Topological R-fects in Chern-Simons theory and 3d gravity

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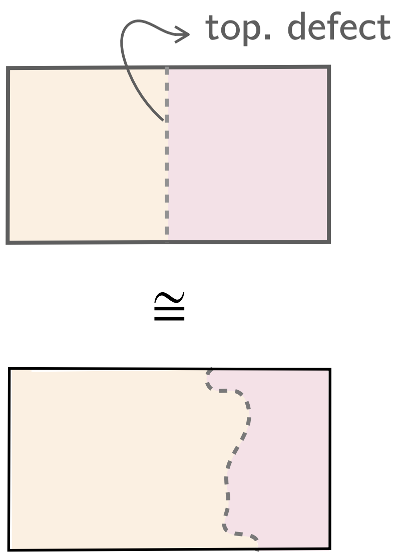
[2410.XXXX] in collaboration with Alex Arvanitakis, Lewis Cole, Daniel Thompson

Corfu Workshop on Noncommutative and Generalized Geometry in String Theory,
Gauge Theory and Related Physical Models
22 Sept. 2024

Topological defects



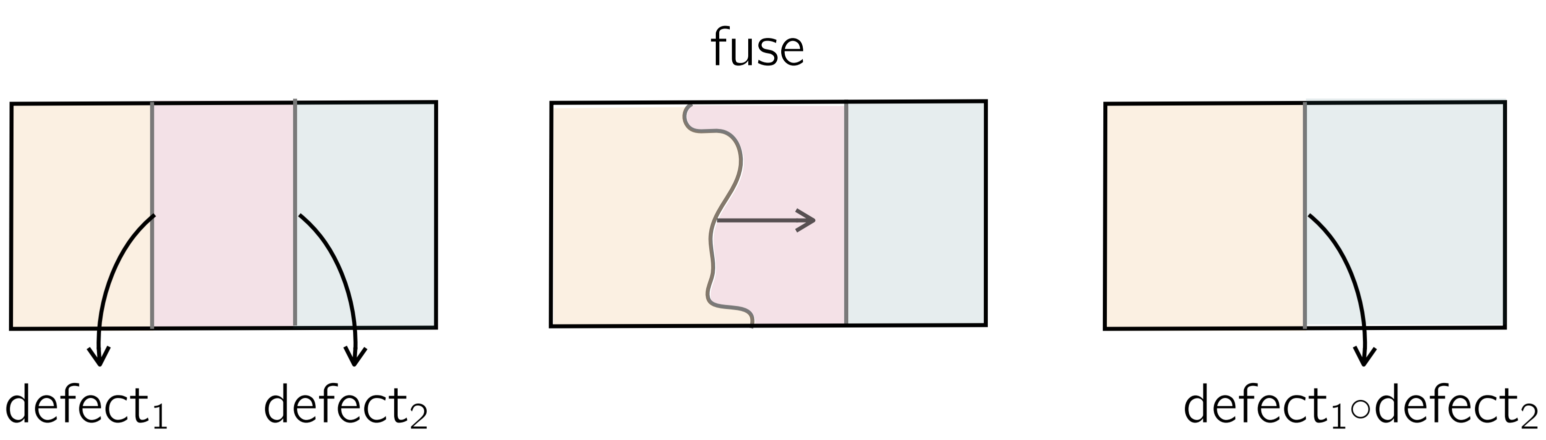
A defect is called topological when the energy momentum tensor

satisfies $T_L = T_R$ & $\bar{T}_L = \bar{T}_R$ at the defect locus

→ “Topological” = can be deformed and moved at no cost

→ Encode dualities and symmetries [Bachas, de Boer, Dijkgraaf, Ooguri, Kapustin, Tikhonov, Fröhlich, Fuchs, Gaberdiel, Runkel, Schweigert, Brunner, Roggenkamp, Carqueville,...]

→ “Fusion” = move and compose topological defects



Motivation/digression

T-duality is an example of **a topology defect** [Fuchs, Gaberdiel, Runkel, Schweigert] [Kapustin, Saulina][Niro, Roumpedakis, Sela]

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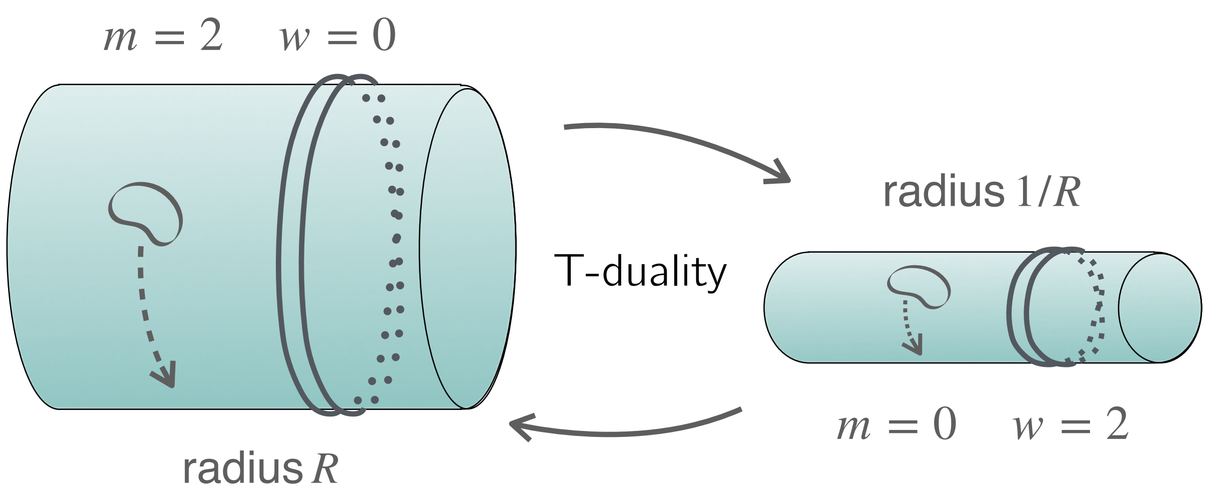

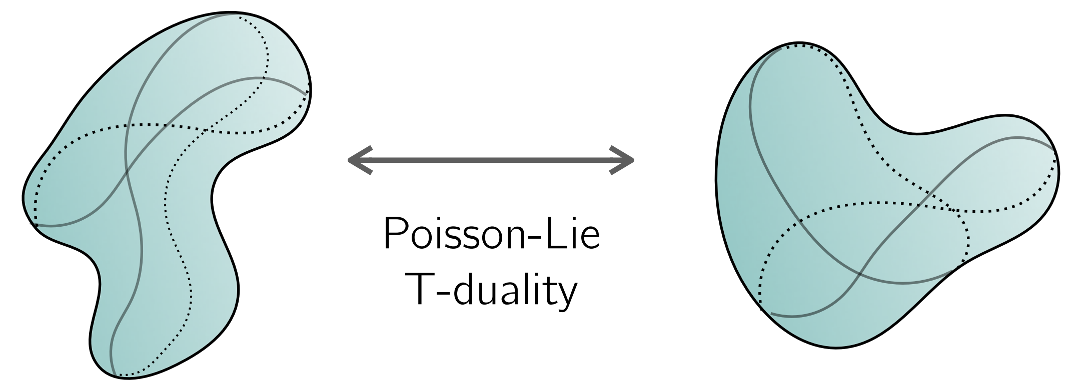

Exact symmetry of string theory

is a topological defect !

SUGRA solution generating technique

.... but is also a topological defect !

[SD, Raml]

Goal: Use the technology of defects and their fusion to understand generalised T-duality

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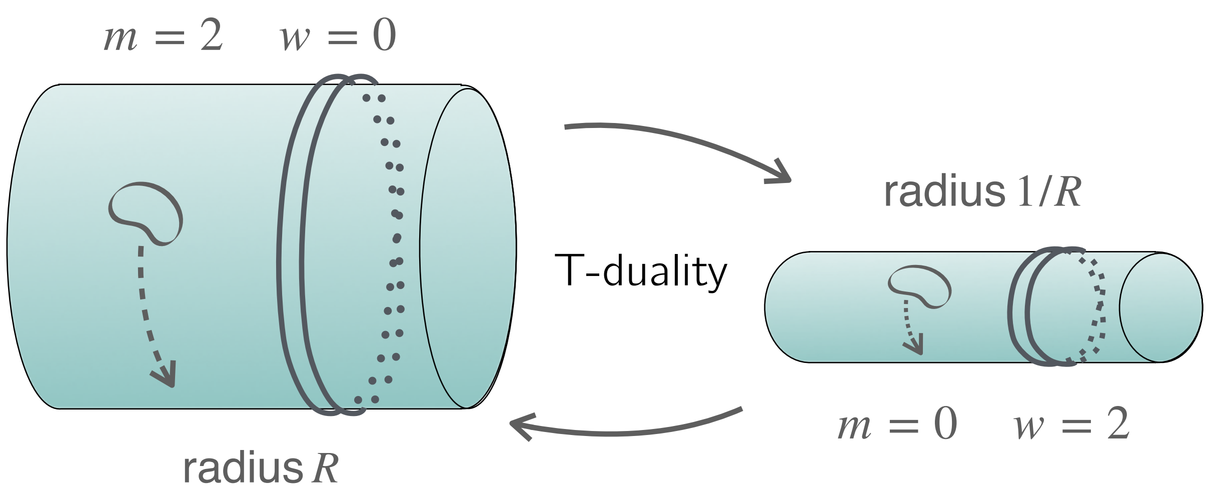

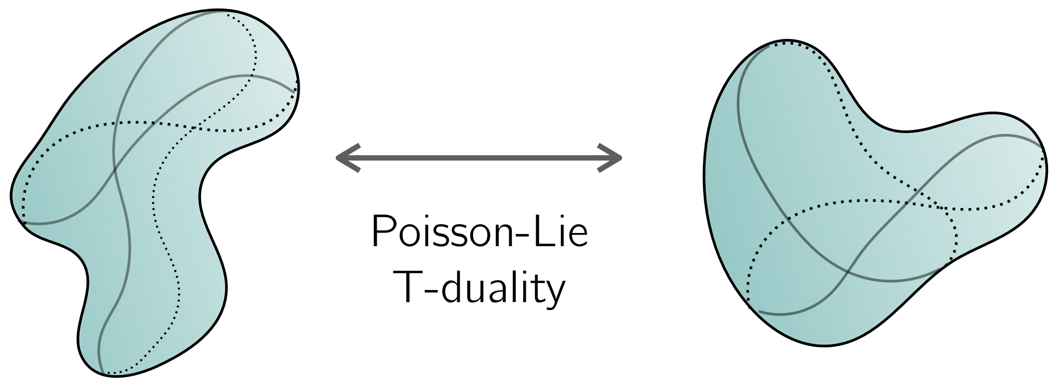

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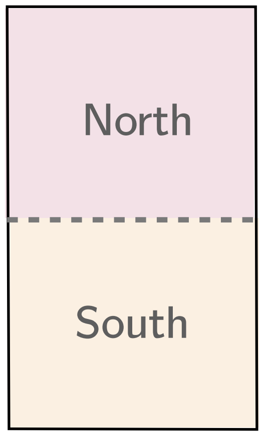
Goal: Use the technology of defects and their fusion to understand generalised T-duality

→ **problem:** fusion remained a difficult question

→ needed a way to construct topological defects for non-Abelian Chern-Simons...

"Holographic picture" and the folding trick



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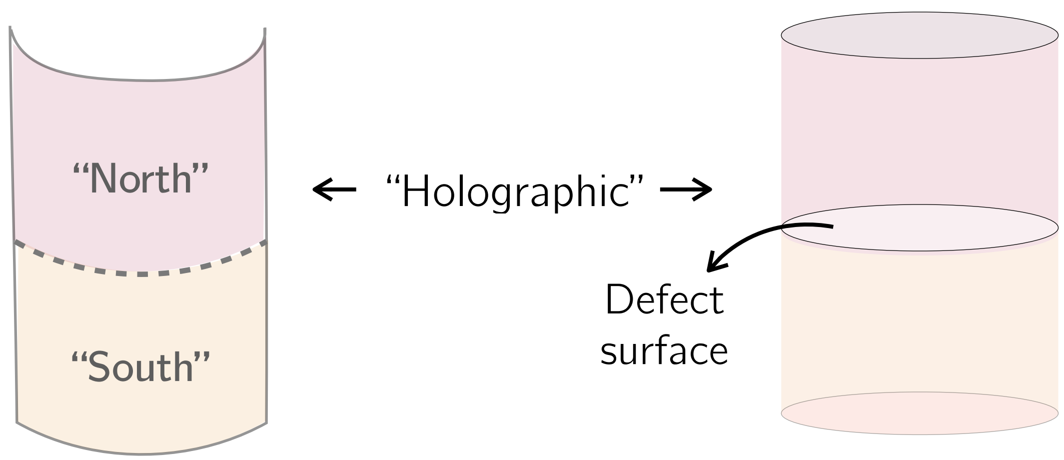

$$S_{\text{unfolded}}[A_N, A_S] = S_{\text{CS}}[A_N] + S_{\text{CS}}[A_S] + S_D[\mathbb{A}]$$

with $A_S \in \mathfrak{u}(1)^d_S\,, \quad A_N \in \mathfrak{u}(1)^d_N$

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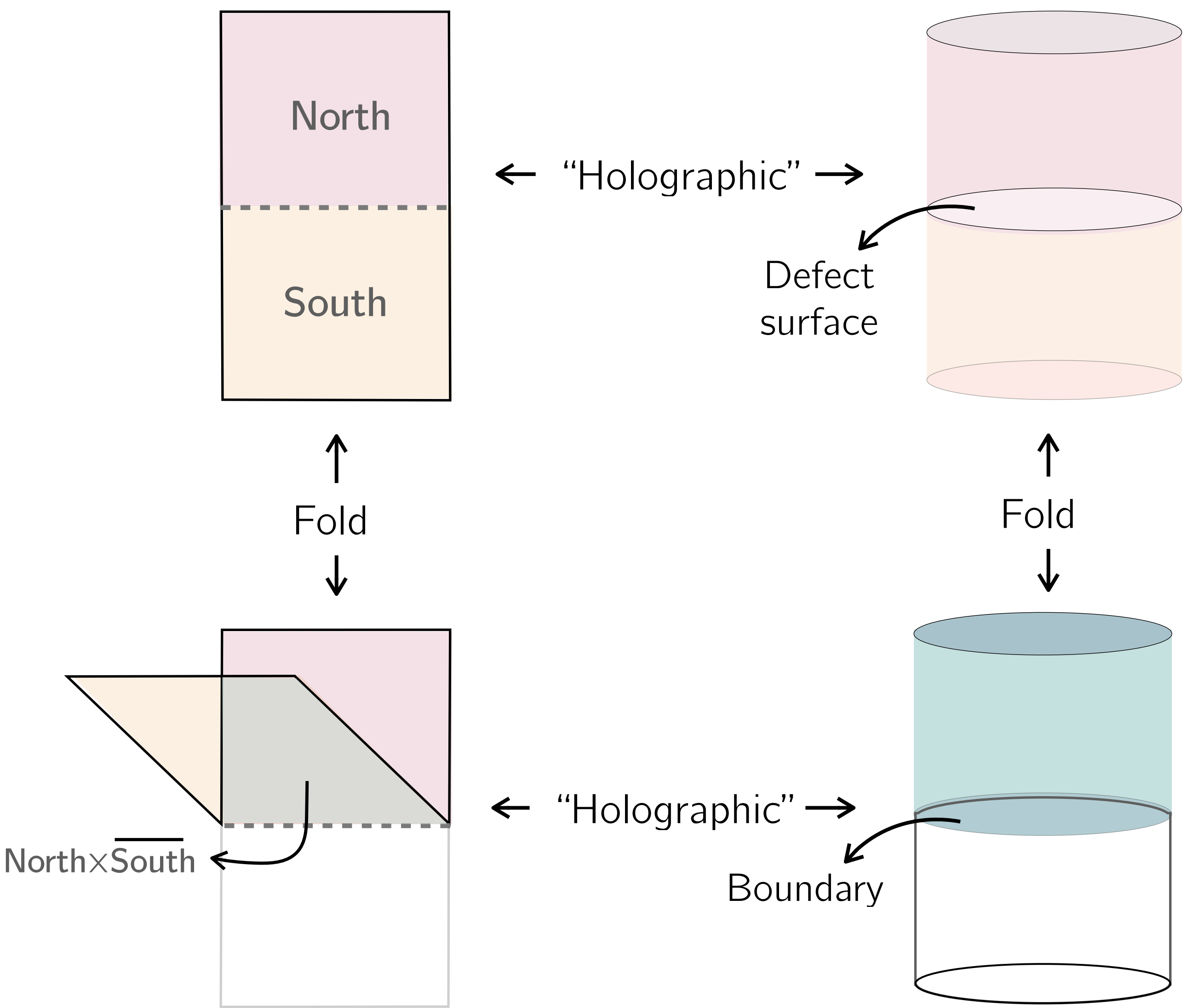

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$$\Big\updownarrow \; =$$

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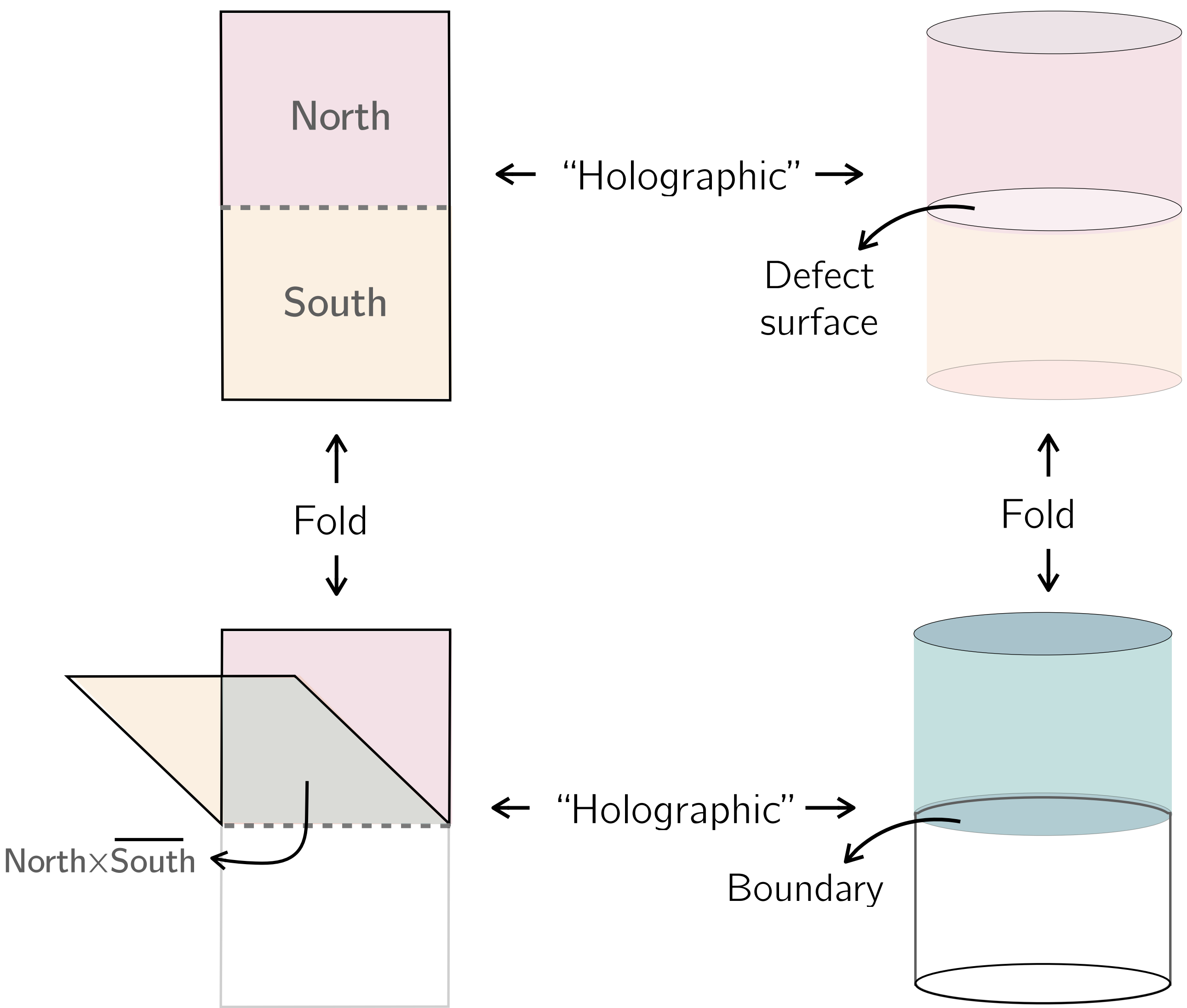

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$$S_{\text{folded}}[A_N, A_S] = k \int_{M_N} \langle\!\langle \mathbb{A}\,, \mathrm{d}\mathbb{A} \rangle\!\rangle + S_D[\mathbb{A}]$$

with $\quad \mathbb{A} = (A_N, A_S) \ \in \ \mathfrak{u}(1)^{2d} = \mathfrak{u}(1)^d_N \oplus \mathfrak{u}(1)^d_S$

$$\langle\!\langle \mathbb{X}\,, \mathbb{Y} \rangle\!\rangle = \langle X_N\,, Y_N \rangle - \langle X_S\,, Y_S \rangle$$

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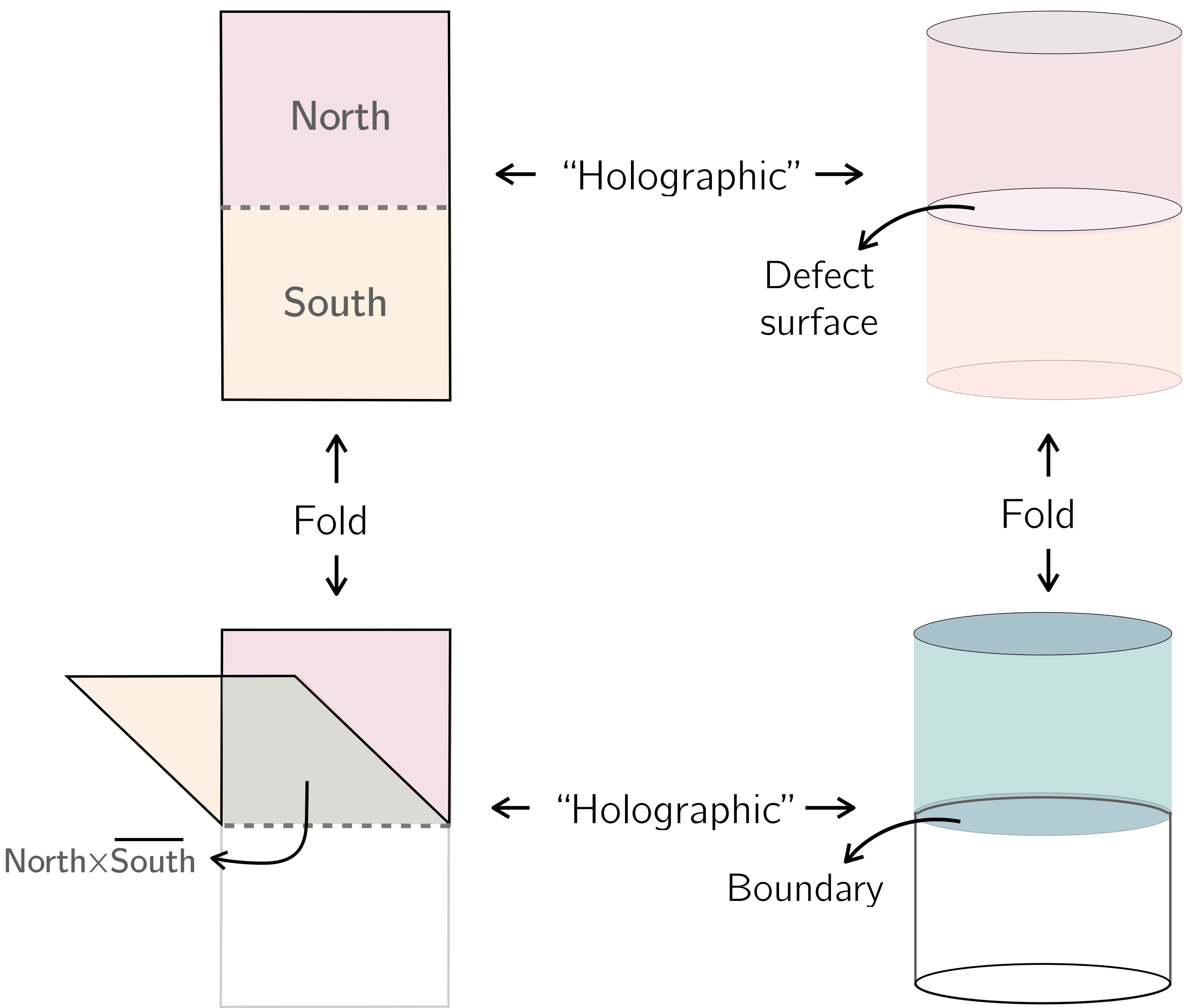

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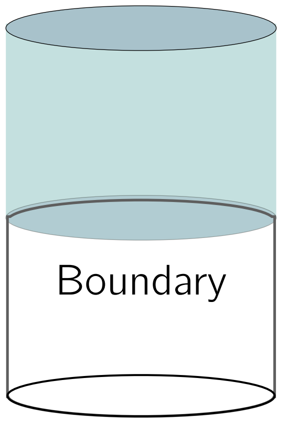
In the 3d Abelian Chern-Simons:

when does Chern-Simons theory admit a topological surface ?

Elegant algebraic answer: [Kapustin, Saulina] Look for Lagrangian subalgebras !

Lagrangians as topological boundary conditions



$$S_{\mathrm{CS}} = k \int_{M_N} \langle\langle \mathbb{A} \,, \mathrm{d}\mathbb{A} \rangle\rangle \qquad \text{where} \quad \langle\langle \mathbb{X} \,, \mathbb{Y} \rangle\rangle = \langle X_N \,, Y_N \rangle - \langle X_S \,, Y_S \rangle$$

with gauge group $\mathfrak{u}(1)^{2d} = \mathfrak{u}(1)^d_N \oplus \mathfrak{u}(1)^d_S$

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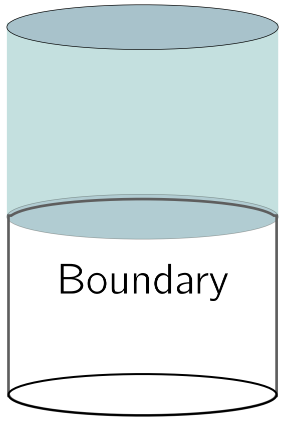

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Varying the action

$$\delta S_{\mathrm{CS}} = 2\,k \int_{M_N} \underbrace{\langle\!\langle \delta\mathbb{A} \,, \mathrm{d}\mathbb{A} \rangle\!\rangle} + k \int_D \underbrace{\langle\!\langle \delta\mathbb{A} \,, \mathbb{A} \rangle\!\rangle}$$

Vanishes by the e.o.m. $F[\mathbb{A}] \equiv \mathrm{d}\mathbb{A} = 0$

requires a boundary cond. that we will take to be topological

Demand that $\mathbb{A}|_D \in$ "Lagrangian subspace S" of $\mathfrak{u}(1)^{2d} = \mathfrak{u}(1)^d_N \oplus \mathfrak{u}(1)^d_S$

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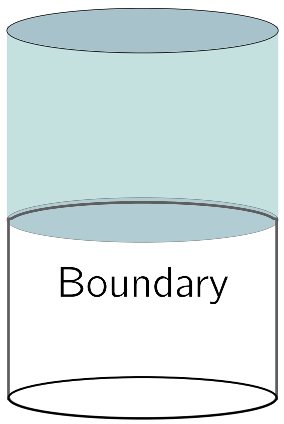

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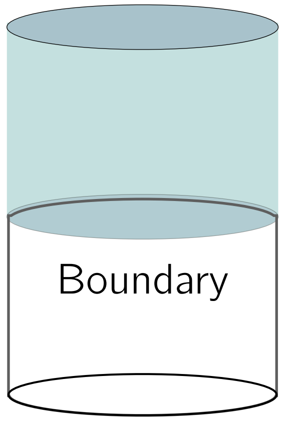
Instead of boundary condition: include the boundary term

projector

$$S = k \int_{M_N} \langle\!\langle \mathbb{A} \,, \mathrm{d}\mathbb{A} \rangle\!\rangle + k \int_D \langle\!\langle \mathbb{A} \,, P_S\, \mathbb{A} \rangle\!\rangle$$

Lagrangians as topological boundary conditions



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For Abelian Chern-Simons, the gauge group is

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Take the two Lagrangians

$$\begin{aligned} \text{diagonal} \quad &: \quad \mathfrak{u}(1)^d_+ = \{(X_N, X_S) \in \mathfrak{u}(1)^{2d} \mid X_N = X_S\} \ , \\ \text{anti-diagonal} \quad &: \quad \mathfrak{u}(1)^d_- = \{(X_N, X_S) \in \mathfrak{u}(1)^{2d} \mid X_N = -X_S\} \end{aligned}$$

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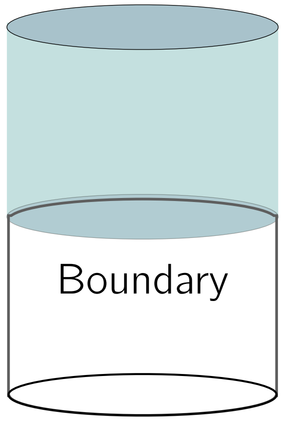

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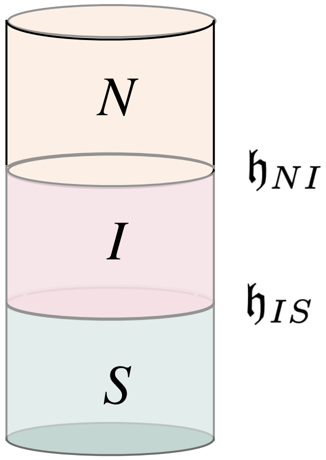

Yields the fusion algebra

$$\mathfrak{u}(1)^d_- \circ \mathfrak{u}(1)^d_- = \mathfrak{u}(1)^d_+ \qquad \mathfrak{u}(1)^d_+ \circ \mathfrak{h} = \mathfrak{h} \,, \qquad \mathfrak{h} \circ \mathfrak{u}(1)^d_+ = \mathfrak{h}$$

That is $\left(\{\mathfrak{u}(1)^d_+ \,, \mathfrak{u}(1)^d_-\} \,, \circ\right) \cong \mathbb{Z}_2$

identity idempotent

Topological surfaces in non-Abelian Chern-Simons

In the 3d non-Abelian Chern-Simons:

when does Chern-Simons theory admit a topological surface ?

$$S_{\mathrm{CS}} = k \int_{M_N} \left(\langle\!\langle \mathbb{A} \,, \mathrm{d}\mathbb{A} \rangle\!\rangle + \frac{1}{3} \langle\!\langle \mathbb{A} \,, [\![\mathbb{A} \,, \mathbb{A}]\!] \rangle\!\rangle \right)$$

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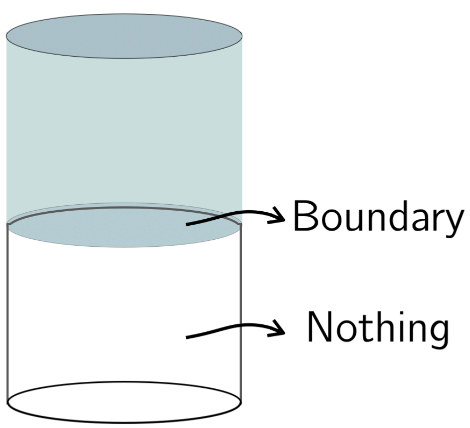

Problem

find Lagrangian subalgebras and corresponding boundary term for the gauge group of "doubled" Chern-Simons

When gauge group is non-Abelian: hard problem

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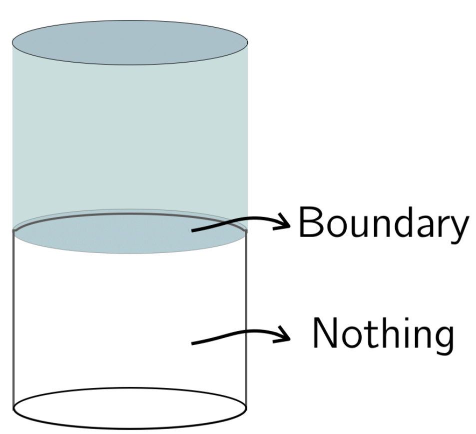

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When gauge group is non-Abelian: hard problem

We showed: [Arvanitakis, Cole, SD, Thompson]

→ Canonical way to construct topological defects

→ Crucial tool: (modified) Yang-Baxter equation

→ Defined and studied their fusion

Lagrangians via the mCYBE: $\mathcal{R}$-defects

Strategy: simplify one's life a little by looking for a subclass of defects

Solve the modified classical Yang-Baxter equation

$$[\mathcal{R}X\,,\mathcal{R}Y] - \mathcal{R}\big([\mathcal{R}X\,,Y] + [X\,,\mathcal{R}Y]\big) + [X\,,Y] = 0 \qquad X, Y \in \mathfrak{g}$$

Yields a Lagrangian subalgebra $\quad \mathfrak{g}_{\mathcal{R}} = \{\big((\mathcal{R}+1)X, (\mathcal{R}-1)X\big) \in \mathfrak{d}\}$

With Lie-bracket $\quad [X\,,Y]_{\mathcal{R}} = [\mathcal{R}X\,,Y] + [X\,,\mathcal{R}Y]$

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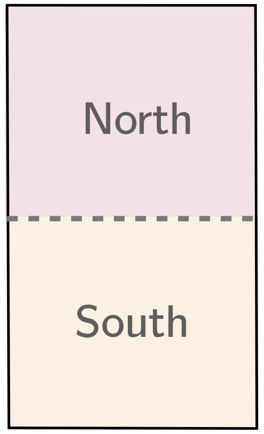
With Lie-bracket $[X\,,Y]_{\mathcal{R}} = [\mathcal{R}X\,,Y] + [X\,,\mathcal{R}Y]$

Called a "bi-algebra" or "Manin triple" $\mathfrak{d} = \mathfrak{g}_{\Delta} \oplus \mathfrak{g}_{\mathcal{R}}$

→ Directly generalises the Abelian case $\mathfrak{u}(1)^{2d} = \mathfrak{u}(1)^{d}_{\Delta} \oplus \mathfrak{u}(1)^{d}_{-}$



→ Technical requirement $\mathfrak{d} = \mathfrak{g}_{\Delta} \oplus \mathfrak{g}_{\mathcal{R}} \cong \mathfrak{g} \oplus \mathfrak{g}$

Effectively specialising to non-compact algebras

Lagrangians via the mCYBE: $\mathcal{R}$-defects

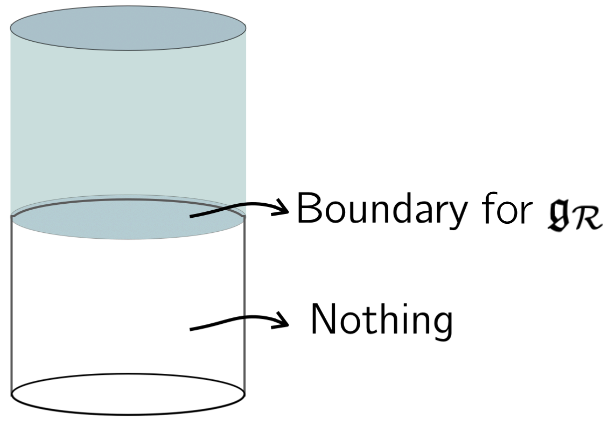
We have a topological boundary condition for the Lagrangian subalgebra

$$\mathfrak{g}_{\mathcal{R}} = \{\big((\mathcal{R}+1)X, (\mathcal{R}-1)X\big) \in \mathfrak{d}\}$$

Since with the R-matrix we can construct a projector

$$\langle\langle \mathbb{A}\,, \mathcal{P}_{\mathcal{R}}\mathbb{A}\rangle\rangle = \langle A_S\,, A_N\rangle + \tfrac{1}{2}\langle A_N - A_S\,, \mathcal{R}(A_N - A_S)\rangle$$

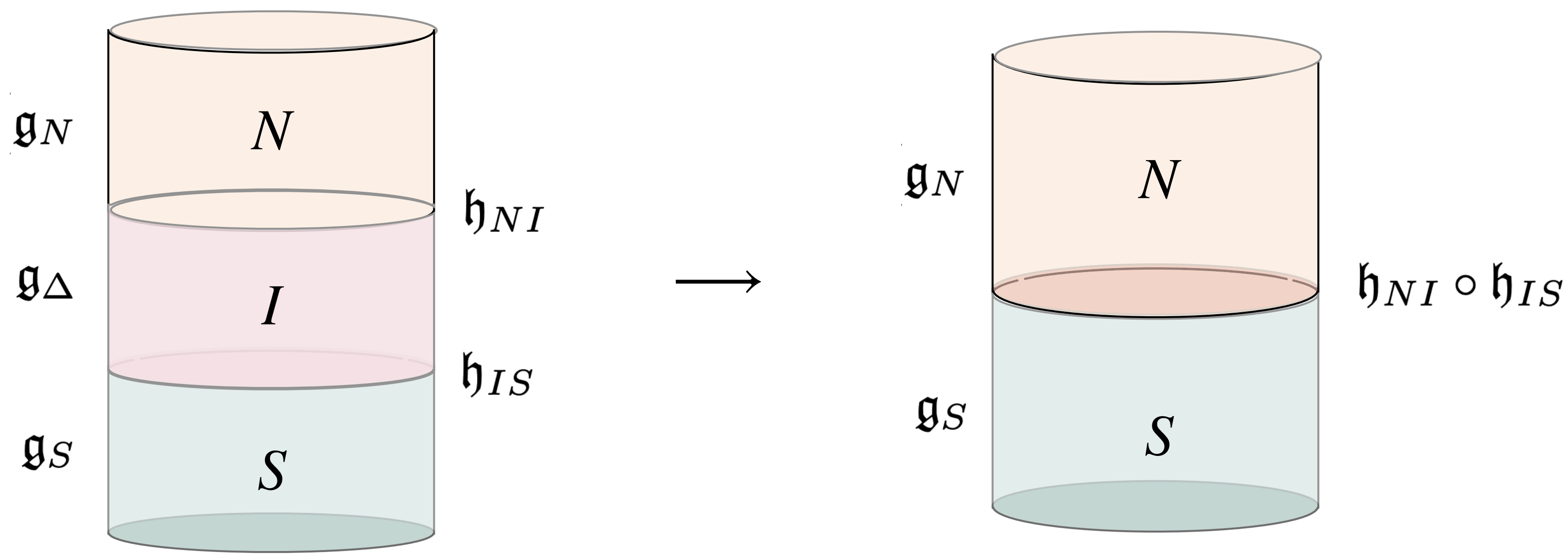
Yielding the (folded) Chern-Simon action



$$S_{\text{folded}} = \int \mathrm{CS}[\mathbb{A}] + \int_D \langle\langle \mathbb{A}\,, \mathcal{P}_{\mathcal{R}}\mathbb{A}\rangle\rangle$$

Fusion at the level of the algebra

[Arvanitakis] [Fuchs, Schweigert, Waldorf] [Burzstyn]



$$\mathfrak{h}_{NI} \circ \mathfrak{h}_{IS} = \Pi_{NS}\Big((\mathfrak{h}_{NI} \oplus \mathfrak{h}_{IS}) \cap (\mathfrak{g}_N \oplus \mathfrak{g}_\Delta \oplus \mathfrak{g}_S) \Big)$$

$$\Downarrow$$

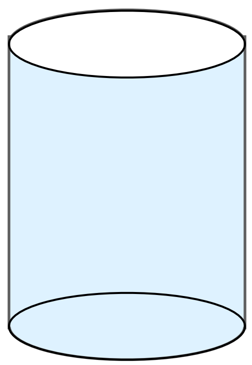
$$\mathfrak{g}_\mathcal{R} \circ \mathfrak{g}_\mathcal{R} = \{ (X_N, X_S) \in \mathfrak{d} \mid X_N^\mathfrak{t} = X_S^\mathfrak{t} \, , X_N^- = 0 \, , X_S^+ = 0 \}$$

→ Proved that $\mathfrak{h}_{NI} \circ \mathfrak{h}_{IS}$ is

- ✓ Lagrangian
- ✓ a subalgebra
- with identity element $\mathfrak{g}_\Delta \circ \mathfrak{h} = \mathfrak{h}$, $\mathfrak{h} \circ \mathfrak{g}_\Delta = \mathfrak{h}$
- reduces to Lagrangian fusion in the Abelian case

AdS_3 gravity

$$S_{\text{grav}} = \underbrace{\frac{1}{16\pi G}\int_M \sqrt{g}(R+2)}_{\text{Einstein-Hilbert action}} + \overbrace{\frac{1}{8\pi G}\int_{\partial M} \mathrm{d}^2x\sqrt{h}K}^{\text{Gibbons-Hawking-York}} - \underbrace{\frac{1}{8\pi G}\int_{\partial M}\sqrt{h}}_{\text{Boundary area counter-term}}$$



AdS_3 gravity a re-formulation in terms of Chern-Simons theories

$$S_{\text{grav}} = S_{\text{CS}}[A] - S_{\text{CS}}[\bar{A}] + S_{\text{bdy}}$$

"folded" Chern-Simons theory for the gauge group $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$

Where the gauge connections are related to the metric vielbein and the soldering form

$$A^a = \omega^a + \frac{1}{\ell}e^a \ , \qquad \tilde{A}^a = \omega^a - \frac{1}{\ell}e^a$$

With respect to the $\mathfrak{sl}(2,\mathbb{R})$ -generators $[L_a, L_b] = (a-b)L_{a+b}$

AdS_3 higher-spin gravity

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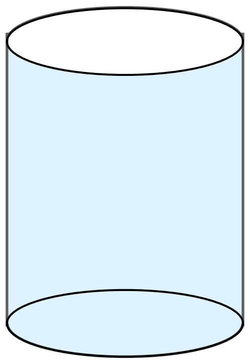

Higher-spin AdS_3 gravity a re-formulation in terms of Chern-Simons theories

$$S_{\text{grav}} = S_{\text{CS}}[A] - S_{\text{CS}}[\bar{A}] + S_{\text{bdy}} \qquad \mathfrak{sl}(N,\mathbb{R}) \oplus \mathfrak{sl}(N,\mathbb{R})$$

"folded" Chern-Simons theory for the gauge group ~~$\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$~~

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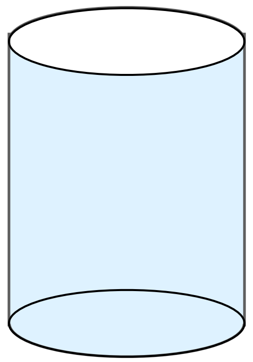
$$A^a = \omega^a + \frac{1}{\ell}e^a\,, \qquad \tilde{A}^a = \omega^a - \frac{1}{\ell}e^a$$

and with spin fields $\qquad \phi_{\mu_1\ldots\mu_{s-1}\mu_s} \sim \mathrm{Tr}\left(e_{(\mu}\ldots e_{\mu_{s-1}}e_{\mu_s)}\right)$

AdS$_3$ gravity

$$S_{\text{grav}} = \underbrace{\frac{1}{16\pi G}\int_M \sqrt{g}(R+2)}_{\text{Einstein-Hilbert action}} + \overbrace{\frac{1}{8\pi G}\int_{\partial M} \mathrm{d}^2x\sqrt{h}K}^{\text{Gibbons-Hawking-York}} - \underbrace{\frac{1}{8\pi G}\int_{\partial M}\sqrt{h}}_{\text{Boundary area counter-term}}$$



AdS_3 gravity a re-formulation in terms of Chern-Simons theories

$$S_{\text{grav}} = S_{\text{CS}}[A] - S_{\text{CS}}[\bar{A}] + S_{\text{bdy}}$$

"folded" Chern-Simons theory for the gauge group $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$

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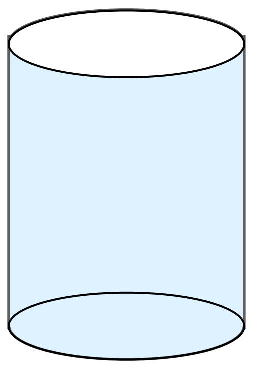

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In the Feffer-Graham gauge the boundary term is [Llabres; Apolo; Ebert, Hijano, Kraus, Monten, Myers]

$$S_{\rm bdy} = \frac{k}{4\pi}\int_{\partial M} \mathrm{tr}\, A\wedge\bar{A} - \frac{k}{2\pi}\int_{\partial M}\mathrm{tr}\left(L_0(A-\bar{A})\wedge(A-\bar{A})\right)$$

L_0: Cartan generator of $\mathfrak{sl}(2,\mathbb{R})$

AdS$_3$ gravity

[Arvanitakis, Cole, SD, Thompson]

The R-boundary term where the R-matrix is the Drinfel'd-Jimbo R-matrix for $\mathfrak{sl}(2,\mathbb{R})$

$$S_{\rm bdy} = \int_{\partial M} \langle A, \bar{A}\rangle - \tfrac{1}{2}\langle \bar{A} - A, \mathcal{R}(\bar{A} - A)\rangle \qquad \mathcal{R}L_0 = 0\,, \qquad \mathcal{R}L_\pm = \pm L_\pm$$

$$= \frac{k}{4\pi}\int_{\partial M} \mathrm{tr}\, A\wedge\bar{A} - \frac{k}{2\pi}\int_{\partial M}\mathrm{tr}\left(L_0(A-\bar{A})\wedge(A-\bar{A})\right)$$

matches precisely the bdy term from GHY

AdS_3 higher-spin gravity

[Arvanitakis, Cole, SD, Thompson]

More generally we can always define **boundary term**

$$S_{\mathrm{bdy}} = \int_{\partial M} \langle A, \bar{A} \rangle - \tfrac{1}{2} \langle \bar{A} - A, \mathcal{R}(\bar{A} - A) \rangle$$

where the R-matrix is the Drinfel'd-Jimbo R-matrix for $\mathfrak{sl}(N, \mathbb{R})$

$$\mathcal{R}H_i = 0 \ , \quad \mathcal{R}E_\alpha = +c\, E_\alpha \ , \quad \mathcal{R}E_{-\alpha} = -c\, E_{-\alpha}$$

→ get a canonical **boundary condition for higher spin AdS_3 gravity**

AdS_3 higher-spin gravity

[Arvanitakis, Cole, SD, Thompson]

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→ get a **canonical boundary condition for higher spin AdS_3 gravity**

For example for $\mathfrak{sl}(3,\mathbb{R})$-gravity

$$\begin{aligned} S_{\text{bdy}} &= \int_{\partial M} \langle A, \bar{A} \rangle - \tfrac{k}{2\pi} \langle \bar{A} - A \, \overset{\wedge}{,} \, \mathcal{R}_{\text{DJ}}(\bar{A} - A) \rangle \\ &= \int_{\partial M} \langle A, \bar{A} \rangle - \frac{k}{4\pi} \int_{\partial M} \text{tr}[(A - \bar{A}) \wedge (A - \bar{A}) L_0] \\ &\qquad\qquad + \frac{k}{64\pi} \int_{\partial M} \text{tr}[(A - \bar{A}) W_{+2}] \wedge \text{tr}[(A - \bar{A}) W_{-2}] \,. \end{aligned}$$

Coincides with the boundary term constructed by [Apolo]

AdS$_3$ gravity

See review [Campoleoni, Fredenhagen]

Gauge transfo A

$$\delta A = \mathrm{d}\lambda + [A, \lambda]$$

$$\updownarrow$$

$$\delta e = \mathrm{d}\xi + [\omega, \xi] + [e, \Lambda]$$

$$\xi = \frac{\ell}{2}\left(\lambda - \tilde{\lambda}\right)$$

diffeos metric

AdS$_3$ gravity

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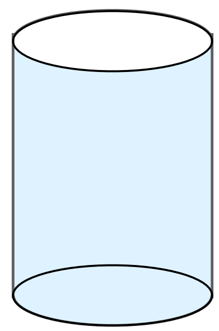

preserving
the bdy conditions
while acting
on the bdy data

Global charges and
symmetry on the bdy theory

$$\{Q(\lambda_1), Q(\lambda_2)\} = Q([\lambda_1, \lambda_2]) + \frac{k}{2\pi}\oint \mathrm{d}\varphi\, \mathrm{tr}(\lambda_1 \partial_\varphi \lambda_2)$$

Celebrated result: Brown-Henneaux boundary conditions $\rightarrow$ two copies of Virasoro

$$A - A_{\mathrm{AdS}} \sim \mathcal{O}(1) \qquad \bar{A} - \bar{A}_{\mathrm{AdS}} \sim \mathcal{O}(1)$$

AdS$_3$ gravity

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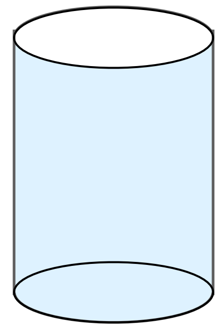

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Instead the R-boundary condition (and restricting to *AAdS$_3$*)

[Campoleoni, Fredenhagen, Raeymaekers]
[Arvanitakis, Cole, SD, Thompson]

$$(\mathcal{R} - 1)A = (\mathcal{R} + 1)\bar{A}$$

$$\downarrow$$

Free boson realisation of a single copy of Virasoro

$\longrightarrow$ Summary

Constructed **topological boundary conditions in non-Abelian Chern-Simons**

- Led to a subclass: R-defects
- Looked into their fusion
- Application to AdS_3 -gravity

$\longrightarrow$ What's next ?

- Can we identify the surface defect for Poisson-Lie T-duality ?
- Does the R-defect boundary make sense in higher spin 3d gravity
- SymTFT description ?

Thank you for your attention !