UNIVERSITÀ DEGLI STUDI DI MILANO

Istituto Nazionale di Fisica Nucleare

Precision Electroweak Physics at future colliders

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Corfù workshop on Standard Model and beyond, August 31st 2024

Outline

Interest in the precision electroweak tests of the Standard Model

Recent developments in the prediction of standard candle processes: fermion-pair production

Prospects towards the completion of full NNLO (QCD + QCDxEW + EW) corrections

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The inclusive production of a fermion pair is a standard candle process both

at LHC (Drell-Yan) $\sigma(pp \to \mu^+\mu^- + X)$

and

at FCC-ee $\sigma(e^+e^- \to \mu^+\mu^- + X)$

the lowest order process, at partonic level, is in both cases $f\bar{f} \to \mu^+\mu^-$: they share very similar computational challenges

The evaluation of NNLO-EW corrections is needed not only at FCC-ee, but already at the LHC or high-intensity facilities !

Motivations

Motivation: statistical precision from small to large fermion-pair invariant masses

Statistical errors

FCC-ee $\sigma(e^+e^- \to \mu^+\mu^- + X)$

arXiv:2206.08326

sqrt(S) (GeV)	luminosity (ab^{-1})	σ (fb)	% error
91	150	$2.17595\ 10^6$	0.0002
240	5	1870.84 ± 0.612	0.03
365	1,5	787.74 ± 0.725	0.09

LHC and HL-LHC $\sigma(pp \to \mu^+\mu^- + X)$

arXiv:2106.11953

bin range (GeV)	% error 140 fb^{-1}	% error 3 ab^{-1}
91-92	0.03	$6\ 10^{-3}$
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

Theoretical systematics

EW input parameters

large QED corrections

increasingly large EW corrections

proton PDFs

increasingly large QCD, QCD-EW and EW corrections

Are we able to reach (at least) the 0.1% precision throughout the whole invariant mass range?

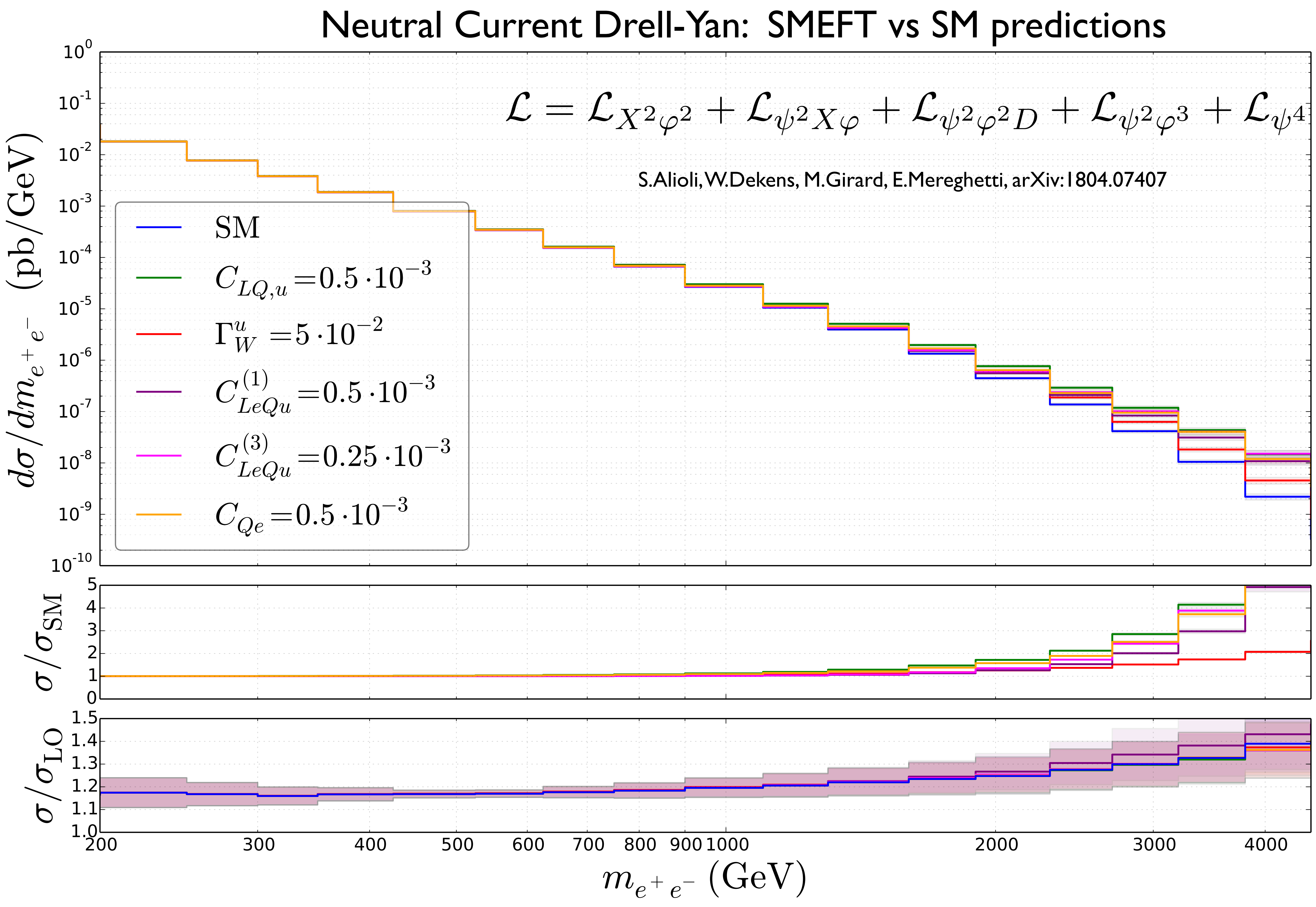
The Drell-Yan case poses the same challenges relevant for FCC-ee

Motivation: impact of higher dimension operators, as a function of the invariant mass

The parameterisation of BSM physics in the SMEFT language can be probed by studying the impact of higher dimension operators as a function of energy.

Deviations from the SM prediction require to answer the question "What is the SM?"

→ SM prediction have to be at the same precision level of the data i.e. (sub) per mille level



Motivation: interplay of precision measurements at Z resonance, low-, and high-energy

The very high precision determination of EW parameters at the Z resonance is a cornerstone of the whole precision program but there is more…

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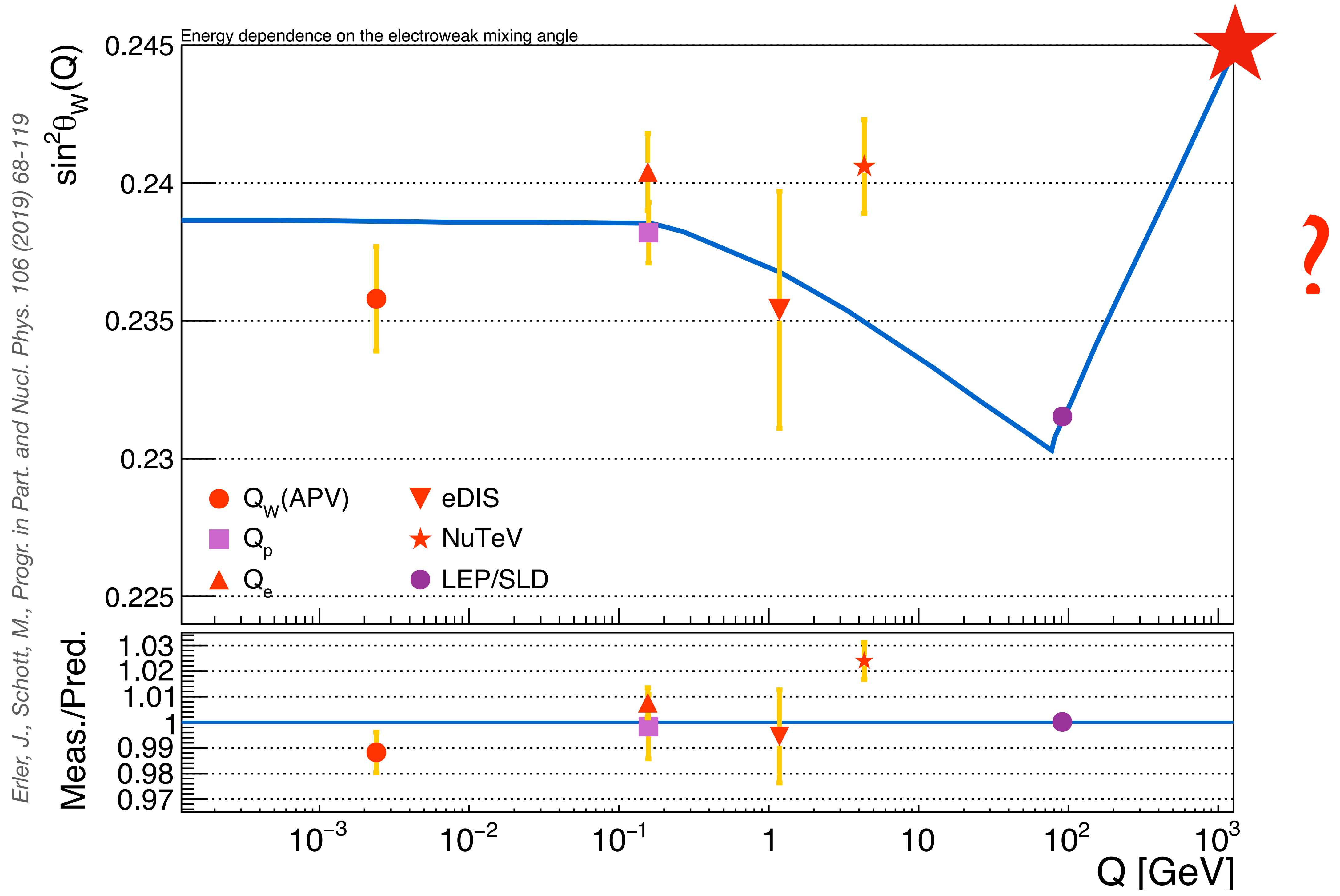
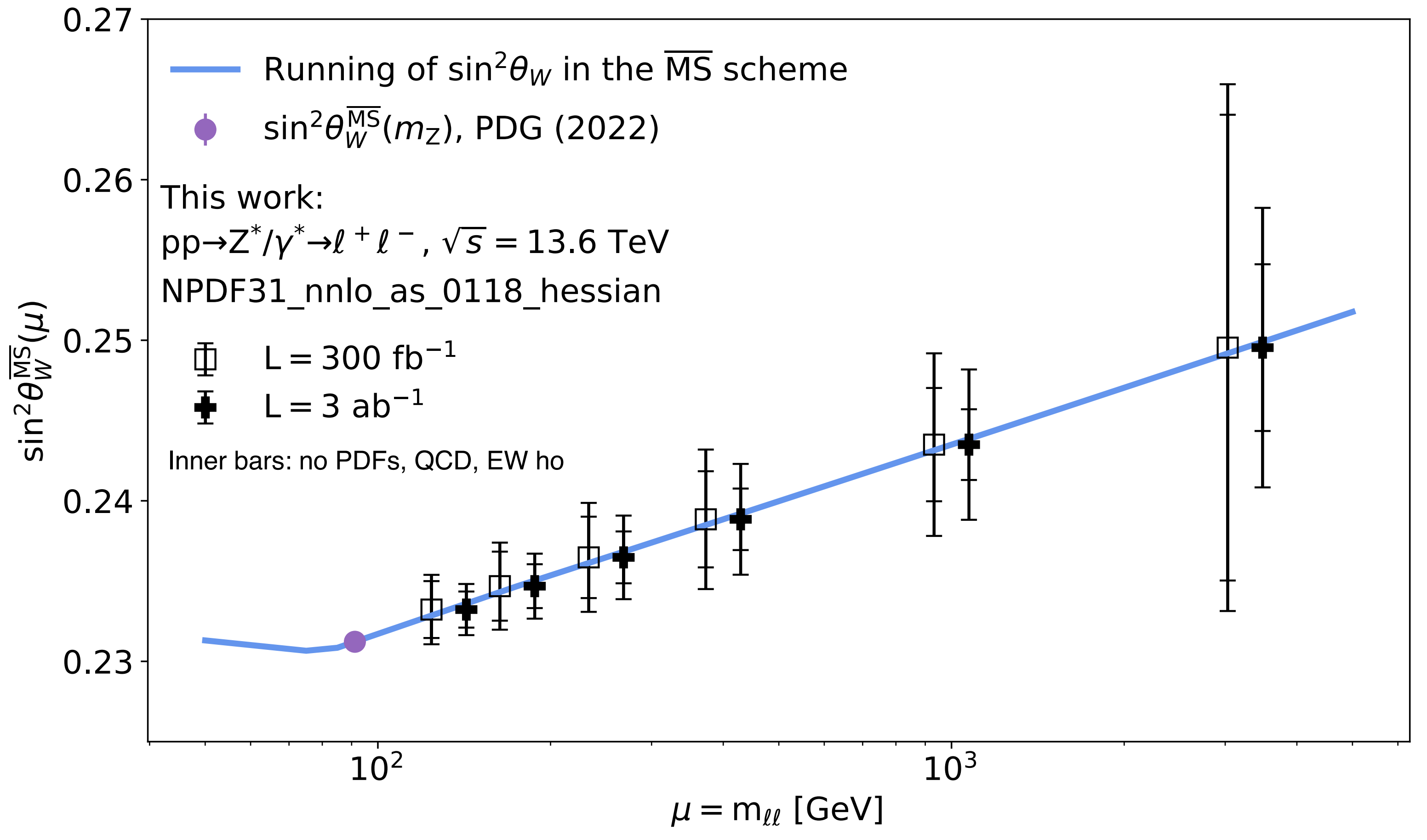
The SM predicts the running of its parameters, like e.g. $\sin^2\hat{\theta}(\mu_R^2)$, with non-trivial features and in turn complementary sensitivity to BSM physics

low-energy (sub-GeV) determinations (P2 in Mainz, Møller at JLab)
high-energy (TeV) determinations (CMS, ATLAS)
offer a stringent test of the SM
complementary to the results at the Z resonance

The running of an MSbar parameter is completely assigned once boundary and matching conditions are specified



Motivation: exploiting simultaneously Z-resonance and high-mass precision



The sensitivity to determine the running of $\sin^2\hat\theta(\mu_R^2)$ at the LHC has been demonstrated in arXiv: 2302.10782

A dedicated POWHEG NCDY version has been implemented for this study, with $\sin^2\hat\theta(\mu_R^2)$ among the input parameters, with NLO-EW renormalisation.
(when fitting the distributions to the data, we can only vary the input parameters of the calculation)

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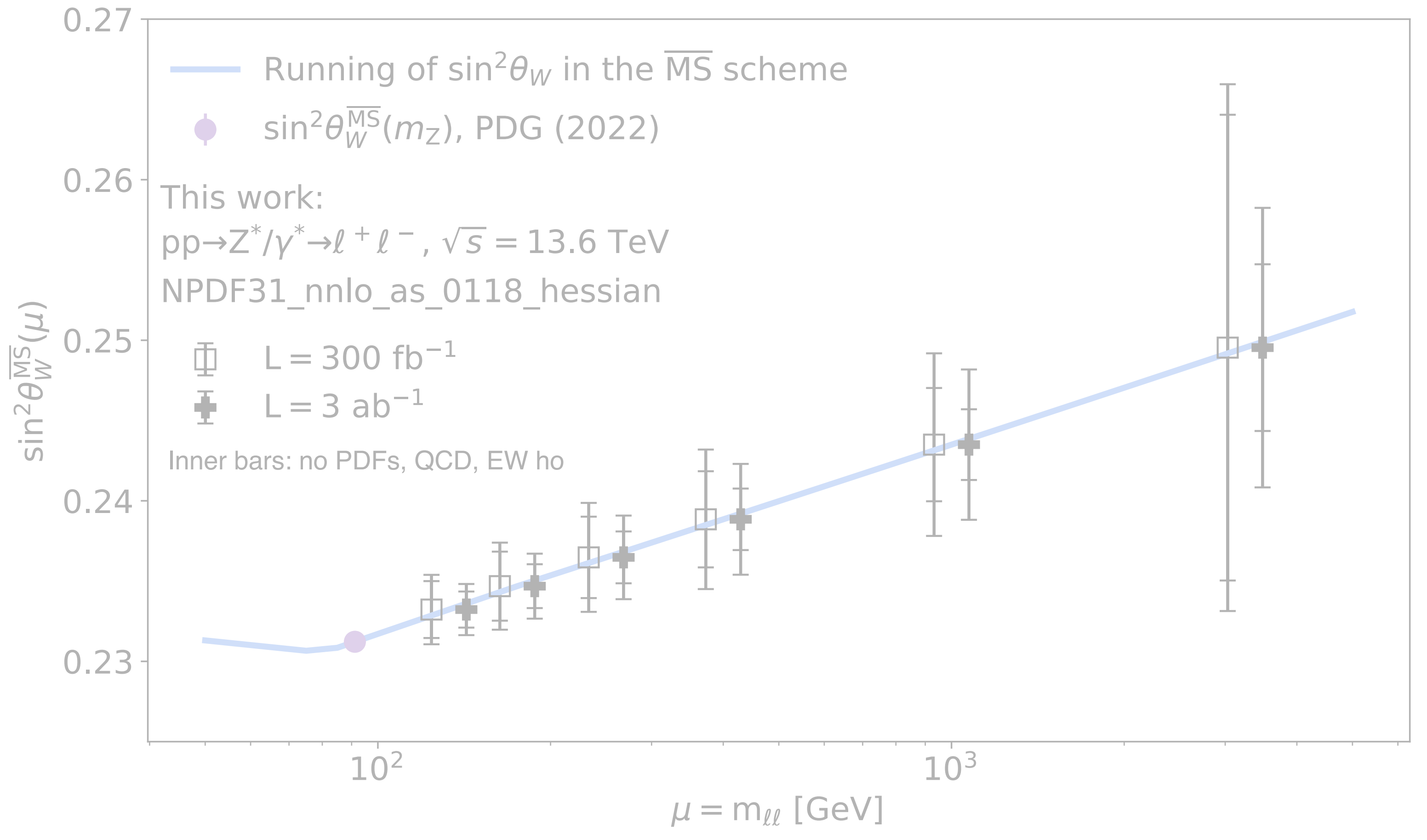

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The determinations of the
- $\sin^2\hat{\theta}(\mu_R^2)$ running
- Wilson coefficients of higher-dimension operators in SMEFT

share a problem:

Missing SM higher-order effects, not related to the coupling definition, may be reabsorbed in these fitting parameters faking a BSM signal

examples: all the QCD corrections, the EW Sudakov logs, the corrections contributing to the electric charge running
→ we need the best SM description of the cross sections, before we move to the interpretation phase in terms of couplings

NNLO-EW corrections (with UV renormalisation) are needed both at the LHC and FCC-ee to tame this potential problem

Testing the Standard Model with the W-boson mass

The W boson mass can be predicted

in terms of the input parameters of the model,

including the quantum effects Standard Model or beyond

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{G_\mu \sqrt{2}\, m_Z^2}\,(1+\Delta r)}\right)$$

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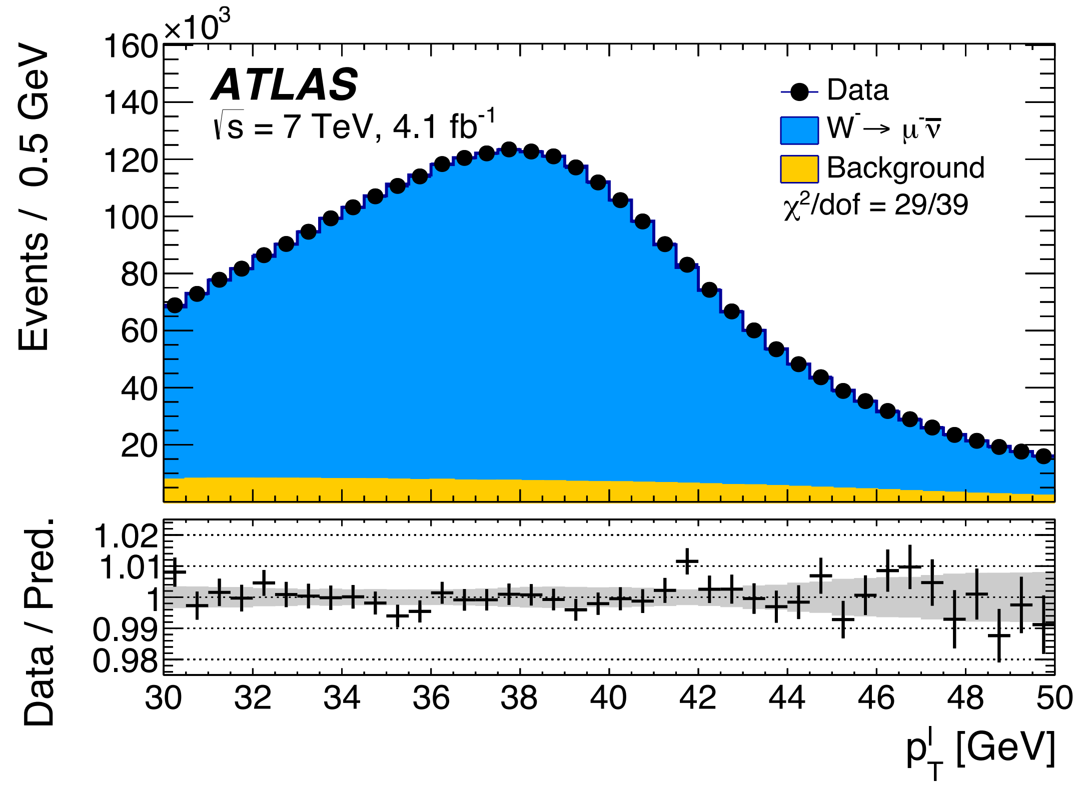

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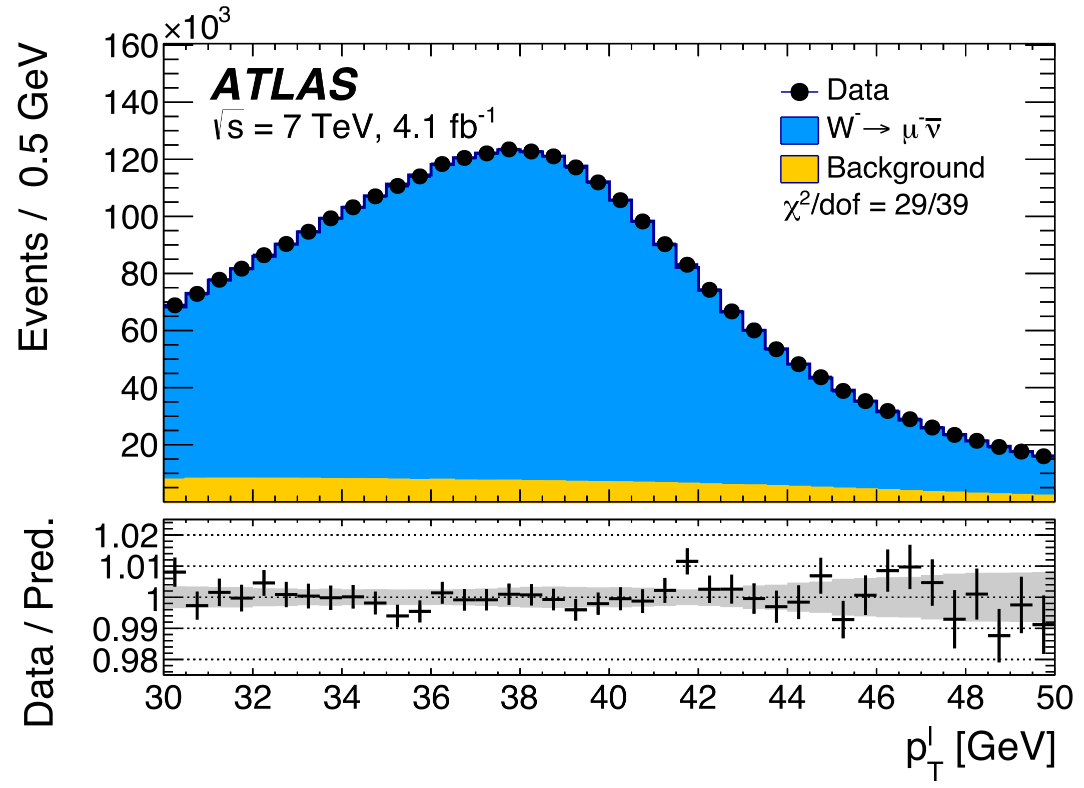
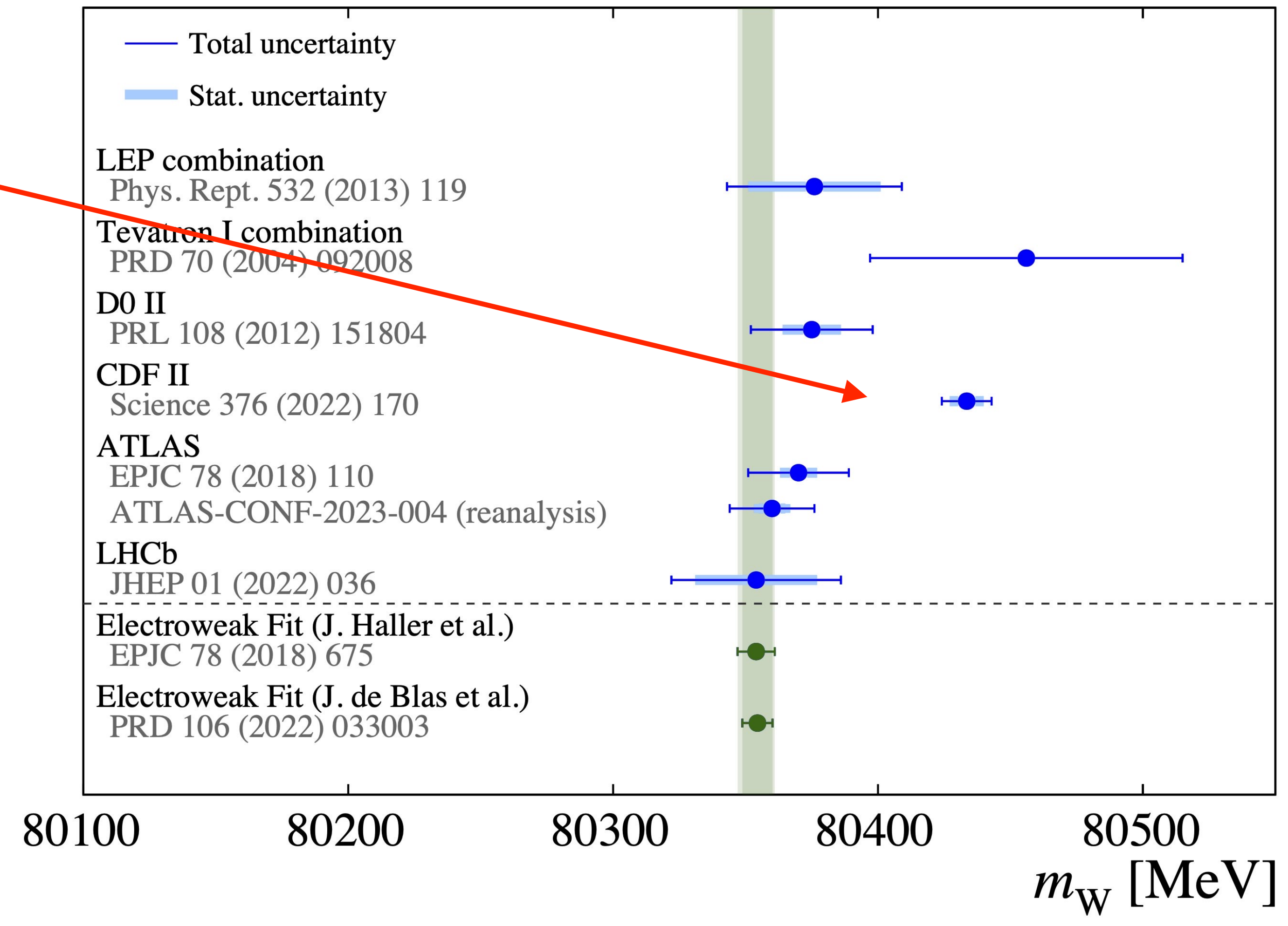


A discrepancy between the Standard Model and experimental values
may hint about the presence of New Physics:
new BSM particles contributing to Δr could explain the difference



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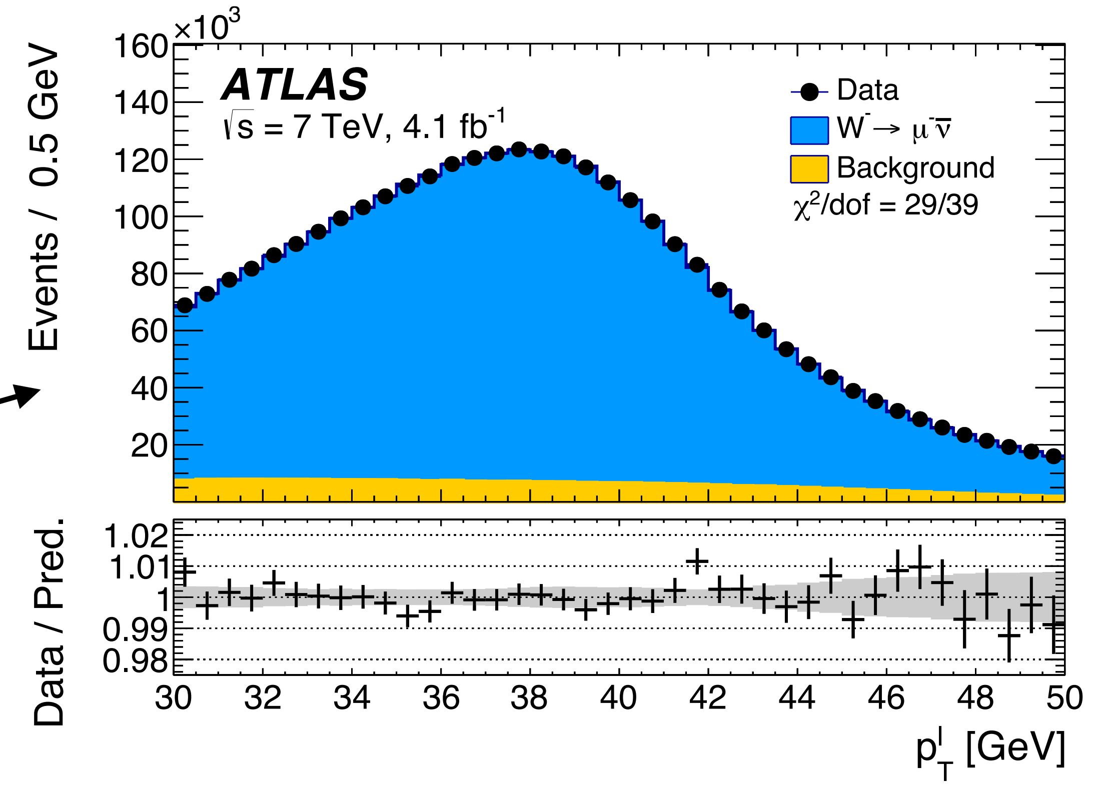

Challenging theoretical calculations are needed for both:
the theoretical predictions and the distributions used to fit the data

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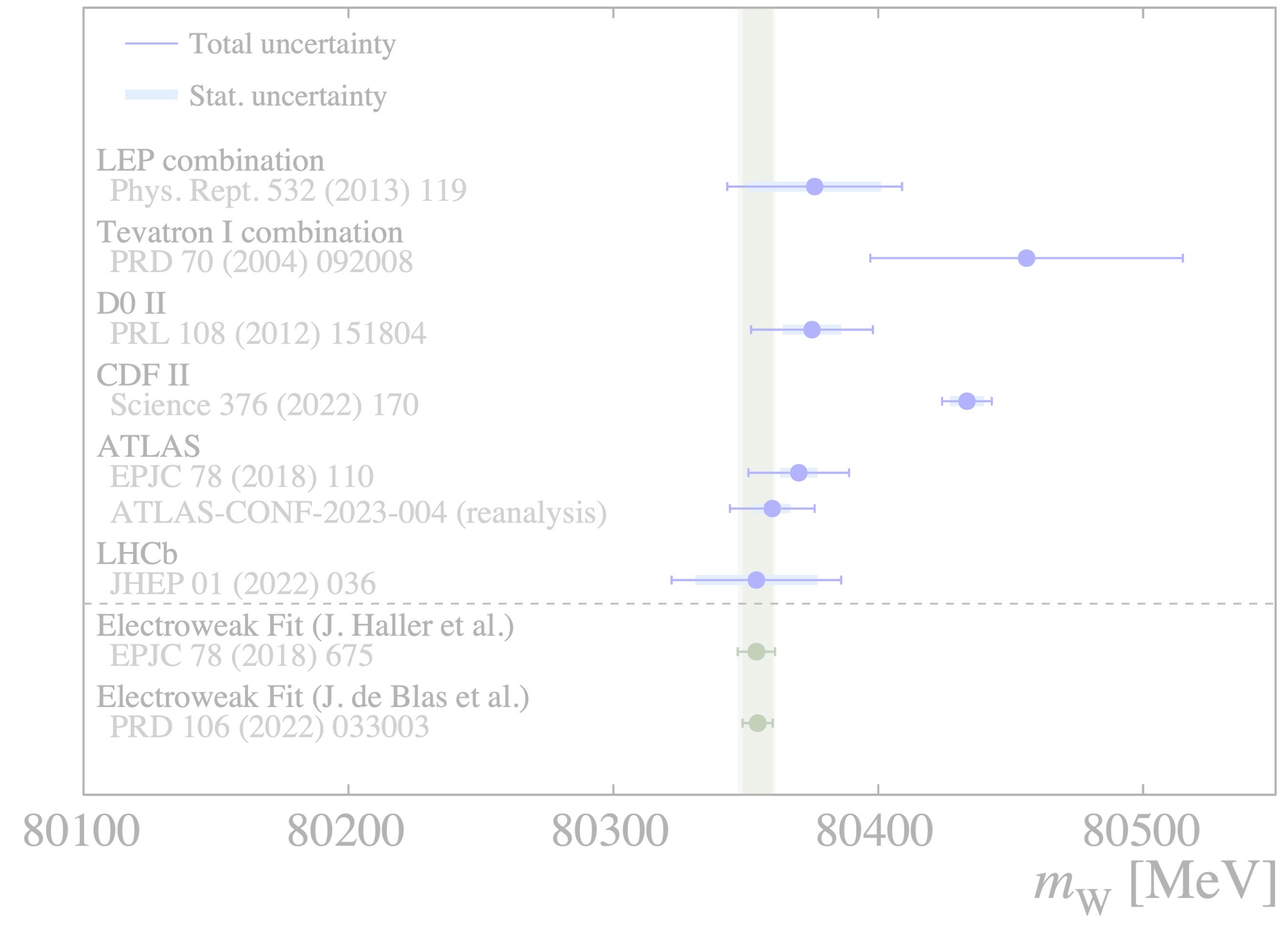


The W boson mass: theoretical prediction

Sirlin, 1980, 1984; Marciano, Sirlin, 1980, 1981;
van der Bij, Veltman, 1984; Barbieri, Ciafaloni, Strumia 1993;
Djouadi, Verzegnassi 1987; Consoli, Hollik, Jegerlehner, 1989;
Chetyrkin, Kühn, Steinhauser, 1995;
Barbieri, Beccaria, Ciafaloni, Curci, Viceré, 1992, 1993; Fleischer, Tarasov, Jegerlehner, 1993;
Degrassi, Gambino, AV, 1996; Degrassi, Gambino, Sirlin, 1997;
Freitas, Hollik, Walter, Weiglein, 2000, 2003;
Awramik, Czakon, 2002; Awramik, Czakon, Onishchenko, Veretin, 2003; Onishchenko, Veretin, 2003

The best available prediction includes
the full 2-loop EW result, leading higher-order EW and QCD corrections,
resummation of reducible terms
Missing 3-loop and 4-loop terms needed to reduce the uncertainties.

$$m_W = w_0 + w_1 dH + w_2 dH^2 + w_3 dh + w_4 dt + w_5 dH dt + w_6 da_s + w_7 da^{(5)}$$

$$dt = [(M_t/173.34\,\mathrm{GeV})^2 - 1]$$

$$da^{(5)} = [\Delta\alpha^{(5)}_{\mathrm{had}}(m_Z^2)/0.02750 - 1]$$

$$dH = \ln\left(\frac{m_H}{125.15\,\mathrm{GeV}}\right)$$

$$dh = [(m_H/125.15\ \mathrm{GeV})^2 - 1]$$

$$da_s = \left(\frac{\alpha_s(m_Z)}{0.1184} - 1\right)$$

	$124.42 \le m_H \le 125.87$ GeV	$50 \le m_H \le 450$ GeV
w_0	80.35712	80.35714
w_1	-0.06017	-0.06094
w_2	0.0	-0.00971
w_3	0.0	0.00028
w_4	0.52749	0.52655
w_5	-0.00613	-0.00646
w_6	-0.08178	-0.08199
w_7	-0.50530	-0.50259

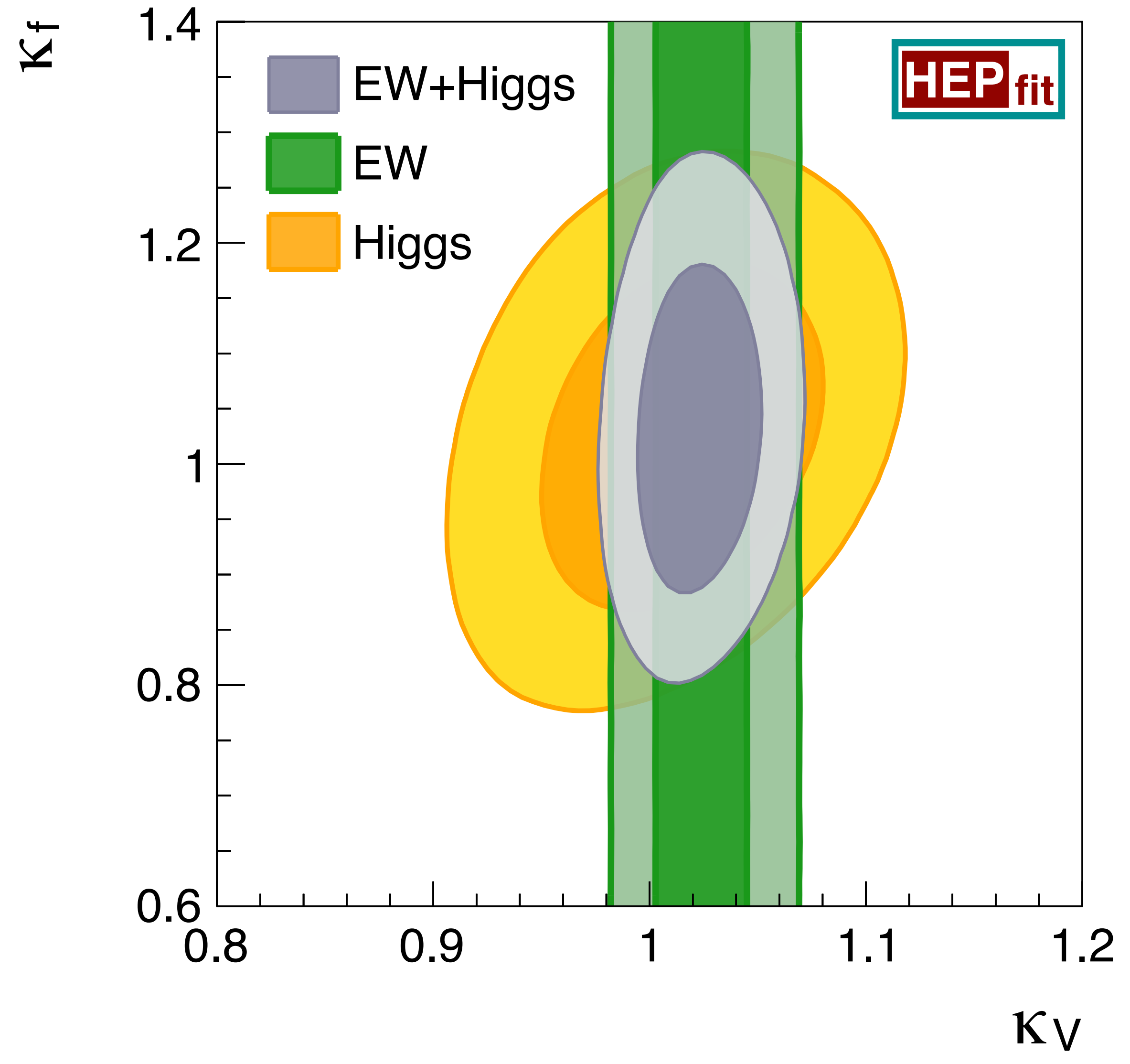
on-shell scheme $m_W^{os} = 80.353 \pm 0.004$ GeV (Freitas, Hollik, Walter, Weiglein)
MSbar scheme. $m_W^{\overline{MS}} = 80.351 \pm 0.003$ GeV (Degrassi, Gambino, Giardino)

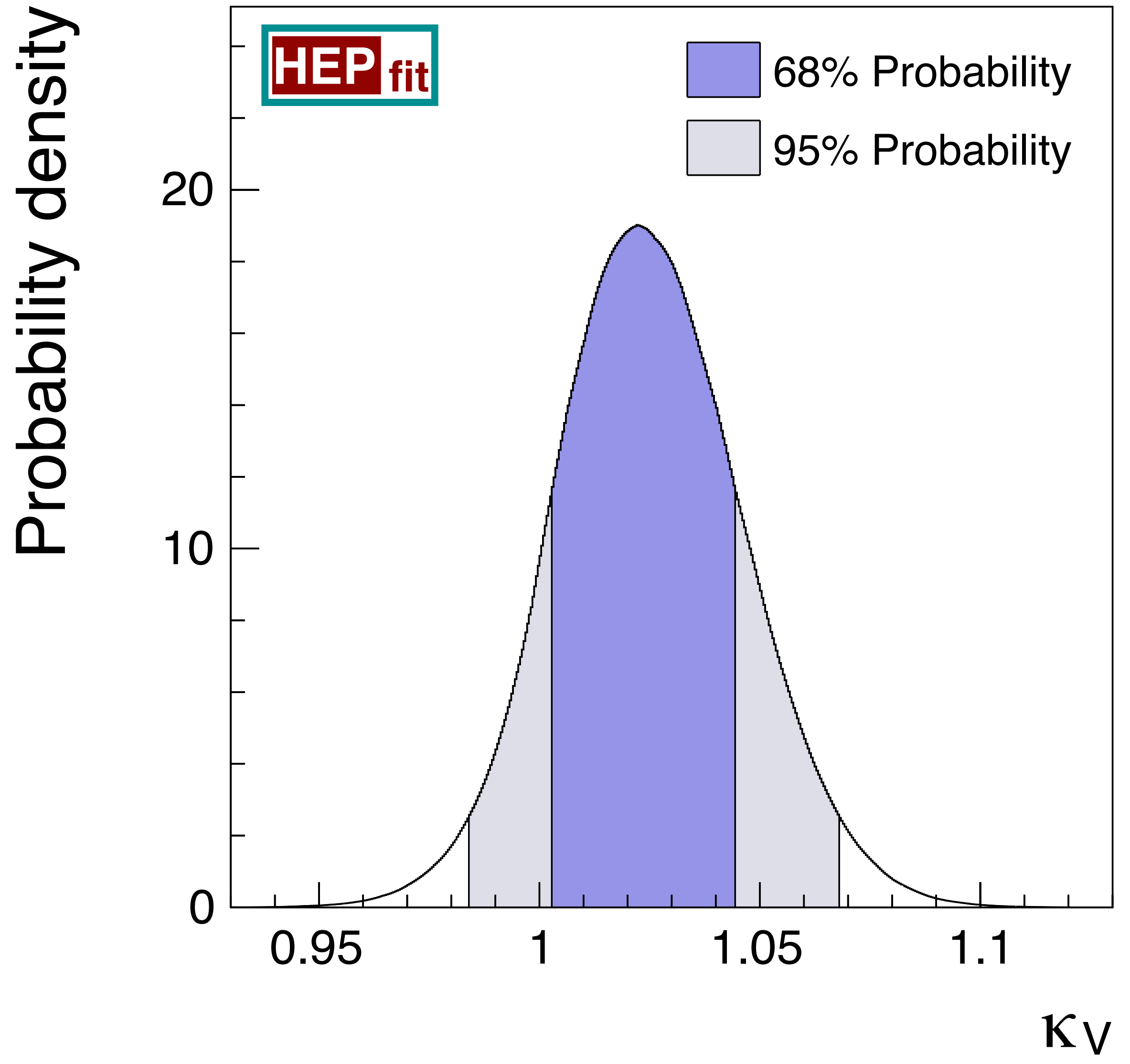
parametric uncertainties $\delta m_W^{par} = \pm 0.005$ GeV due to the $(\alpha, G_\mu, m_Z, m_H, m_t)$ values

still far from the 1 MeV needed for FCC-ee

Relevance of new high-precision measurement of EW parameters

de Blas et al, arXiv:1608.01509





$$\mathcal{L}_{\rm Eff} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}}\mathcal{L}_d = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \qquad [\mathcal{O}_i] = d \longrightarrow \left(\frac{q}{\Lambda}\right)^{d-4}$$

Effects suppressed by $q = v, E < \Lambda$

Λ: Cut-off of the EFT

$$\mathcal{O}_{\phi WB} = \phi^\dagger \sigma_a \phi B^{\mu\nu} W^a_{\mu\nu}$$

EWSB →

$v^2 B^{\mu\nu} W^3_{\mu\nu}$ gauge boson masses

$vh B^{\mu\nu} W^3_{\mu\nu}$ h→ZZ, γ γ

$$M_W^2 = M_Z^2 c^2 \left[1 - \frac{c^2}{c^2 - s^2}\left(\frac{1}{2} C_{\phi D} + 2\frac{s}{c} C_{\phi WB} + \frac{s^2}{c^2}\Delta_{G_\mu} \right)\frac{v^2}{\Lambda^2} \right]$$

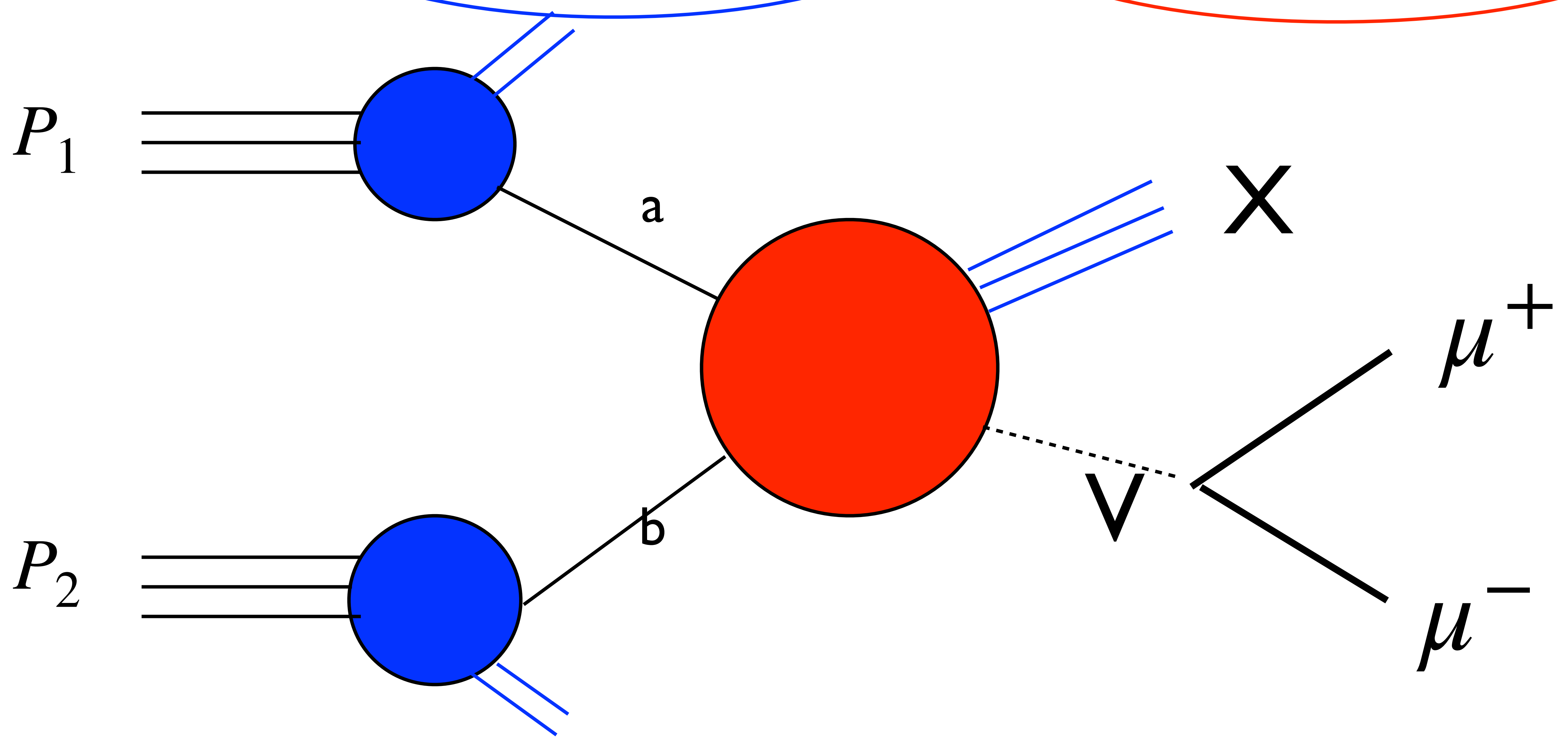
A precise measurement of m_W and $\sin^2\theta_{eff}$ constrains several dim-6 operators contributing to Higgs and gauge interaction vertices.

Today still one of the strongest constraints

Computational framework

Factorisation theorems and the cross section in the partonic formalism

$$\sigma(P_1,P_2;m_V) = \sum_{a,b}\int_0^1 dx_1 dx_2\, f_{h_1,a}(x_1,M_F) f_{h_2,b}(x_2,M_F)\, \hat\sigma_{ab}(x_1P_1,x_2P_2,\alpha_s(\mu),M_F)$$



Particles $P_{1,2}$ can be protons (→ Drell-Yan @ LHC) or leptons (→ FCC-ee, muon collider)

The partonic content of the scattering particles can be expressed in terms of PDFs (see D.Marzocca's talk)

proton PDFs: ABM, CT18, MSHT,NNPDF,… lepton PDFs: Frixione et al. arXiv:1911.12040

The partonic scattering can be computed in perturbation theory, in the full QCD+EW theory, exploiting the theoretical progress in QCD, in the understanding of its IR structure

Factorisation theorems guarantee the validity of the above picture up to power correction effects

The Drell-Yan cross section in a fixed-order expansion

Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} +$$

Altarelli, Ellis, Martinelli (1979)

$$\alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} +$$

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

$$\alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} +$$

$$\alpha_s^3\,\sigma^{(3,0)} + \ldots$$

C.Duhr, B.Mistlberger, arXiv:2111.10379

still missing
Sudakov high-energy approximations

Neutral Current

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2022)
F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, (2022)

New!!! Charged-current 2-loop amplitude

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2024)

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New!!! Charged-current 2-loop amplitude — T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2024)

The resummation of QCD and QED corrections is another crucial topic not covered here

Mixed QCD-EW corrections to the Drell-Yan processes

Strong boost of the activities in the theory community in the last 4 years! (references not covering the Monte Carlo developments)

→ mathematical and theoretical developments and computation of universal building blocks

- 2-loop virtual Master Integrals with internal masses

U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193, R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581, M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491, S.Hasan, U.Schubert, arXiv:2004.14908, M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130

- New methods to solve the Master Integrals

M.Hidding, arXiv:2006,05510, D.X.Liu, Y.-Q. Ma, arXiv:2201.11669, T.Armadillo, R.Bonciani, S.Devoto, N.Rana,AV, arXiv: 2205.03345

- Altarelli-Parisi splitting functions including QCD-QED effects

D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612

- renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

→ on-shell Z and W production as a first step towards full Drell-Yan

- pole approximation of the NNLO QCD-EW corrections

S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016, 2401.15682

- analytical total cross section including NNLO QCD-QED and NNLO QED corrections

D. de Florian, M.Der, I.Fabre, arXiv:1805.12214

- ptZ distribution including QCD-QED analytical transverse momentum resummation

L. Cieri, G. Ferrera, G. Sborlini, arXiv:1805.11948

- fully differential on-shell Z production including exact NNLO QCD-QED corrections

M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428

- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections

R. Bonciani, F. Buccioni, R.Mondini, AV, arXiv:1611.00645, R. Bonciani, F. Buccioni, N.Rana, I.Triscari, AV, arXiv:1911.06200, R. Bonciani, F. Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections

F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221, A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671,

Mixed QCD-EW corrections to the Drell-Yan processes

Strong boost of the activities in the theory community in the last 4 years! (references not covering the Monte Carlo developments)

→ complete Drell-Yan

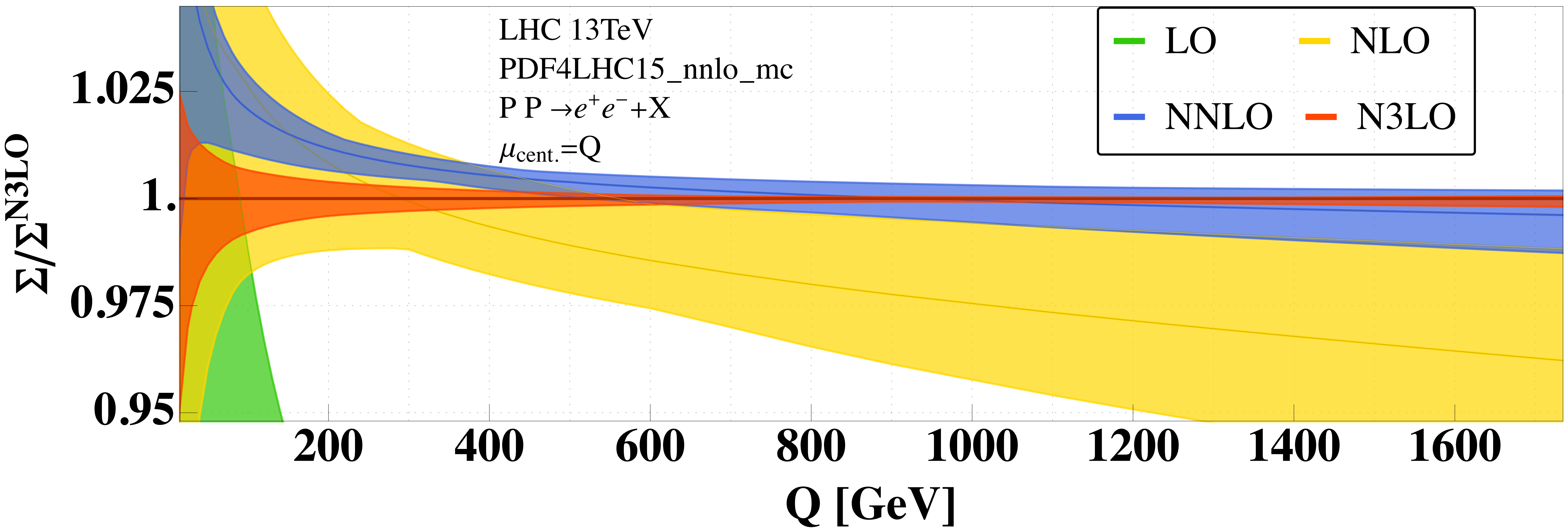
- neutrino-pair production including NNLO QCD-QED corrections
  L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315
- 2-loop NC and CC amplitudes
  M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918 , T.Armadillo, R.Bonciani, S.Devoto, N.Rana,AV, arXiv: 2201.01754, 2405.00612
- NNLO QCD-EW corrections to charged-current DY (2-loop contributions in pole approximation).
  L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539
- NNLO QCD-EW corrections to neutral-current DY
  R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, N.Rana, F.Tramontano, AV, arXiv:2102.12539, F. Buccioni, F. Caola, H.A.Chawdhry, F.Devoto, M.Heller, A.V.Manteuffel, K.Melnikov, R.Roentsch, C.Signorile-Signorile, arXiv:2203.11237

→ mixed QCD-QED resummation

- initial-state corrections
  L. Cieri,G.Ferrera, G.Sborlini,, arXiv:1805.11948, A.Autieri, L. Cieri,G.Ferrera, G.Sborlini,, arXiv:2302.05403
- initial and final state corrections
  L.Buonocore, L'Rottoli, P.Torrielli, arXiv:2404.15112

QCD results: lepton-pair invariant mass

C.Duhr, B.Mistlberger, arXiv:2111.10379



Thanks to the N3LO-QCD results for the Drell-Yan cross section, scale variation band at the few per mille level at any Q

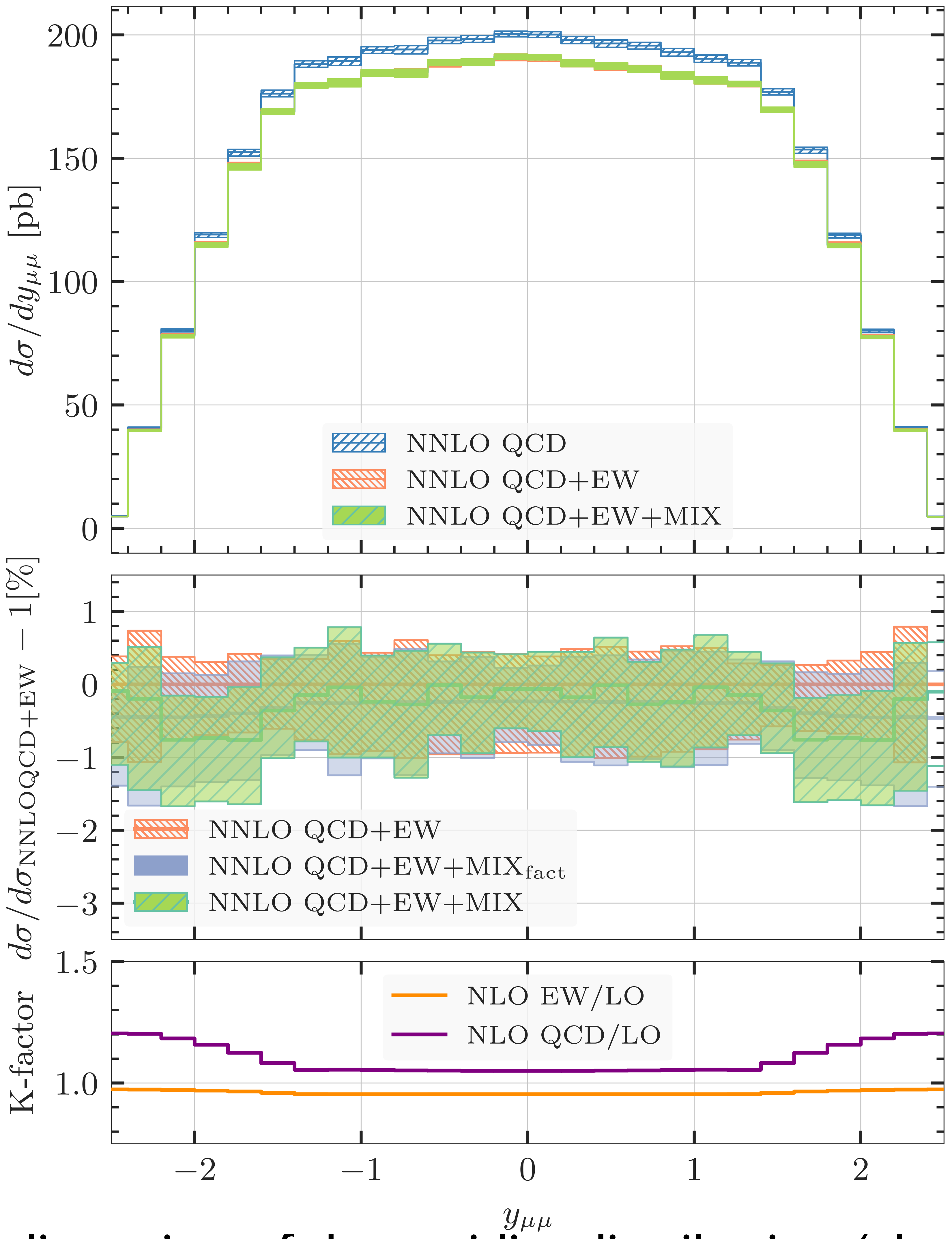
The PDFs are not yet at N3LO

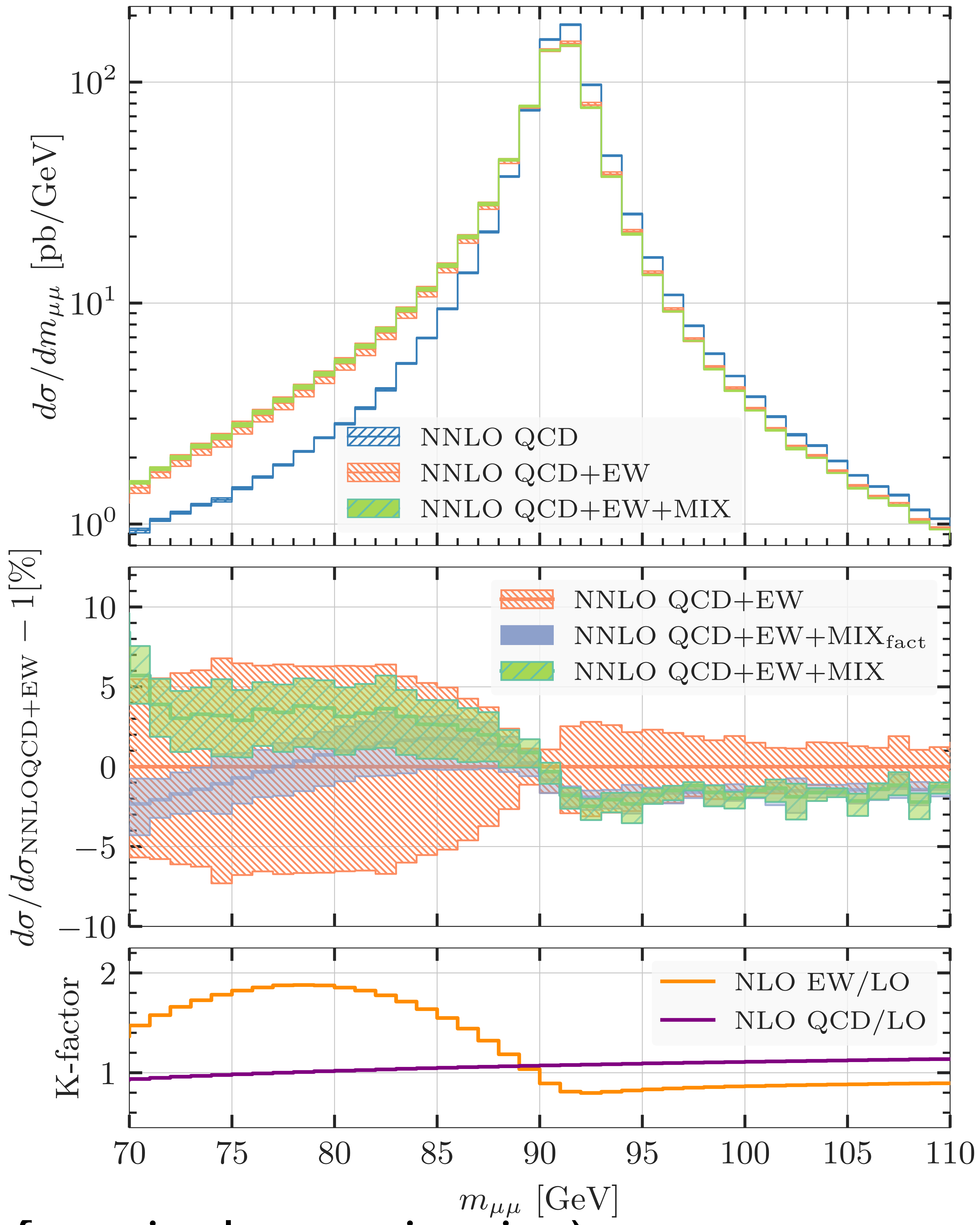
This is promising, in view of the program of searches for deviation from the SM in the TeV range

What about NNLO QCD-EW and NNLO-EW corrections ?

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953 , Phys.Rev.Lett. 128 (2022) 1, 012002 and work in preparation





Non-trivial distortion of the rapidity distribution (absent in the naive factorised approximation)

Large effects below the Z resonance (the factorised approximation fails) → impact on the $\sin^2\theta_{eff}$ determination

O(-1.5%) effects above the resonance → ongoing precision studies in the CERN EW WG

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

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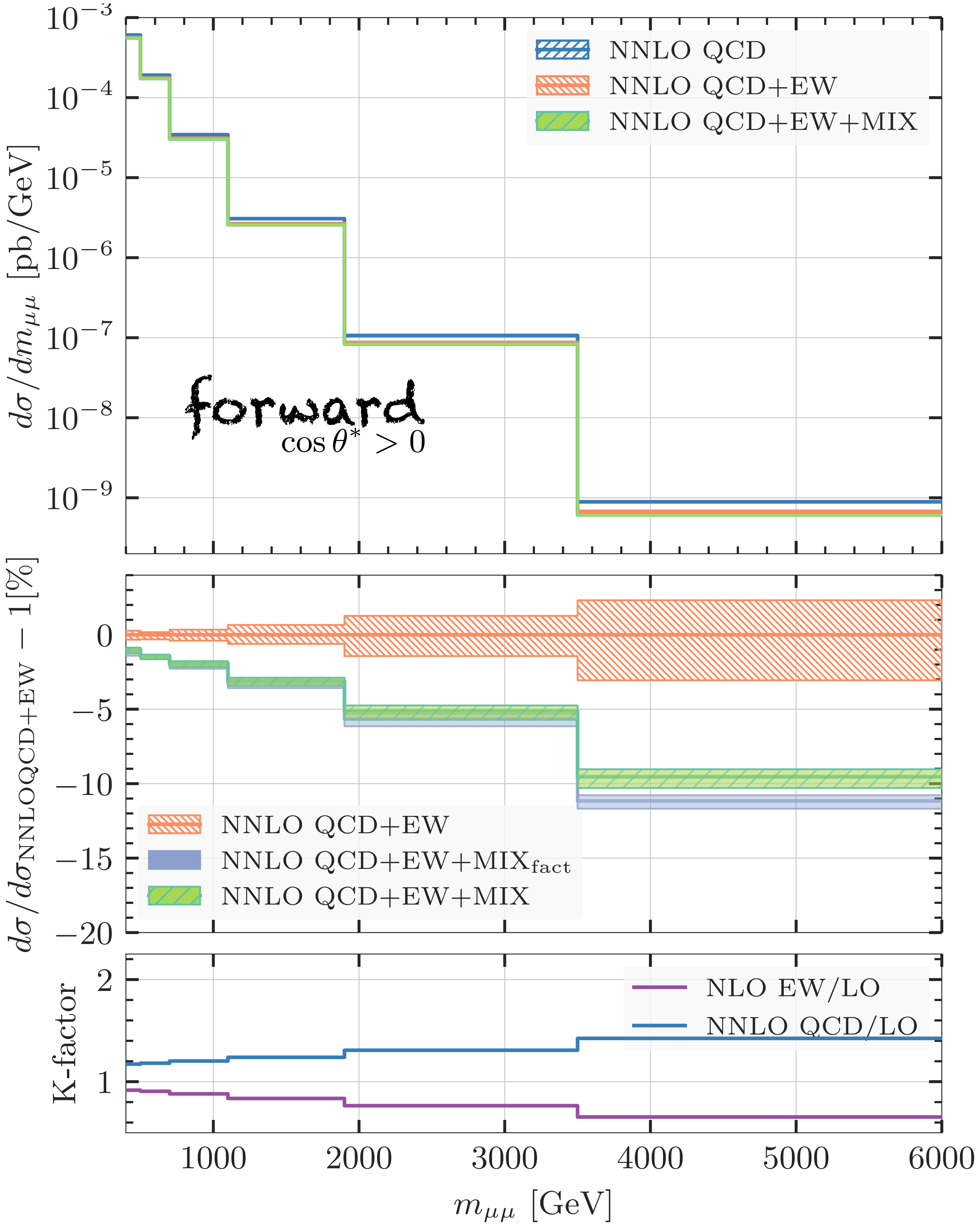
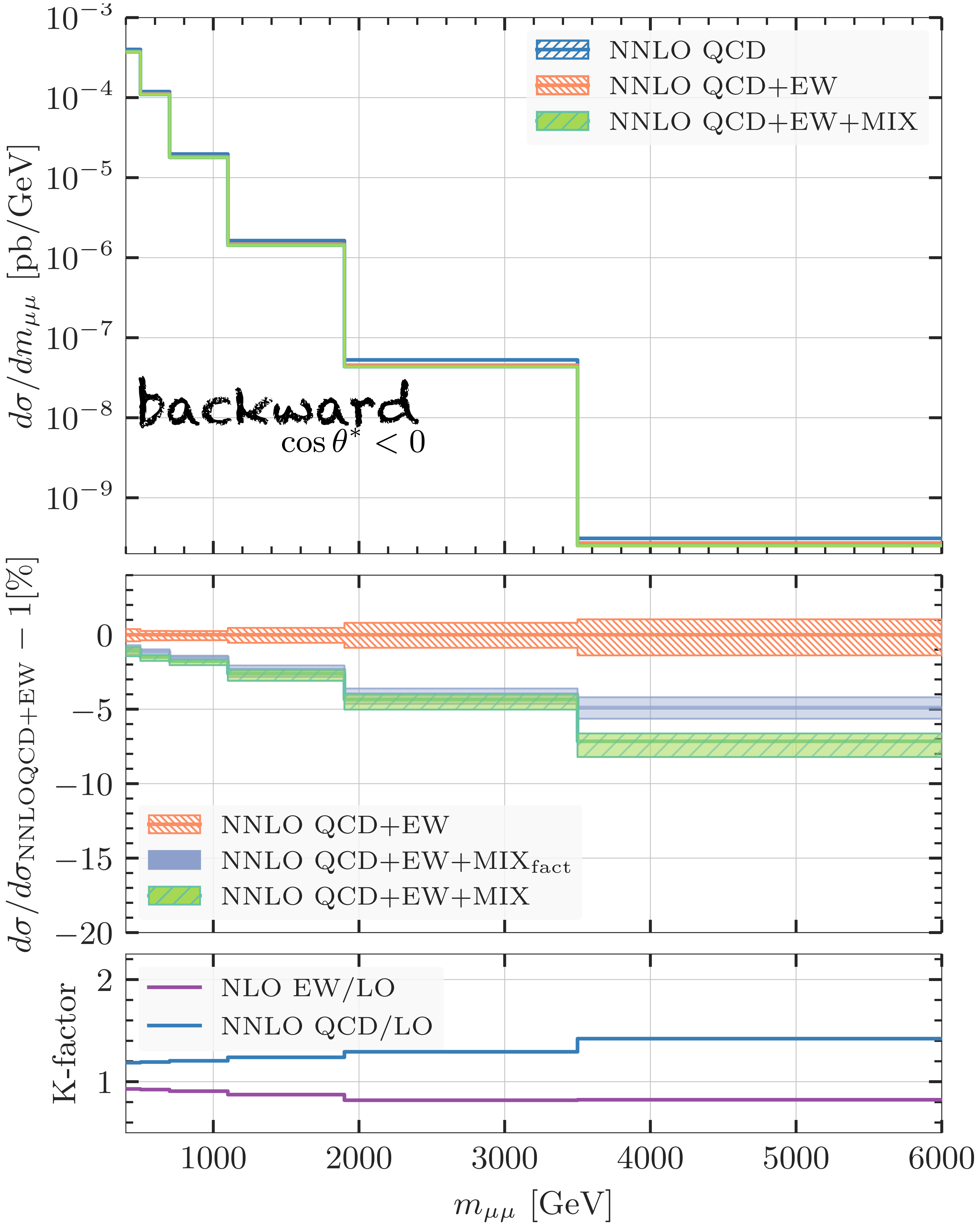




Negative mixed NNLO QCD-EW effects (-3% or more) at large invariant masses,

absent in any additive combination → impact on the searches for new physics

Charged Current Drell-Yan: NNLO QCD-EW results with approximated 2-loop virtual corrections

L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539

Exact LO, NLO (QCD+EW), NNLO QCD corrections
are combined with mixed QCD-EW corrections

Partonic subprocesses with 1 and 2 additional partons
are evaluated exactly at NLO and LO respectively

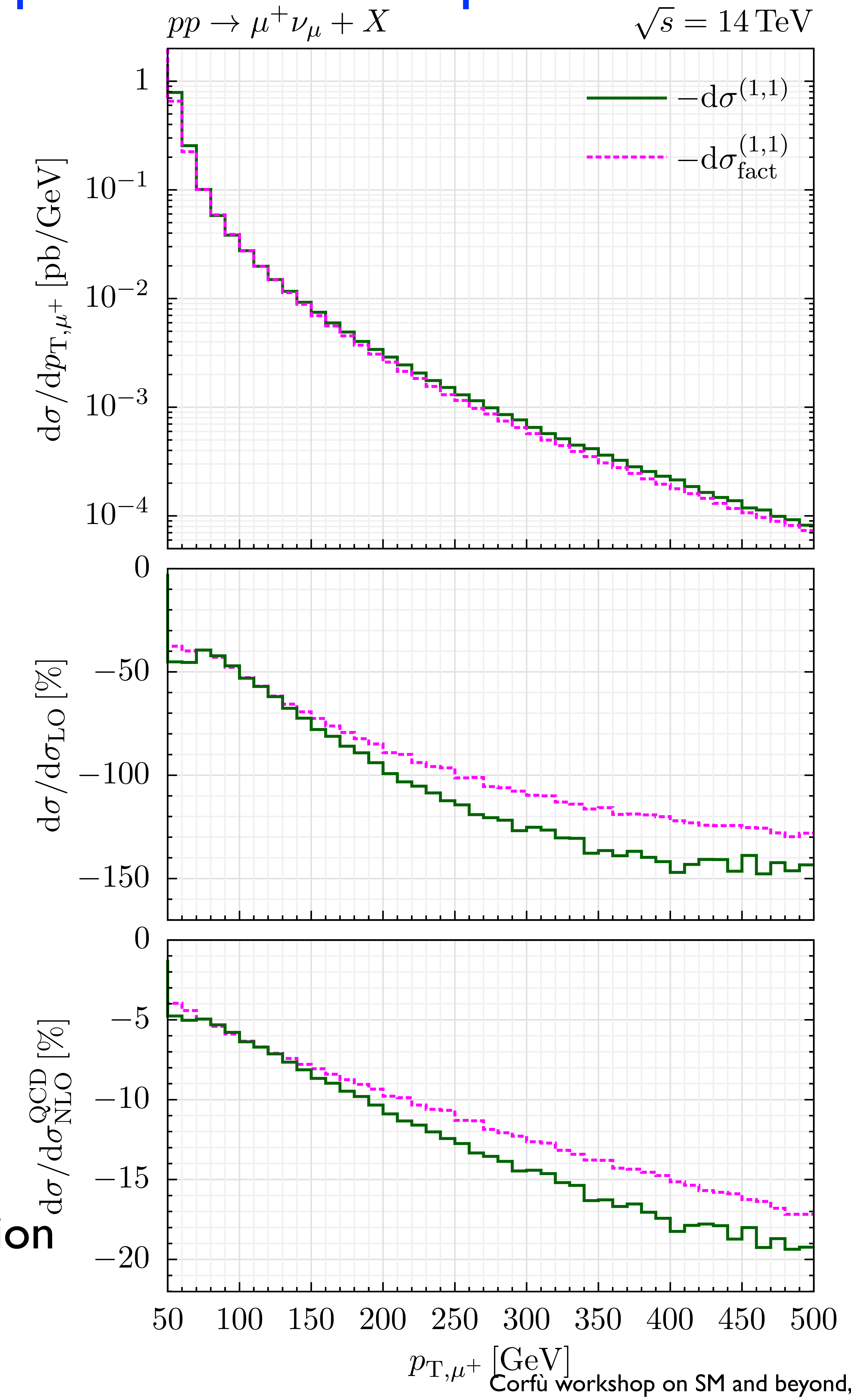
The 2-loop virtual corrections to $q\bar{q}' \to \ell\nu_\ell$ treated in pole approximation

Accurate description of the charged lepton $p_\perp^\ell$ spectrum,
dominated by the (exact) real radiation effects
resonant configurations

The factorisation of QCD and EW corrections is not accurate at large $p_\perp^\ell$

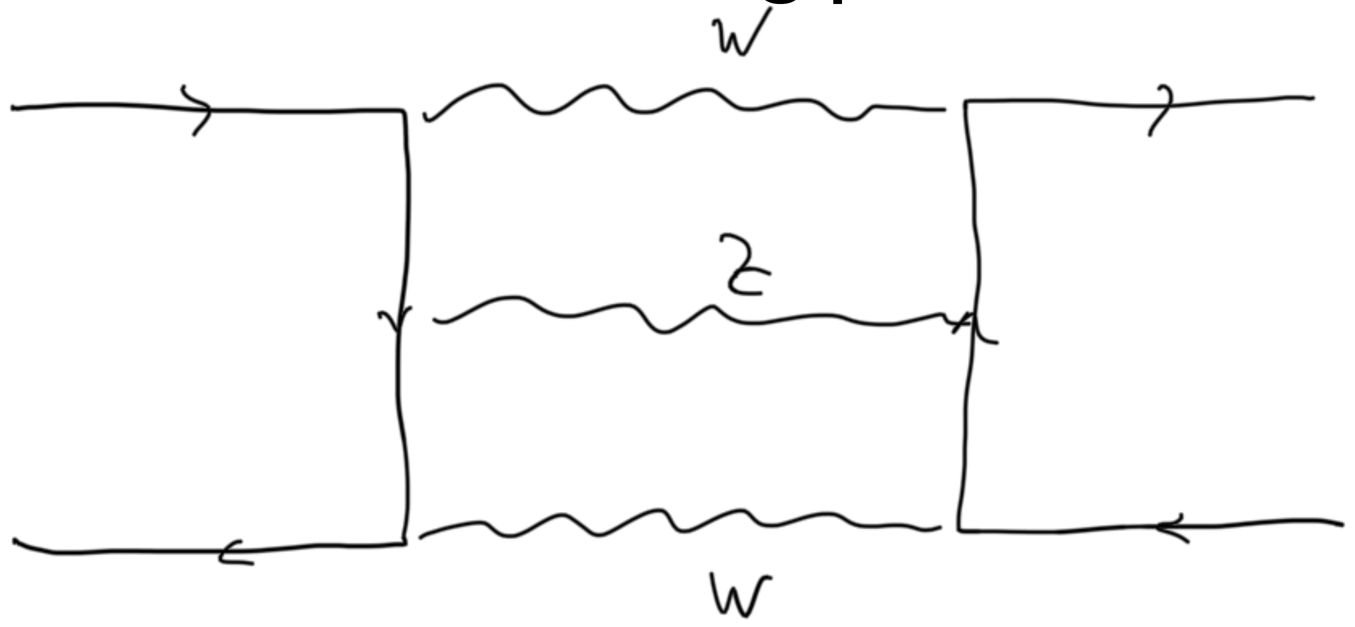
The lepton-pair transverse mass might receive large non-negligible
2-loop virtual corrections at large mass, poorly described in pole approximation

→ new results !



Need for a full NNLO-EW calculation to reduce the uncertainties to sub-percent level

The NNLO-EW corrections to scattering processes are still today one of the frontiers in QFT



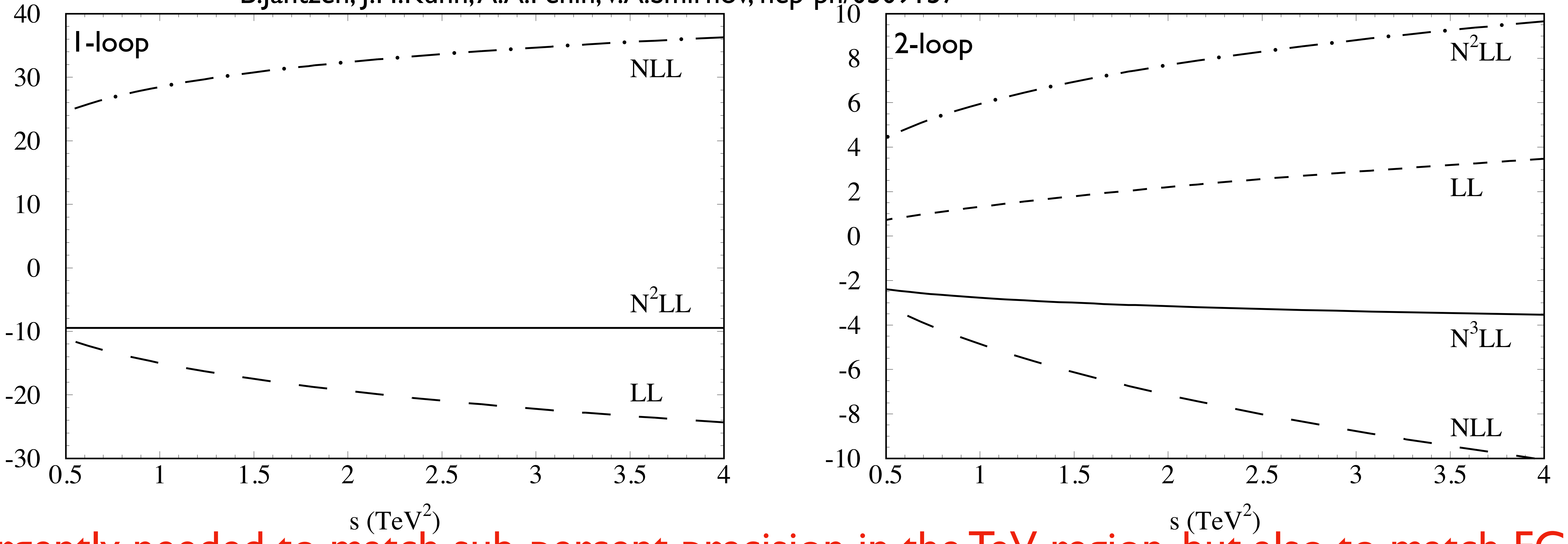
The NNLO-EW corrections could modify in a non-trivial way the large-mass/momentum tails of the distributions

Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections

At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$

The size of the constant term is not trivial

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157



corrections to $e^+e^- \to q\bar{q}$ due to EW Sudakov logs

urgently needed to match sub-percent precision in the TeV region, but also to match FCC-ee precision

Evaluation of the exact NNLO QCD-EW corrections

Neutral-Current Drell-Yan

The Neutral Current Drell-Yan cross section in the SM: perturbative expansion

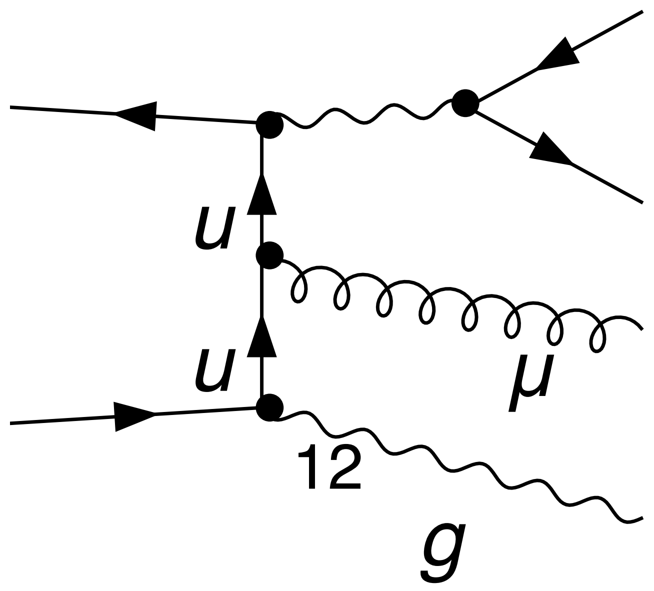
$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} + \alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} + \alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} + \alpha_s^3\,\sigma^{(3,0)} + \dots$$

$$\sigma(h_1 h_2 \to l\bar{l} + X) = \sum_{i,j=q\bar{q},g,\gamma} \int dx_1\,dx_2\,f_i^{h_1}(x_1,\mu_F) f_j^{h_2}(x_2,\mu_F)\,\hat{\sigma}(ij \to l\bar{l} + X)$$

$\sigma^{(1,1)}$ requires the evaluation of the xsecs of the following processes, including photon-induced

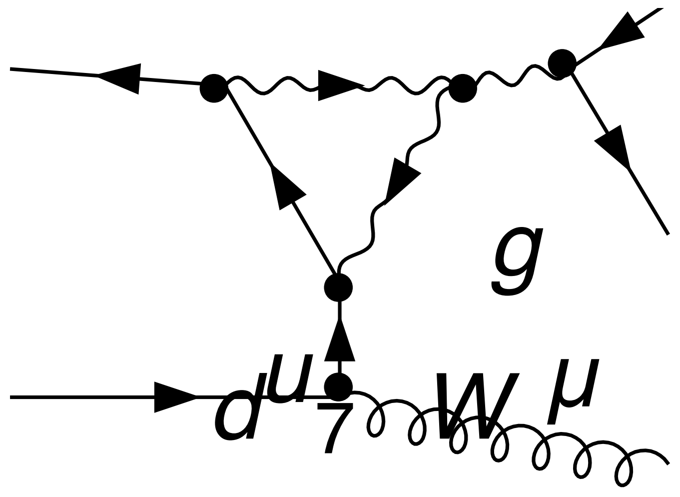
0 additional partons	$q\bar{q} \to l\bar{l},\ \gamma\gamma \to l\bar{l}$	including virtual corrections of $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha), \mathcal{O}(\alpha\alpha_s)$
1 additional parton	$q\bar{q} \to l\bar{l}g,\ qg \to l\bar{l}q$	including virtual corrections of $\mathcal{O}(\alpha)$
	$q\bar{q} \to l\bar{l}\gamma,\ q\gamma \to l\bar{l}q$	including virtual corrections of $\mathcal{O}(\alpha_s)$
2 additional partons	$q\bar{q} \to l\bar{l}g\gamma, qg \to l\bar{l}q\gamma, q\gamma \to l\bar{l}qg, g\gamma \to l\bar{l}q\bar{q}$	
	$q\bar{q} \to l\bar{l}q\bar{q},\ q\bar{q} \to l\bar{l}q'\bar{q}',\ qq' \to l\bar{l}qq',\ q\bar{q}' \to l\bar{l}q\bar{q}',\ qq \to l\bar{l}qq$	at tree level

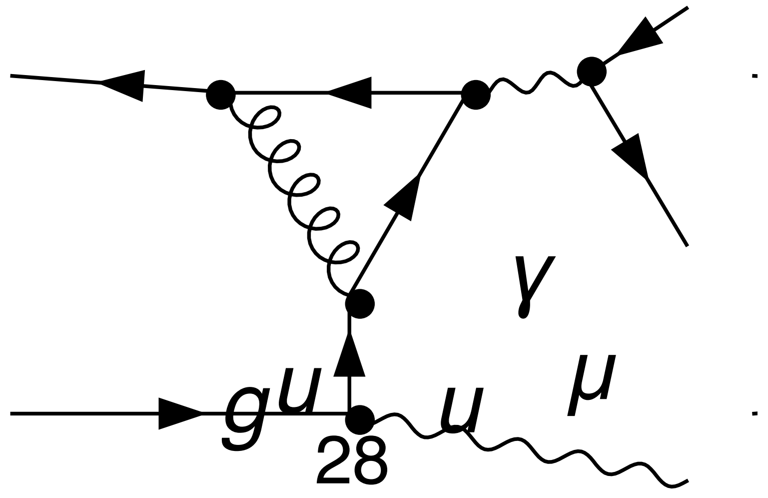
Different kinds of contributions at $\mathcal{O}(\alpha\alpha_s)$ and corresponding problems



double-real contributions

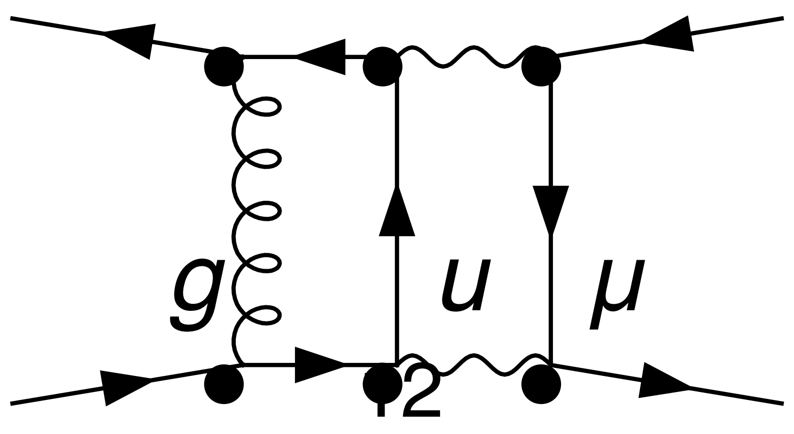
amplitudes are easily generated with OpenLoops
IR subtraction
care about the numerical convergence when aiming at 0.1% precision

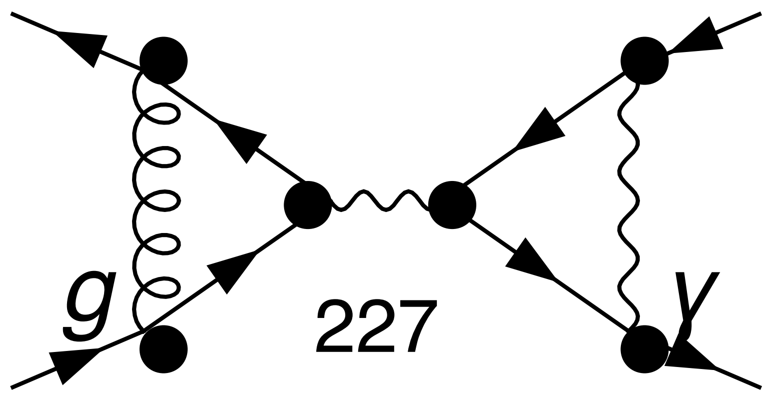




real-virtual contributions

amplitudes are easily generated with OpenLoops or Recola
1-loop UV renormalisation and IR subtraction
care about the numerical convergence when aiming at 0.1% precision





double-virtual contributions

generation of the amplitudes
γ_5 treatment
2-loop UV renormalization
solution and evaluation of the Master Integrals
subtraction of the IR divergences
numerical evaluation of the squared matrix element

General structure of the inclusive cross section and the q_T-subtraction formalism

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation

(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of final-state emitters (heavy quarks in QCD, leptons in EW)

(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

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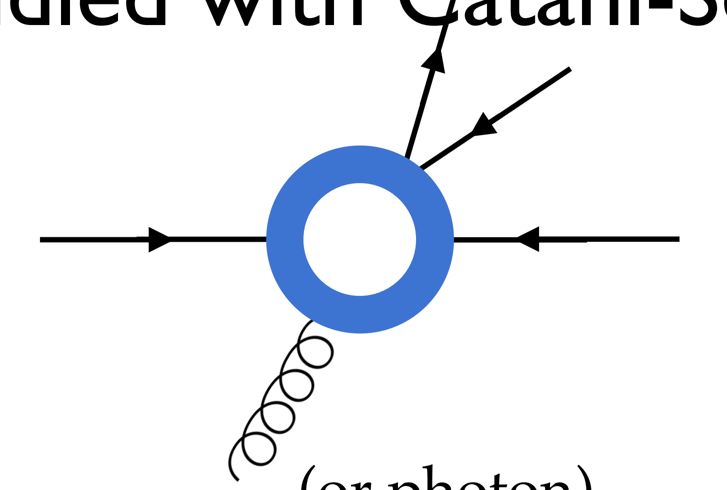
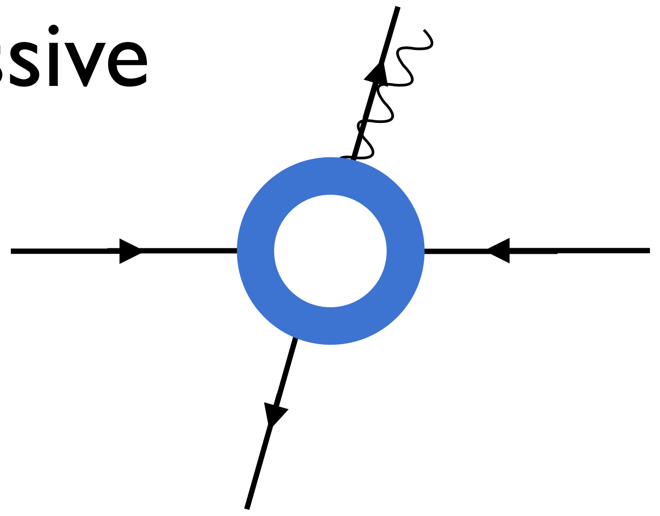
the gauge-boson phase space is split into $q_T = 0$ and $q_T > 0$ regions $\qquad r_{cut} = q_T^{cut}/Q$

for ISR, if $q_T > 0$ the emitted parton is always resolved
and the process under study receives only NLO corrections
which can be handled with Catani-Seymour dipoles


(or photon)

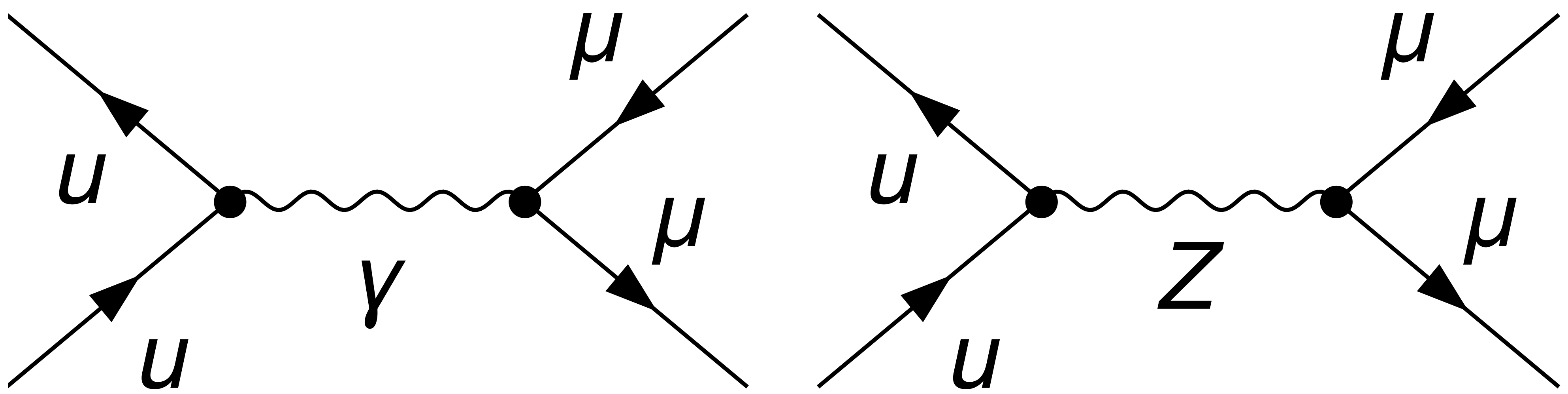
in the FSR case, with $q_T > 0$,
the emitted parton is always resolved
only if the emitter is massive



the final state consists of a pair of massive leptons (treated as bare) to regulate the collinear (mass) singularities

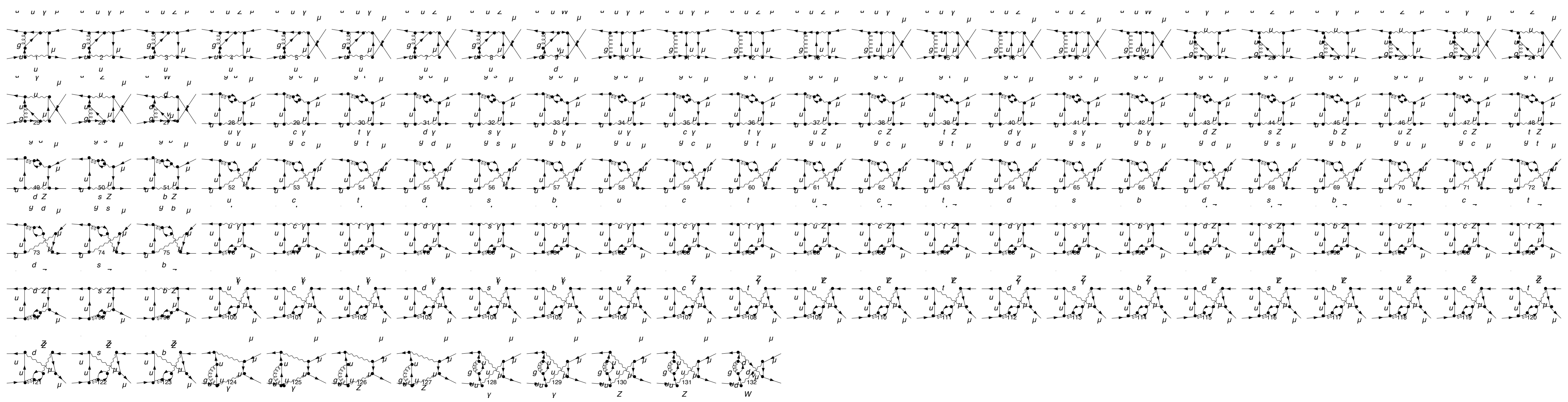
The double virtual amplitude: generation of the amplitude

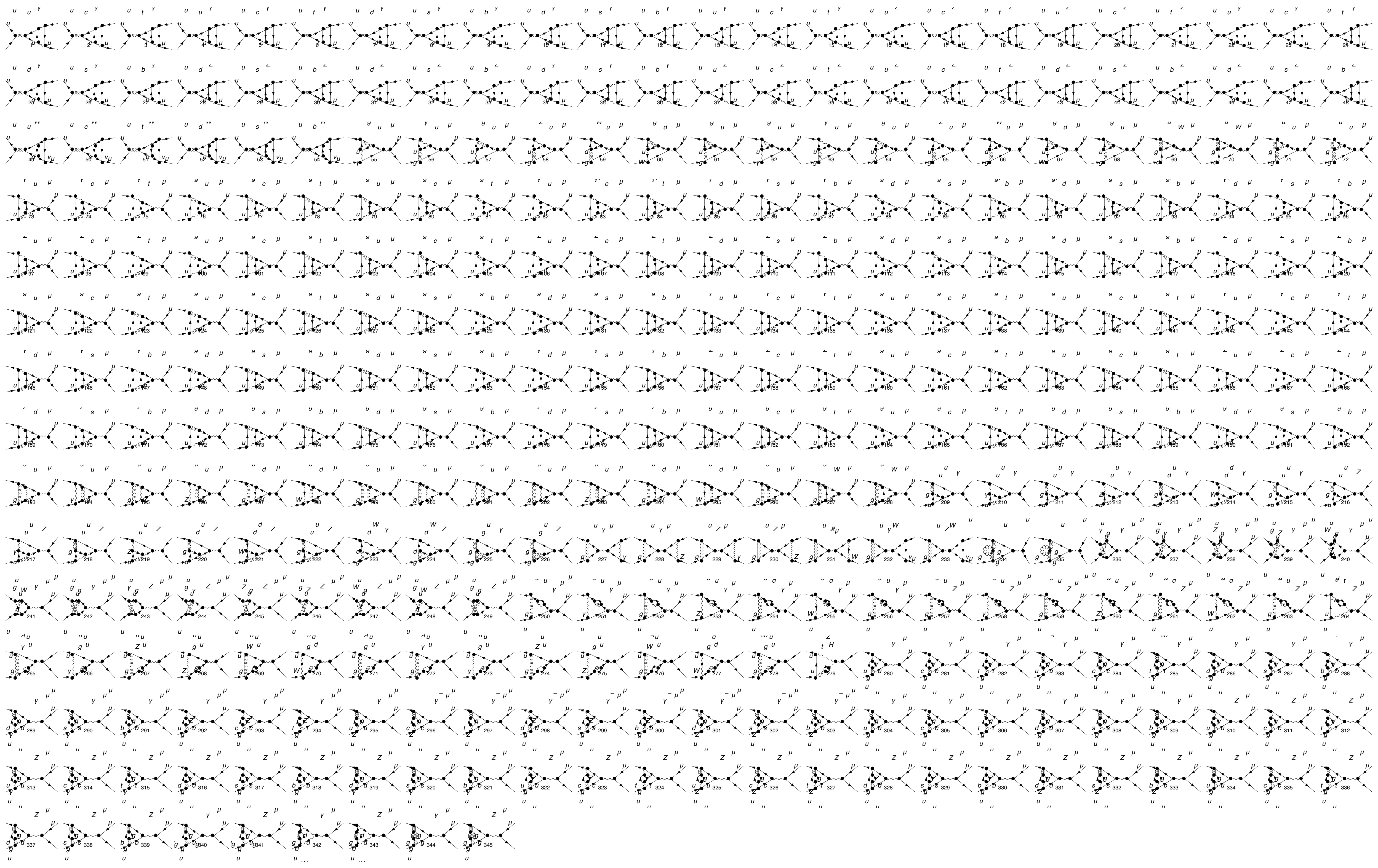
$$\mathcal{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$$



$$\mathcal{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$$

O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)





Structure of the double virtual amplitude

$$2\mathrm{Re}\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^{\dagger}\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathscr{I}_i(s,t,m;\varepsilon)$$

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The coefficients c_i are rational functions of the invariants, masses and of ε

Their size can rapidly "explode" in the GB range

→ careful work to identify the patterns of recurring subexpressions, keeping the total size in the O(1-10 MB) range

Abiss Mathematica package

Structure of the double virtual amplitude

$$2\mathrm{Re}\left(\mathcal{M}^{(1,1)}(\mathcal{M}^{(0,0)})^{\dagger}\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathcal{I}_i(s,t,m;\varepsilon)$$

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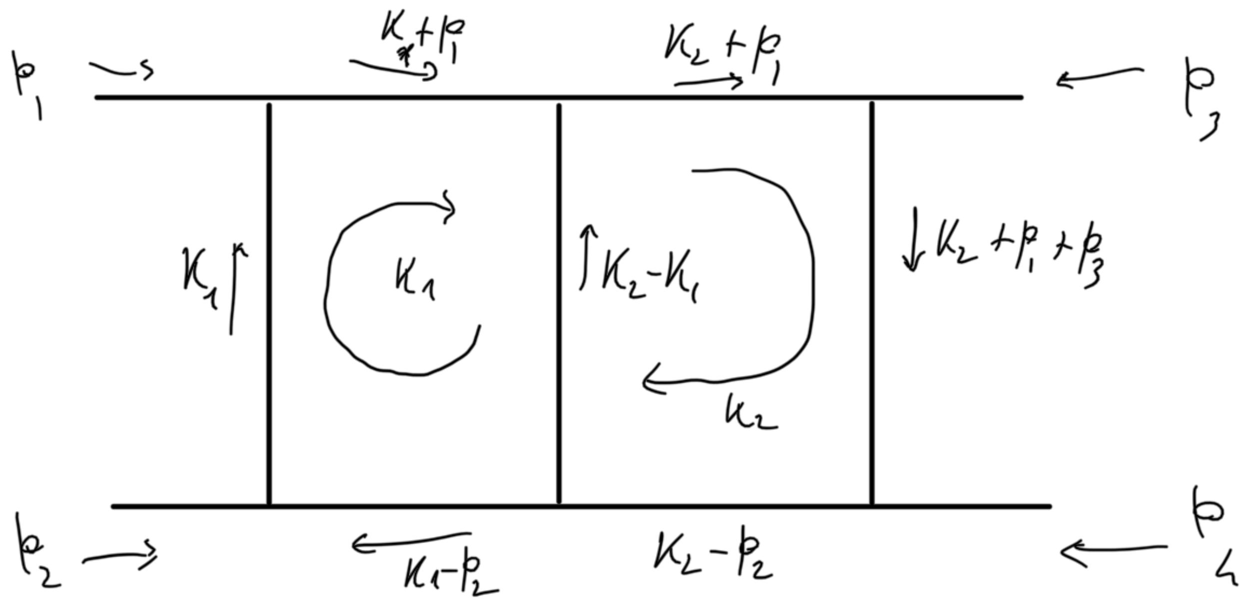
The Feynman Integrals $\mathcal{I}_i$ are one of the major challenges in the evaluation of the virtual corrections

$$\mathcal{I}(p_i\cdot p_j;\vec{m}) = \int\frac{d^nk_1}{(2\pi)^n}\int\frac{d^nk_2}{(2\pi)^n}\frac{1}{[k_1^2-m_0^2]^{\alpha_0}\,[(k_1+p_1)^2-m_1^2]^{\alpha_1}\ \ldots\ [(k_1+k_2+p_j)^2-m_j^2]^{\alpha_j}\ \ldots\ [(k_2+p_l)^2-m_l^2]^{\alpha_l}}$$



The complexity of the solution grows with the number of energy scales (masses and invariants) upon which it depends

The double virtual amplitude: reduction to Master Integrals

The complexity of the MIs depends on the number of energy scales
MIs relevant for the QCD-QED corrections, with massive final state

Bonciani, Ferroglia,Gehrmann, Maitre, Studerus., arXiv:0806.2301, 0906.3671

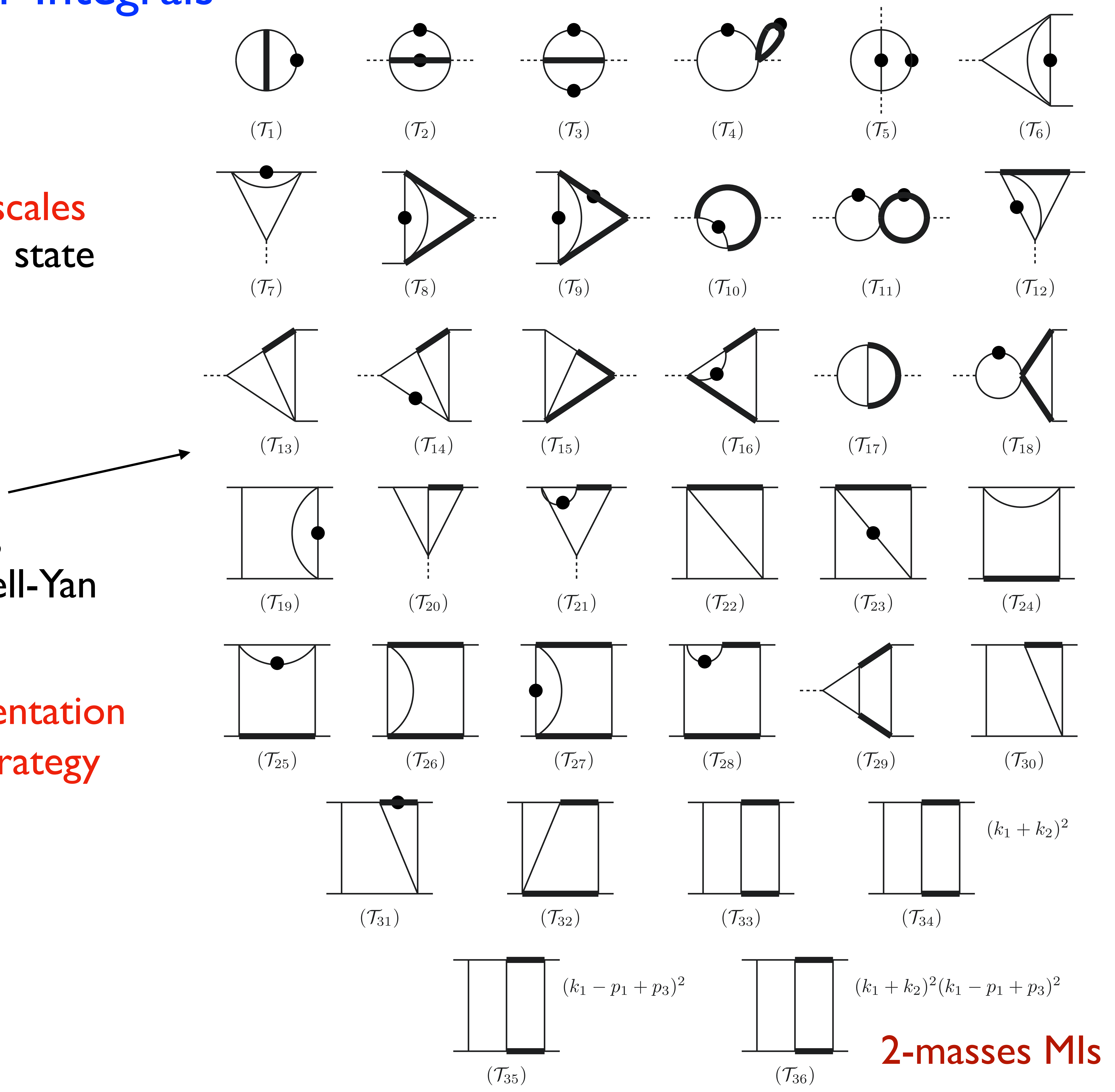
MIs with 1or 2 internal mass relevant for the EW form factor

Aglietti, Bonciani, hep-ph/0304028, hep-ph/0401193

31 MIs with 1 mass and 36 MIs with 2 masses including boxes,
relevant for the QCD-weak corrections to the full NC Drell-Yan

Bonciani, Di Vita, Mastrolia, Schubert., arXiv:1604.08581

In the 2-mass case, 5 box integrals in Chen-Goncharov representation
→ problematic numerical evaluation→ need an alternative strategy



cfr. also Heller, von Manteuffel, Schabinger,
arXiv:1907.00491 for a representation of the MIs in terms of GPLs
arXiv:2012.05918 for a description of the 2-loop virtual amplitude

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations.

The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.

The package `DiffExp` by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.

But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (`SeaSyde`)

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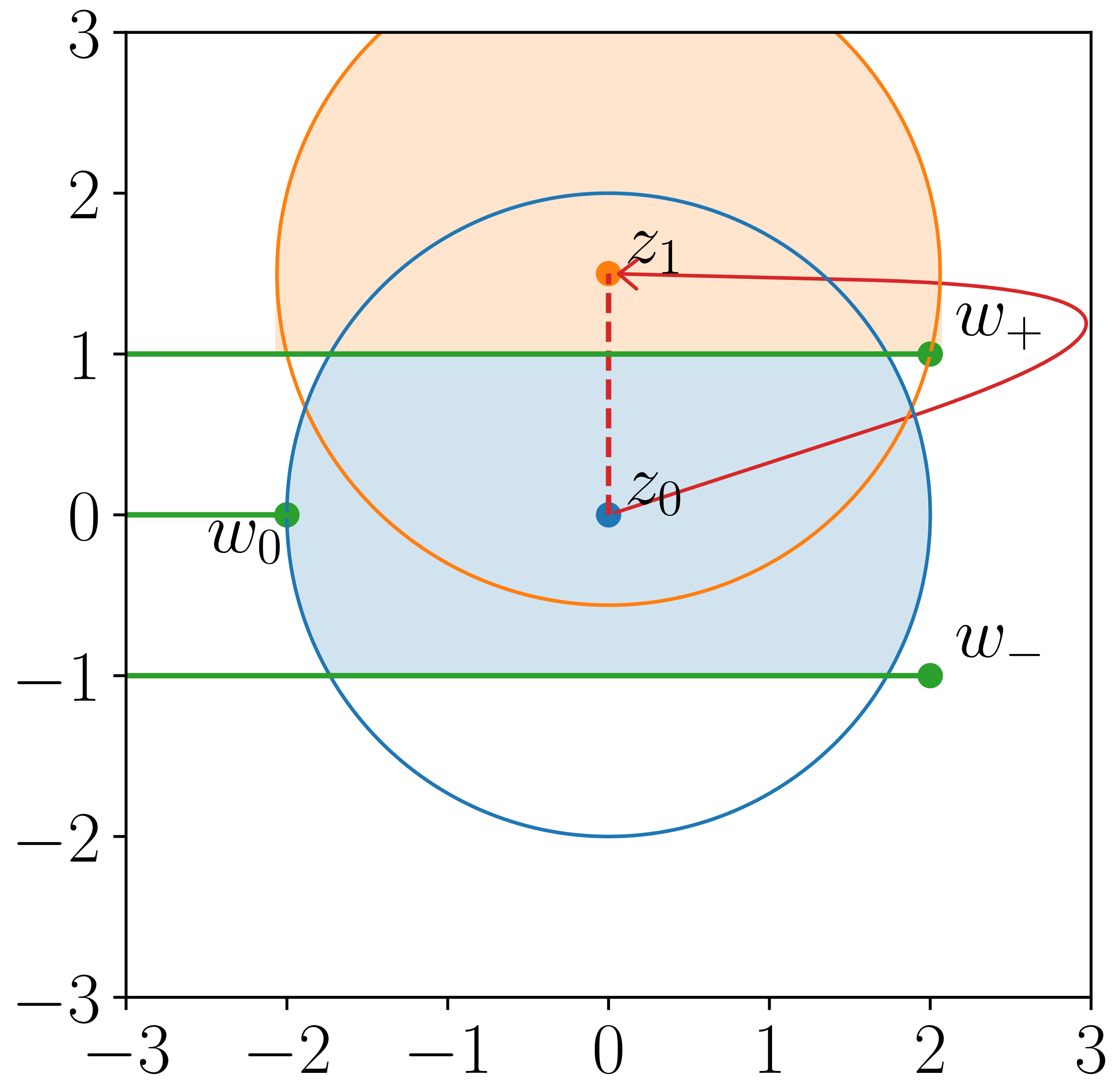
We implemented the series expansion approach, for arbitrary complex-valued masses,
working in the complex plane of each kinematical variable, one variable at a time

Complete knowledge about the singular structure of the MI
can be read directly from the differential equation matrix

The solution can be computed with an arbitrary number of significant digits,
but not in closed form → semi-analytical



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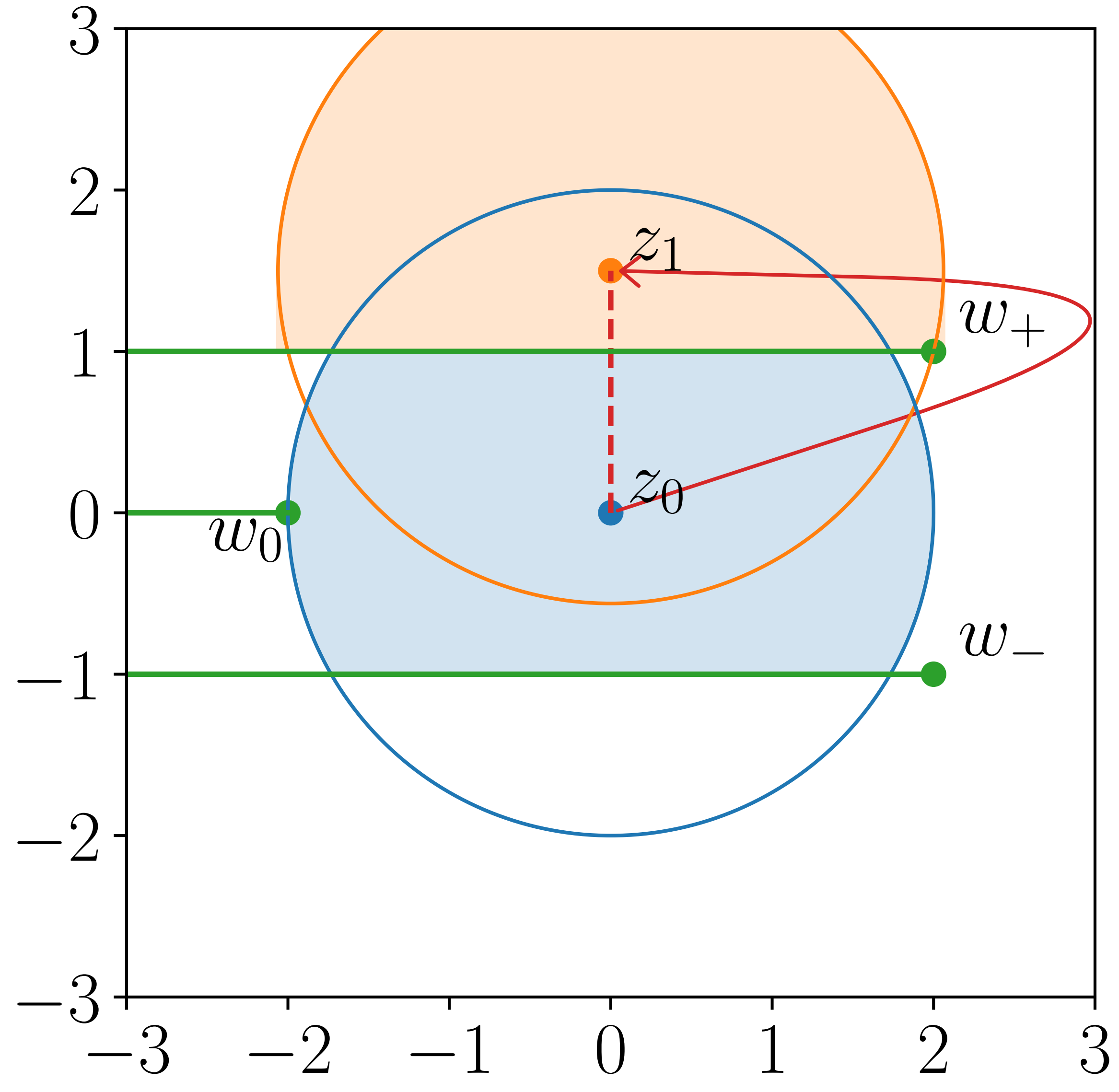


Numerical evaluation of the hard coefficient function

The interference term $2\mathrm{Re}\langle \mathscr{M}^{(1,1),fin} | \mathscr{M}^{(0,0)} \rangle$ contributes to the hard function $H^{(1,1)}$

After the subtraction of all the universal IR divergences, it is a finite correction

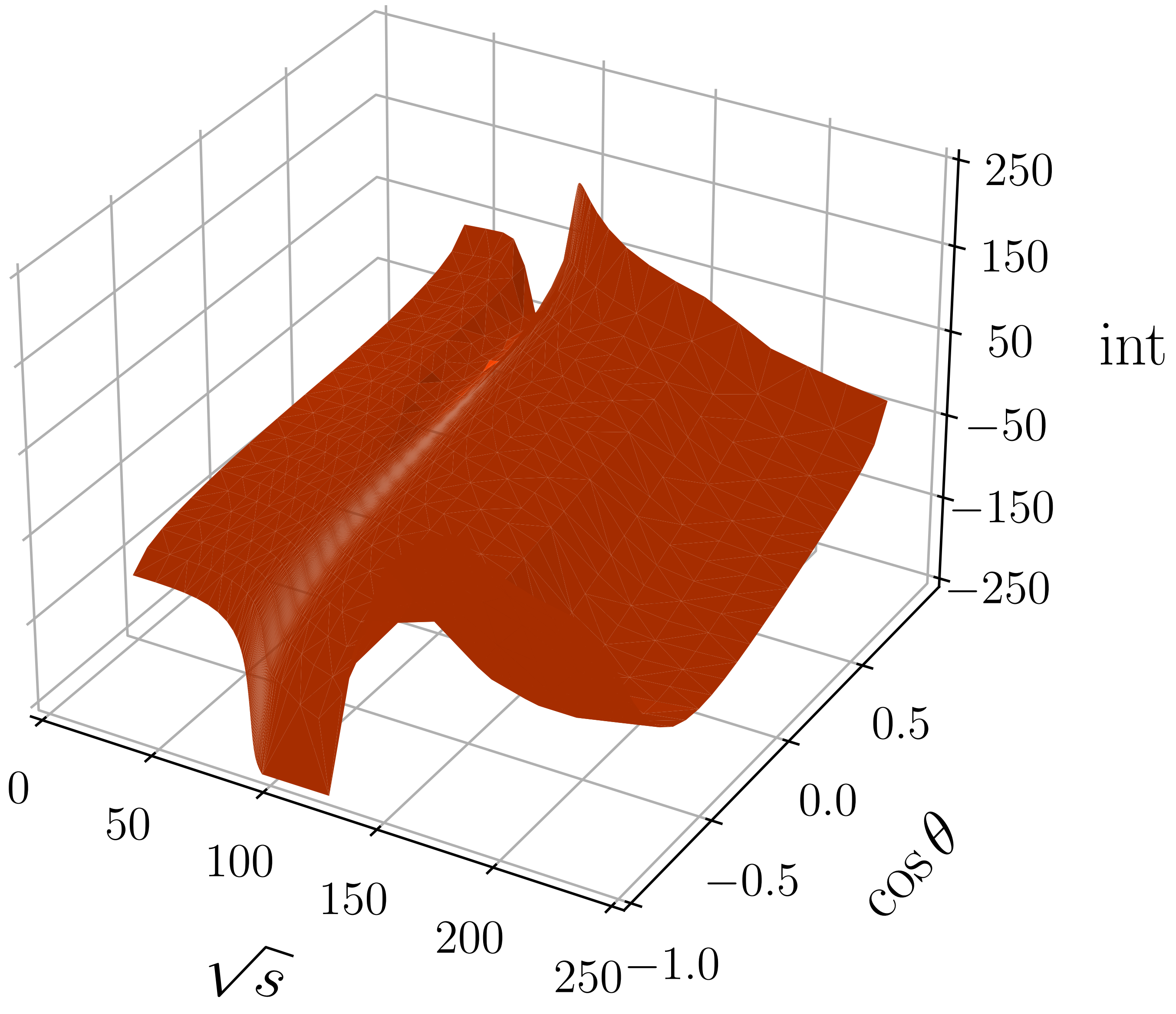
It has been published in arXiv:2201.01754 and is available as a Mathematica notebook

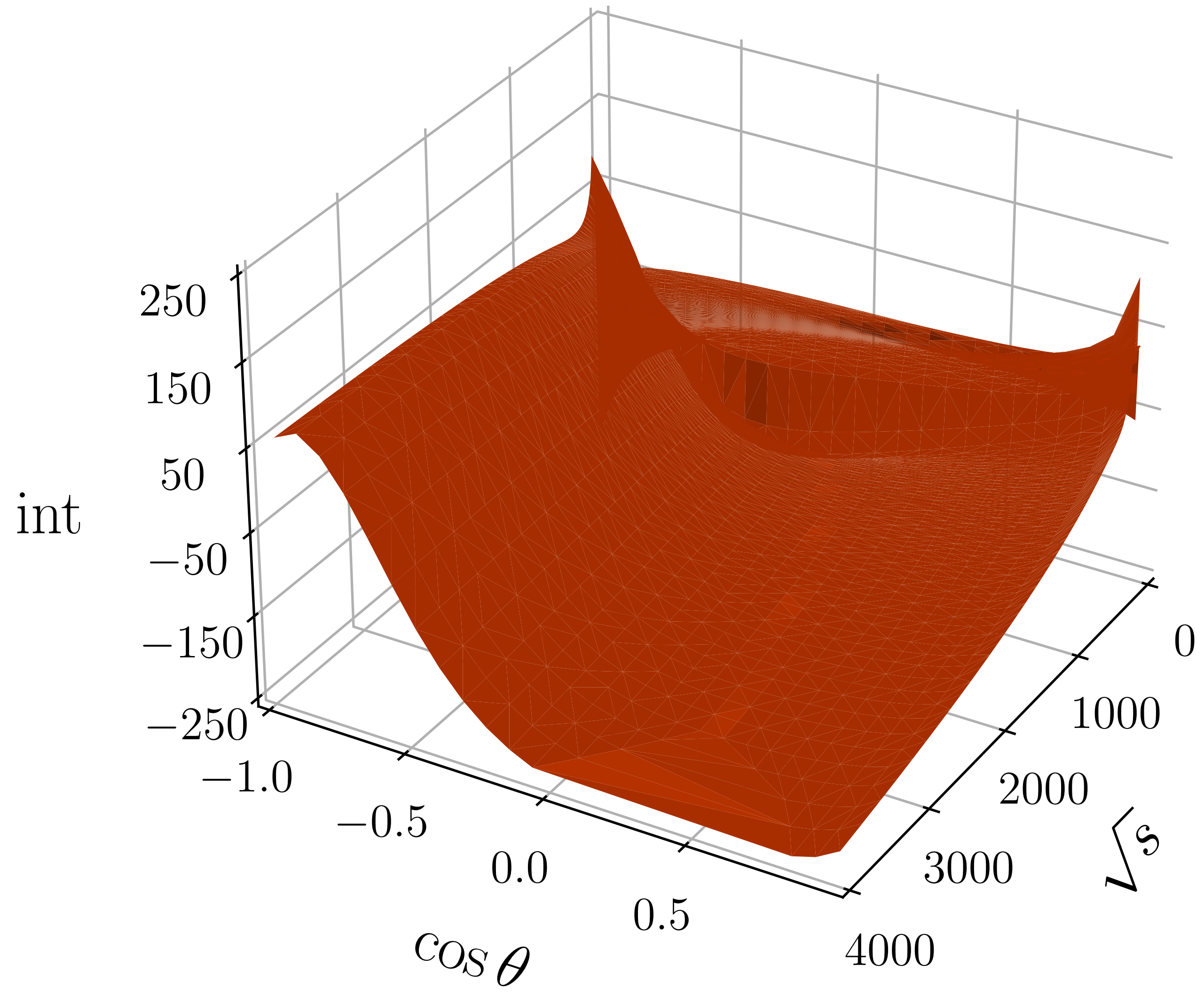
Several checks of the MIs performed with Fiesta, PySecDec and AMFlow

A numerical grid has been prepared for all the 36 MIs, with GiNaC and SeaSyde (T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345) , covering the whole $2 \to 2$ phase space in (s,t) (3250 points) , in O(12 h) on one 32-cores machine

→ values at arbitrary phase space points obtained with excellent accuracy via interpolation, with negligible evaluation time

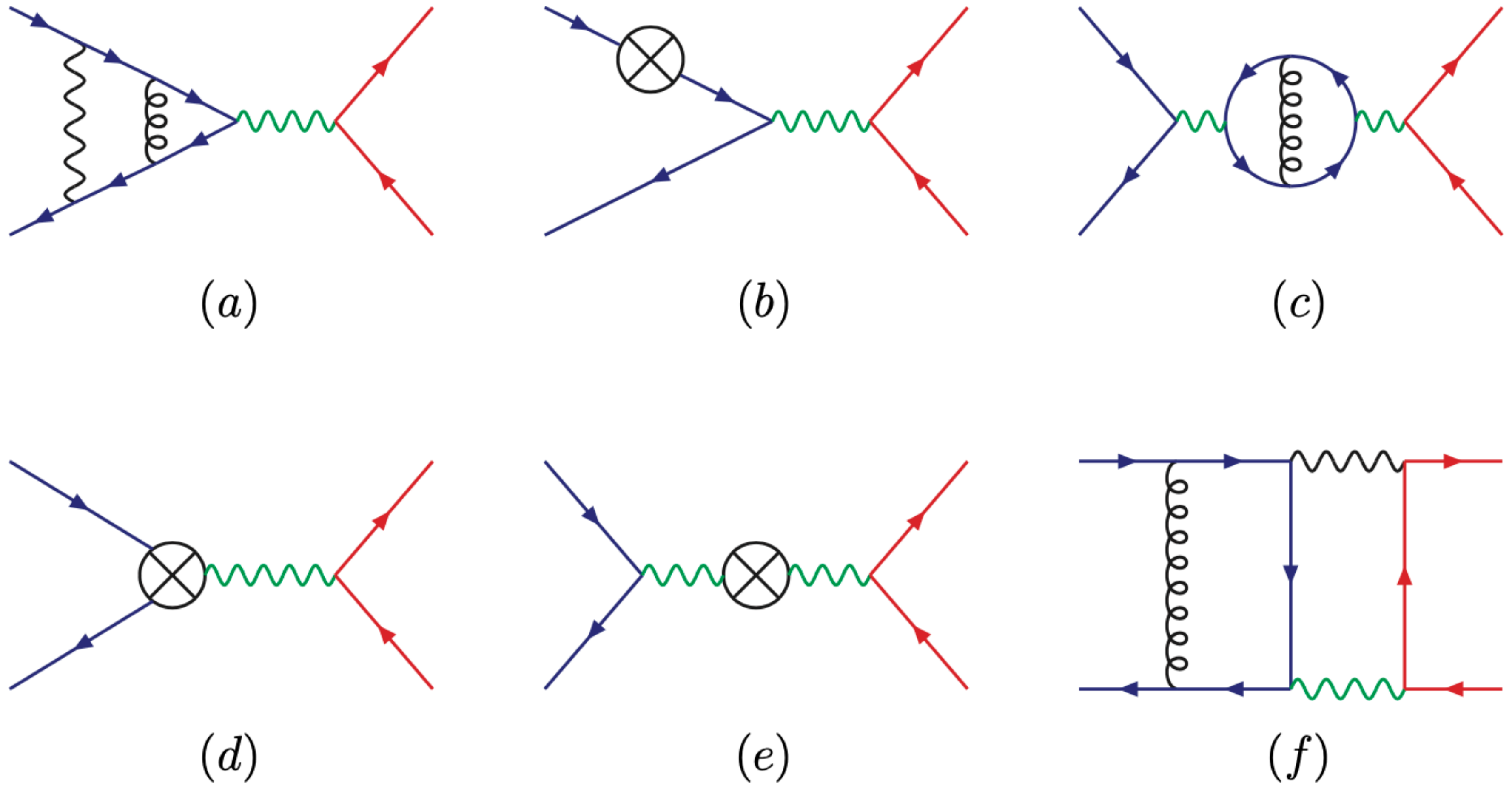
in units $\frac{\alpha}{\pi}\frac{\alpha_s}{\pi}\sigma_0$

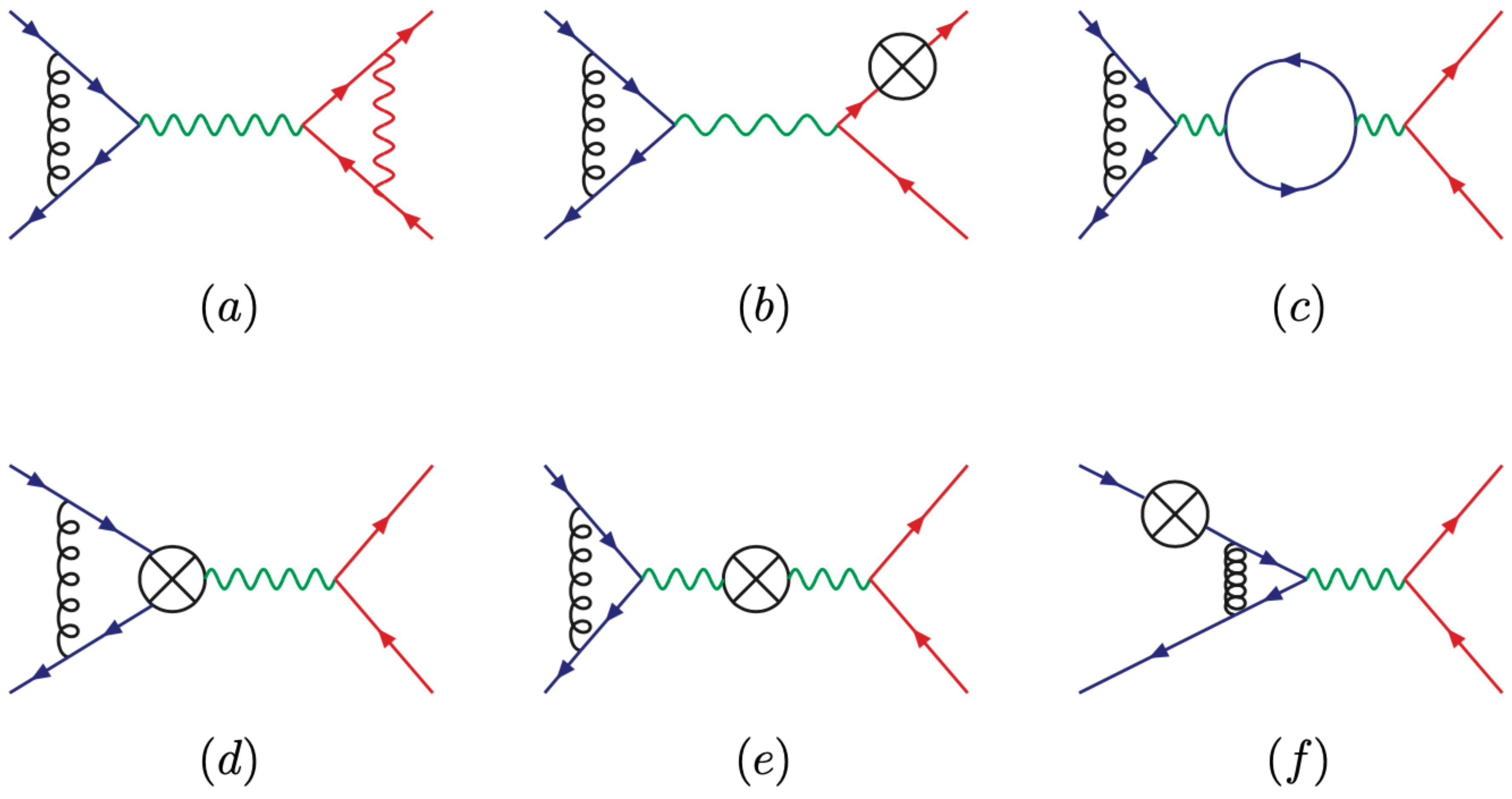




Exact 2-loop virtual QCD-EW corrections to Charged-Current Drell-Yan

2-loop virtual QCD-EW corrections to the Charged-Current Drell-Yan in the SM





The Charged-Current process is mediated by a *W* exchange

For a general lepton-pair invariant mass,
there is no general gauge invariant separation
of initial- and final-state photonic corrections,
at variance with the NC DY case

We consider a massive final-state lepton,
yielding mass logarithms instead of collinear poles in dim.reg.

The presence of two weak bosons with different masses (*W* and *Z*)
is a new challenge for the solution of the Feynman integrals

Large number of terms → increased automation level

Subtraction of the IR divergences from the 2-loop amplitude

we identify QCD-QED (poles up to $1/\varepsilon^4$) and QCD-weak (poles up to $1/\varepsilon^2$ with cumbersome coefficients) diagrams

$$|\mathcal{M}^{(1,0),fin}\rangle = |\mathcal{M}^{(1,0)}\rangle - \mathcal{I}^{(1,0)}|\mathcal{M}^{(0)}\rangle\,,$$

standard NLO-QCD subtraction

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle - \mathcal{I}^{(0,1)}|\mathcal{M}^{(0)}\rangle\,.$$

NLO-EW subtraction, with massive leptons

$$|\mathcal{M}^{(1,1),fin}\rangle = |\mathcal{M}^{(1,1)}\rangle - \mathcal{I}^{(1,1)}|\mathcal{M}^{(0)}\rangle - \tilde{\mathcal{I}}^{(0,1)}|\mathcal{M}^{(1,0),fin}\rangle - \tilde{\mathcal{I}}^{(1,0)}|\mathcal{M}^{(0,1),fin}\rangle\,.$$

$$\mathcal{I}^{(1,0)} = \left(\frac{s}{\mu^2}\right)^{-\epsilon} C_F \left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3+2i\pi) + \zeta_2\right),$$

$$\mathcal{I}^{(0,1)} = \left(\frac{s}{\mu^2}\right)^{-\epsilon} \left[\frac{Q_u^2+Q_d^2}{2}\left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3+2i\pi) + \zeta_2\right) + \frac{4}{\epsilon}\Gamma_l^{(0,1)}\right]$$

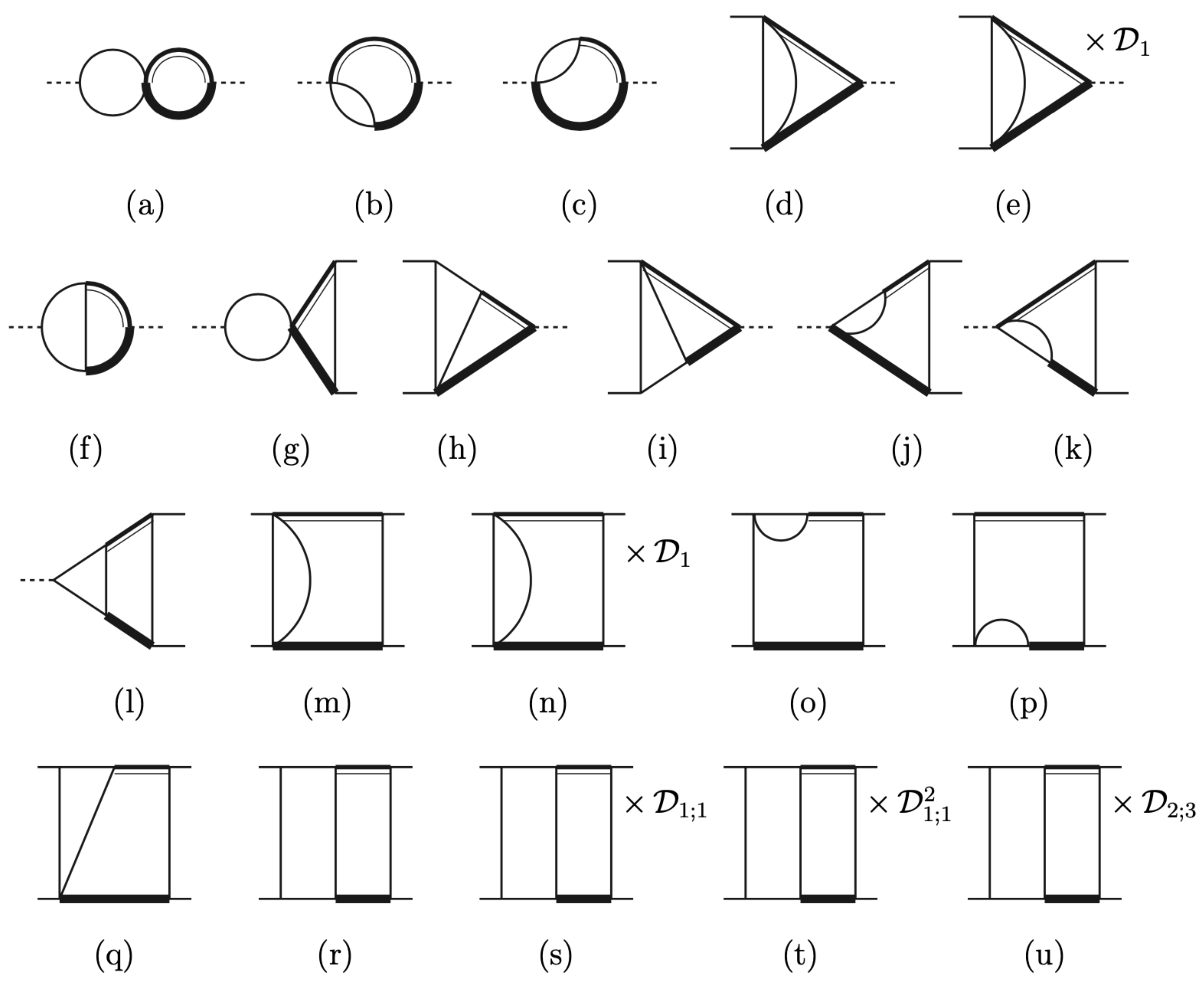
$$\Gamma_l^{(0,1)} = -\frac{1}{4}\Big[Q_l^2\,(1-i\pi) + Q_l^2 \log\left(\frac{m_l^2}{s}\right) + \\ + 2Q_uQ_l\log\left(\frac{(2p_1\cdot p_4)}{s}\right) - 2Q_dQ_l\log\left(\frac{(2p_2\cdot p_4)}{s}\right)\Big]$$

$$\mathcal{I}^{(1,1)} = \left(\frac{s}{\mu^2}\right)^{-2\epsilon} C_F\Big[\frac{Q_u^2+Q_d^2}{2}\left(\frac{4}{\epsilon^4} + \frac{1}{\epsilon^3}(12+8i\pi) + \frac{1}{\epsilon^2}(9-28\zeta_2+12i\pi) ++ \frac{1}{\epsilon}\left(-\frac{3}{2}+6\zeta_2-24\zeta_3-4i\pi\zeta_2\right)\right) \\ + \left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3+2i\pi)+\zeta_2\right)\frac{4}{\epsilon}\,\Gamma_l^{(0,1)}\Big]$$

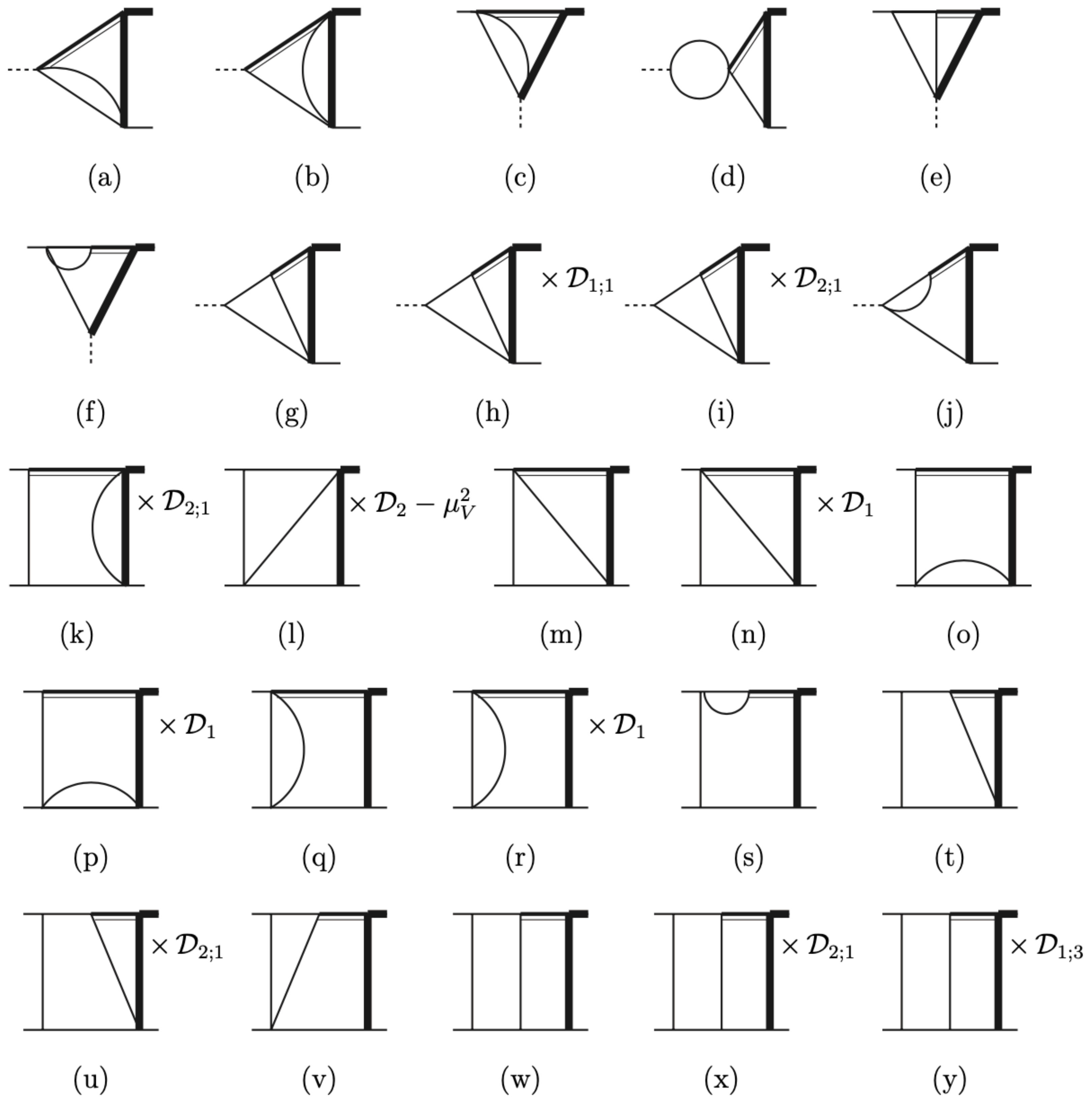
The analytical check of the cancellation of the IR poles in the QCD-weak sector is one very demanding test of the calculation.

In CC-DY for the first time we achieved a completely numerical check of the cancellation of all the IR poles

2-loop virtual QCD-EW corrections to CC DY: new Master Integrals



Master Integrals with two different internal masses

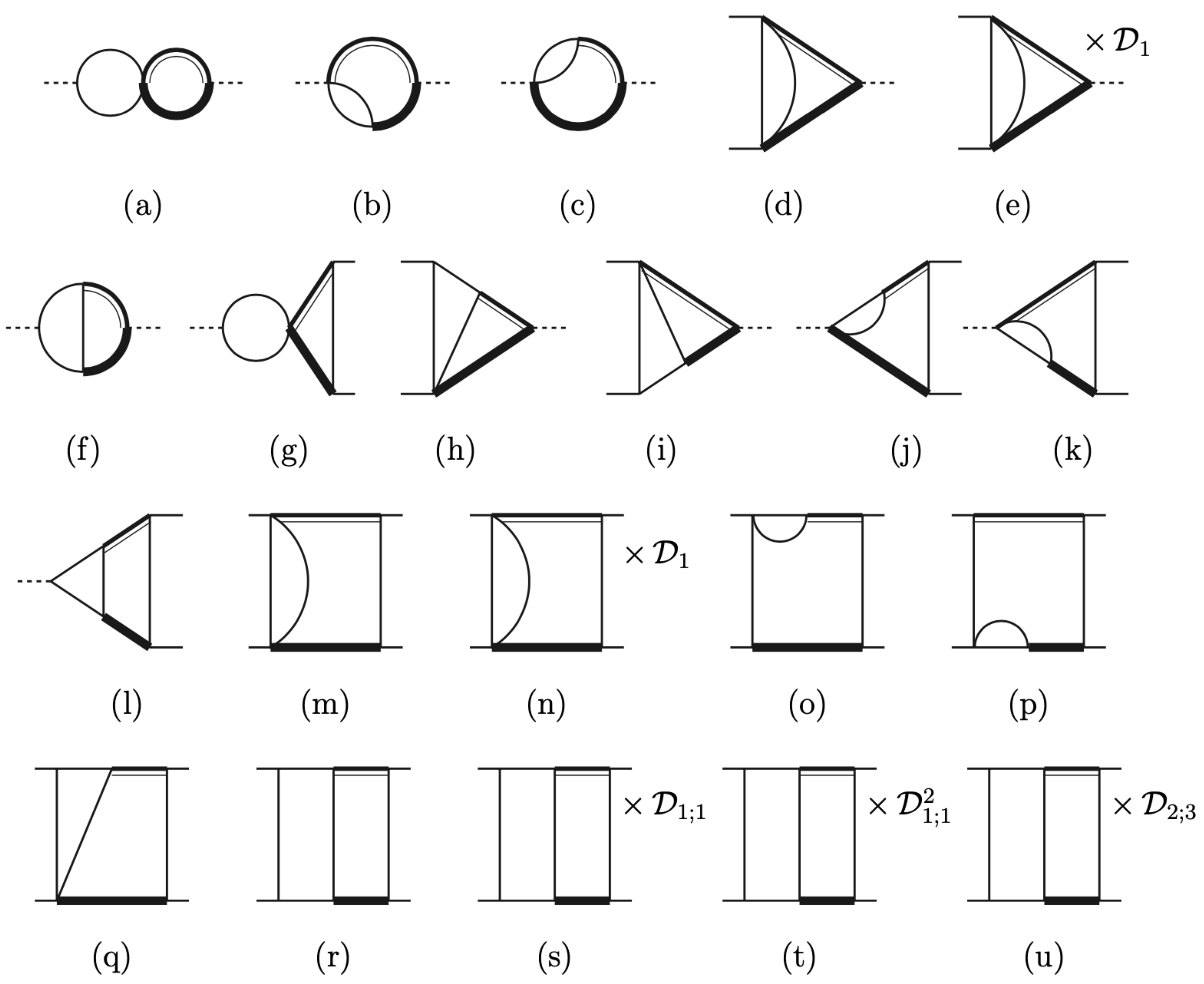


Master Integrals with
one W and one internal massive lepton lines

Automated workflow

- All the terms in the amplitude are reduced to Master Integrals with Abiss+KIRA
- The differential equations are written with LiteRed
- The Boundary Conditions are computed with AMFlow
- The Master Integrals are computed with SeaSyde

2-loop virtual QCD-EW corrections to CC DY: new Master Integrals



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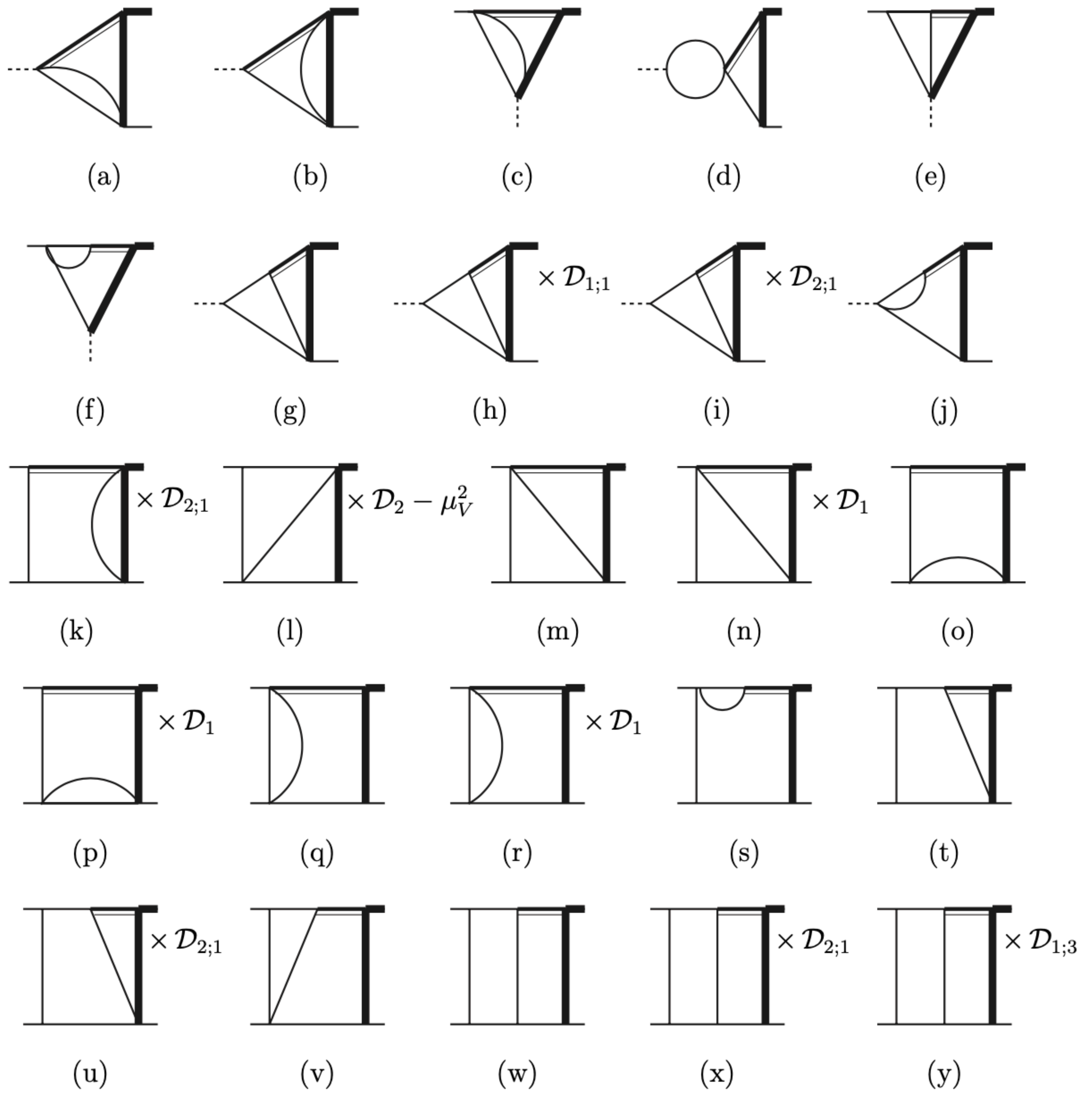

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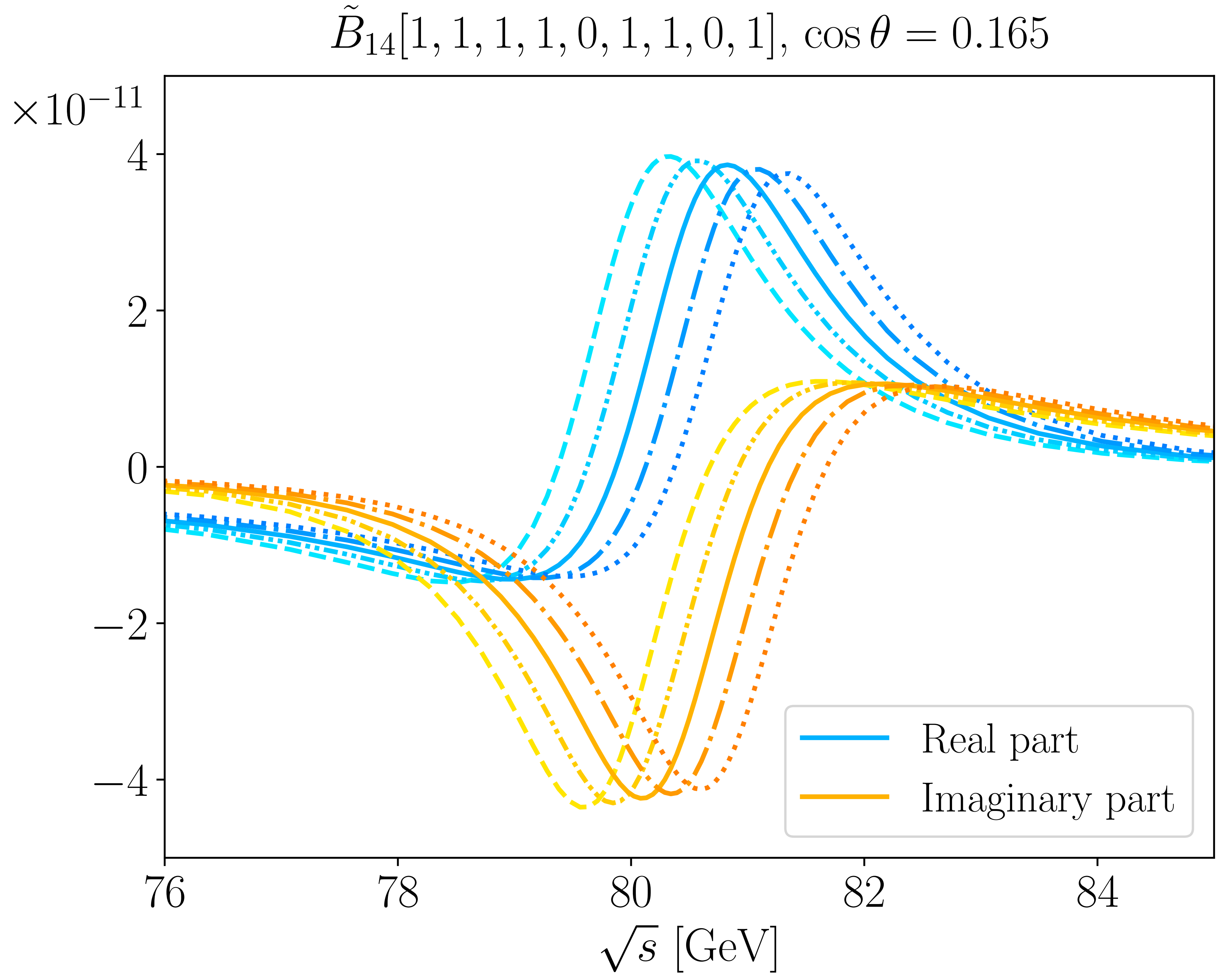
useful to tackle NNLO-EW corrections
→ relevant at LHC and later at FCC-ee

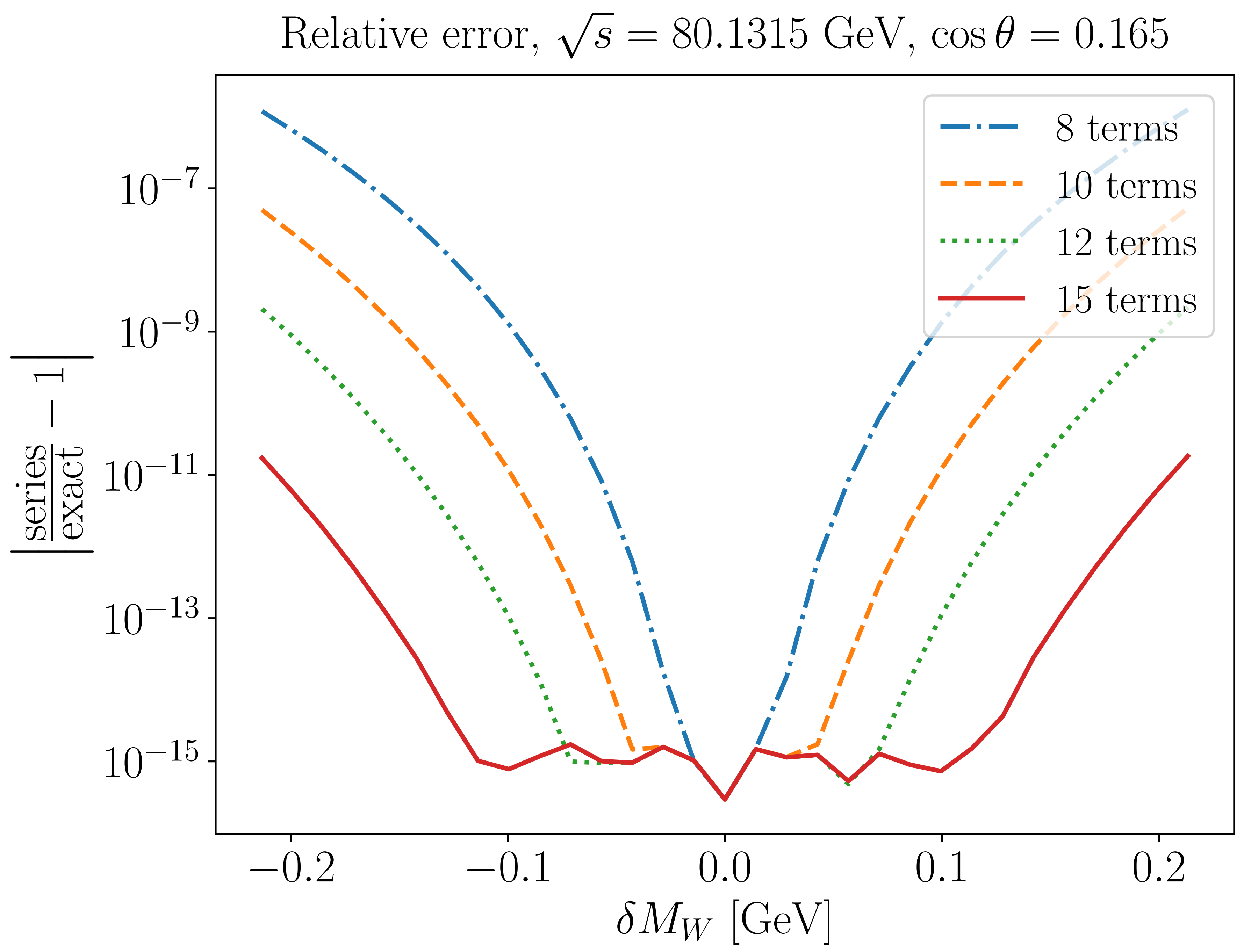
Fast numerical evaluation with arbitrary W-mass values

The Master Integrals can be solved at different (s, t) values, yielding a numerical grid, for a given value $\overline{m}_W$ of the W boson mass.
→ very efficient and accurate in Monte Carlo simulations

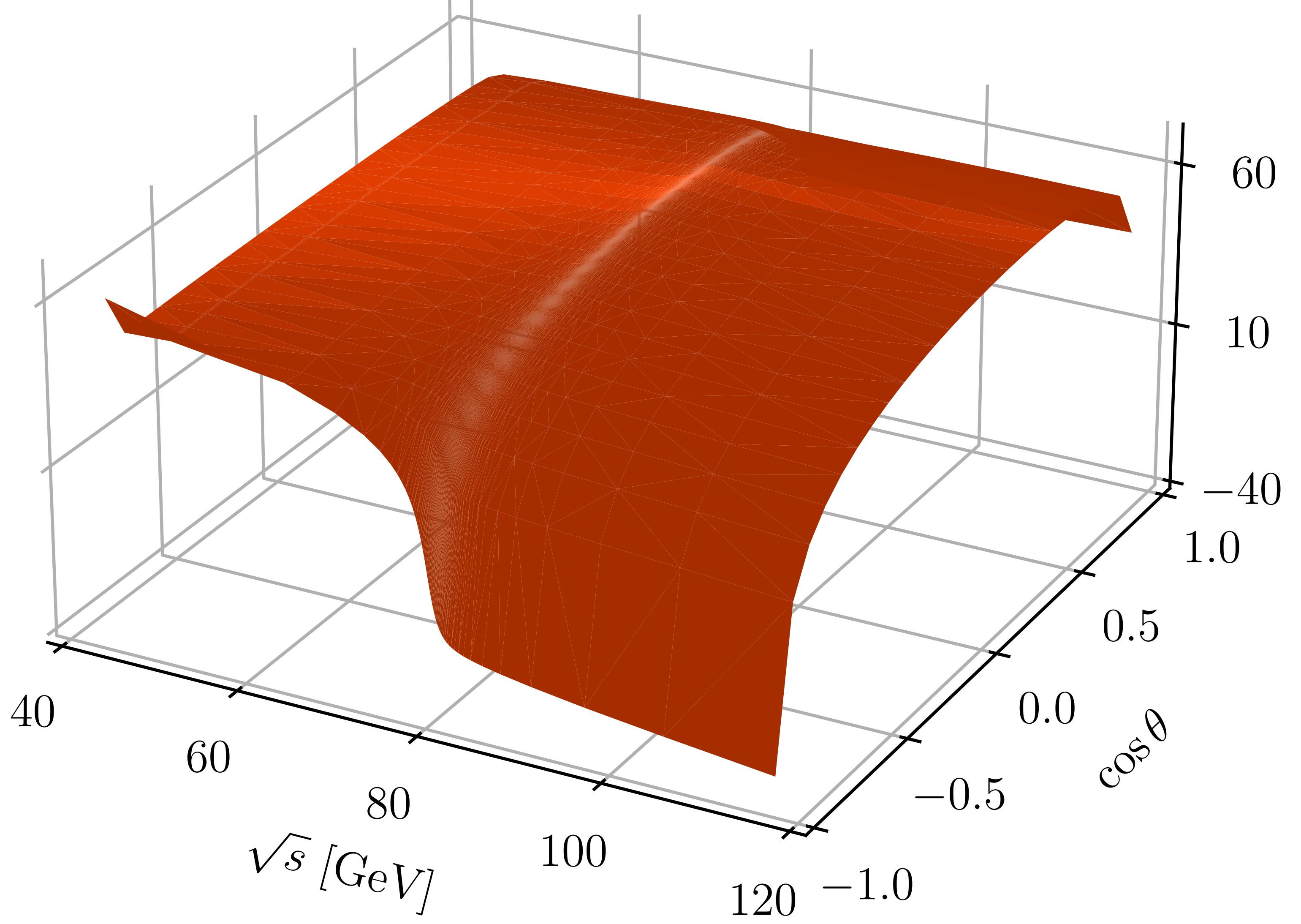
The differential equations with respect to the internal W mass
can be solved via the series expansion approach, yielding as a solution a power series in $\delta m_W = m_W - \overline{m}_W$,
taking as BCs the first grid with $\overline{m}_W$.

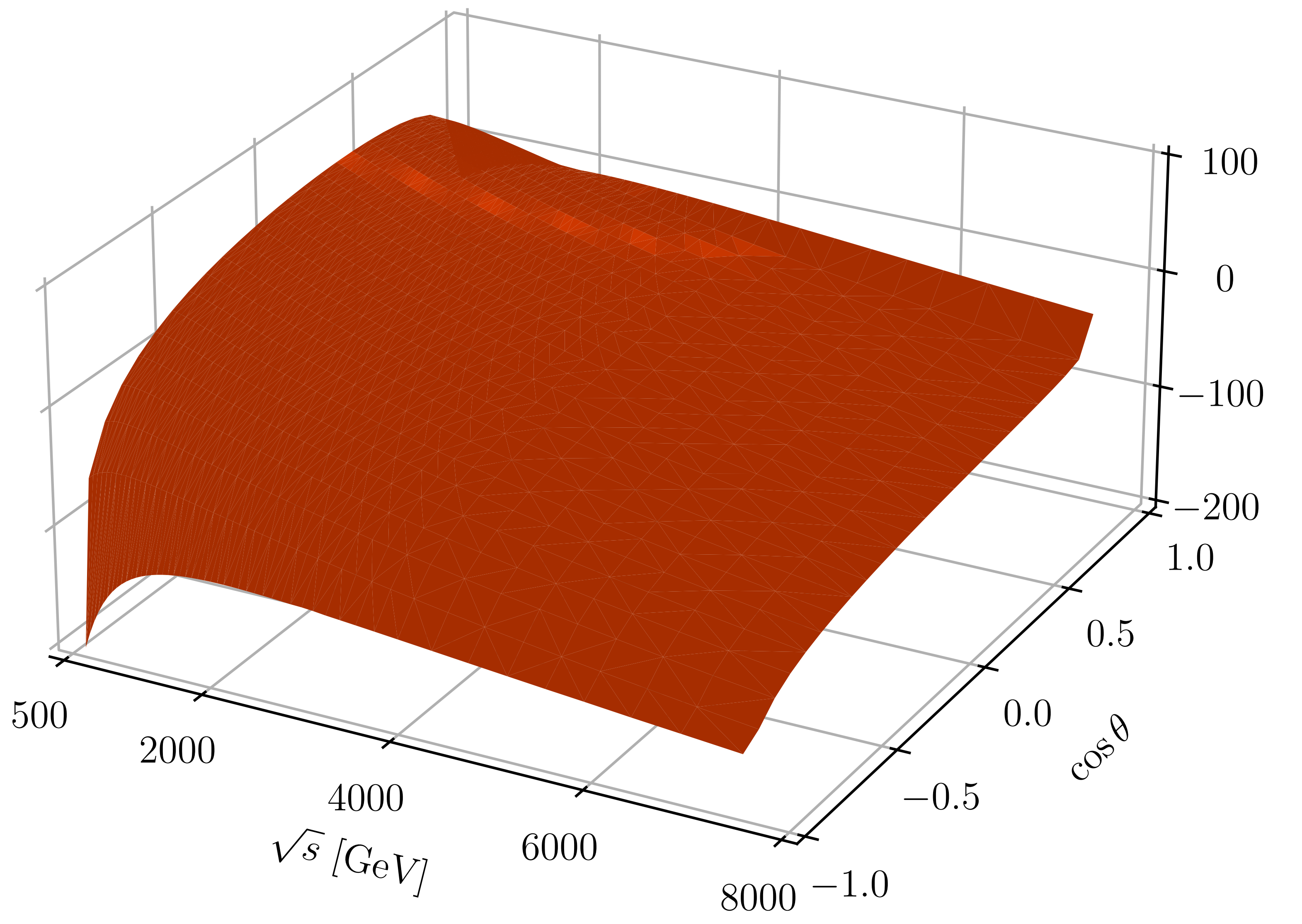
Our final 2-loop virtual result is cast, at every phase-space point, as a power series in δm_W,
which can be evaluated in a negligible amount of time, to give the actual grid, for any m_W choice





Finite 2-loop exact QCD-EW virtual corrections to Charged-Current Drell-Yan

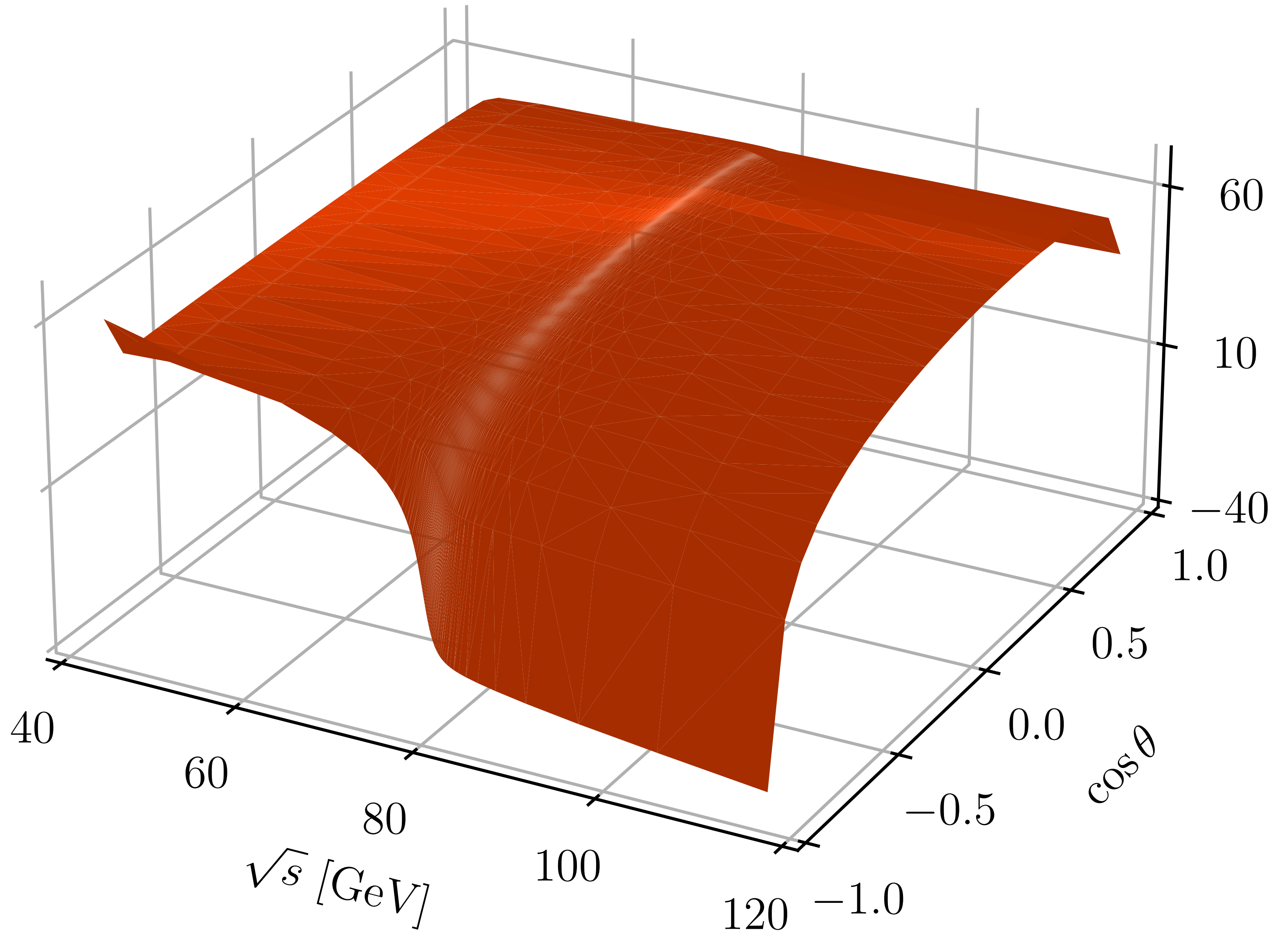


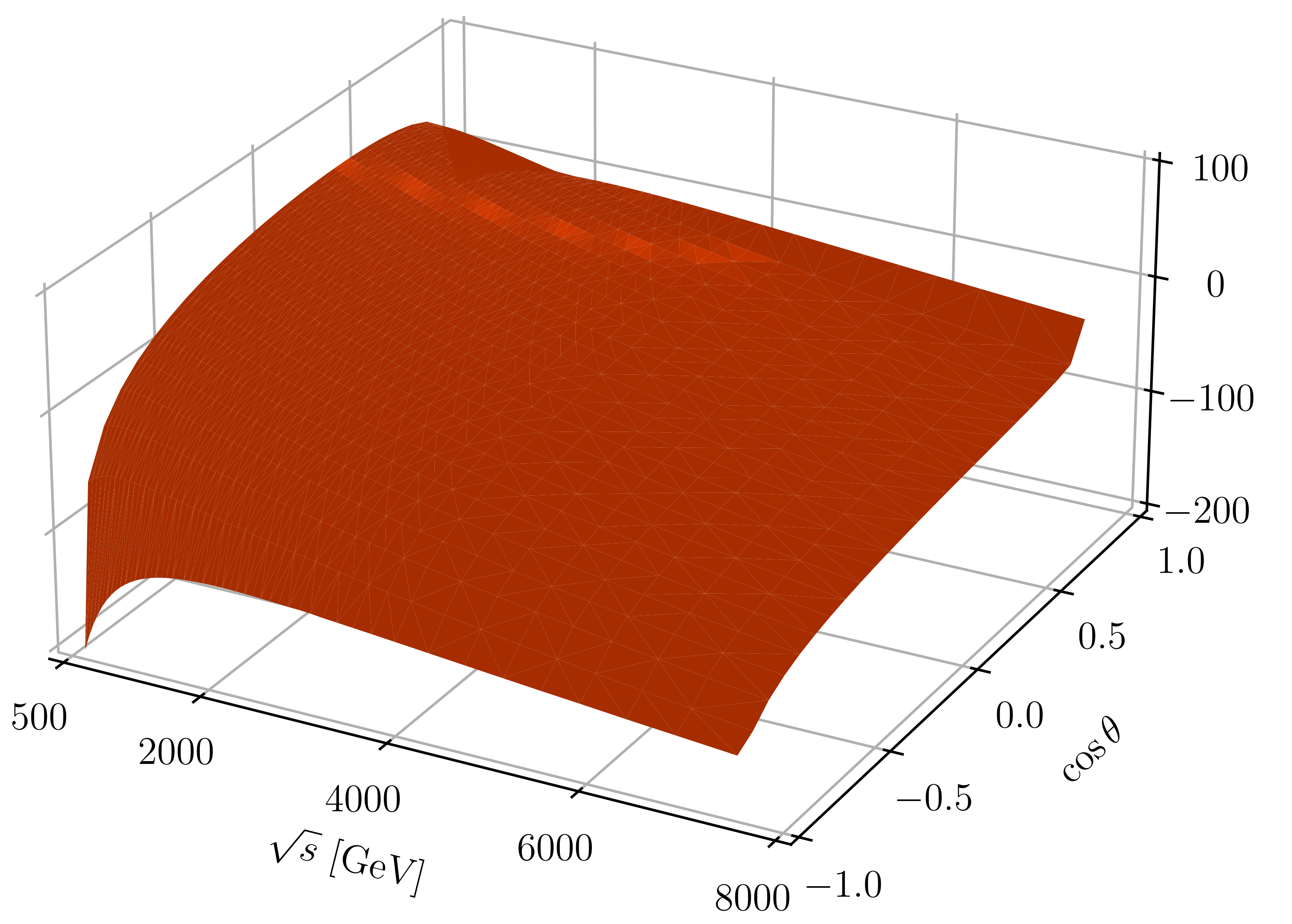


in units $\frac{\alpha}{\pi}\frac{\alpha_s}{\pi}\sigma_0$

- Expected large effects at large transverse masses, analogously to the NC DY case
- Improved theoretical stability in PDFs determination at (sub)percent level
- Relevance in the discussion of the W resonance region, when matching fixed-order and QCD-QED resummation → m_W fit

Finite 2-loop exact QCD-EW virtual corrections to Charged-Current Drell-Yan





in units $\frac{\alpha}{\pi}\frac{\alpha_s}{\pi}\sigma_0$

- Expected large effects at large transverse masses, analogously to the NC DY case
- Improved theoretical stability in PDFs determination at (sub)percent level
- Relevance in the discussion of the W resonance region, when matching fixed-order and QCD-QED resummation → m_W fit

- In the evaluation of the corrections to CC DY we have not optimised the choice of the Master Integrals
  → the diff.eqs. systems are not triangular (like in the NC DY case) but they are generic coupled systems

  SeaSyde is able to handle such systems, achieving a relative precision of 10^{-14} (or higher) at every phase-space point

  Potential limitations: the size of the diff.eqs. system can lead to long evaluation time
  Computing the full CC DY grid for LHC applications (3250 points in (s,t)) requires 3 weeks on one 26-core machine

Phenomenological impact

Towards a tool matching QCD+QED resummation with NNLO QCD-EW fixed order

- The exact NNLO QCD-EW corrections yield large effects at large transverse/invariant masses → BSM searches
- m_W determination (see M.Boonekamp's talk)

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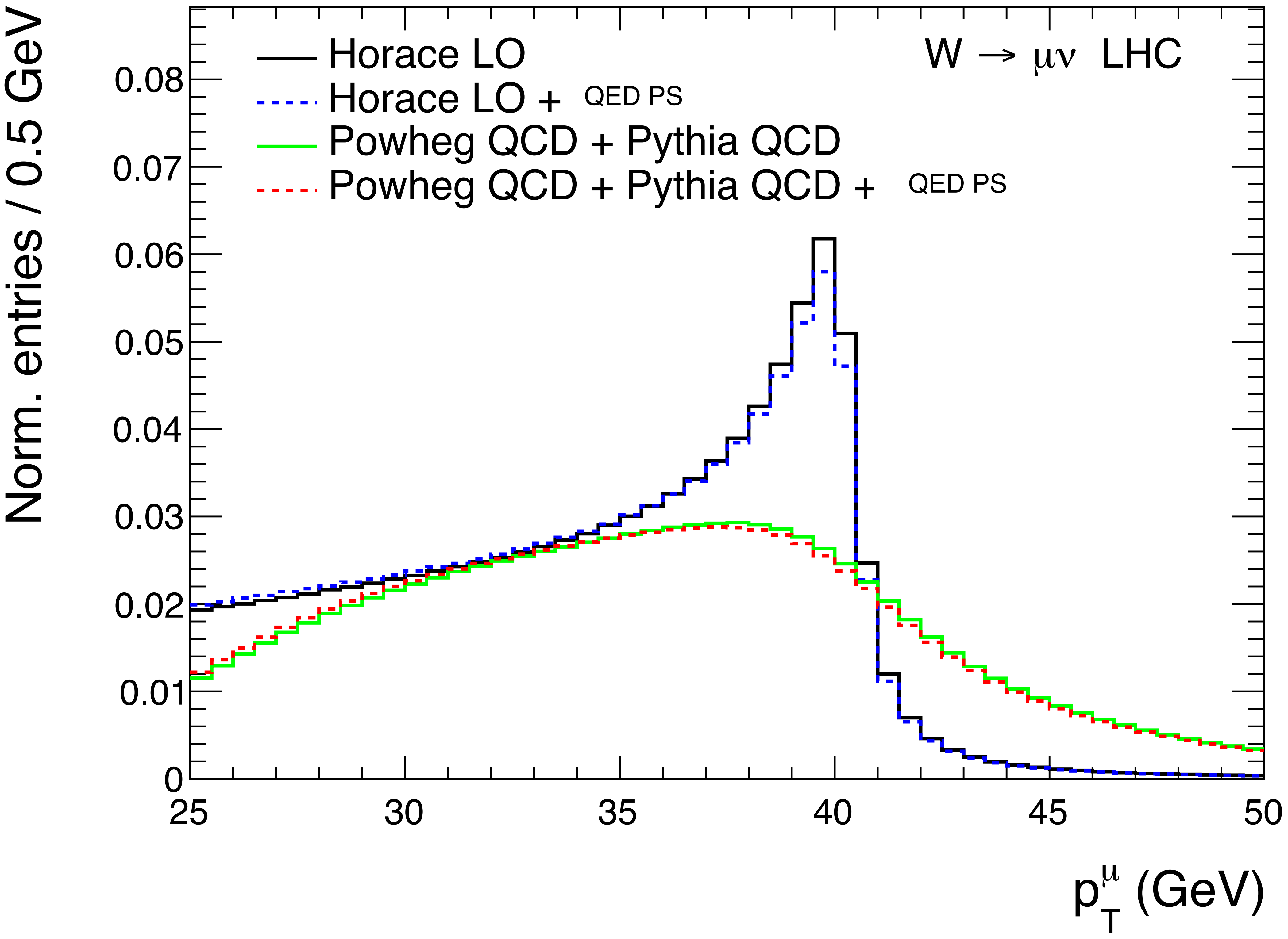

POWHEG simulation NLO QCD+EW +QCDPS + QEDPS

	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{\text{PS}}$ Pseudodata accuracy	QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{\text{PS}}$	Pythia	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{\text{PS}}$	Photos	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{\text{PS}}$two-rad	Pythia	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{\text{PS}}$two-rad	Photos	-88.6±0.6	-370±3	-39.2±0.6	-159±2

Huge impact of QED and mixed QCD-QED corrections in the m_W determination

What is the theoretical uncertainty on this estimated shift ? e.g. what would be the difference using POWHEG vs MC@NLO ?

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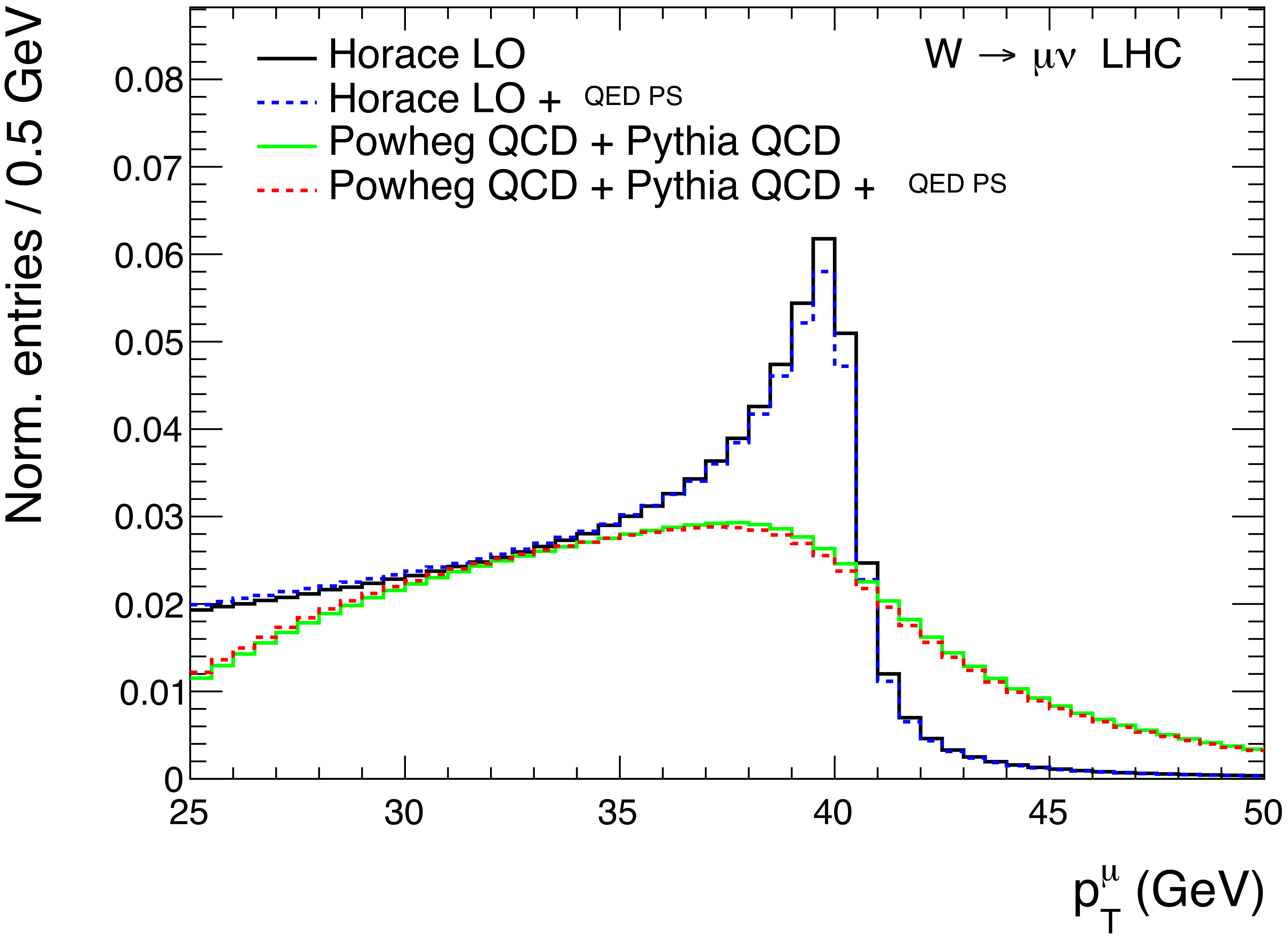

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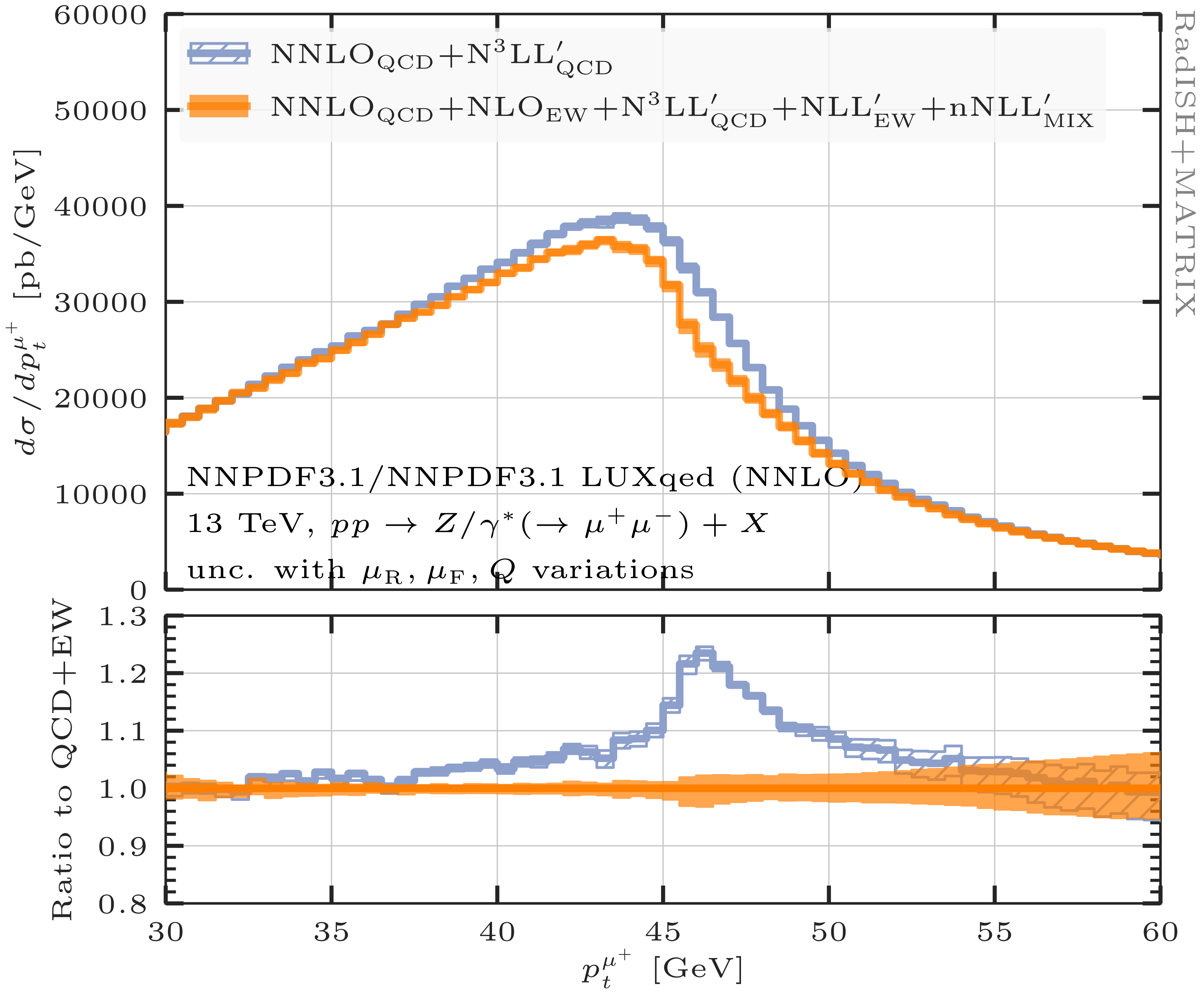
Huge impact of QED and mixed QCD-QED corrections in the m_W determination

What is the theoretical uncertainty on this estimated shift ? e.g. what would be the difference using POWHEG vs MC@NLO ?

with NNLO QCD-EW results we can fix the dominant source of ambiguity

Towards a tool matching QCD+QED resummation with NNLO QCD-EW fixed order

L.Buonocore, L.Rottoli, P.Torrielli, arXiv:2404.15112



Joined QCD-QED resummation in the RadIsh formulation at N3LL'-QCD + NLL'-EW + nNLL'-mixed accuracy including QED effects from all charged legs

Non-trivial interplay of QCD and EW corrections

Missing final step : Matching with the exact $\mathcal{O}(\alpha\alpha_s)$ corrections needed to reach full NNLL-mixed

→ Reliable estimate of the reduced residual theoretical uncertainties

Conclusions

Precision

- The NNLO (QCD + QCDxEW + EW) corrections are needed to match the final HL-LHC precision

  Steady progress is pushing the frontier of NNLO calculations from QCD-EW to full EW

  These results will be the core of the calculations needed at the FCC-ee
  to describe fermion-pair production in the whole energy range

The Standard Model benchmark

- The availability of these corrections will establish the SM benchmark with precision comparable to the data

  → increase the significance of an observed deviation, as a function of energy → relevant to SMEFT studies

The determination of the lagrangian parameter

- For example, the extraction of $\sin^2\hat{\theta}(\mu_R^2)$ at high-masses shows the LHC potential
  but also the potential biases induced by neglecting SM higher-order effects

  → any BSM study must be done on top of the best SM results (NNLO-EW?) to avoid fake conclusions

Thank you

Towards the NNLO-EW corrections to $\sigma(f\bar{f} \to \mu^+\mu^- + X)$

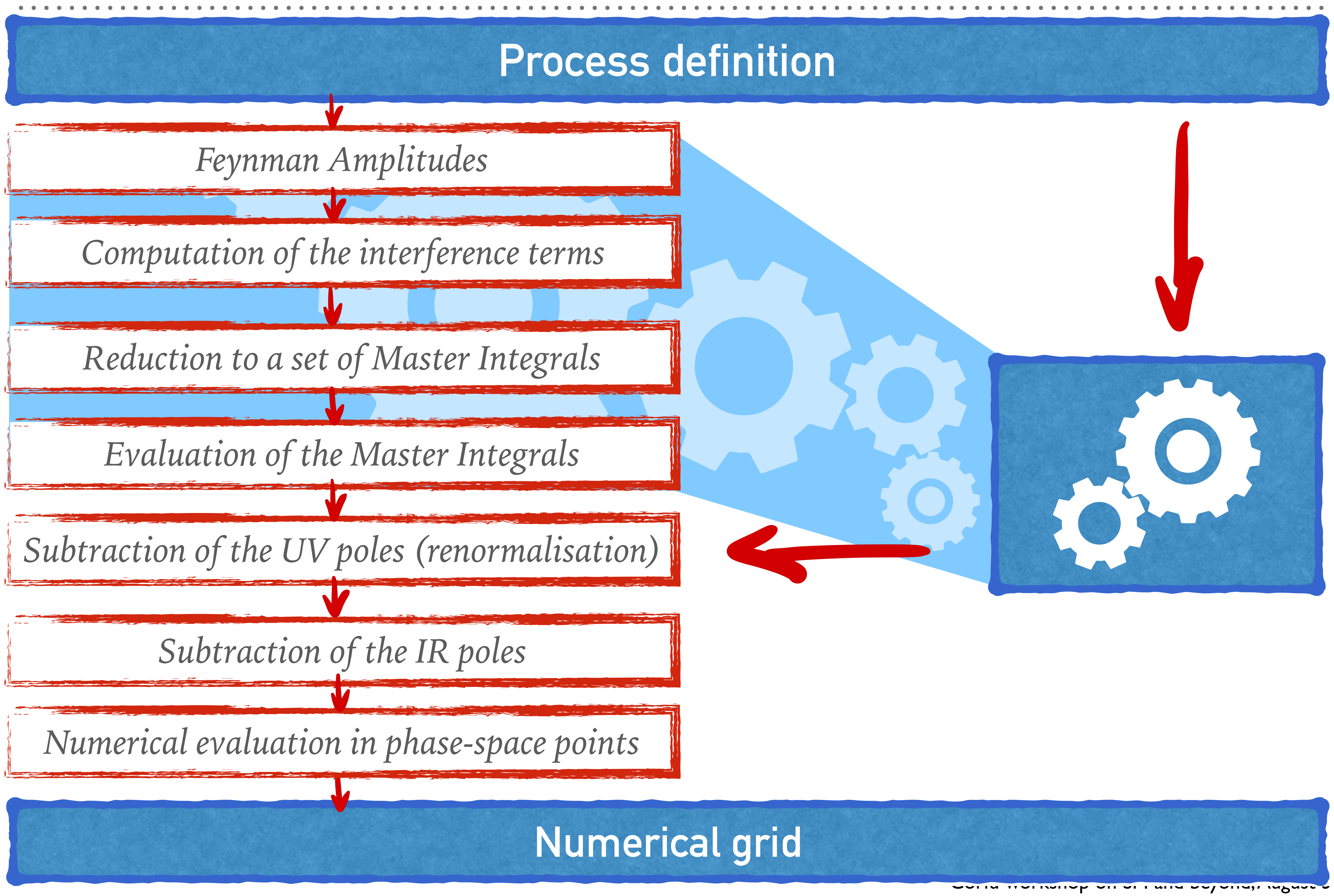
The NNLO QCD-EW corrections to Drell-Yan are an excellent playground for many of these problems

T.Armadillo, R.Bonciani, S.Devoto N.Rana,, AV, arXiv:2201.01754

→ in turn, directly relevant for $e^+e^- \to q\bar{q} + X$

STRUCTURE OF A LOOP COMPUTATION

courtesy of Simone Devoto

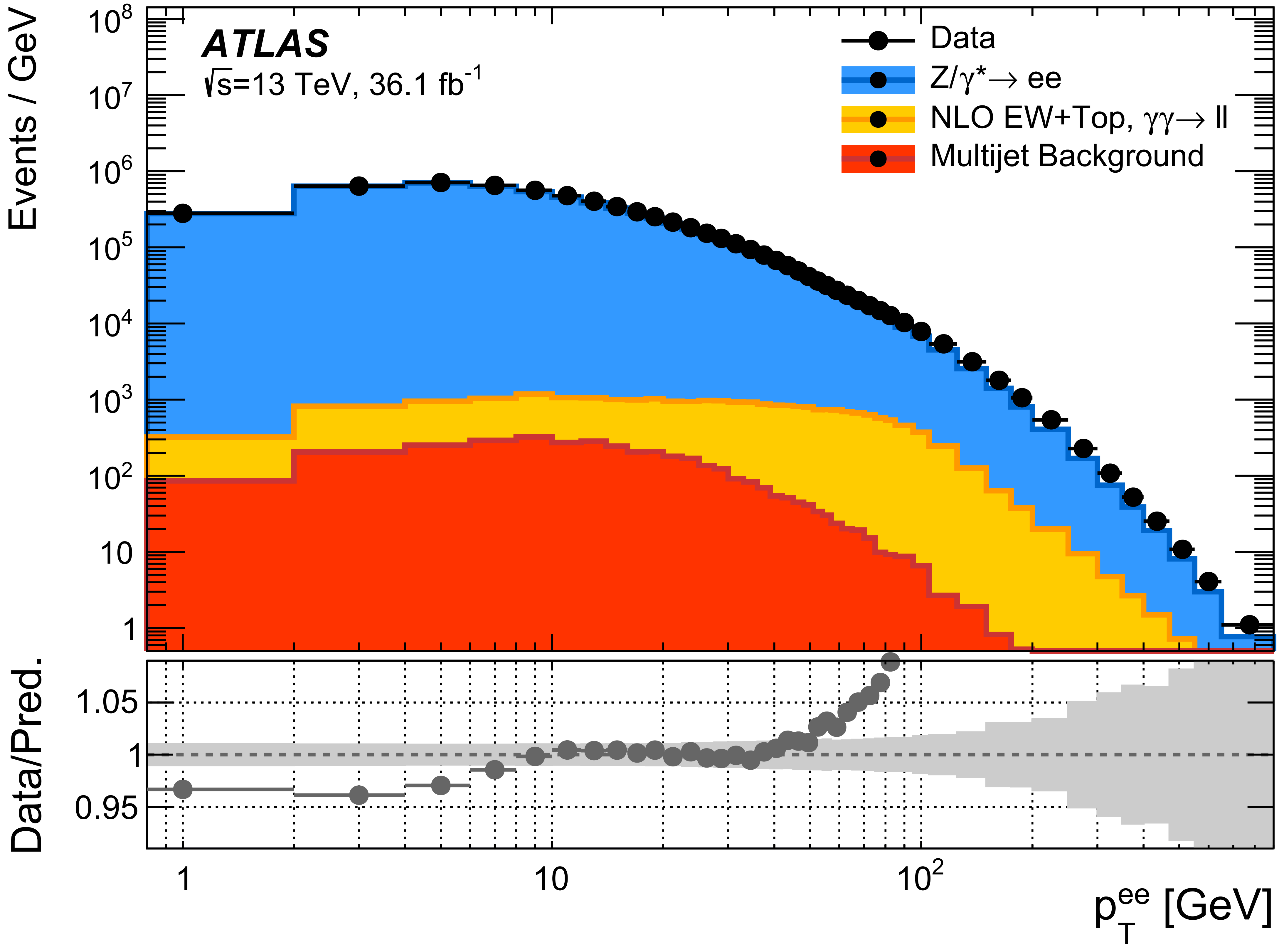


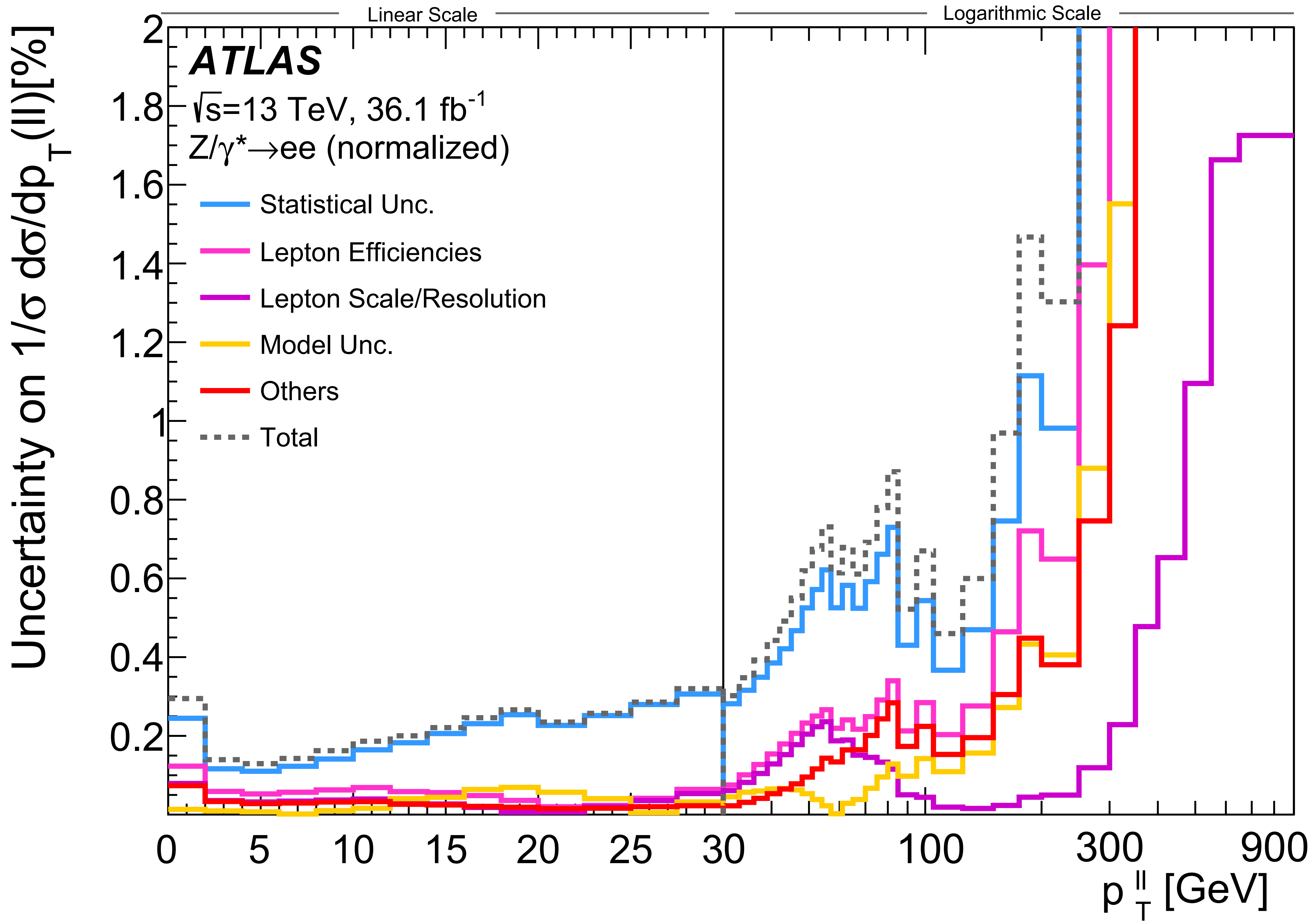
Lepton-pair transverse momentum distribution

- A crucial role in QCD tests and precision EW measurements (m_W in particular) is played by the $p_\perp^{\ell^+\ell^-}$ distribution
- The impressive experimental precision is a formidable test of the theory predictions, QCD in first place
- At per mille level higher-order QCD resummation matched with fixed order corrections

  non-perturbative QCD effects and heavy quarks corrections — are relevant

  EW corrections





At CERN the EW WG has a subgroup scrutinising the predictions of this observable by different collaborations

The q_T-subtraction and the residual cut-off dependency

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

When $q_T/Q > r_{cut}$ the double-real and the real-virtual contributions, subtracted with CS dipoles, are finite

$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

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Logarithmic sensitivity on r_{cut} in the double unresolved limit $\int d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{cut} + c_0 + \mathcal{O}(r_{cut}^m)$

The counterterm removes the IR sensitivity to the cutoff variable $\int \left(d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)}\right) \sim c_0 + \mathcal{O}(r_{cut}^m)$

→ we need small values of the cutoff

→ explicit numerical tests to quantify the bias induced by the cutoff choice (cfr. Buonocore, Kallweit, Rottoli, Wiesemann, arXiv:2111.13661 Camarda, Cieri, Ferrera, arXiv:2111.14509)

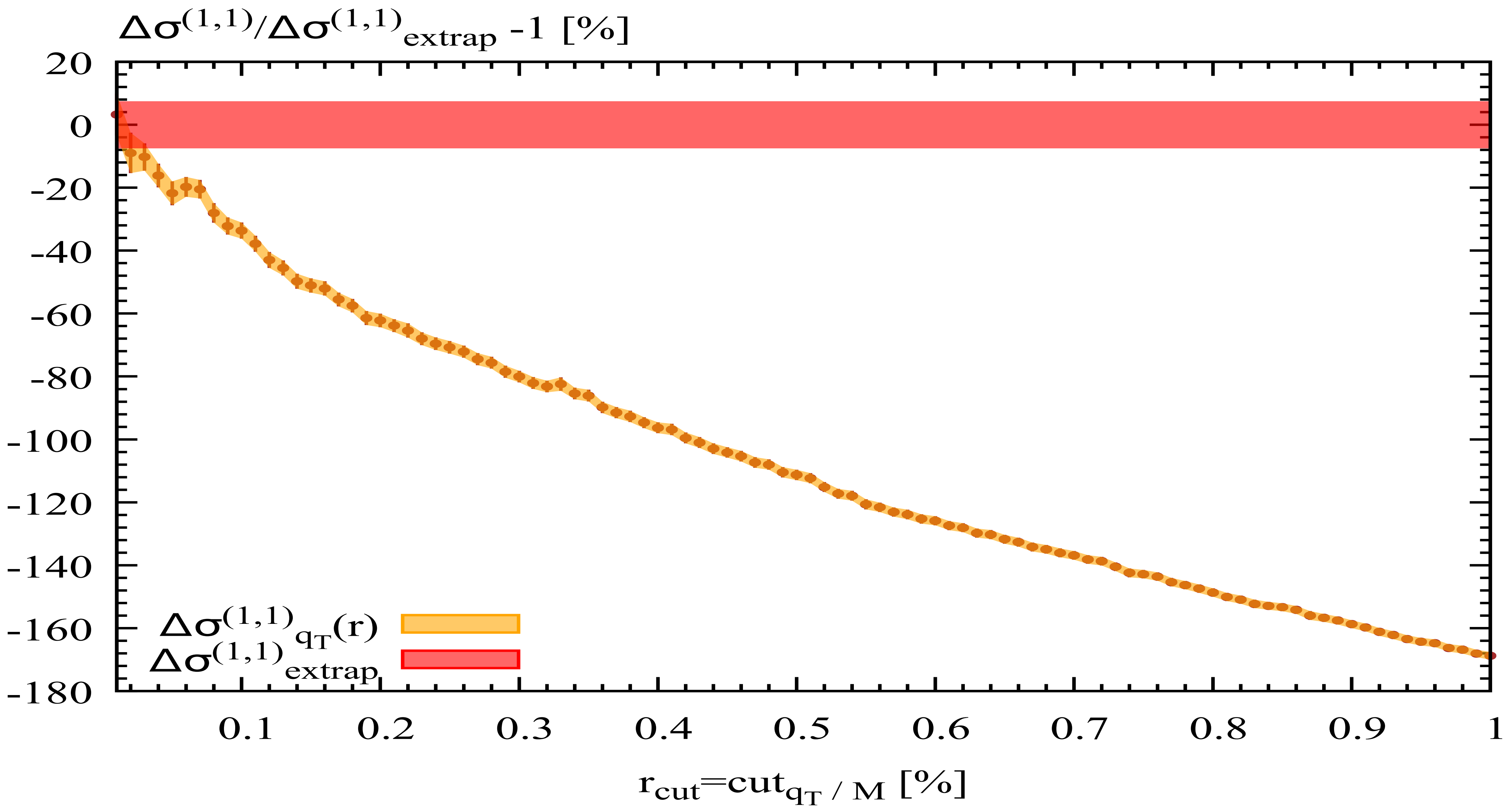
we can fit the r_{cut} dependence and extrapolate in the $r_{cut} \to 0$ limit

Dependence on r_{cut} of the NNLO QCD-EW corrections to NC DY

courtesy of S.Kallweit

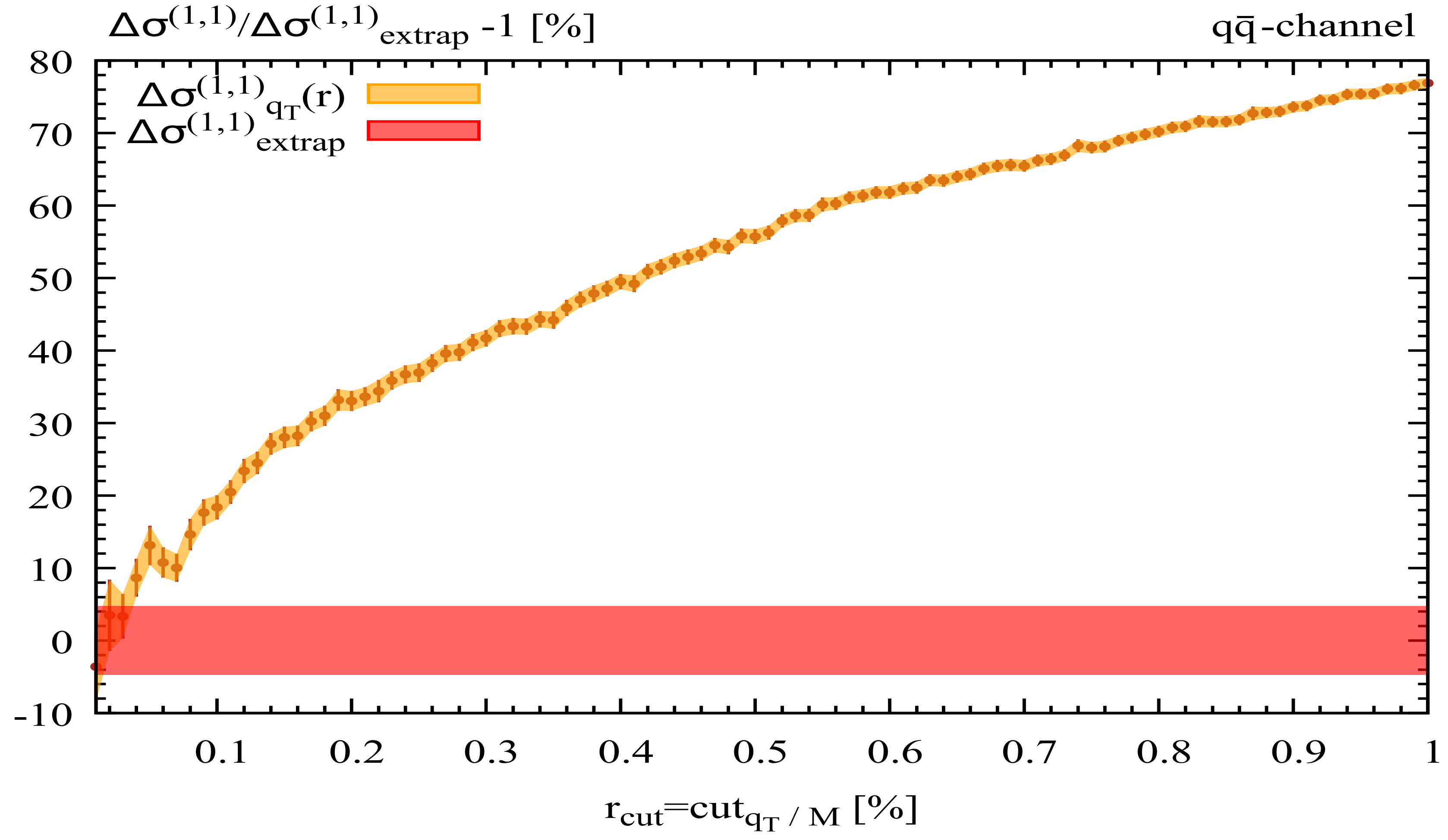
Symmetric-cut scenario

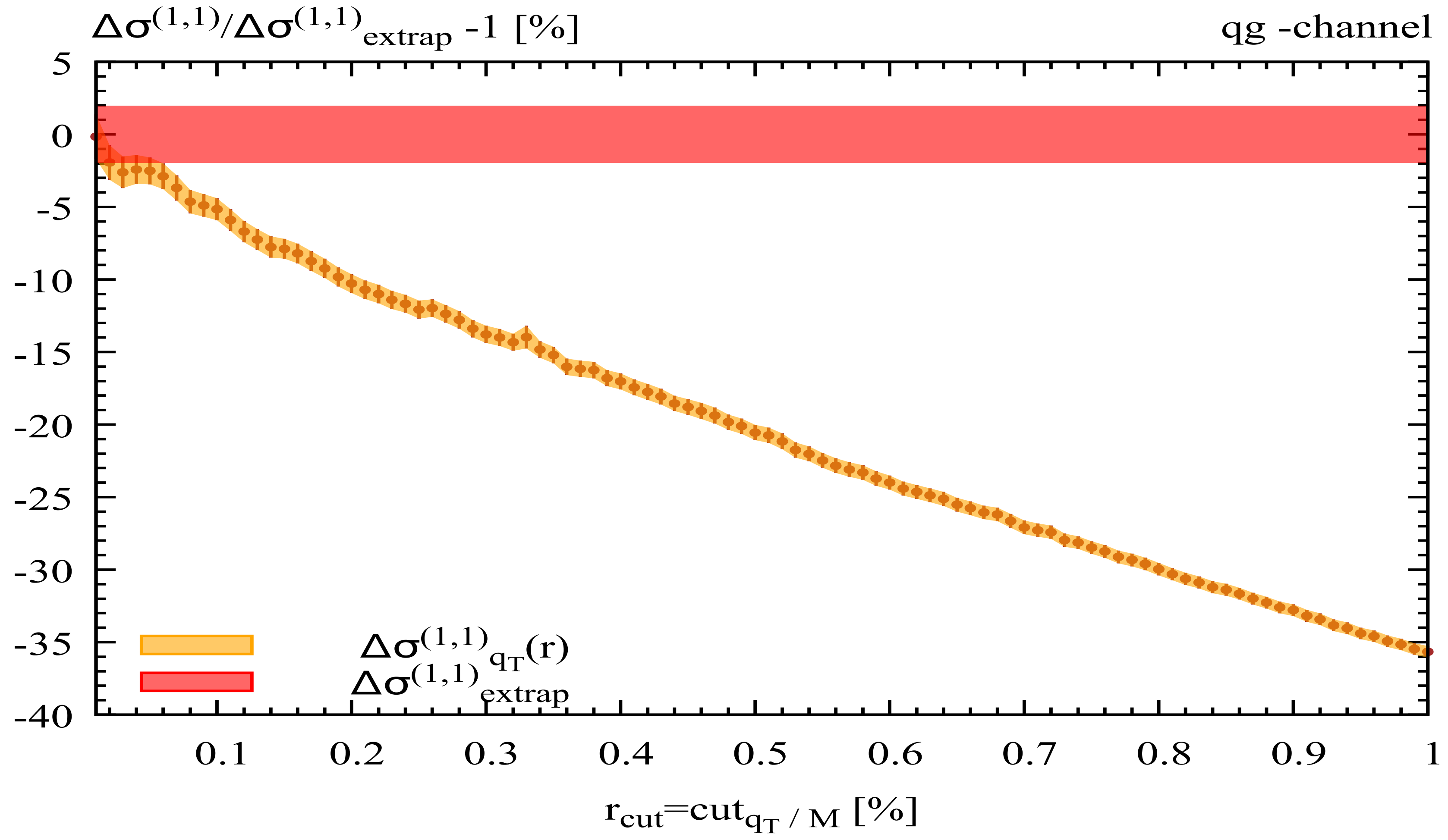
$p_{T,\ell^\pm} > 25$ GeV $\quad y_{\ell^\pm} < 2.5 \quad m_{\ell\ell} > 50$ GeV



- **large power corrections in r_{cut} for mixed corrections**
  - explained by overall small size of corrections, and in parts also by cancellation between partonic channels
- **by far less dramatic dependence at level of cross sections**
  - better than permille precision at inclusive level

Splitting into partonic channels





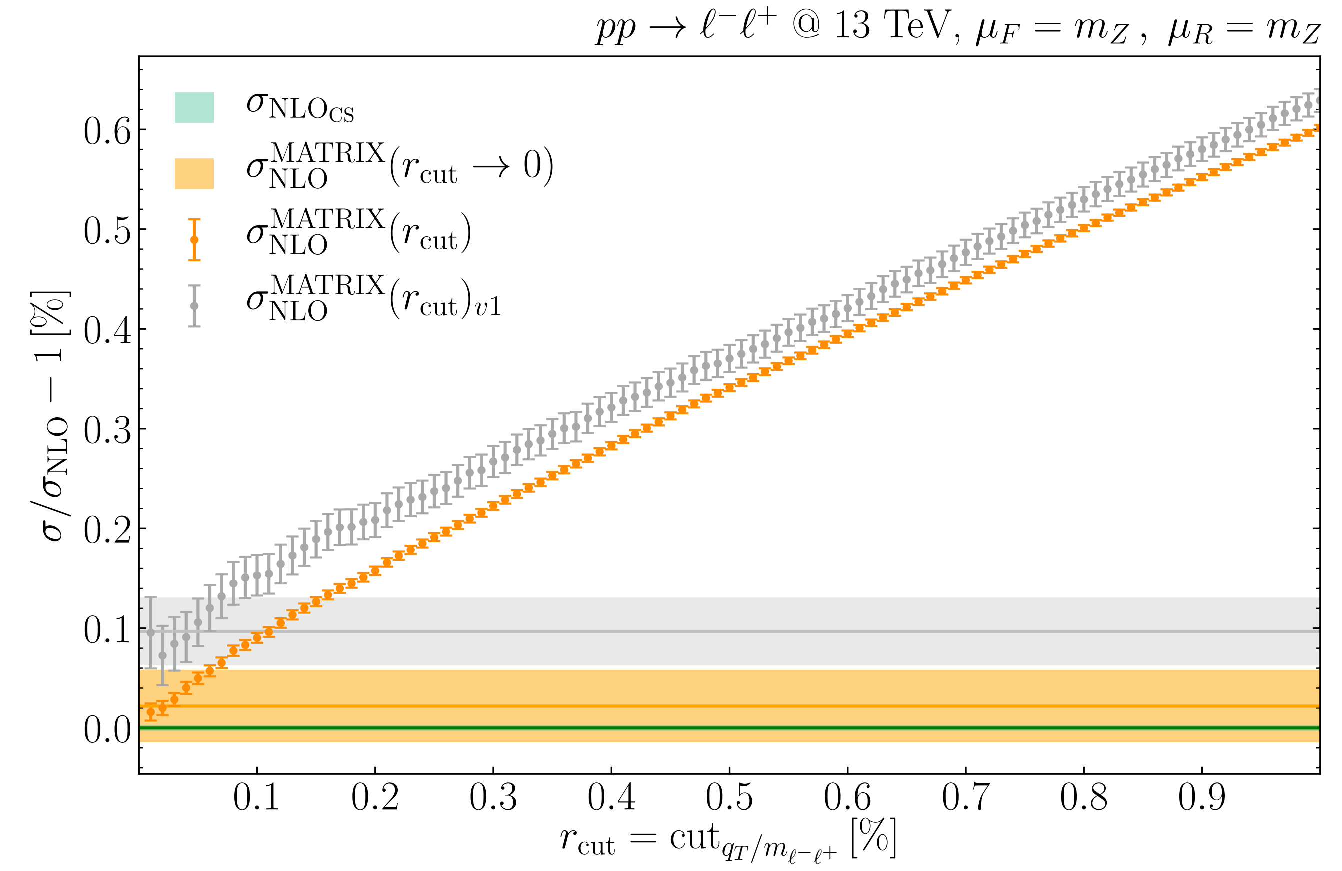
The q_T-subtraction and the residual cut-off dependency in different acceptance setups

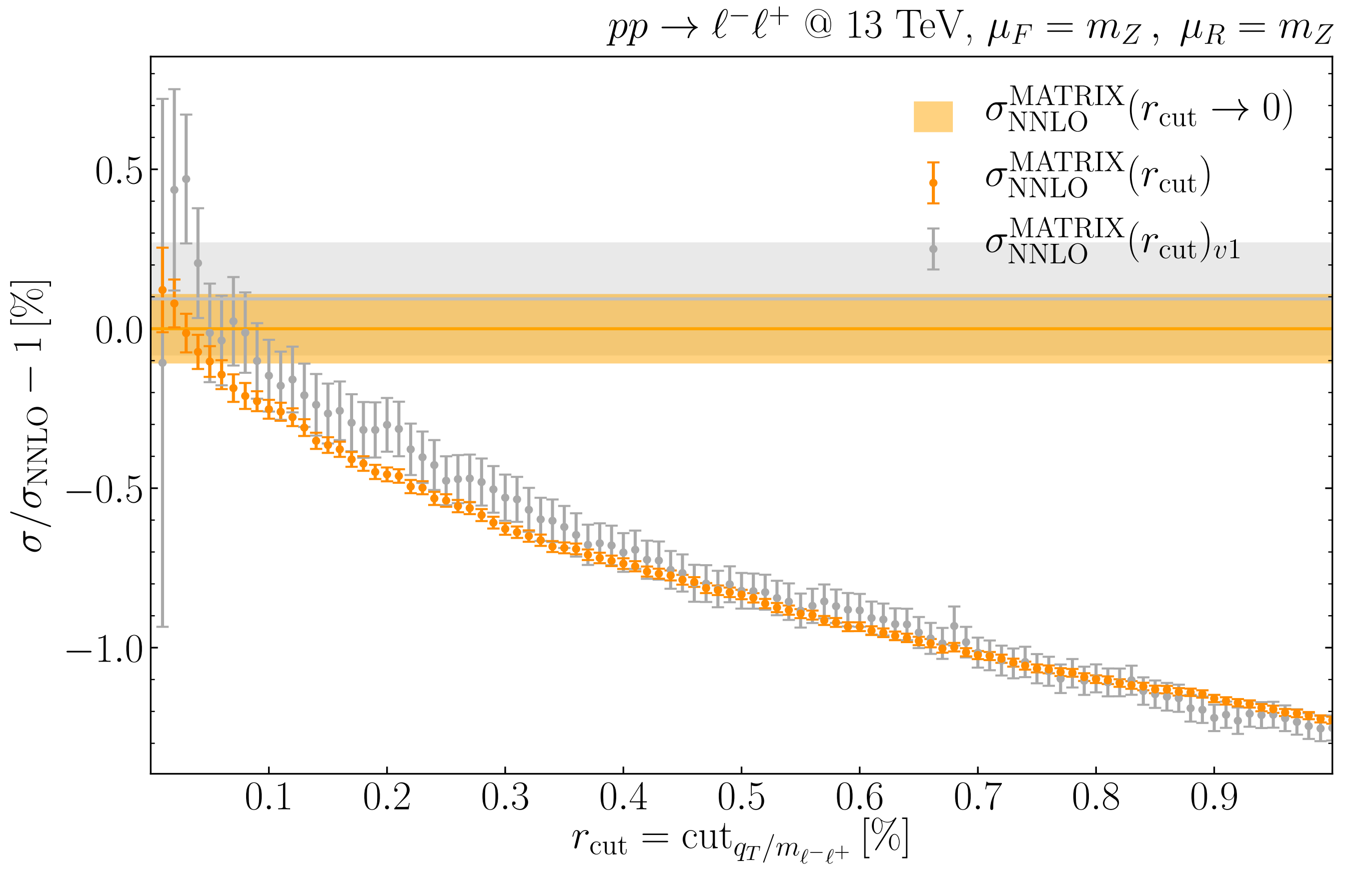
courtesy of S.Kallweit

(cfr. Buonocore, Kallweit, Rottoli, Wiesemann, 2111.13661)

Symmetric cuts

- $p_{\mathrm{T},\ell^\pm} > 25$ GeV

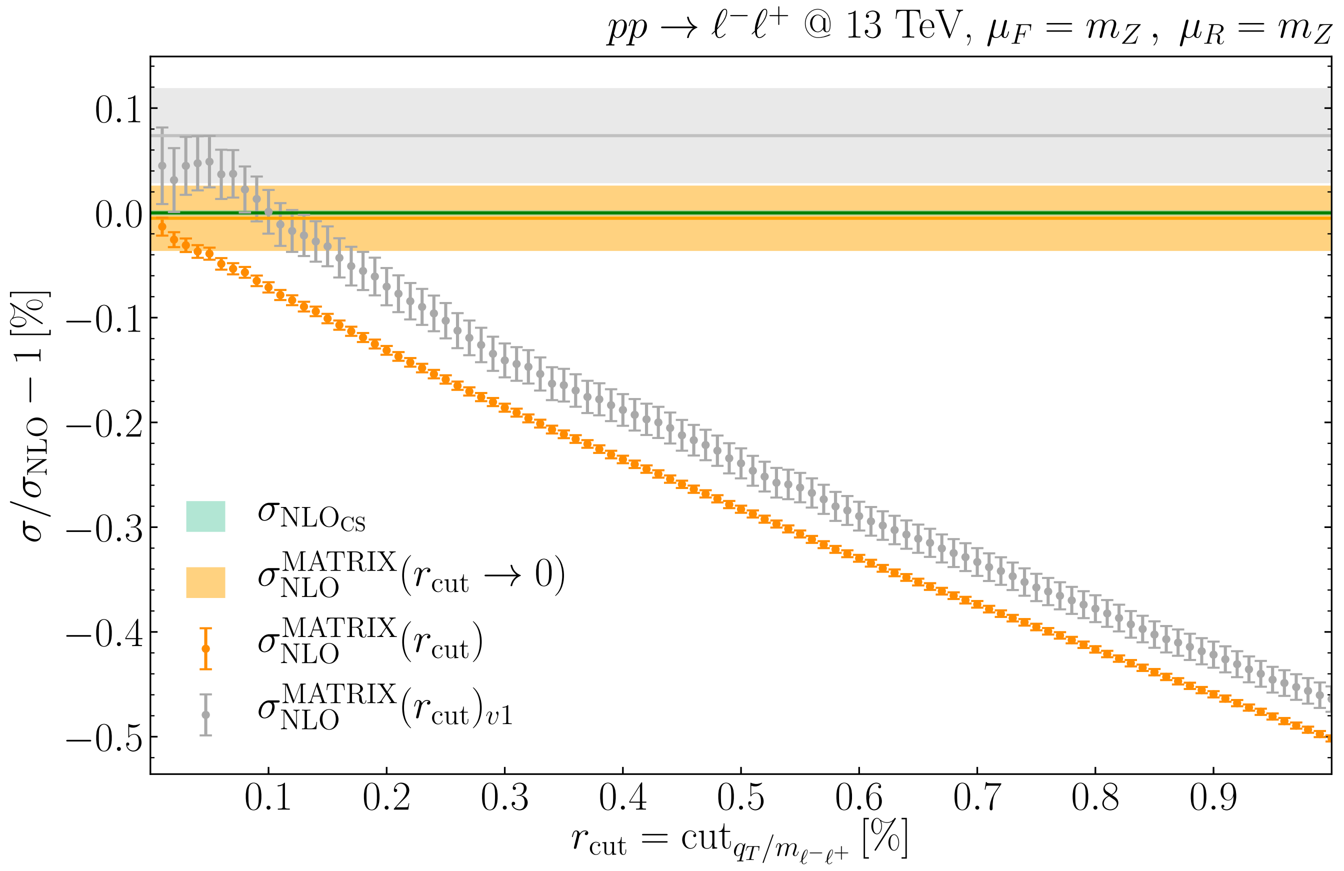


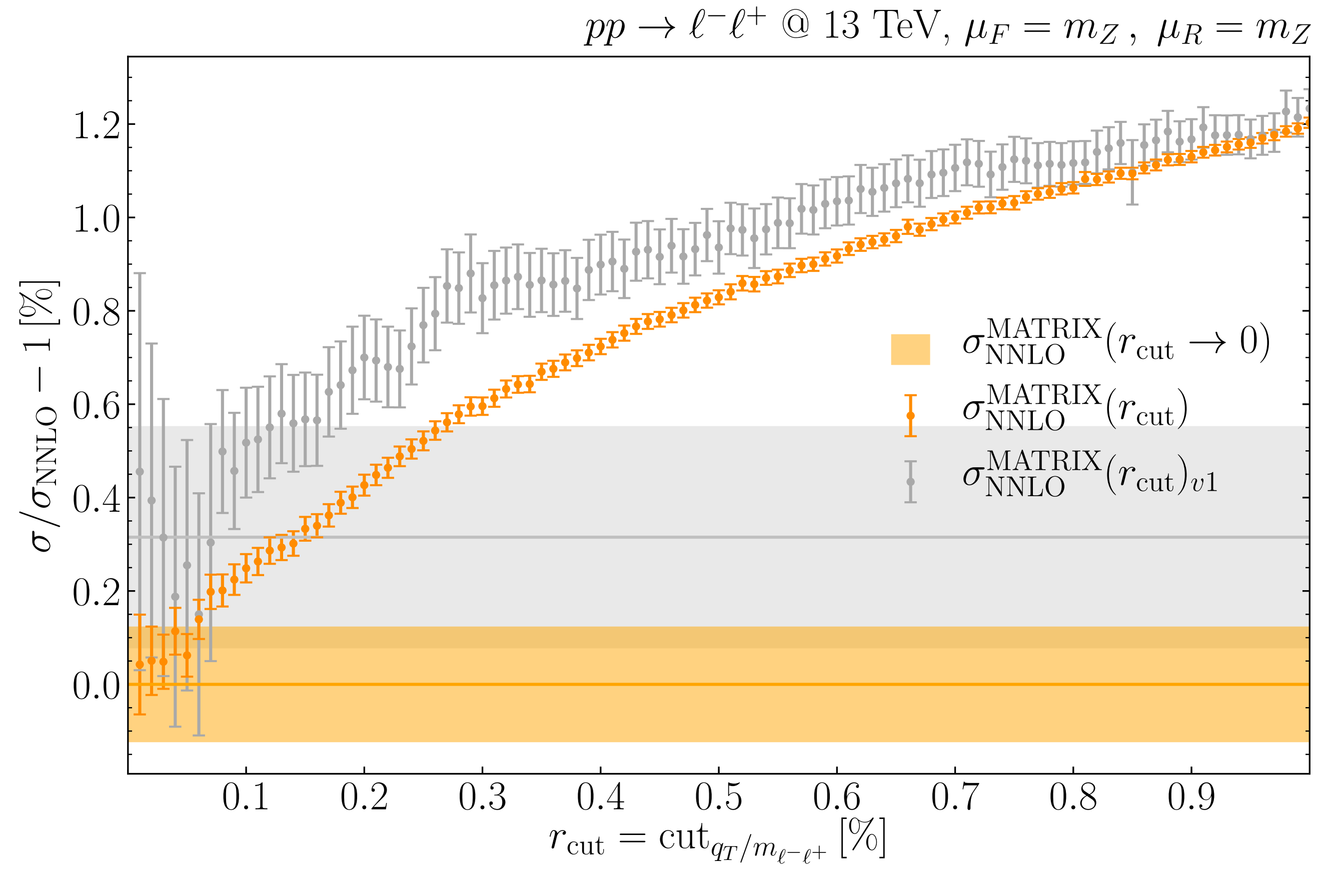


➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ_1 and ℓ_2

- $p_{\mathrm{T},\ell_1} > 25$ GeV $p_{\mathrm{T},\ell_2} > 20$ GeV

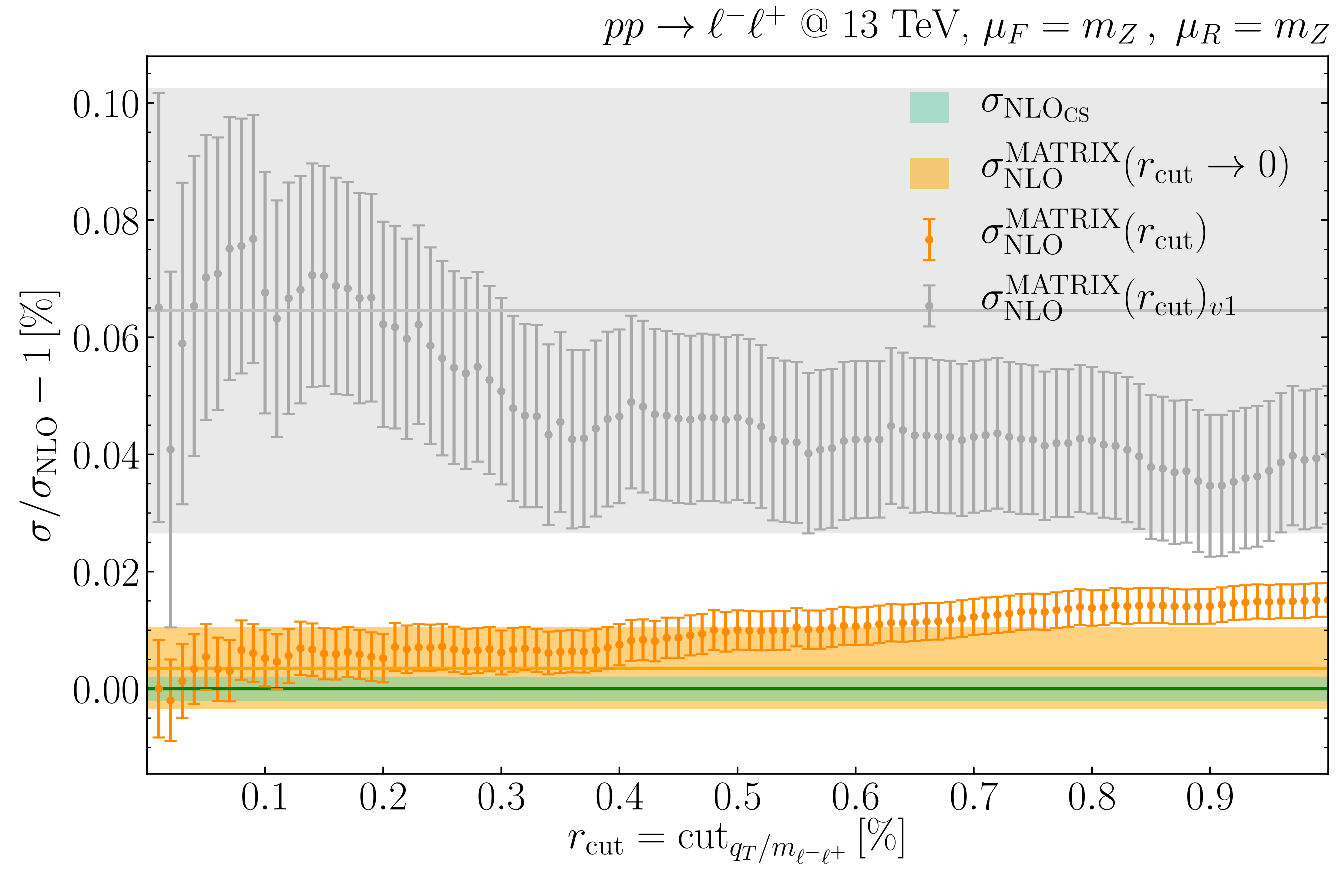


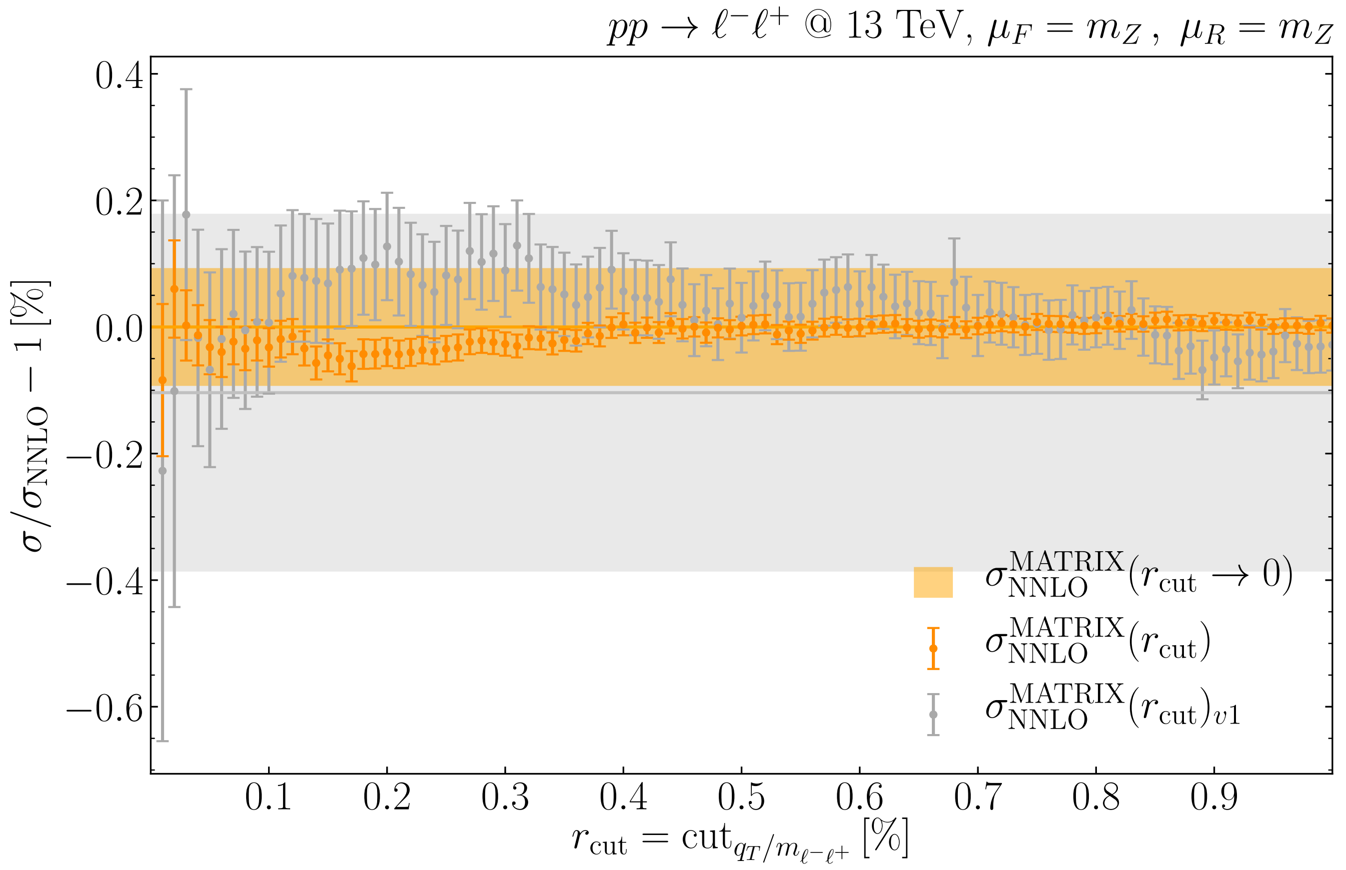


➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ^+ and ℓ^-

- $p_{\mathrm{T},\ell^+} > 25$ GeV $p_{\mathrm{T},\ell^-} > 20$ GeV



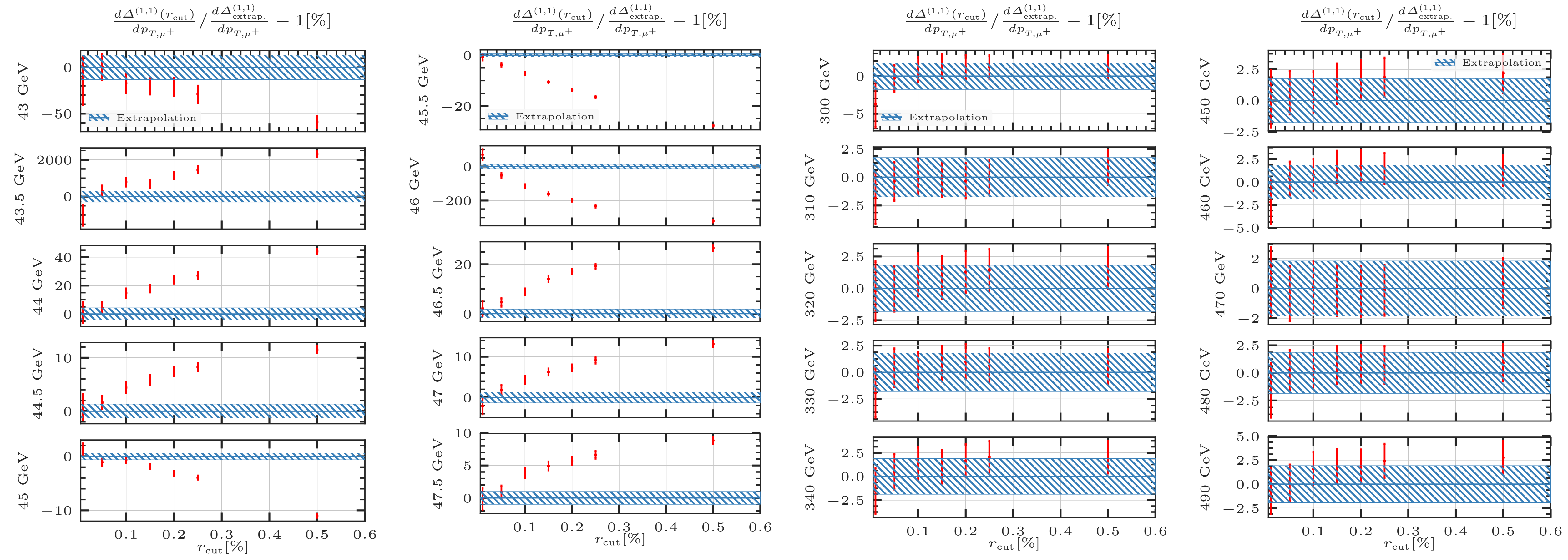


➥ no significant dependence on r_{cut}

Differential sensitivity to r_{cut}

Binwise $r_{\rm cut}$ dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

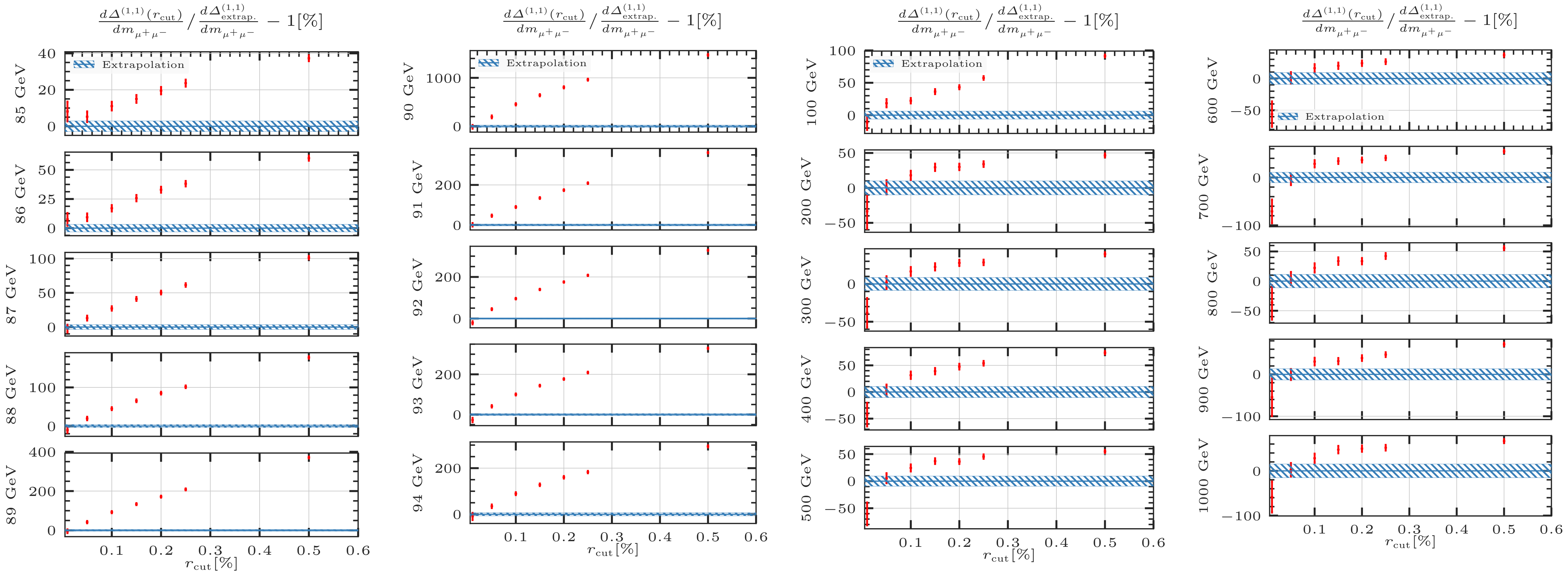
Differential distribution in p_{T,μ^+}: peak (left panels) and tail (right panels) regions



- large $r_{\rm cut}$ dependence in particular around the peak of the distribution, and typically precision of $\lesssim 3\%$ on the relative mixed QCD–EW corrections (artificially large where corrections are basically zero)

Binwise $r_{\rm cut}$ dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

Differential distribution in $m_{\mu^+\mu^-}$: peak (left panels) and tail (right panels) regions



- quite large $r_{\rm cut}$ dependence throughout, and lower numerical precision of $\lesssim 10\%$ on the relative mixed QCD–EW corrections (but still permille-level precision at the level of cross sections))

The hard-virtual coefficient

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

$$\mathscr{H}^{(1,1)} = H^{(1,1)}\, C_1\, C_2$$

The process independent collinear functions C_1, C_2 are known up to N3LO

The process dependent hard function H is defined upon subtraction of the universal IR contributions

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$$2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}^{(1,1)} \rangle = \sum_{k=-4}^{0} \varepsilon^k f_i(s,t,m)$$

after UV renormalisation the poles are only of IR origin

$$|\mathscr{M}_{fin}\rangle \equiv (1 - I)|\mathscr{M}\rangle \qquad H \propto \langle \mathscr{M}_0 | \mathscr{M}_{fin} \rangle$$

$$H^{(1,0)} = \frac{2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}_{fin}^{(1,0)} \rangle}{|\mathscr{M}^{(0,0)}|^2}, \qquad H^{(0,1)} = \frac{2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}_{fin}^{(0,1)} \rangle}{|\mathscr{M}^{(0,0)}|^2}, \qquad H^{(1,1)} = \frac{2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}_{fin}^{(1,1)} \rangle}{|\mathscr{M}^{(0,0)}|^2}$$

NLO-QCD | NLO-EW | NNLO QCD-EW

The double virtual amplitude: UV renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

Complex mass scheme

$$\mu_{W0}^2 = \mu_W^2 + \delta\mu_W^2, \quad \mu_{Z0}^2 = \mu_Z^2 + \delta\mu_Z^2, \quad e_0 = e + \delta e$$

$$\frac{\delta s^2}{s^2} = \frac{c^2}{s^2}\left(\frac{\delta\mu_Z^2}{\mu_Z^2} - \frac{\delta\mu_W^2}{\mu_W^2}\right)$$

the mass counterterms are defined at the complex pole of the propagator

the weak mixing angle is complex valued $c^2 \equiv \mu_W^2/\mu_Z^2$

BFG EW Ward identity → cancellation of the UV divergences combining vertex and fermion WF corrections

The bare couplings of Z and photon to fermions in the (G_μ, μ_W, μ_Z) input scheme are given by

$$\frac{g_0}{c_0} = \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left[1 - \frac{1}{2}\Delta r + \frac{1}{2}\left(2\frac{\delta e}{e} + \frac{s^2 - c^2}{c^2}\frac{\delta s^2}{s^2}\right)\right] \equiv \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left(1 + \delta g_Z^{G_\mu}\right)$$

$$g_0 s_0 = \sqrt{4\sqrt{2}G_\mu\mu_W^2 s^2}\left[1 + \tfrac{1}{2}\left(-\Delta r + 2\tfrac{\delta e}{e}\right)\right] \equiv e_{ren}^{G_\mu}\left(1 + \delta g_A^{G_\mu}\right)$$

Gauge boson renormalised propagators

$$\Sigma_{R,T}^{AA}(q^2) = \Sigma_T^{AA}(q^2) + 2\,q^2\,\delta g_A$$

$$\Sigma_{R,T}^{ZZ}(q^2) = \Sigma_T^{ZZ}(q^2) - \delta\mu_Z^2 + 2\,(q^2 - \mu_Z^2)\,\delta g_Z$$

$$\Sigma_{R,T}^{AZ}(q^2) = \Sigma_T^{AZ}(q^2) - q^2\,\frac{\delta s^2}{sc}$$

$$\Sigma_{R,T}^{ZA}(q^2) = \Sigma_T^{ZA}(q^2) - q^2\,\frac{\delta s^2}{sc},$$

After the UV renormalisation, the singular structure is entirely due to IR soft and/or collinear singularities

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$\mathrm{Tr}(\gamma_\alpha \ldots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \ldots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \ldots \right)$$

If a_1 is evaluated in a non-consistent way,
then poles might not cancel and the finite part of the xsec might have a spurious contribution

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If a_1 is evaluated in a non-consistent way,
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- 't Hooft-Veltman treat γ_5 (anti)commuting in (4) $n-4$ dimensions preserving the cyclicity of the traces
  (one counterterm is needed)
- Kreimer treats γ_5 anticommuting in n dimensions, abandoning the cyclicity of the traces (→ need of a starting point)

- Heller, von Manteuffel, Schabinger verified that the IR-subtracted squared matrix element are identical in the two approaches

- we adopted the naive anticommuting prescription (Kreimer); we use $\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$ to compute traces with one γ_5
  - we computed the 2-loop amplitude and, independently, the IR subtraction term; both depend on the prescription chosen
  - the cancellation of all the lowest order poles is checked (and non trivial)
  - absence of fermionic triangles because of colour conservation

Differential equations and IBPs

- Not all the Feynman integrals in one amplitude are independent
  → exploit Integration-by-parts (IBP) and Lorentz identities to reduce to a basis of independent Master Integrals

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_1^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0}\, [(k_1+p_1)^2 - m_1^2]^{\alpha_1}\, \ldots\, [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j}\, \ldots\, [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_2^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0}\, [(k_1+p_1)^2 - m_1^2]^{\alpha_1}\, \ldots\, [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j}\, \ldots\, [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

- Henn's conjecture (2013): if a change of basis exists which leads to $d\vec{J}(\vec{s};\varepsilon) = \varepsilon \tilde{\mathbf{A}}(\vec{s}) \cdot \vec{J}(\vec{s};\varepsilon)$
  then the solution is expressed in terms of iterated integrals (Chen integral representation)
  depending only on the results at previous orders in the ε expansion

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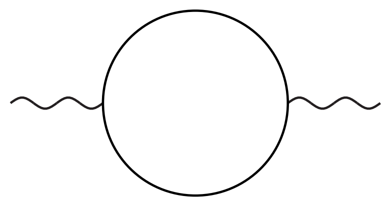
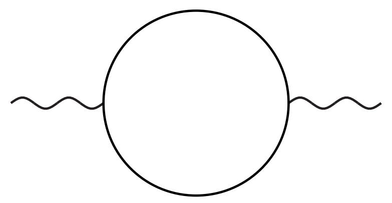
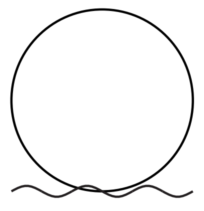
$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_2^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0} \, [(k_1+p_1)^2 - m_1^2]^{\alpha_1} \, \ldots \, [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j} \, \ldots \, [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

- The independent Master Integrals (MIs) satisfy a system of first-order linear differential equations
  with respect to each of the kinematical invariants / internal masses

When considering the complete set of MIs, the system can be cast in homogeneous form: $d\vec{I}(\vec{s};\varepsilon) = \mathbf{A}(\vec{s};\varepsilon)\cdot\vec{I}(\vec{s};\varepsilon)$

$\frac{d}{dk^2}$  $+ \frac{1}{2}\left[\frac{1}{k^2} - \frac{(D-3)}{(k^2+4m^2)}\right]$  $= -\frac{(D-2)}{4m^2}\left[\frac{1}{k^2} - \frac{1}{(k^2+4m^2)}\right]$ 

- Henn's conjecture (2013): if a change of basis exists which leads to $d\vec{J}(\vec{s};\varepsilon) = \varepsilon\tilde{\mathbf{A}}(\vec{s})\cdot\vec{J}(\vec{s};\varepsilon)$
  then the solution is expressed in terms of iterated integrals (Chen integral representation)
  depending only on the results at previous orders in the ε expansion

A Simple Example

$$\begin{cases} f'(x) + \frac{1}{x^2-4x+5} f(x) = \frac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f'_{hom}(x) = \sum_{k=0}^{\infty} (k+r)\, c_k\, x^{(k+r-1)}$$

$$\begin{cases} r c_0 = 0 \\ \frac{1}{5} c_0 + c_1 (r+1) = 0 \\ \frac{4}{25} c_0 + \frac{1}{5} c_1 + c_2 (2+r) = 0 \\ \ldots \end{cases}$$

$$f_{hom}(x) = 5 - x - \frac{3}{10} x^2 + \frac{11}{150} x^3 + \ldots$$

Expanded around $x' = 0$

$$f_{part}(x) = f_{hom}(x) \int_0^x dx' \, \frac{1}{(x'+2)} f_{hom}^{-1}(x')$$

$$= \frac{1}{2} x - \frac{7}{40} x^2 + \frac{2}{75} x^3 + \ldots$$

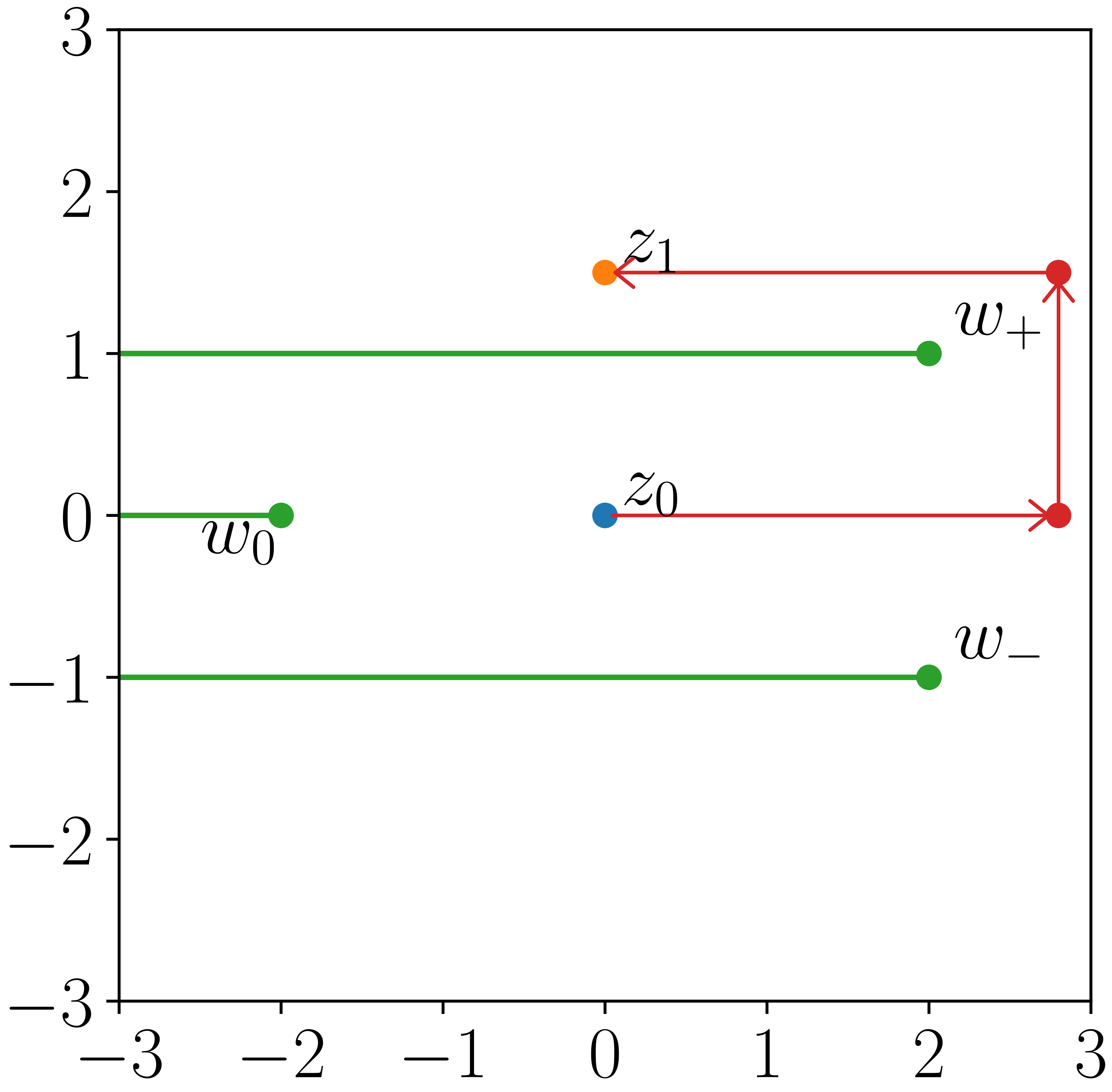
$$f(x) = f_{part}(x) + C f_{hom}(x)$$

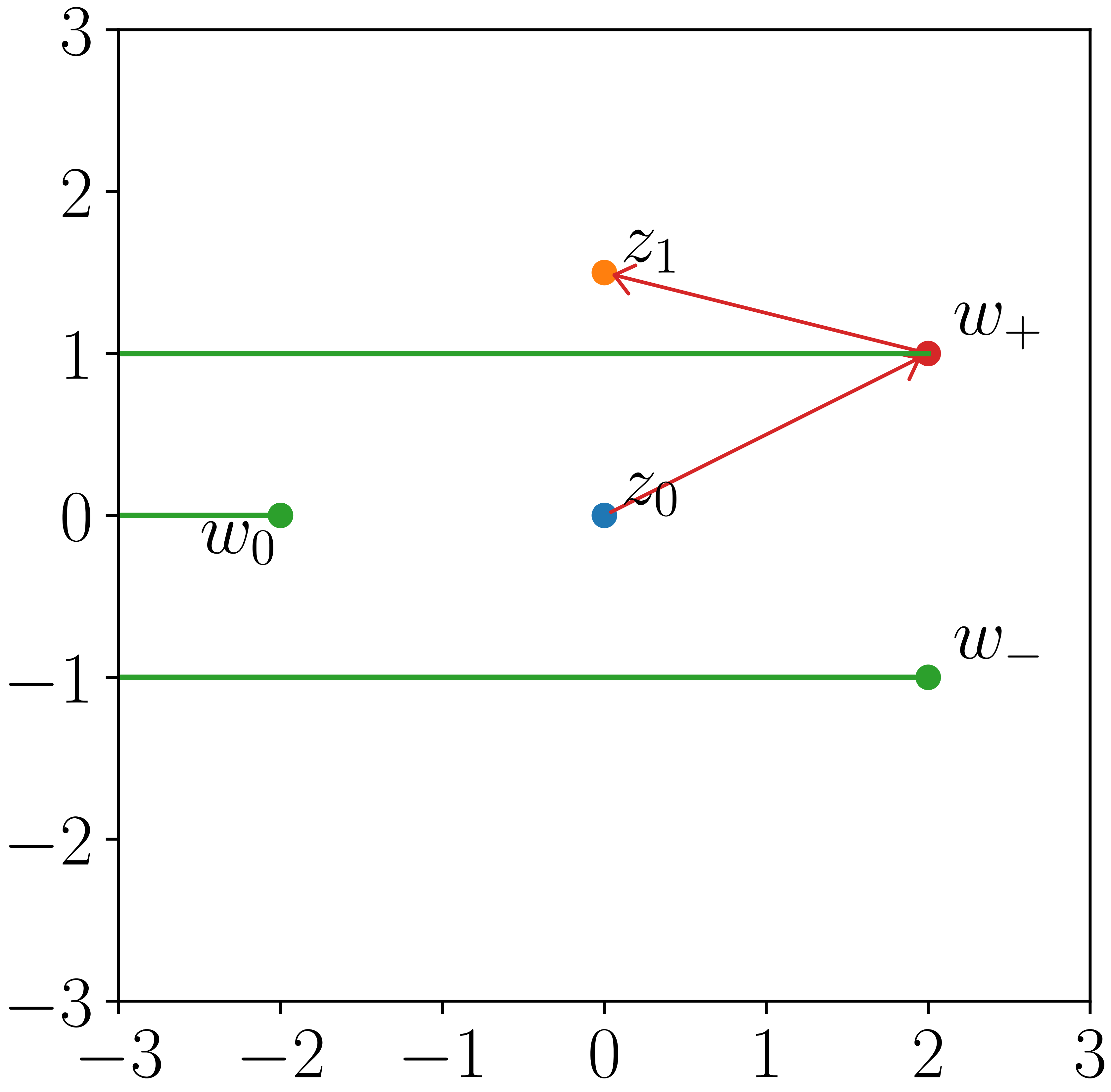
$$f(0) = 1 \rightarrow C = \frac{1}{5}$$

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

- **Taylor expansion**: **avoids** the singularities;
- **Logarithmic expansion**: uses the singularities as **expansion points**.
- Logarithmic expansion has larger convergence radius but requires longer evaluation time. **We use Taylor expansion as default**.



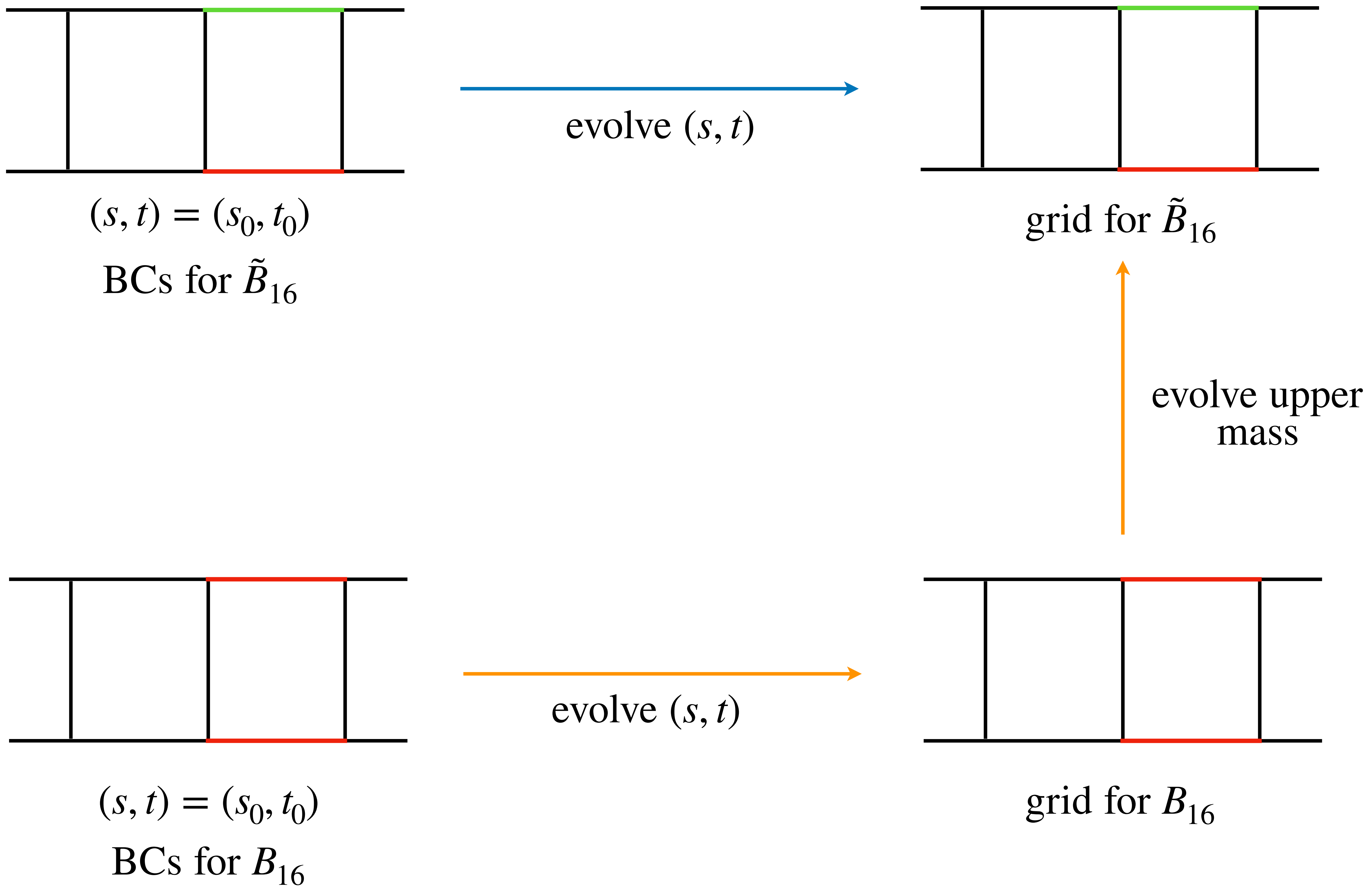


Exploiting the flexibility of the Differential Equations approach

The CC-DY Master Integrals can be evaluated with two different approaches:

- compute the BCs with AMFlow and then solve the differential equations in the invariants s and t

- use the results of the NC DY process as BCs (two equal internal masses, arbitrary s and t) then solve the differential equation in the mass parameter from (m_Z, m_Z) to (m_W, m_Z)



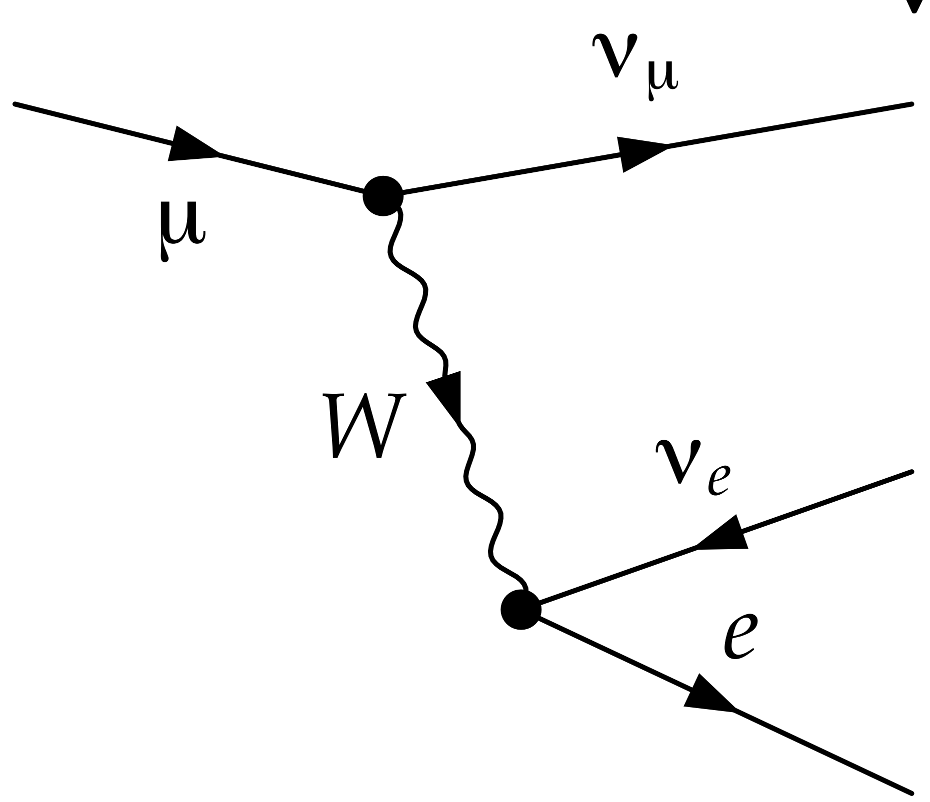
Perfect agreement of the two approaches

The W boson mass: theoretical prediction

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}(\alpha,\, G_\mu,\, m_Z; m_H; m_f; CKM)$$

$$\rightarrow \text{we can compute } m_W$$

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8 m_W^2}\,(1+\Delta r)$$



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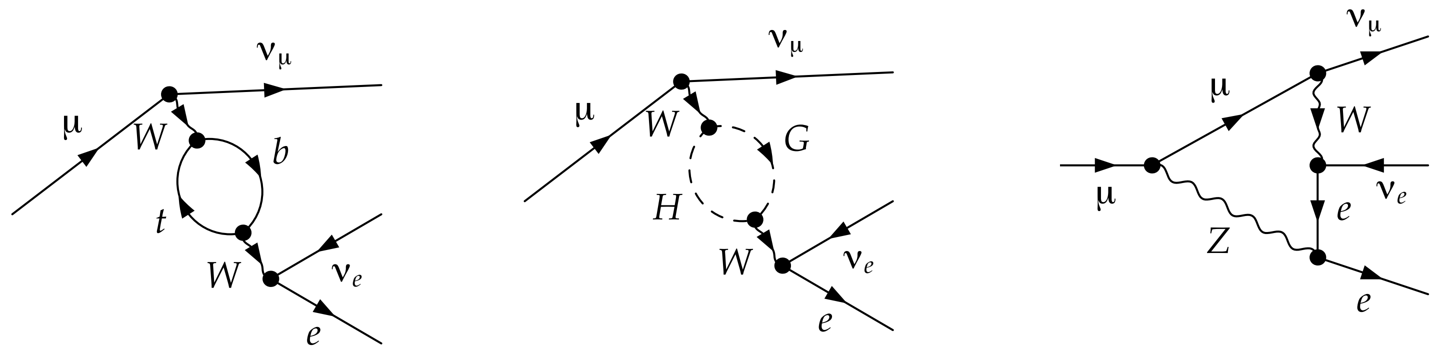


+ ....

$$\Delta r \;=\; \Delta r(\alpha, G_\mu, m_Z, m_H; m_f; CKM)$$

$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

The W boson mass: theoretical prediction

<u>on-shell scheme:</u> dominant contributions to Δr

$$\Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\rm rem}$$

$$\Delta\alpha = \Pi^{\gamma}_{\rm ferm}(M_Z^2) - \Pi^{\gamma}_{\rm ferm}(0) \quad \to \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} = 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} \quad \textit{[one-loop]} \quad \sim \frac{m_t^2}{v^2} \sim \alpha_t$$

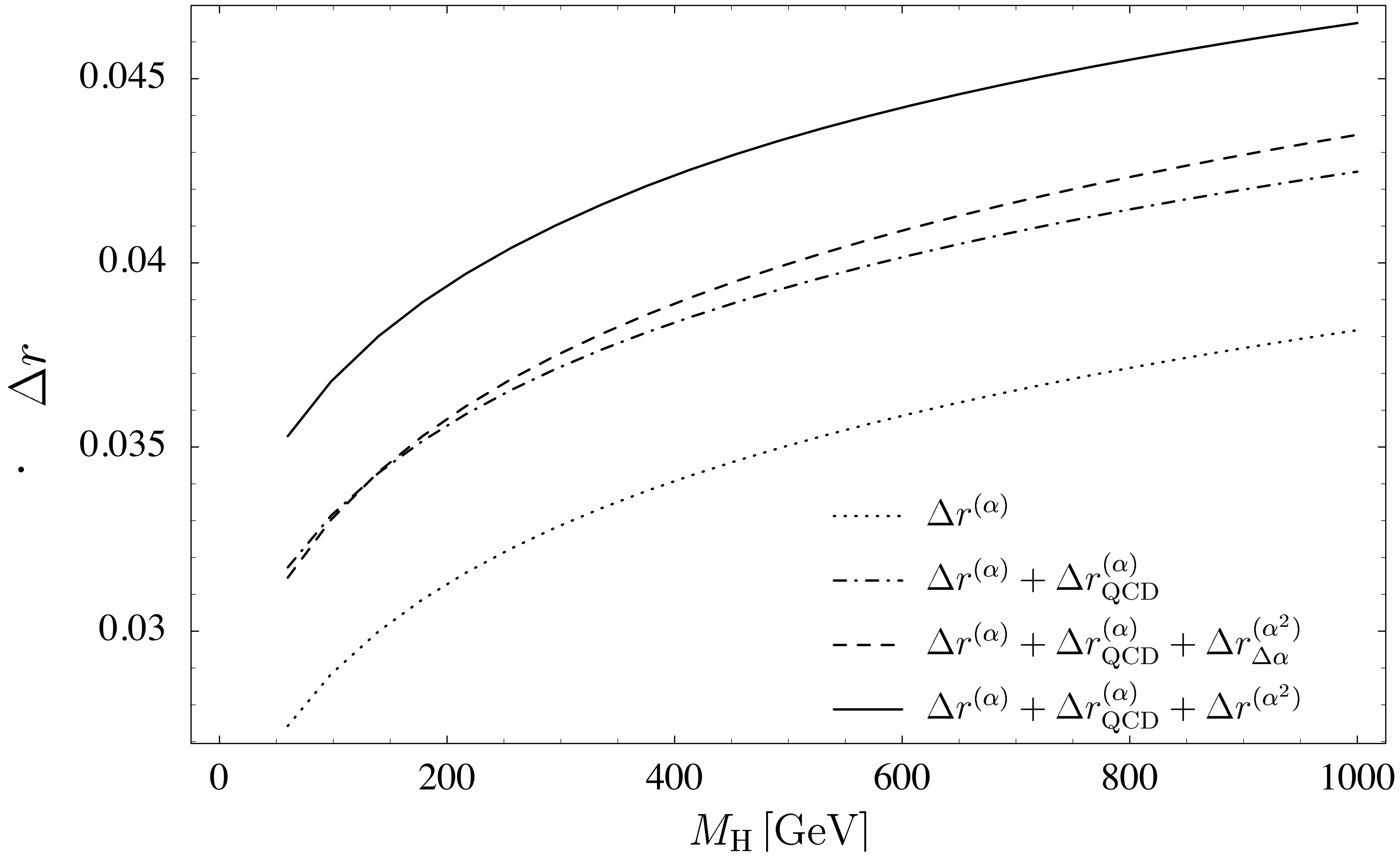
beyond one-loop order: $\sim \alpha^2,\ \alpha\alpha_t,\ \alpha_t^2,\ \alpha^2\alpha_t,\ \alpha\alpha_t^2,\ \alpha_t^3, \ldots$

reducible higher order terms from $\Delta\alpha$ and $\Delta\rho$ via

$$1+\Delta r \ \to \ \frac{1}{(1-\Delta\alpha)\left(1+\frac{c_w^2}{s_w^2}\Delta\rho\right)+\cdots}$$

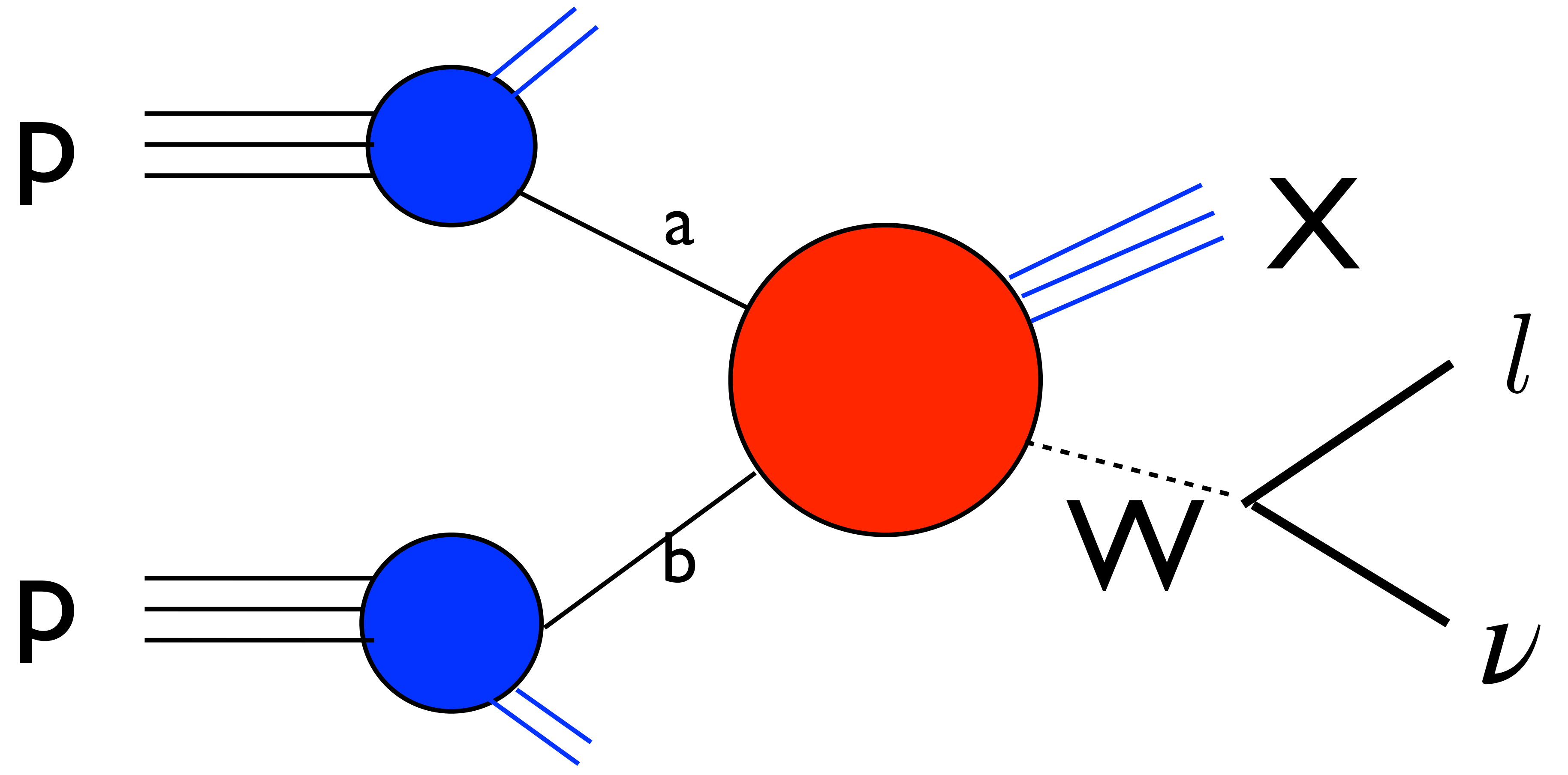
$$\rho = 1+\Delta\rho \ \to \ \frac{1}{1-\Delta\rho}$$

effects of higher-order terms on Δr



(Consoli, Hollik, Jegerlehner)

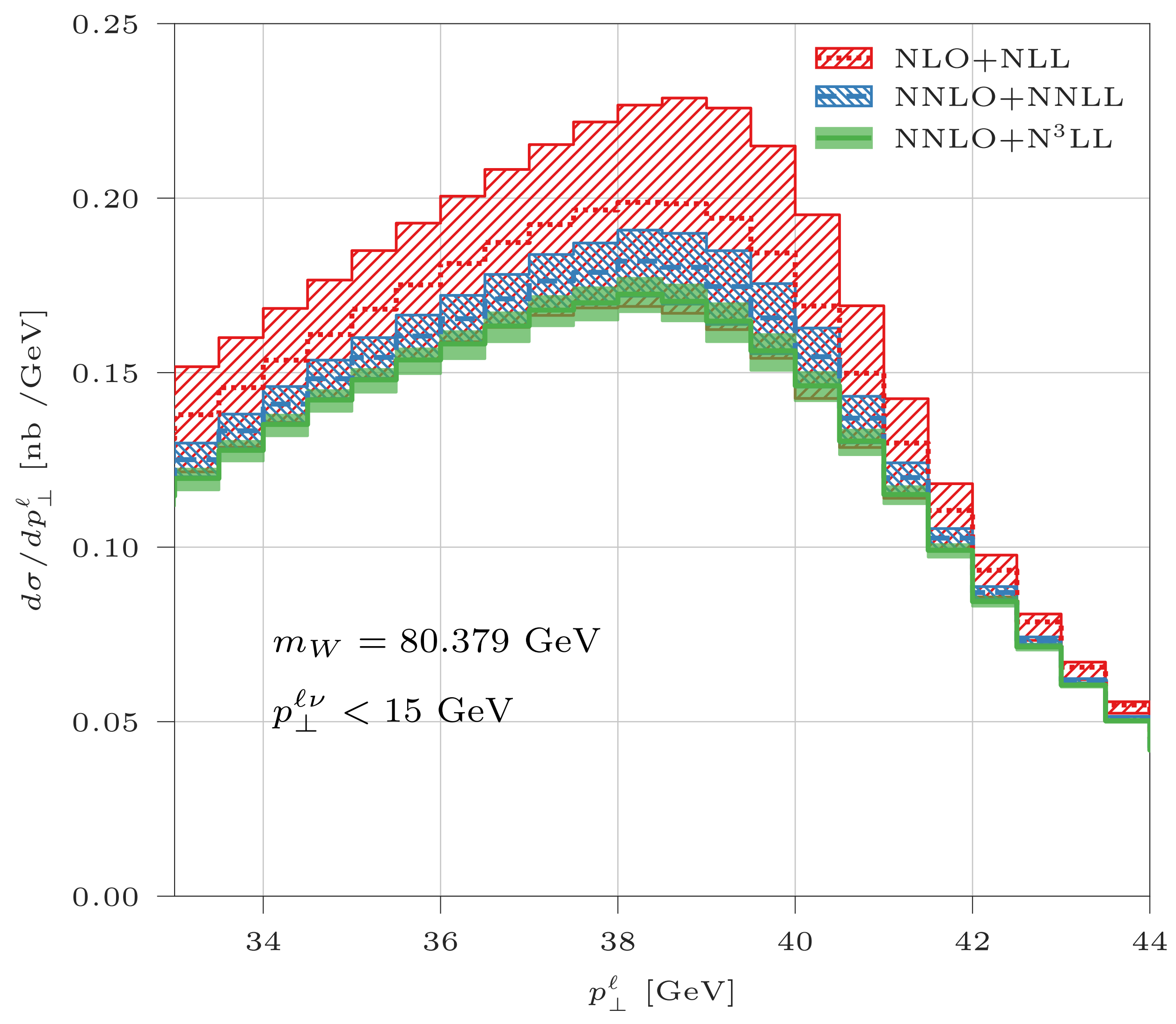
m_W determination at hadron colliders



- In charged-current DY,
  it is NOT possible to reconstruct the lepton-neutrino invariant mass
  Full reconstruction is possible (but not easy) only in the transverse plane

- A generic observable has a linear response to an m_W variation
  With a goal for the relative error of 10^{-4}, the problem seems to be unsolvable

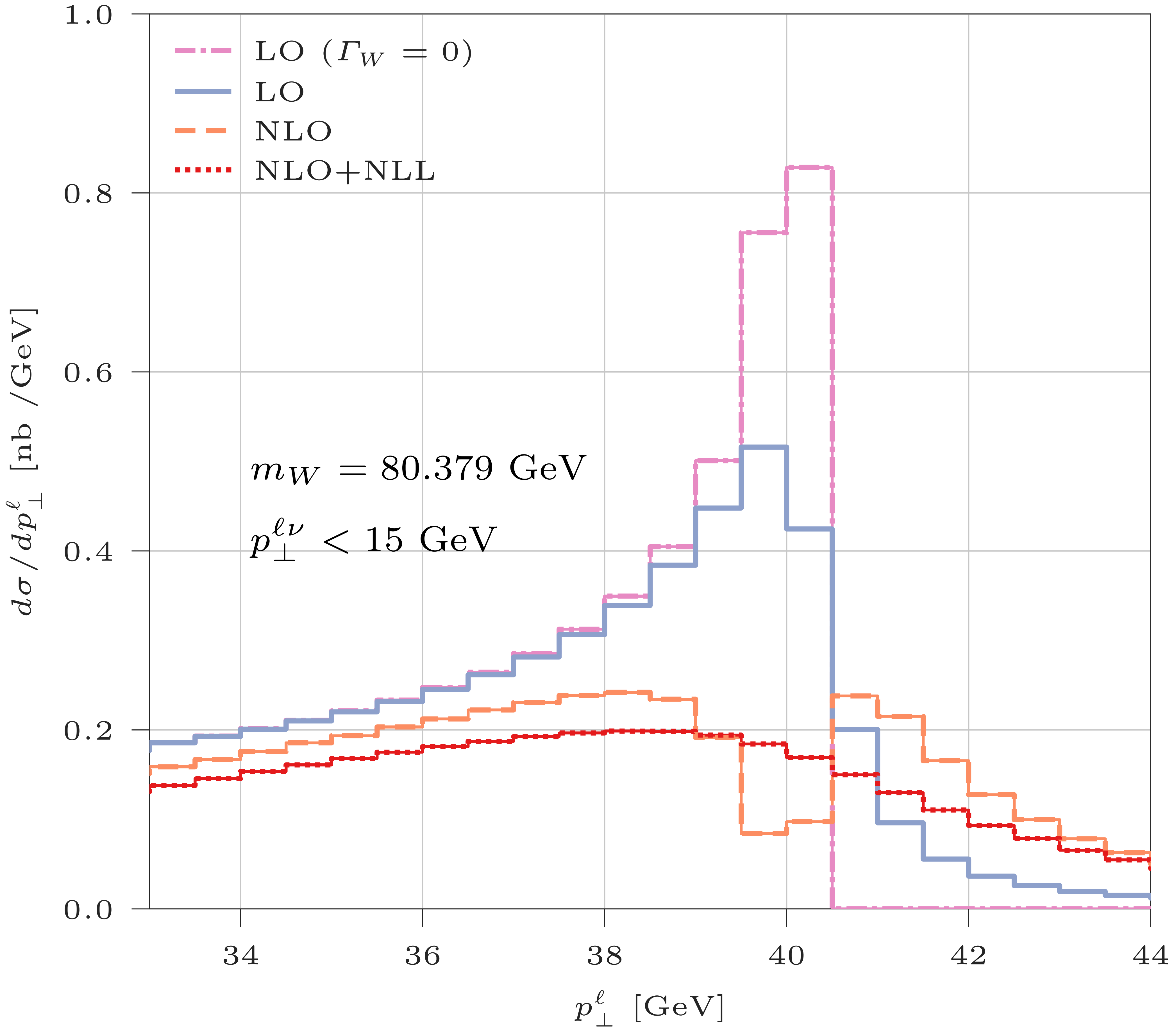
- m_W extracted from the study of the shape of the $p_\perp^l, M_\perp$ and $E_\perp^{miss}$ distributions in CC-DY
  thanks to the jacobian peak that enhances the sensitivity to m_W



$$\frac{d}{dp_\perp^2} \rightarrow \frac{2}{s}\frac{1}{\sqrt{1-4p_\perp^2/s}}\frac{d}{d\cos\theta} \sim \frac{d}{dp_\perp^2} \rightarrow \frac{2}{s}\frac{1}{\sqrt{1-4p_\perp^2/m_W^2}}\frac{d}{d\cos\theta}$$

→ enhanced sensitivity at the 10^{-3} level ($p_\perp^l$ distribution)
or even at the 10^{-2} level ($M_\perp$ distribution)

The lepton transverse momentum distribution in charged-current Drell-Yan



The lepton transverse momentum distribution has a jacobian peak induced by the factor $1/\sqrt{1-\frac{s}{4p_\perp^2}}$.

When studying the W resonance region, the peak appears at $p_\perp \sim \frac{m_W}{2}$

Kinematical end point at $\frac{m_W}{2}$ at LO

The decay width allows to populate the upper tail of the distribution

Sensitivity to soft radiation → double peak at NLO-QCD

The QCD-ISR next-to-leading-log resummation broadens the distribution and cures the sensitivity to soft radiation at the jacobian peak.

In the $p_\perp^\ell$ spectrum the sensitivity to m_W and important QCD features are closely intertwined

m_W determination at hadron colliders: template fitting

Given one experimental kinematical distribution

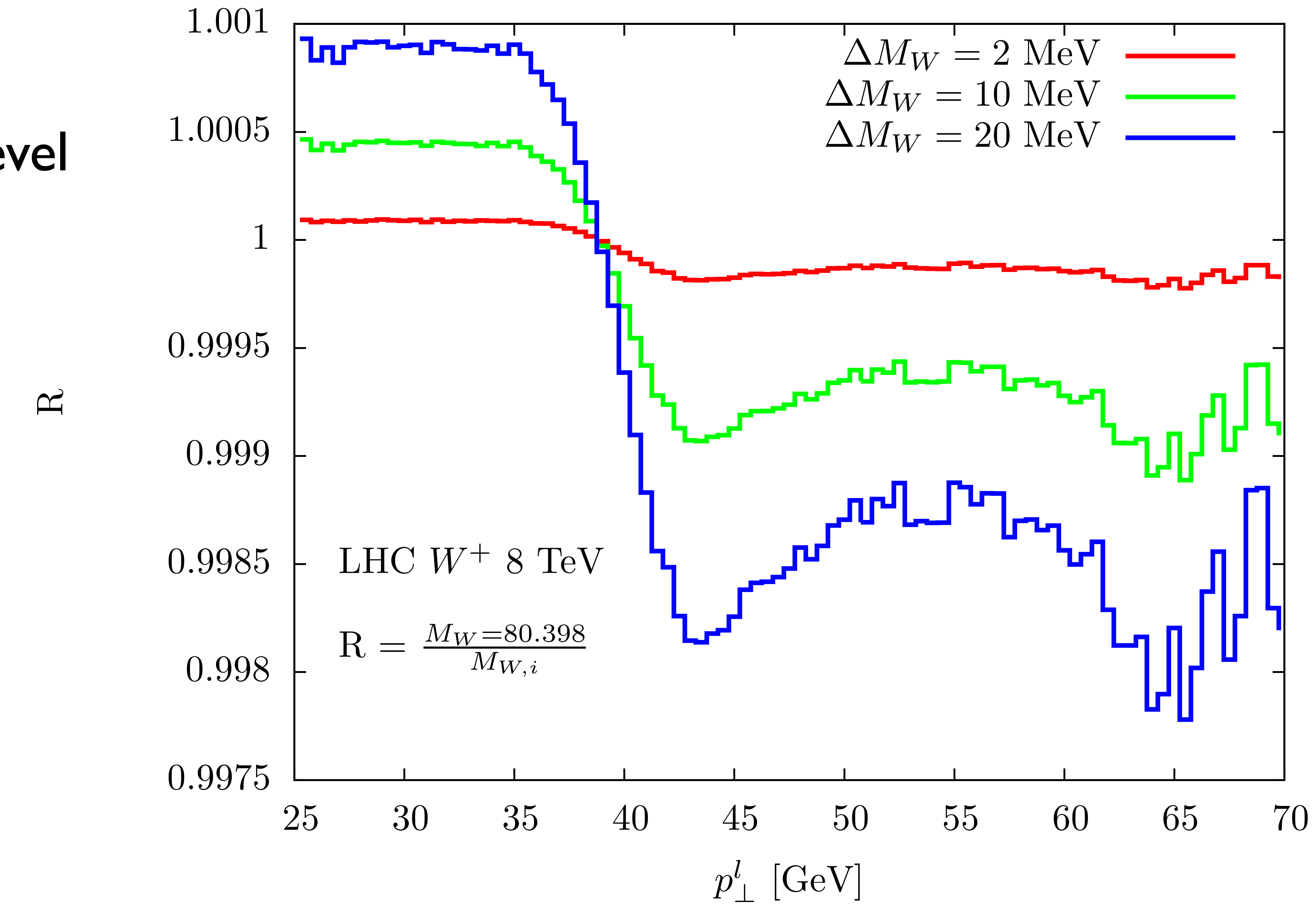
- we compute the corresponding theoretical distribution for several hypotheses of one Lagrangian input parameters (e.g. m_W)
- we compute, for each $m_W^{(k)}$ hypothesis, a χ_k^2 defined in a certain interval around the jacobian peak (fitting window)
- we look for the minimum of the χ^2 distribution

The m_W value associated to the position of the minimum of the χ^2 distribution is the experimental result

A determination at the 10^{-4} level requires a control over the shape of the distributions at the per mille level

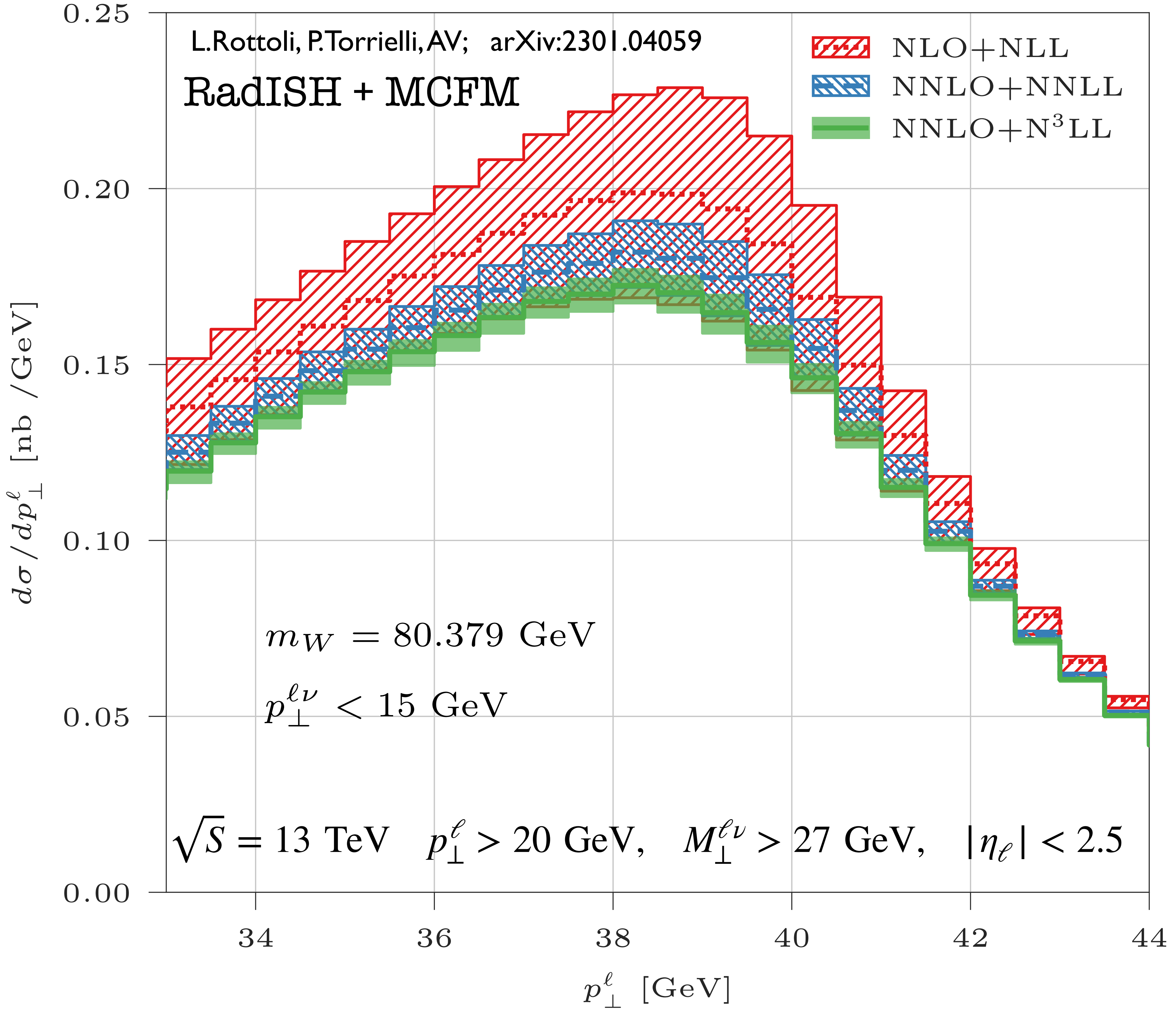
The theoretical uncertainties of the templates contribute to the theoretical systematic error on m_W

- higher-order QCD
- non-perturbative QCD
- PDF uncertainties
- heavy quarks corrections
- EW corrections



Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality



Scale variation of the NNLO+N3LL prediction for ptlep
provides a set of equally good templates
but the width of the uncertainty band is at the few percent level
a factor 10 larger than the naive estimate would require !

→ data driven approach
a Monte Carlo event generator is tuned to the data in NCDY ($p_\perp^Z$)
for one QCD scale choice

↓

the same parameters are then used to prepare the CCDY templates

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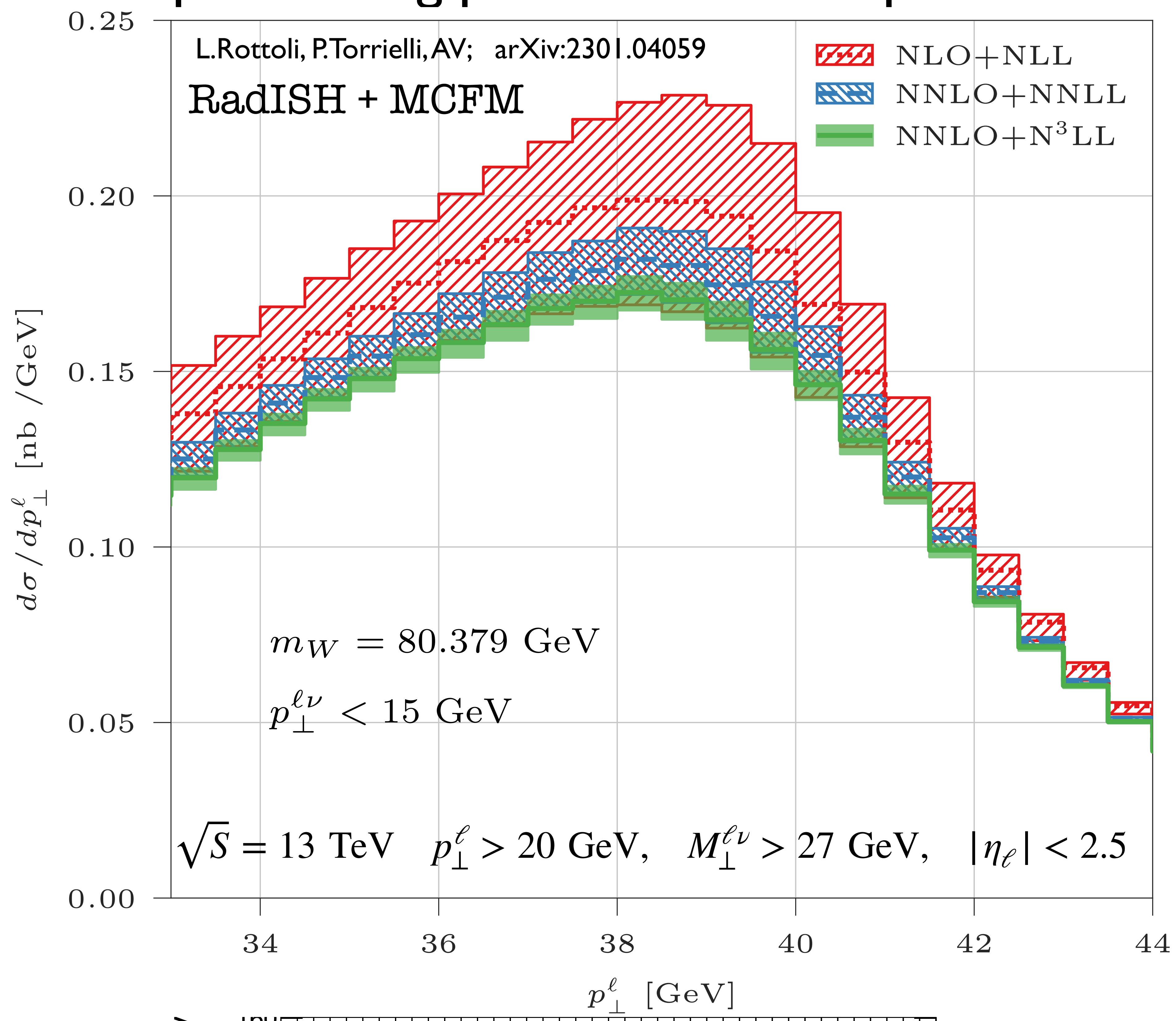

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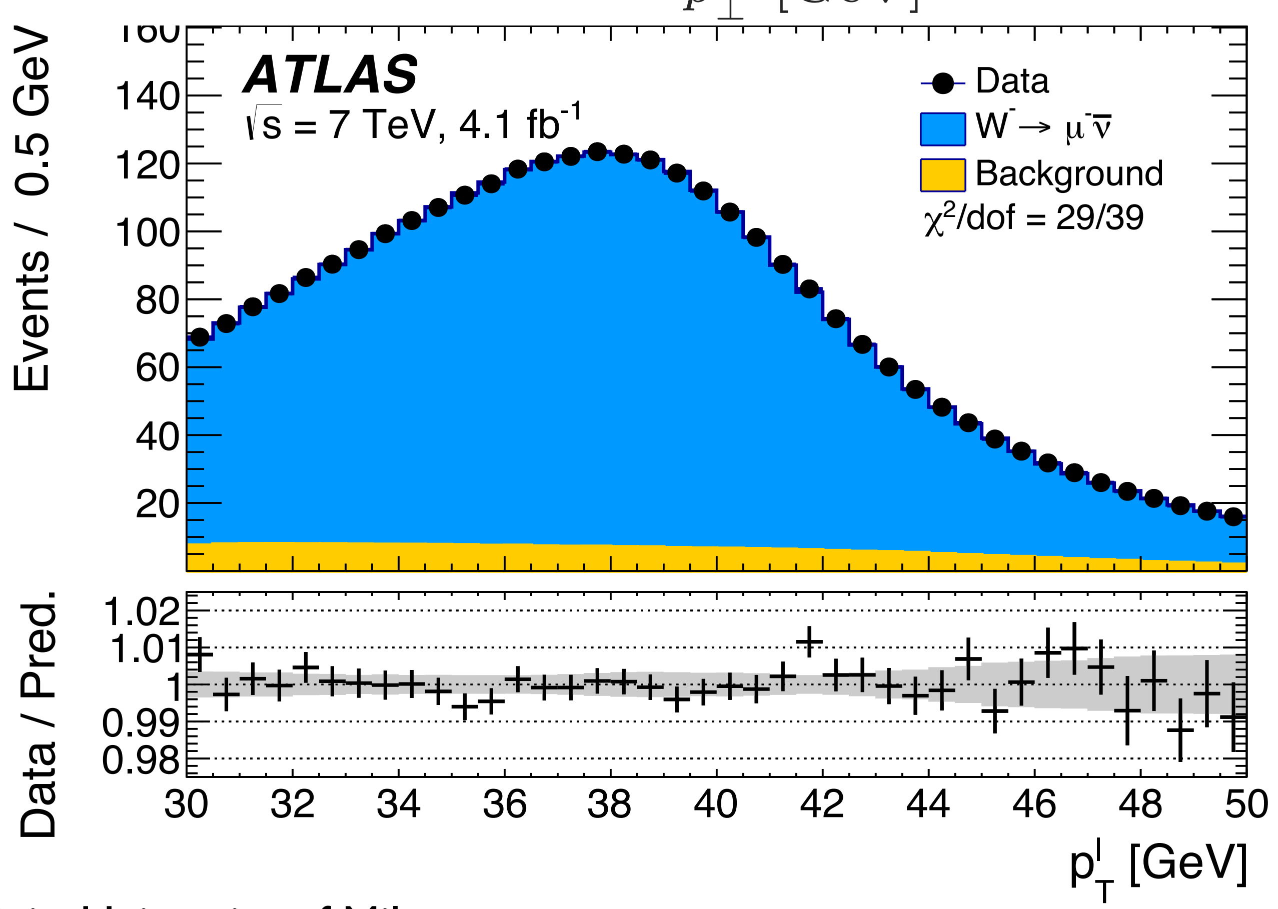
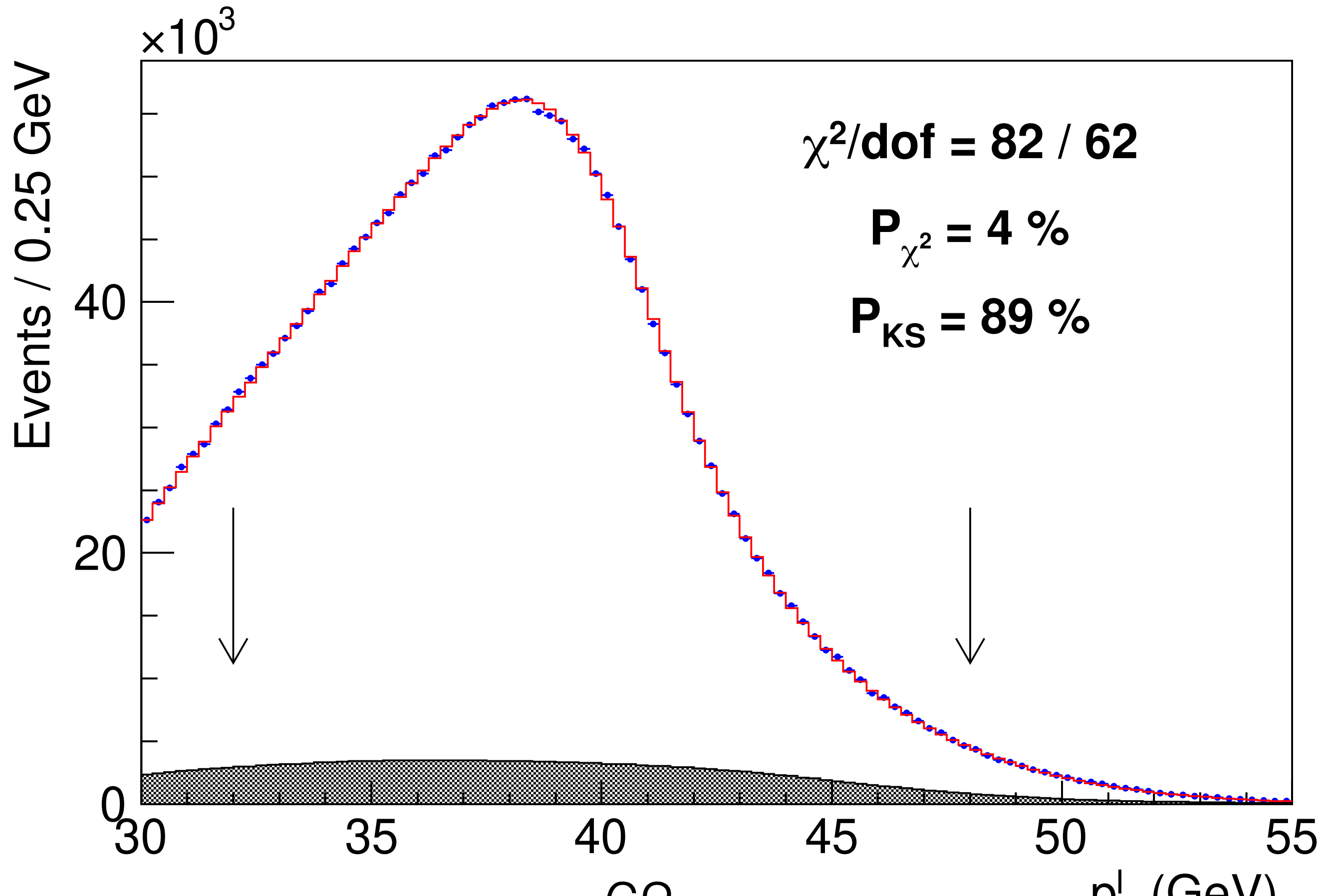
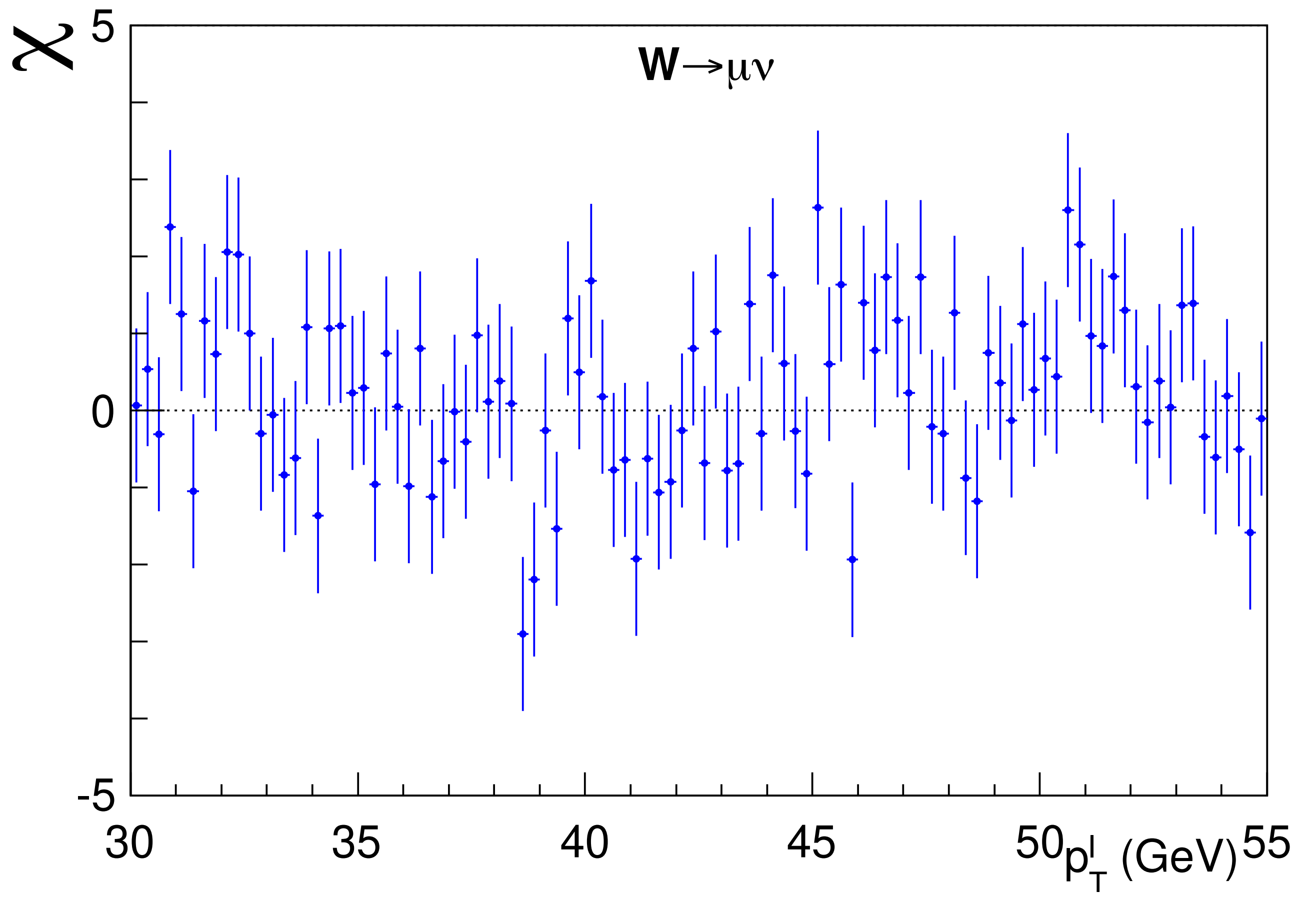


CDF collaboration, Scince 376, 170-176 (2022)





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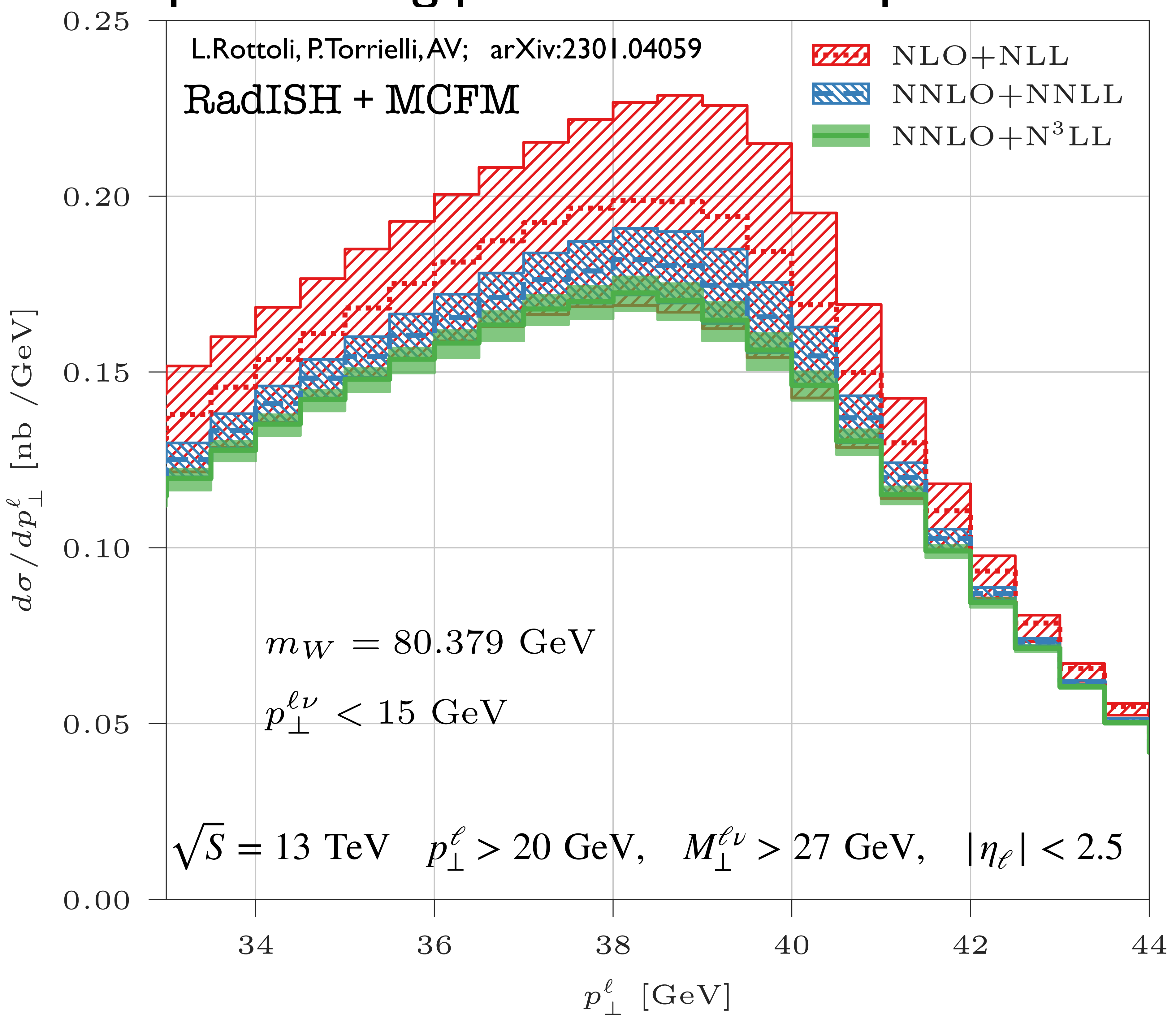

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↓

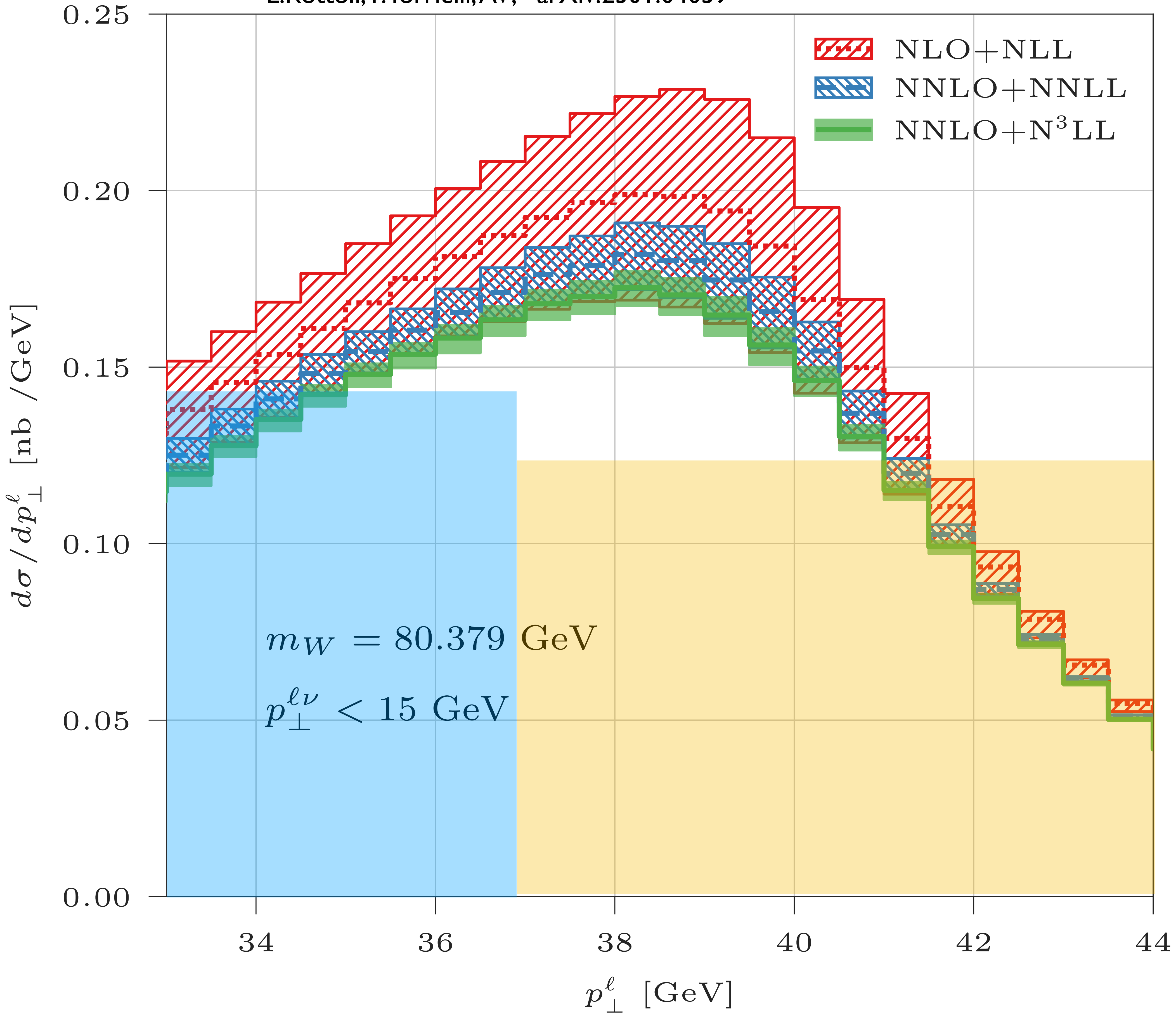
the same parameters are then used to prepare the CCDY templates

A data driven approach improves the accuracy of the model (i.e. its ability to describe the data)
does not improve the precision of the model (the intrinsic ambiguities in the model formulation)

What are the limitations of the transfer of information from NCDY to CCDY ?

The jacobian asymmetry $\mathcal{A}_{p_\perp^\ell}$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



$$L_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell,\min}}^{p_\perp^{\ell,\mathrm{mid}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}, \qquad U_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell,\mathrm{mid}}}^{p_\perp^{\ell,\max}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

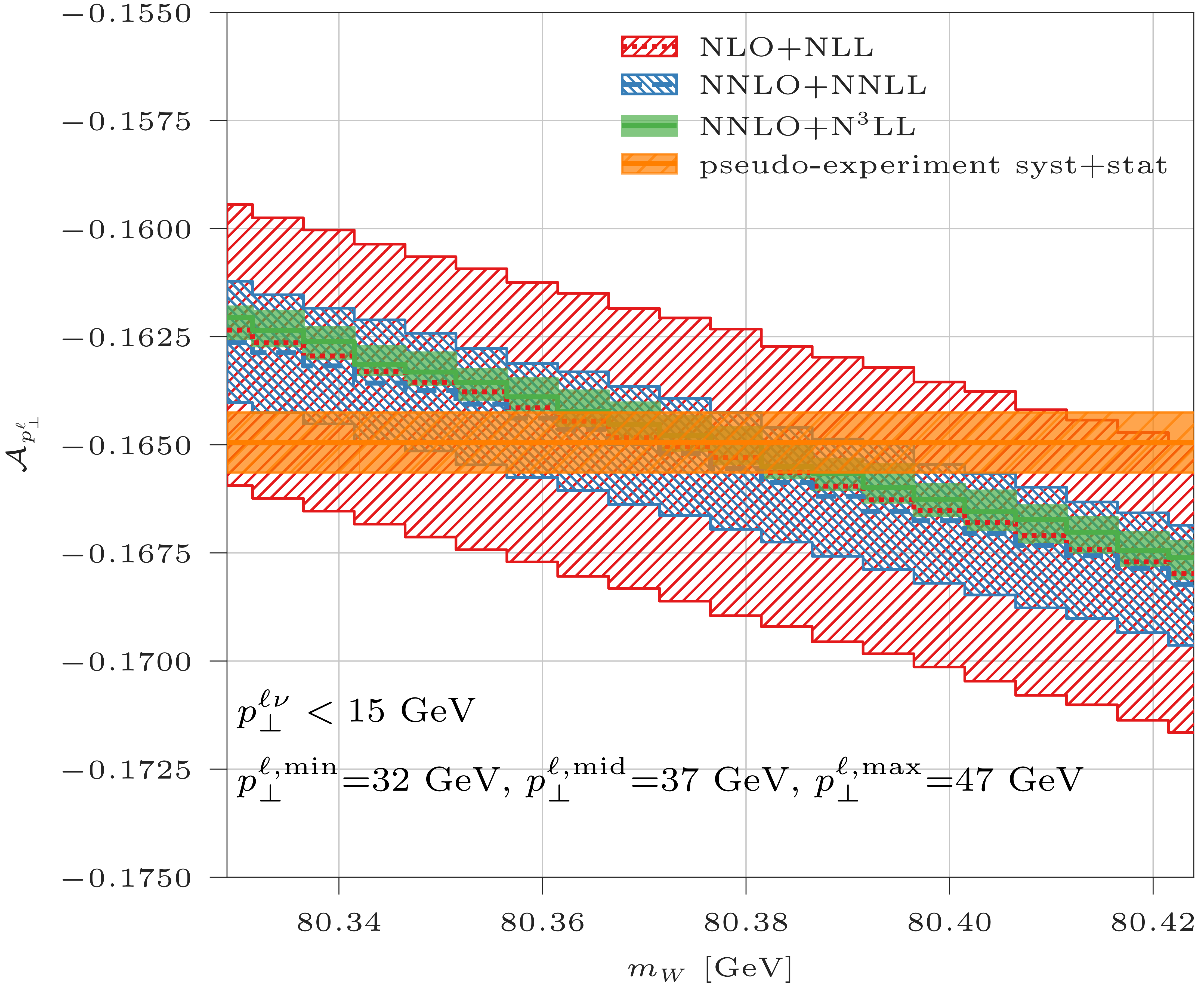
$$\mathcal{A}_{p_\perp^\ell}(p_\perp^{\ell,\min}, p_\perp^{\ell,\mathrm{mid}}, p_\perp^{\ell,\max}) \equiv \frac{L_{p_\perp^\ell} - U_{p_\perp^\ell}}{L_{p_\perp^\ell} + U_{p_\perp^\ell}}$$

The asymmetry is an observable (i.e. it is measurable via counting): its value is one single scalar number
It depends only on the edges of the two defining bins

Increasing m_W shifts the position of the peak to the right $\rightarrow$ Events migrate from the blue to the orange bin
$\rightarrow$ The asymmetry decreases

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$ as a function of m_W

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



The asymmetry $\mathscr{A}_{p_\perp}$ has a linear dependence on m_W, stemming from the linear dependence on the end-point position

The slope of the asymmetry expresses the sensitivity to m_W , in a given setup $(p_\perp^{\ell,min}, p_\perp^{\ell,mid}, p_\perp^{\ell,max})$

The slope is the same with every QCD approximation (factorization of QCD effects, perturbative and non-perturbative)

The "large" size of the two bins $\mathcal{O}(5-10)$ GeV leads to
- small statistical errors
- excellent stability of the QCD results (inclusive quantity)
- ease to unfold the data to particle level (m_W combination)

The experimental value and the theoretical predictions can be directly compared (m_W from the intersection of two lines)

The main systematics on the two fiducial cross sections is related to the lepton momentum scale resolution