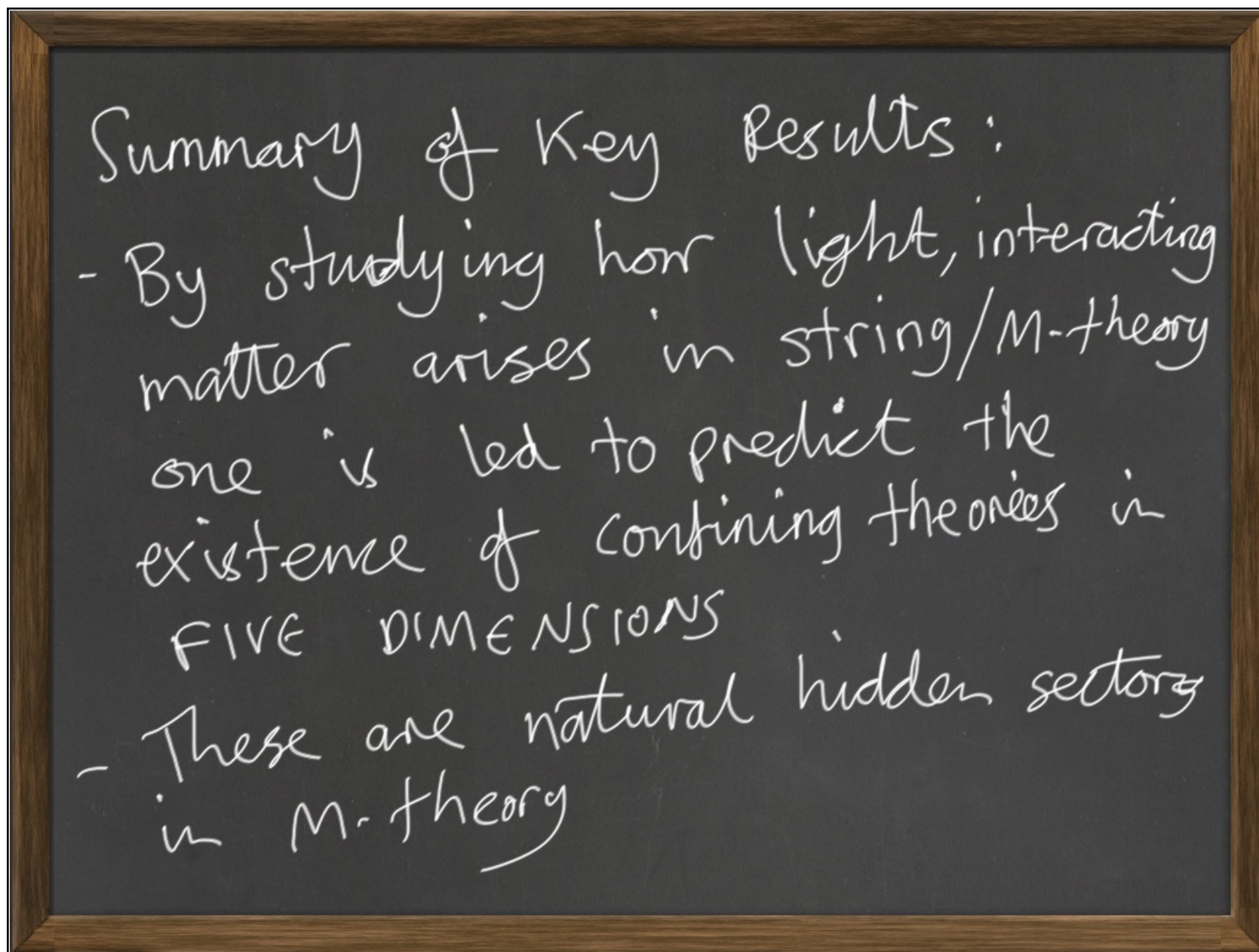


Work based on

anXiv: 2407.03171



## Outline

- Overview of Extra Dimensions framework in string/M-theory
- Light interacting matter and special singularities. Example of 7d Yang-Mills
- 5d (super) Yang-Mills and 5d CFT from geometry

At low energies, when  $M^{10,1}$  is smooth,  
M-theory is well described by a  
field theory: 11d supergravity.

Three fields: metric,  $g_{11}$ ; 3-form  $C$   
Rarita-Schwinger field  
 $\psi \in (SM \otimes TM)$ .

When  $C = \psi = 0$ , Euler-Lagrange eq<sup>s</sup> are

$$\text{Ric}(g) = 0$$

So Ricci flat metrics play a key role

Since our Universe is apparently four dimensional, we <sup>( $M^{3,1}$ )</sup> are interested in

$$(M^{10,1}, g_{11}) = (X^7 \times M^{3,1}, g(X) + \eta(M^{3,1}))$$

where  $X^7$  is "small" and compact and

$$\text{Ric}(g(X^7)) = 0 = \text{Ric}(\eta(M^{3,1}))$$

So compact Ricci flat manifolds play a key role in M-theory.

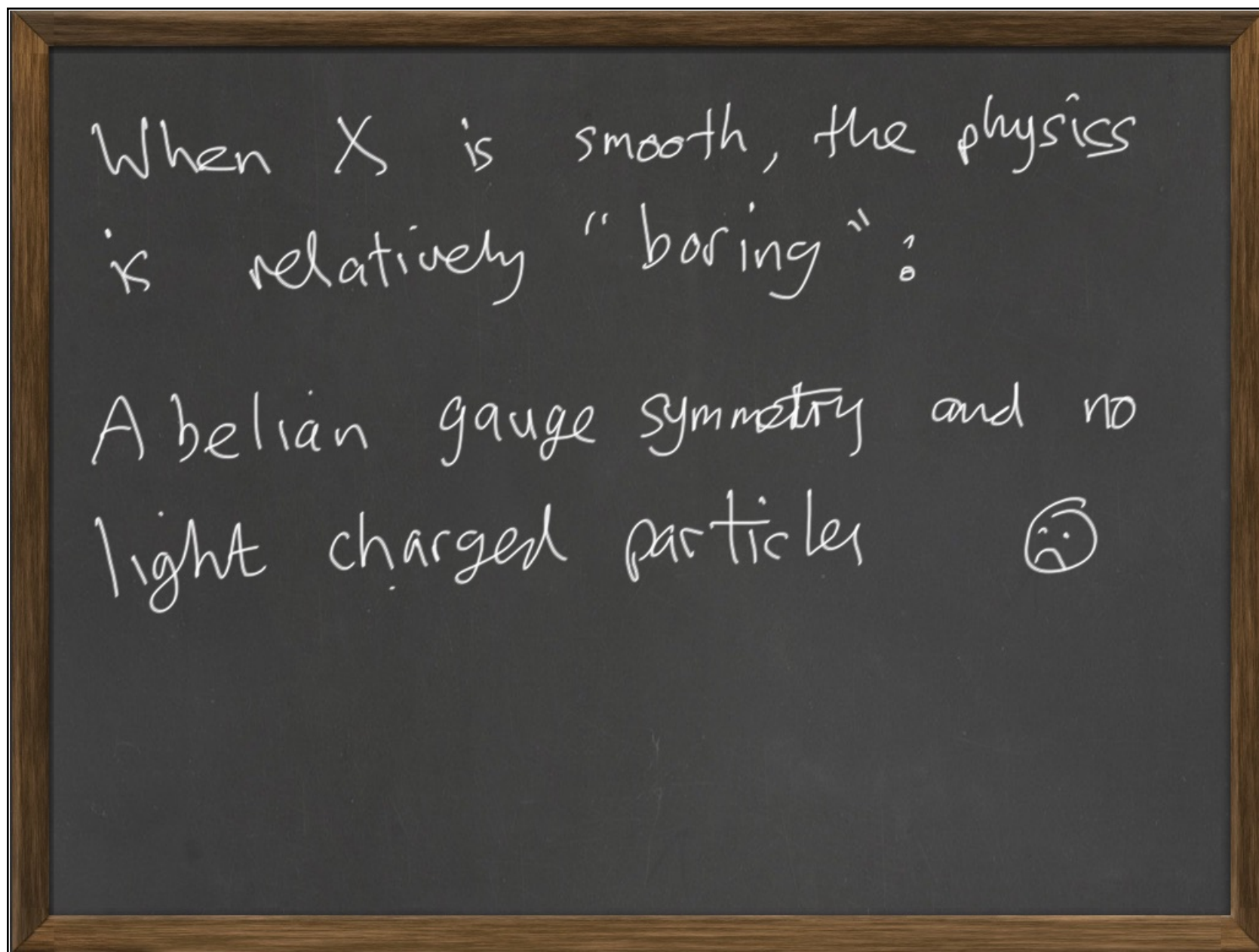
More generally, consider  $M^{10,1} = X^n \times \mathbb{R}^{10-n,1}$ .

Only known examples of compact, irreducible Ricci flat manifolds have special holonomy.

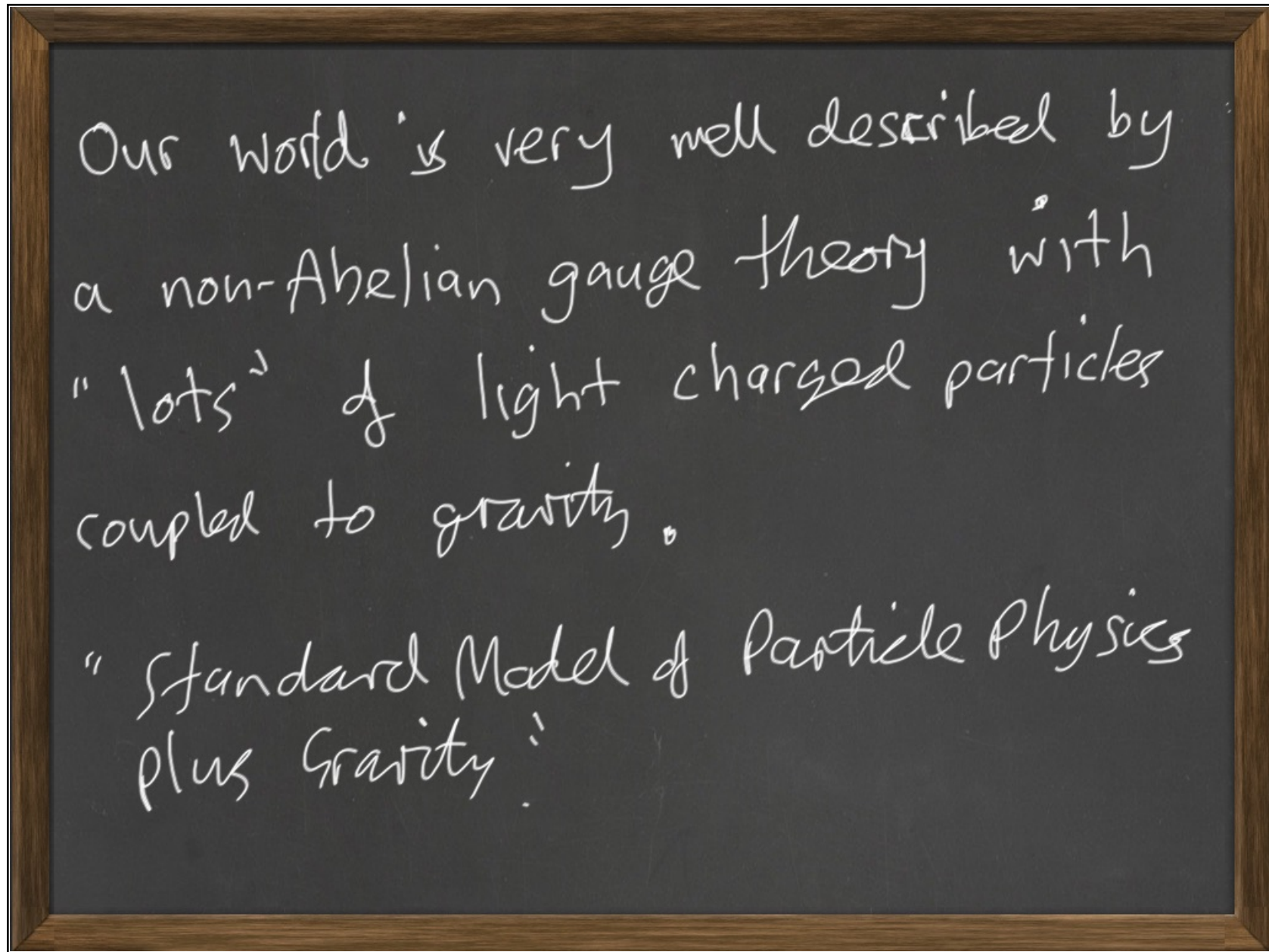
$n$	$Hol(g)$	$Ric(g(x))=0$
$n$	$SO(n)$	?
$n=2k$	$SU(k)$	✓
$n=4k$	$Sp(k)$	✓
$n=7$	$G_2$	✓
$n=8$	$Spin(7)$	✓

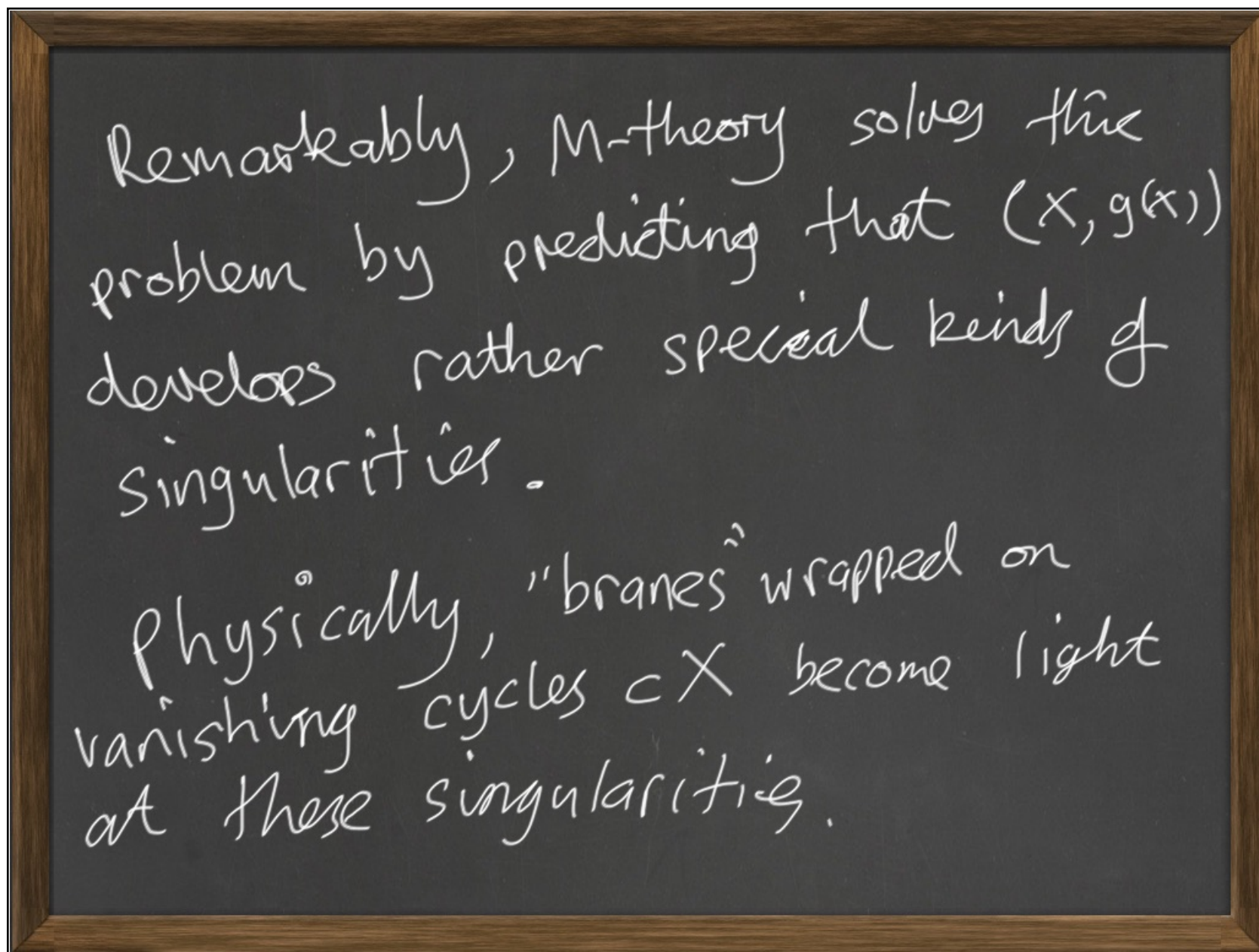
(BSA 2019: compact stable, Ricci flat mflds have special holonomy)

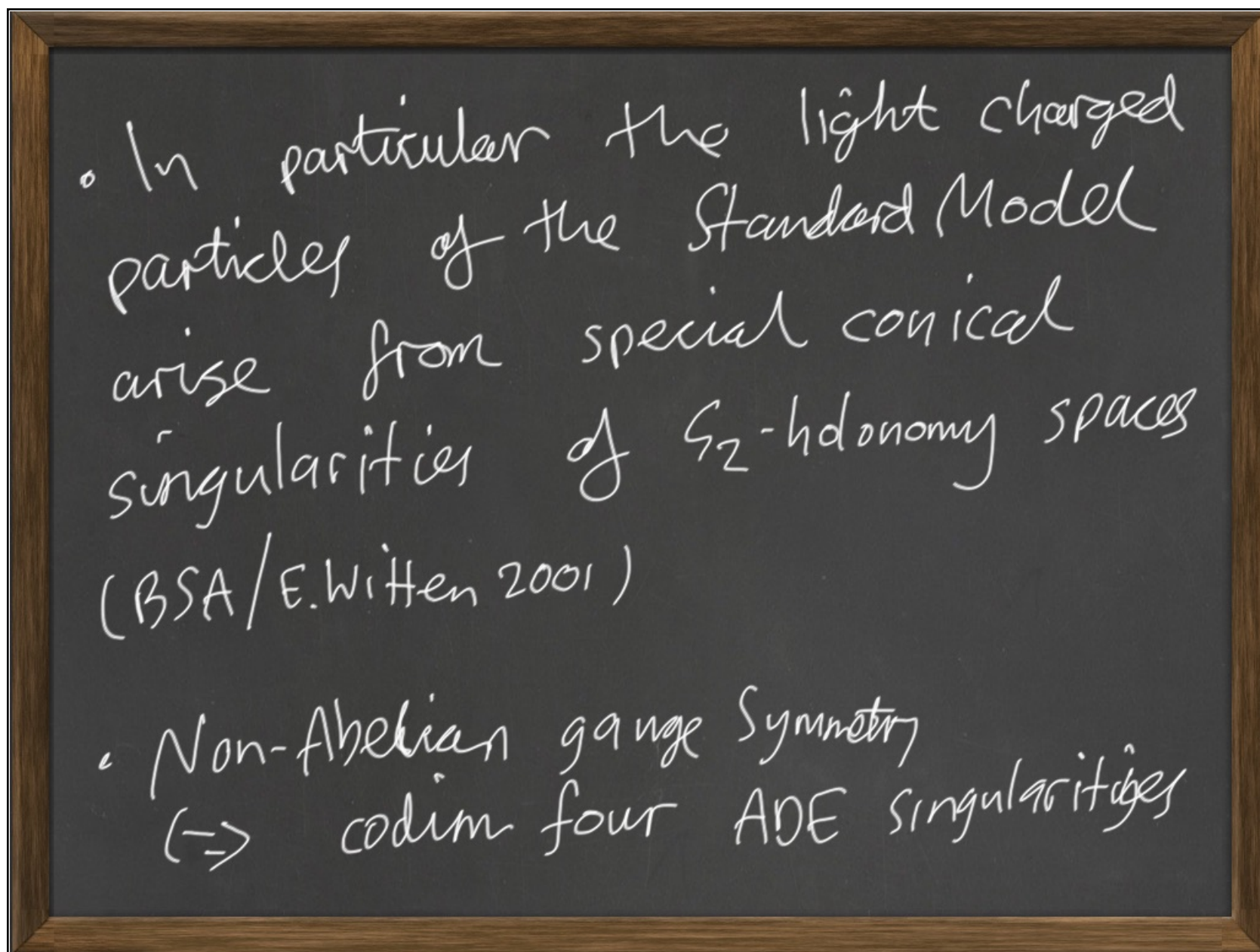
← Calabi-Yau











More generally one is interested in Special Holonomy Cones as local models of the singularity of  $X$ .

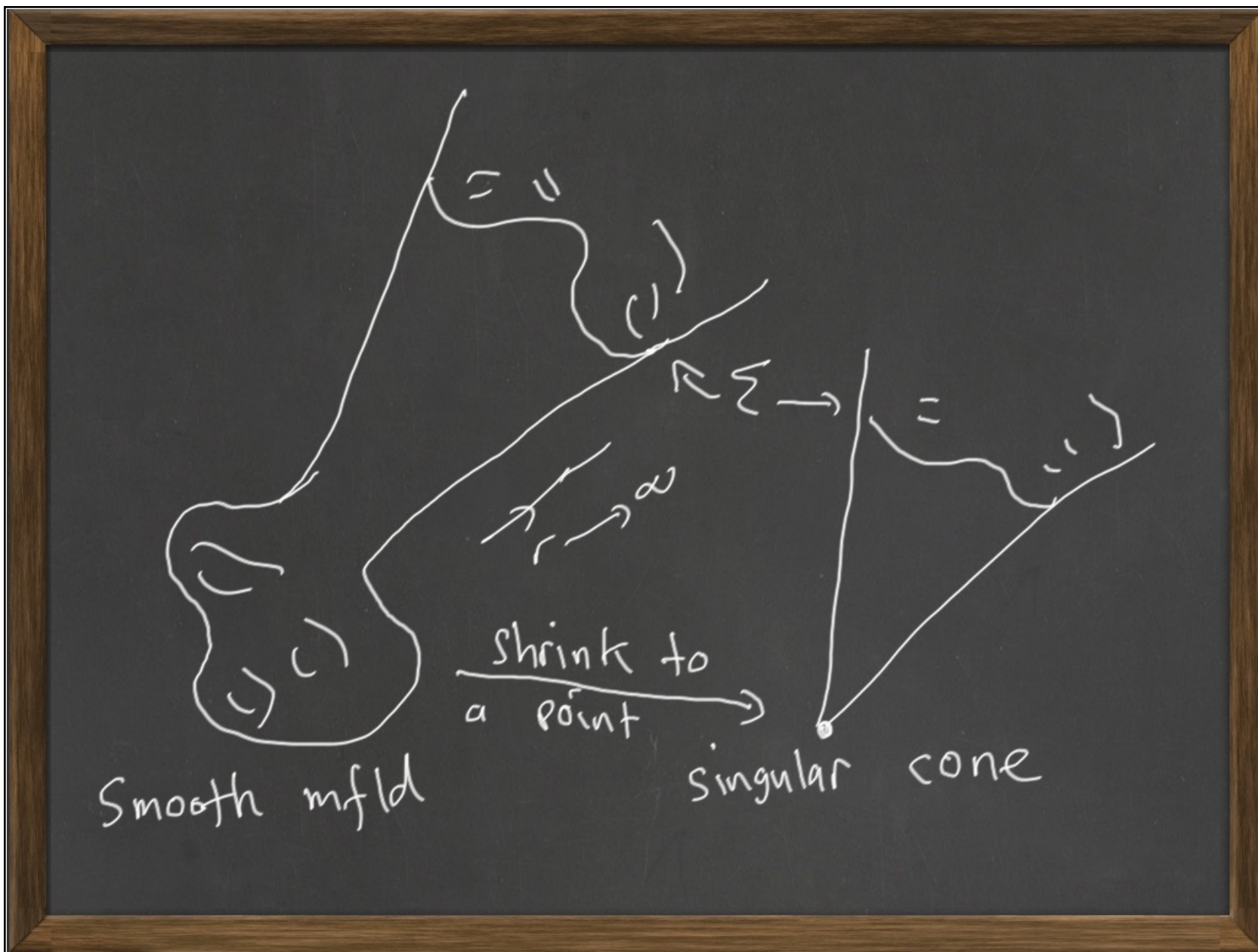
$n$	$\text{Hol}(g(x))$	Physical theory in $d$ -dimensions
4	$SU(2)$	$d=7$ gauge theory
6	$SU(3)$	5d Conformal
7	$G_2$	"All" 4d theories

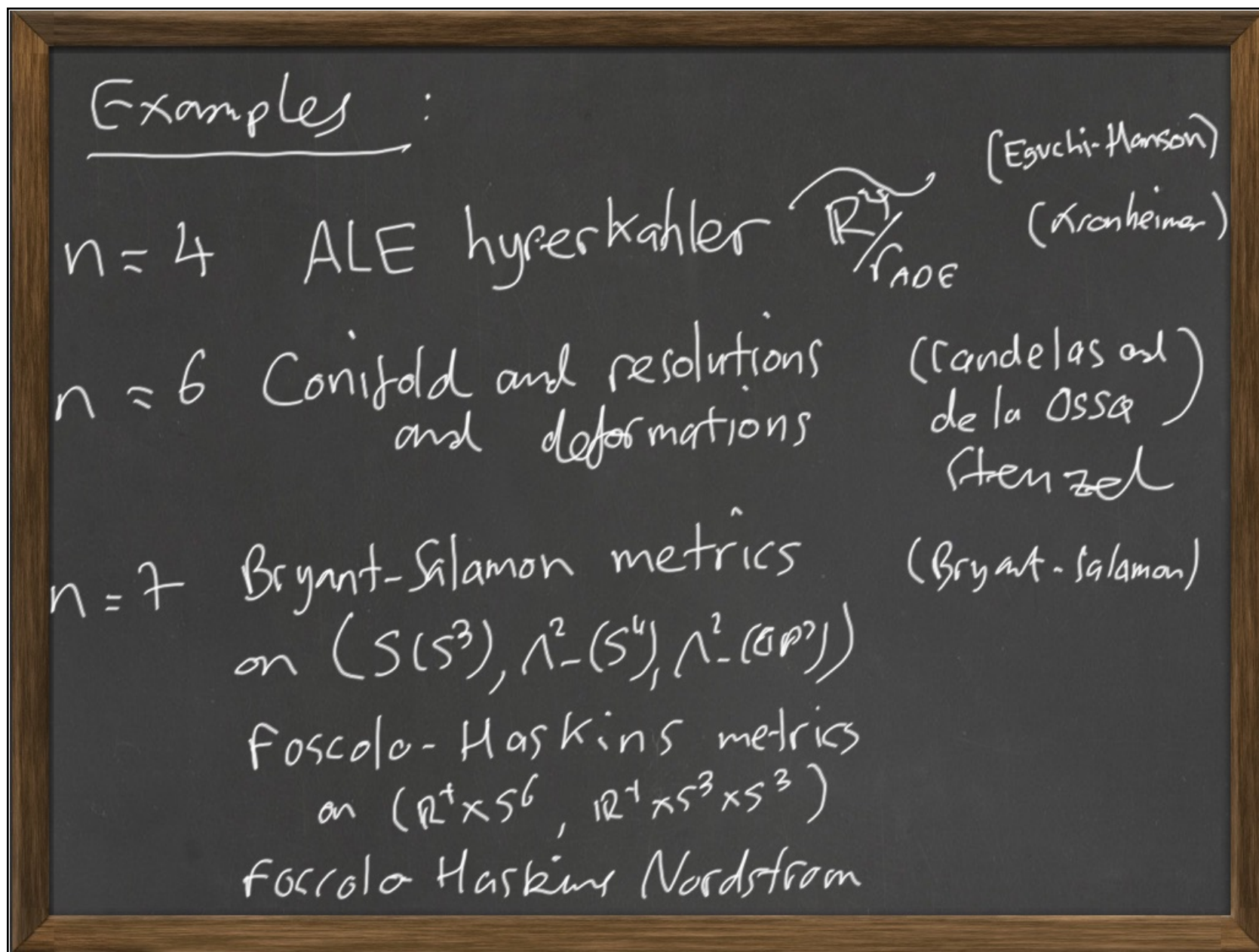
Def<sup>h</sup>: A conical metric  $g(x) \equiv dr^2 + r^2 g(\Sigma)$   
on  $X = \mathbb{R}^+ \times \Sigma$

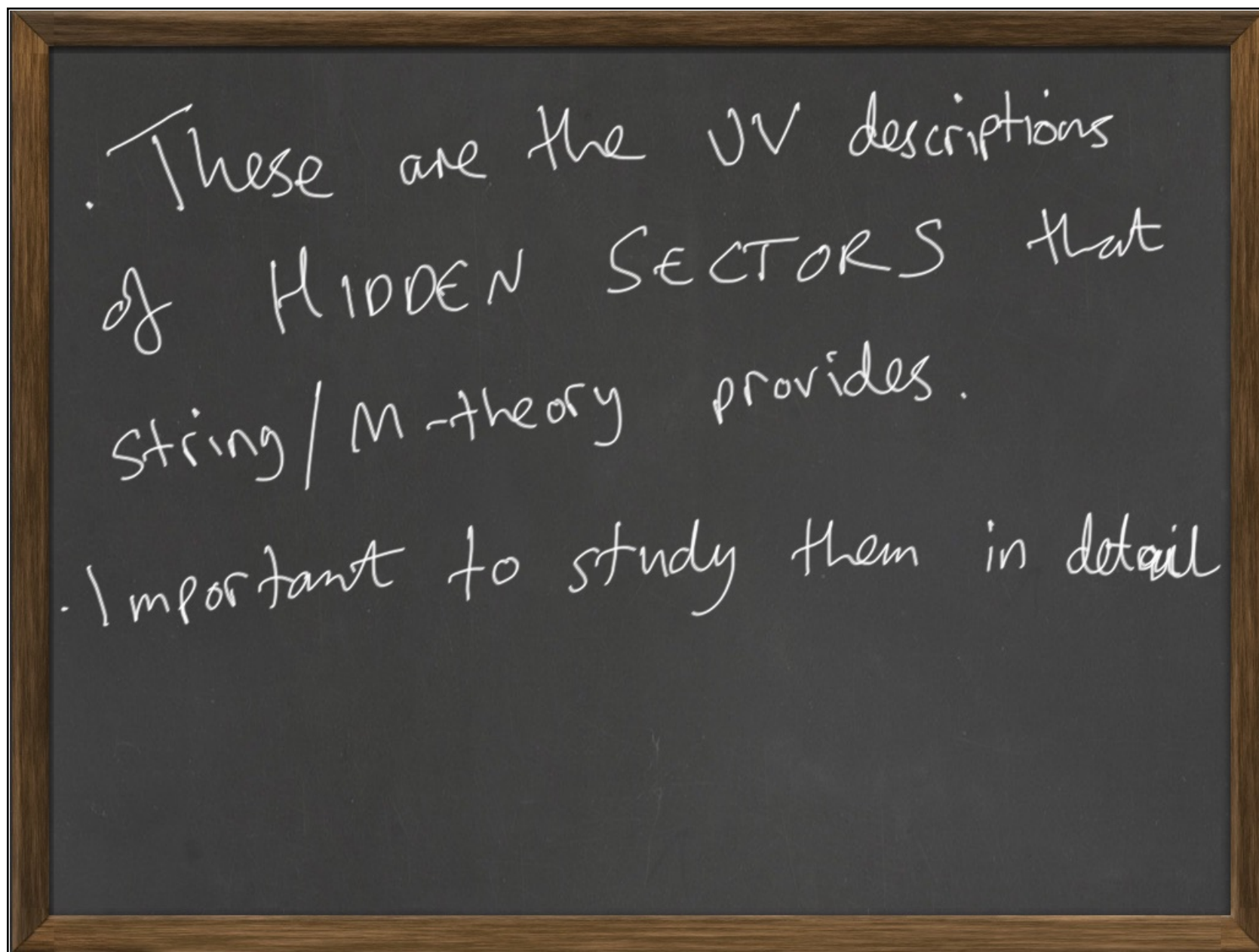
If  $\text{Hol}(g(x)) = \text{SU}(k), \text{Sp}(k), \text{S}_2$  or  $\text{Spin}(7)$

then  $(X, g(x))$  is said to be a special  
holonomy cone. (Scale symmetry  $r \rightarrow \alpha r$ )

More generally one is interested in  
smooth, complete, asymptotically conical  
special holonomy manifolds. (AC)





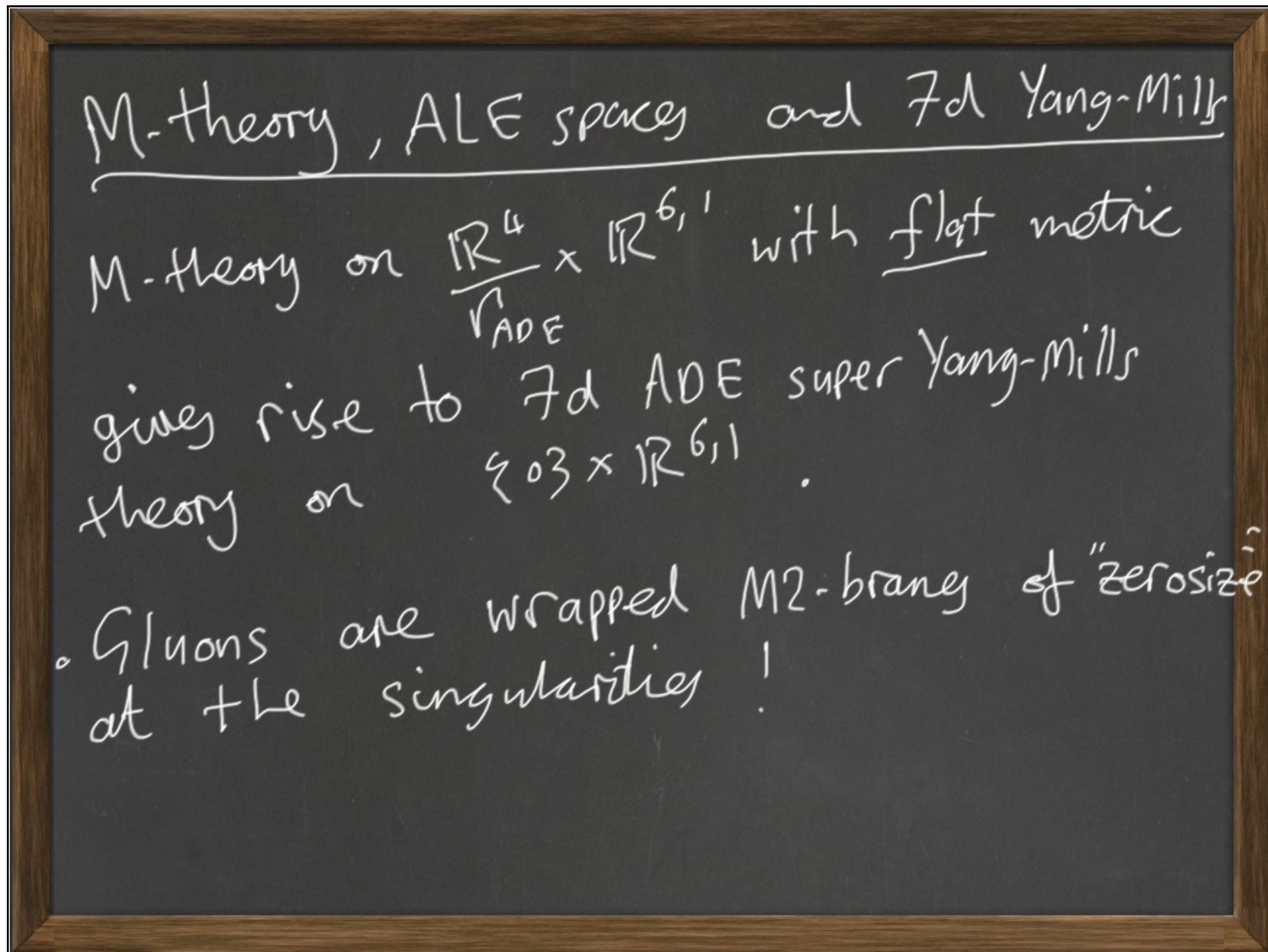


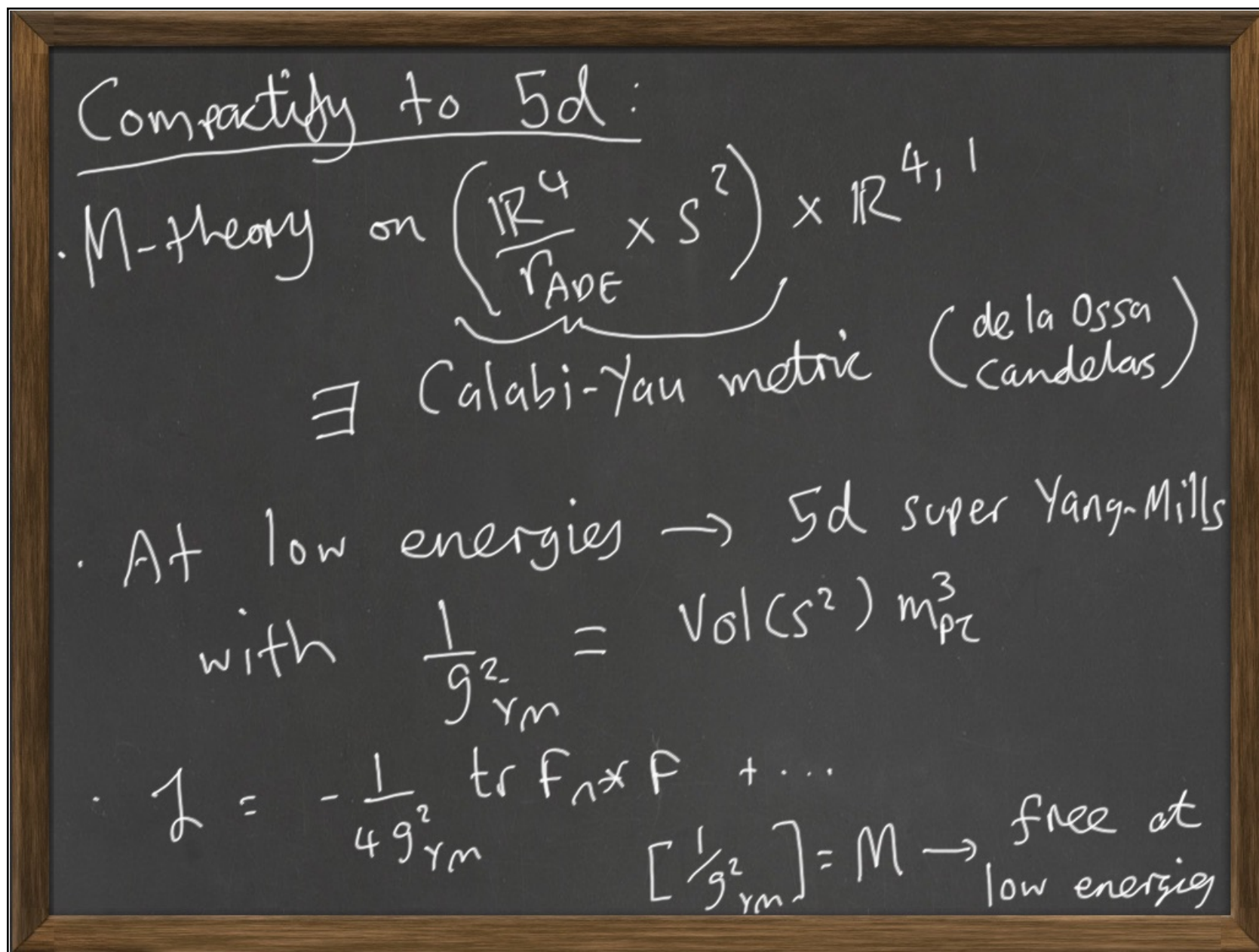


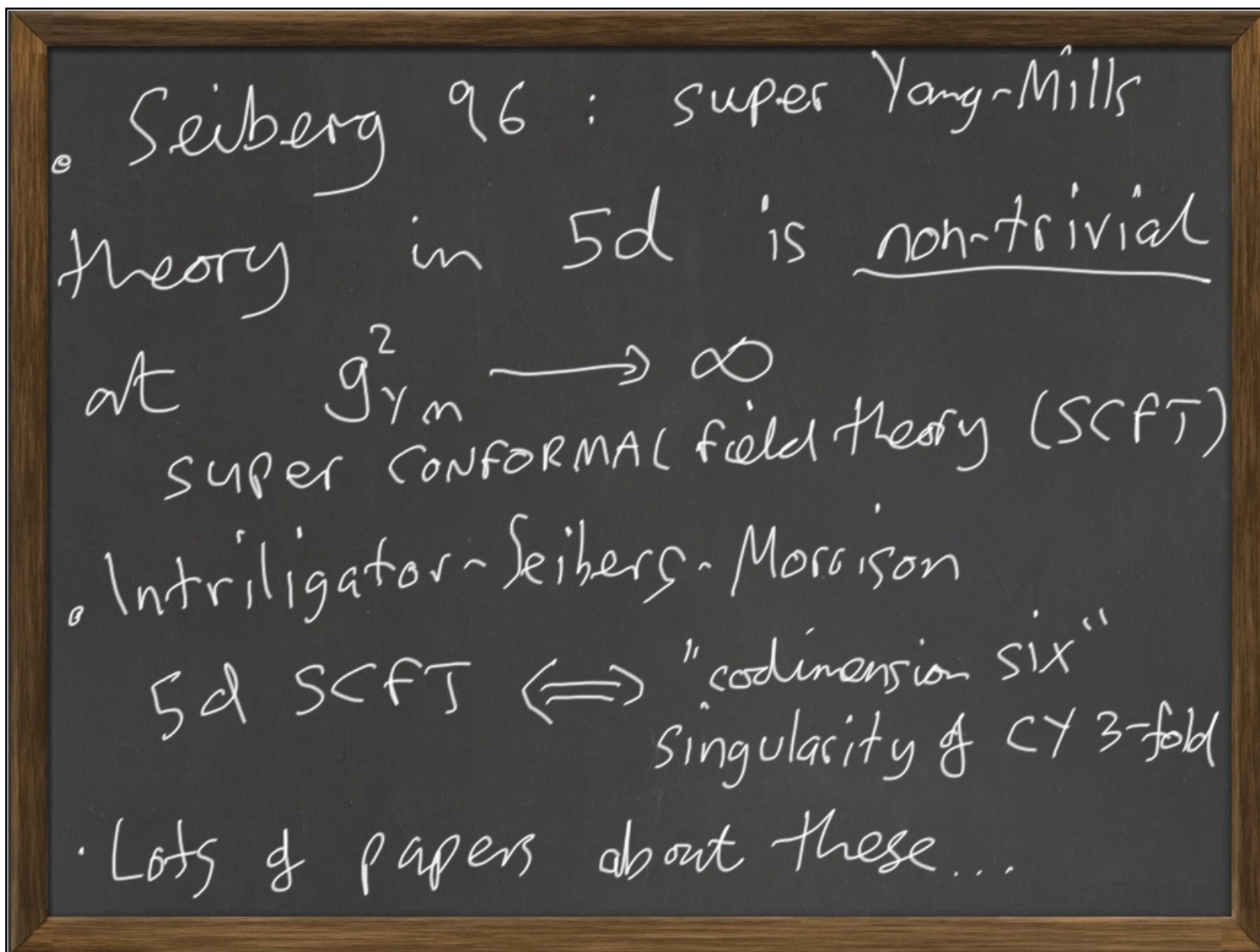
Physically its also interesting to consider  
finite quotients of these examples:

ie  $AC$  special Holonomy orbifolds

(eg codim four ADE singularities  
 $\Leftrightarrow$  non-Abelian gauge fields )  
(like the Standard Model)







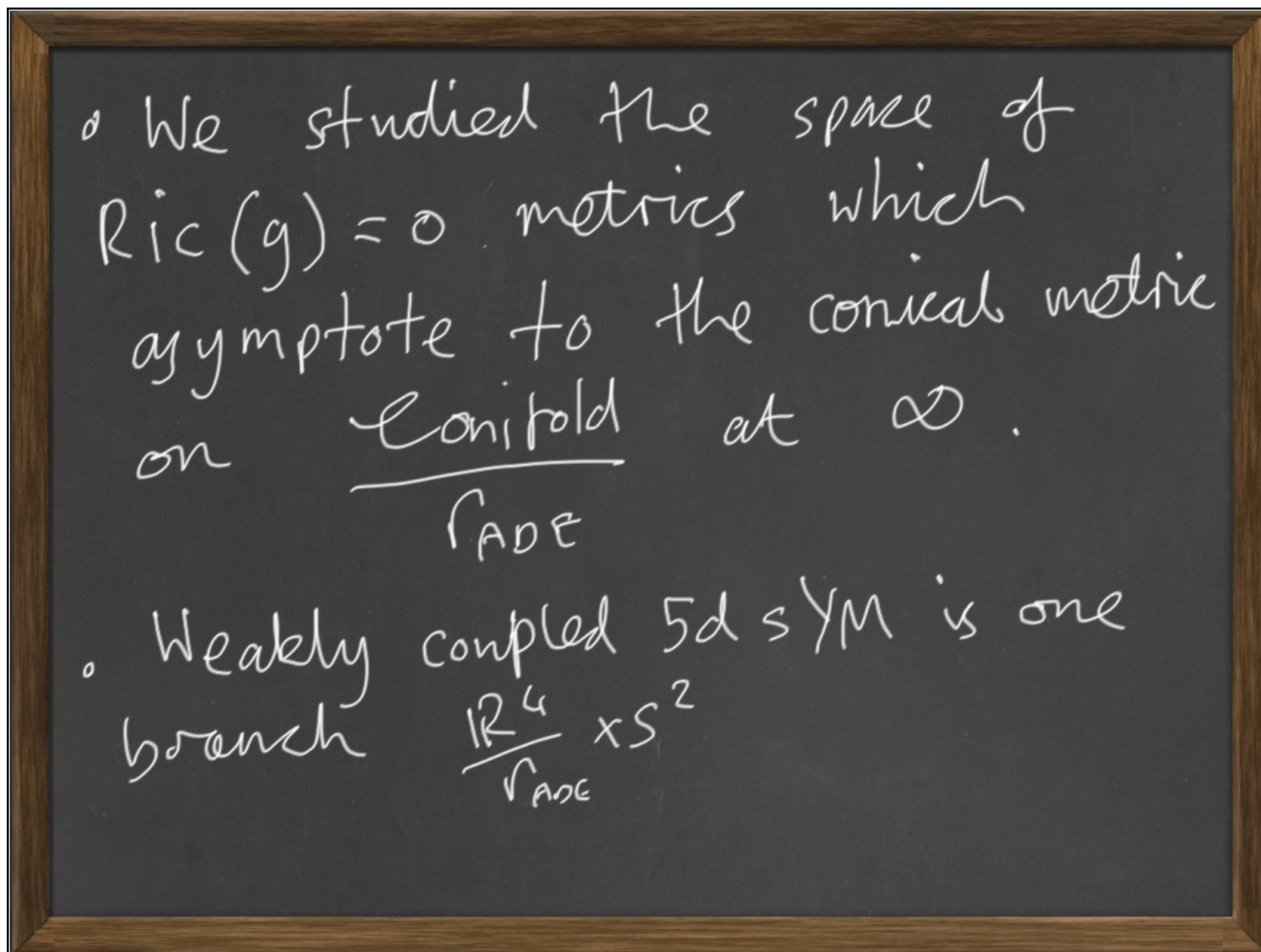
Strong Coupling singularity  $\frac{1}{g_{YM}^2} \rightarrow 0$

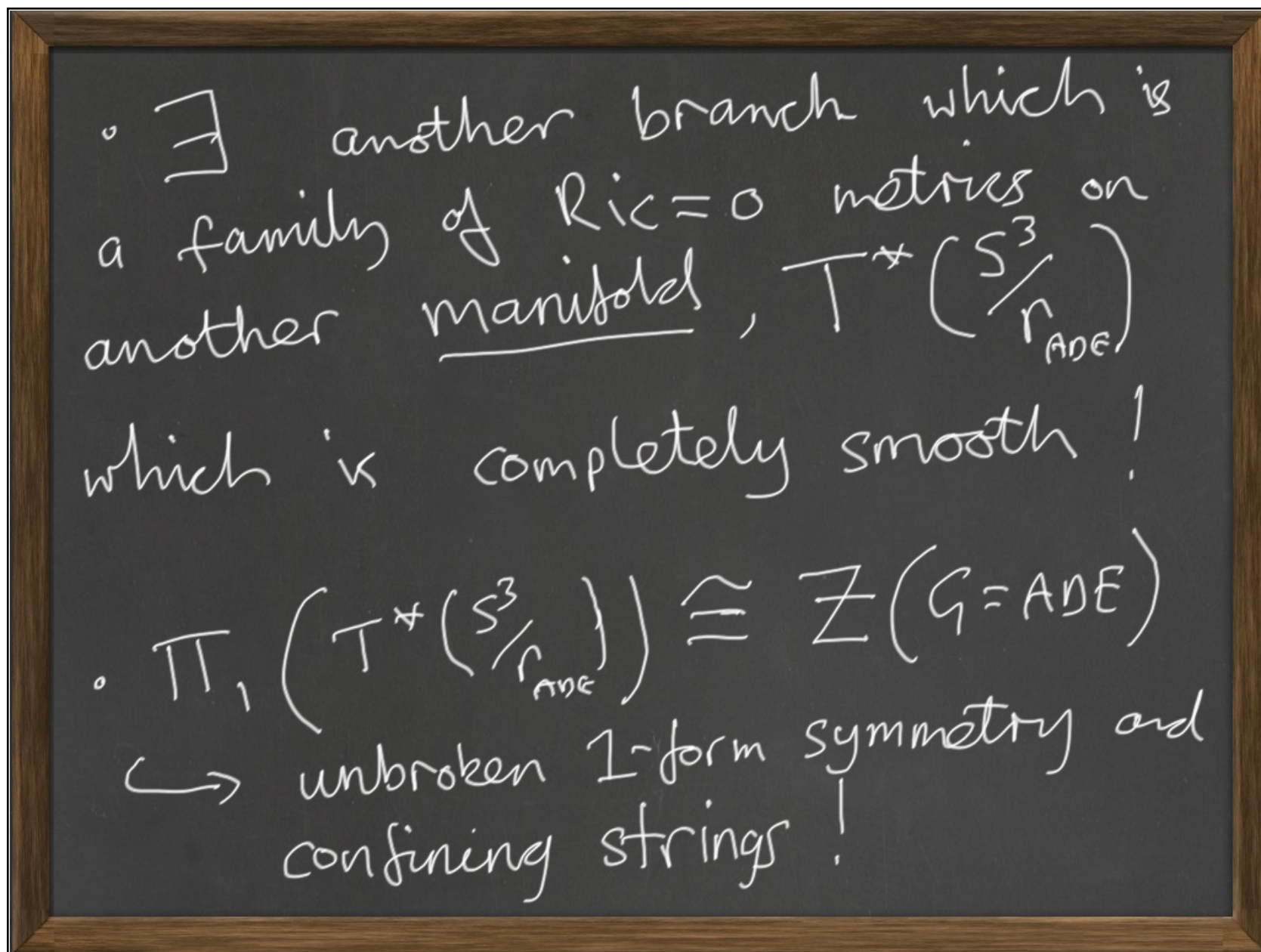
$\text{Vol}(S^2) \rightarrow 0$

And  $\left( \frac{\mathbb{R}^4}{r_{\text{ADE}}} \times S^2 \right) \rightarrow \left( \frac{\text{Conifold}}{r_{\text{ADE}}} \right)$

↑  
Conical singularity  
and  
5d CFT

• Shows that 5d SCFTs arise as  
HIDDEN SECTORS in M-theory





These strings are M2-branes  
which wrap the incontractible  
loops in  $T^*(S^3/r_{\text{AdF}})$ .

They are stable.

Moreover  $\Rightarrow$  there are NO  
massless particles (needing calculation)

$\Rightarrow$  CONFINEMENT and MASS GAP in 5d



