The IR contribution is zero: $\sim q^3$ (Poissonian)
- IGW does not affect the super horizon primordial GWs.

Radiation exchange

*Initial spectrum is enhanced by exchanging radiation.

(d) 

$$= \left(\ln \frac{\tau}{\tau_{\mathrm{R}}} - 1 + \frac{\tau_{\mathrm{R}}}{\tau} \right) P_h^{\mathrm{tree}}$$

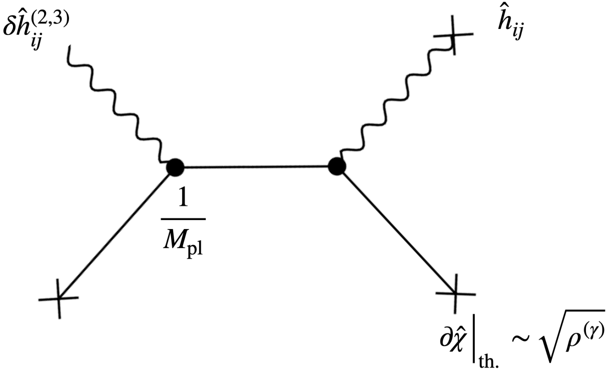
- Reheating time (initial time of RD): τ_R
- Thermal correction dominates over the tree graph: $\ln \frac{\tau}{\tau_{\mathrm{R}}} = \mathcal{O}(10)$
- IR (super horizon) spectrum varies.

Questions?

*Radiation exchange is weird because:

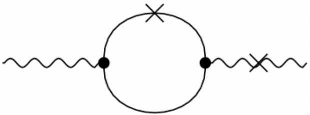
1. Super horizon spectrum enhances. Violation of causality?
2. Graviton interactions should be tiny. Why do we have a big effect?
3. Is perturbation theory valid?→Work in progress!

*Usually, gravitons are optically thin. Why the interaction looks big?



*BG Friedmann equation: $3M_{\rm pl}^2 H^2 = \rho_\gamma$.

Violation of causality?

(d) 

*Diagrams are constructed from causal propagators (based on a local Lagrangian).

Summary

1. Interactions between GWs and fundamental fields in a radiation fluid are considered.
2. These interactions have been missing in the standard framework where the mean-field approximation (MFA) is implicit.
3. We went beyond the MFA using the in-in formalism.
4. The inflationary GW power spectrum is modified at 1-loop order by the thermal effect, even at super horizon scale.
5. 1-loop effect is comparable to the tree level since innumerous thermally excited fields contribute.

Future dirrections

1. Any possibility of cancellation?
2. 2-loop graphs?
3. The same effect on ζ?
4. Isocurvature?