

QCD CORRECTIONS IN A CLASS OF SPONTANEOUS CP -VIOLATING MODELS

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We present a study of QCD corrections in a class of spontaneous CP -violating models. We find that the Higgs fields which are responsible for the CP -violating transitions should be one order of magnitude heavier than what is expected from tree order estimates. This implies so large a self-coupling of the Higgs fields that it makes the use of perturbation expansion questionable.

1. Introduction. The theoretical understanding of the CP -violation phenomenon, first seen in the neutral K system, still remains an open challenge. Thus, although many phenomenologically acceptable models have been proposed so far, a complete theory is still lacking. Among such proposals we mention the Kobayashi–Maskawa (KM) mechanism [1]^{#1}, models based upon left–right symmetries [2], Higgs induced interactions [4,5], horizontal interactions [6] and supersymmetry [7]^{#1}. However, current experimental results have put several of the above models under strain [1]^{#1}. More accurate experiments will be needed before a definite pattern emerges.

Here we shall reexamine a class of models, in which the electroweak lagrangian, being CP -conserving, breaks down spontaneously and the source of CP violation is the neutral Higgs particle exchange, which in addition induces flavour-changing neutral currents (FCNC). The strength of FCNC is constrained by the

smallness of the K_L – K_S mass difference and experimental limits on other flavour-changing interactions. Such constraints could naturally be satisfied by extending the principle of natural flavour conservation (NFC) to the Higgs sector. The requirement of spontaneous CP violation and NFC has been shown [5] to lead to real KM matrix elements leaving the theory with only one source of CP violation. However, this attractive possibility is confronted in a large class of models with the experimental data on the value of ϵ (for a possible way out see ref. [8]). Thus the requirement of NFC due to Higgs exchange should be relaxed. Consequently, Higgs particles responsible for such transitions must be very heavy, a fact which could make the use of perturbation expansion questionable because of the resulting large self-coupling of the Higgs field. The purpose of the present work is the calculation of the leading order QCD corrections to the flavour-changing interactions, and the resulting reevaluation of the Higgs masses.

2. The model. Our analysis is quite model-independent as will be seen soon. However, we find the discussion more instructive within a specific model while the generalization to any other of the same class is self-evident.

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#1 For status reports see ref. [2].

We shall be using the model proposed by Branco and Sanda [9]. According to this model, one introduces three Higgs doublets in the standard $SU(2)_L \times U(1)$ model and imposes a Z_3 symmetry which lead to the following Yukawa couplings:

$$\mathcal{L}_Y = \sum_{j,k} (g_{jk} \bar{\psi}_{jL} \phi_k n_{kR} + h_{jk} \bar{\psi}_{jL} \tilde{\phi}_k p_{kR}) + \text{h.c.}, \quad (1)$$

where $\tilde{\phi}_k = i\sigma_2 \phi_k^*$. The $\psi_{jL} = (p_j, n_j)_L$ are $SU(2)$ doublets, while the right-handed components are singlets. The Yukawa couplings have been chosen to be real, so that CP is a good symmetry of the lagrangian. The form (1) of the Yukawa couplings is dictated by the following Z_3 symmetry:

$$\begin{aligned} \phi_k &\rightarrow \exp(i2\pi k/3)\phi_k, & n_{kR} &\rightarrow \exp(-i2\pi k/3)n_{kR}, \\ p_{kR} &\rightarrow \exp(i2\pi k/3)p_{kR}. \end{aligned} \quad (2)$$

After spontaneous symmetry breaking the neutral components of the Higgs doublets acquire VEVs

$$\langle \phi_k^0 \rangle = (v_k/\sqrt{2})e^{i\theta_k}, \quad (3)$$

and one obtains the following mass terms of the up- and down-quarks:

$$\begin{aligned} \sum_{j,k} \frac{1}{\sqrt{2}} (\bar{n}_{jL} g_{jk} v_k e^{i\theta_k} n_{kR} + \bar{p}_{jL} h_{jk} v_k e^{-i\theta_k} p_{kR}) \\ + \text{h.c.} \end{aligned} \quad (4)$$

One can easily notice that the phases introduced by the VEV, since they depend on the right-handed quark field indices only, can be removed by adjusting the phases of these right-handed fields. The mass matrices can be diagonalized by means of the following transformations:

$$\begin{aligned} n_{jK}^L &= (O_L^d)_{jm} d_{mL}, & p_{jL} &= (O_L^u)_{jm} u_{mL}, \\ n_{kR} &= (K O_R^d)_{kn} d_{nR}, & p_{kR} &= (K^{-1} O_R^u)_{kn} u_{nR}, \end{aligned} \quad (5)$$

where $O_{R,L}^{d,u}$ denote orthogonal matrices and $K_{kl} = \delta_{kl} e^{-i\theta_k}$. The weak left-handed charged current is given by

$$J_{\mu L} = \bar{u}_{iL} \gamma_\mu (O_c)_{ij} d_{jL}, \quad (6)$$

where $O_c = (O_L^u)^T O_L^d$. Expression (6) shows that the vector gauge interactions are CP invariant. The only source of CP nonconservation is the Higgs interaction. There are two pairs of charged Higgs fields which

in general lead to an effective CP -violating interaction due to box-diagram graphs involving one and two charged Higgs bosons. Since the charged Higgs are expected to be as heavy as the neutral ones then the dominant contribution to CP violation in the K system comes from tree order exchanges of five neutral Higgses which are left as physical particles after the spontaneous symmetry breaking. (For details on SSB see ref. [9].) These give neutral Higgses couple in general to flavour-changing neutral currents. Let us consider their contribution to the $K_L - K_S$ mass difference.

A general $\Delta S = 2$ effective interaction lagrangian, generated by Higgs exchange, is given by

$$\mathcal{L}_{\Delta S=2} = \sum_{a=1}^5 -\frac{G_F}{\sqrt{2}M_a^2} (\xi_1^a \bar{s}_L d_R + \xi_2^a \bar{s}_R d_L)^2 + \text{h.c.}, \quad (7)$$

where M_a are the masses of the five neutral Higgses and ξ_1, ξ_2 have dimensions of mass and are functions of the mixing angles entering in O_L^d, O_R^d and in the re-definition of the Higgs fields after SSB.

According to the above effective lagrangian the lowest order graphs responsible for the $\bar{s}d \leftrightarrow \bar{d}s$ transition are shown in fig. 1 corresponding to the amplitude

$$\begin{aligned} \mathcal{M}_0 = -\frac{G_F}{\sqrt{2}} \sum_{a=1}^5 \frac{1}{M_a^2} [(\xi_1^a)^2 (\bar{s}_L d_R)(\bar{s}_L d_R) \\ + (\xi_2^a)^2 (\bar{s}_R d_L)(\bar{s}_R d_L) + 2\xi_1 \xi_2 (\bar{s}_R d_L)(\bar{s}_L d_R)]. \end{aligned} \quad (8)$$

3. *QCD corrections.* We are interested in determining the effect of QCD interactions to the above process. The following six dimension-4 quark operators appearing in the operator product expansion (OPE) are relevant

$$O_1^R = (\bar{s}_L d_R)(\bar{s}_L d_R), \quad O_2^R = (\bar{s}_L \lambda^a d_R)(\bar{s}_L \lambda^a d_R), \quad (9)$$

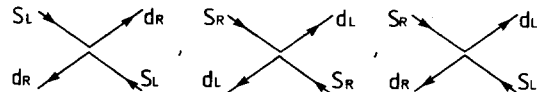


Fig. 1. Lowest order graphs responsible for the $\bar{s}d \leftrightarrow \bar{d}s$ transition.

$$O_1^L = (\bar{s}_R d_L)(\bar{s}_R d_L), \quad O_2^L = (\bar{s}_R \lambda^a d_L)(\bar{s}_R \lambda^a d_L),$$

$$O_1^{LR} = (\bar{s}_R d_L)(\bar{s}_L d_R), \quad O_2^{LR} = (\bar{s}_R \lambda^a d_L)(\bar{s}_L \lambda^a d_R),$$

(9 cont'd)

where $\lambda^a (a = 1, \dots, 8)$ are the usual SU(3) colour matrices ($\text{Tr} \lambda^a \lambda^b = \frac{1}{2} \delta^{ab}$). Higher order gluon exchange to the above operator will generate the following structure

$$O_3^{R(L)} = (\bar{s}_{L(R)} \sigma^{\mu\nu} d_{R(L)}) (\bar{s}_{L(R)} \sigma_{\mu\nu} d_{R(L)}),$$

$$O_3^{LR} = (\bar{s}_R \sigma^{\mu\nu} d_L) (\bar{s}_L \sigma_{\mu\nu} d_R),$$

$$O_4^{R(L)} = (\bar{s}_{L(R)} \sigma^{\mu\nu} \lambda^a d_{R(L)}) (\bar{s}_{L(R)} \sigma_{\mu\nu} \lambda^a d_{R(L)}),$$

$$O_4^{LR} = (\bar{s}_R \sigma^{\mu\nu} \lambda^a d_L) (\bar{s}_L \sigma_{\mu\nu} \lambda^a d_R), \quad (10)$$

where $\sigma^{\mu\nu} = \frac{1}{2} i [\gamma_\mu, \gamma_\nu]$. However, the operators $O_3^{R(L)}$ and $O_4^{R(L)}$ can be expressed as linear combinations of the above operators $O_1^{R(L)}$ and $O_2^{R(L)}$ by means of the following Fierz-rearrangement formulae

$$(\sigma^{\mu\nu} a) (\sigma_{\mu\nu} a) = 8(a) \otimes (a) - 4(a)(a),$$

$$(\lambda^a) (\lambda^a) = \frac{4}{3}(I) \otimes (I) - \frac{1}{3}(\lambda^b) \otimes (\lambda^b),$$

$$(I)(I) = \frac{1}{3}(I) \otimes (I) + 2(\lambda^a) \otimes (\lambda^a), \quad (11)$$

where $a = \frac{1}{2}(1 - \gamma_5)$ and I is the unit colour matrix. The symbol $(A)(A)$ means $(A)_{ij}(A)_{kj}$ while $(A) \otimes (A)$ means $(A)_{ij}(A)_{kj}$. Obviously the same relations are valid for $a = \frac{1}{2}(1 + \gamma_5)$.

Taking into account the fermionic nature of the spinor fields, one finally obtains

$$O_3^{R(L)} = -\frac{20}{3} O_1^{R(L)} - 16 O_2^{R(L)},$$

$$O_4^{R(L)} = -\frac{32}{9} O_1^{R(L)} - \frac{4}{3} O_2^{R(L)}. \quad (12)$$

Hence the systems $(O_1^{R(L)}, O_2^{R(L)})$ are separately closed under renormalization.

For O_3^{LR} and O_4^{LR} the situation is even simpler. The relations

$$(\sigma^{\mu\nu} a) (\sigma_{\mu\nu} a') = 8(a) \otimes (a') - 4(\gamma^\lambda a) (\gamma_\lambda a'),$$

$$(\gamma^\mu a) \otimes (\gamma_\mu a') = 2(a')(a),$$

$$(\gamma^\mu a') \otimes (\gamma_\mu a) = 2(a)(a'), \quad (13)$$

where $a' = \frac{1}{2}(1 + \gamma_5)$, show explicitly that $O_1^{LR} O_2^{LR}$

are multiplicatively renormalized. Moreover, since O_2^{LR} does not appear in the lowest order it can safely be dropped.

The effective lagrangian takes the form

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_{a=1}^5 \frac{1}{M_a^2}$$

$$\times \{ \xi_1^{a^2} [C_1^R \langle O_1^R(\mu^2, g^2, p^2) \rangle + C_2^R \langle O_2^R(\mu^2, g^2, p^2) \rangle] + \xi_2^{a^2} [C_1^L \langle O_1^L(\mu^2, g^2, p^2) \rangle + C_2^L \langle O_2^L(\mu^2, g^2, p^2) \rangle] + 2 \xi_1^a \xi_2^a C_1^{LR} \langle O_1^{LR}(\mu^2, g^2, p^2) \rangle \} + \text{h.c.}, \quad (14)$$

where g^2, μ^2 and p^2 are the QCD coupling, the renormalization point and an infrared cut-off respectively. The C -coefficients depend on μ^2, g^2, M^2 . All the operator matrix elements are to be evaluated between hadronic states.

Note that the Higgs-fermion couplings are strongly renormalized by the colour forces. However, since the same couplings are being used to fermion mass spectrum through SSB, these renormalizations can be fixed from the beginning. Thus, whenever we use g_y we shall tacitly assume that strong corrections have already been taken into account.

The renormalization group equations (RGE) that the C -coefficients obey are

$$[\mu \partial / \partial \mu + b(g) \partial / \partial g + \gamma_J^T] C^J = 0, \quad J = R, L, LR, \quad (15)$$

where $b(g)$ is the β -function associated with QCD coupling constant and γ_R^T, γ_L^T and γ_{LR}^T are the transpose matrices of the anomalous dimensions of the operators $(O_1^R, O_2^R), (O_1^L, O_2^L)$ and the anomalous dimension of O_1^{LR} , respectively. The lowest order anomalous dimension matrices can be found by evaluating the infinite part of the one-loop diagrams shown in fig. 2. The anomalous dimension matrix of the operators O_1^L and O_2^L is the same as the corresponding matrix of the operator O_1^R and O_2^R . Working in the Landau



Fig. 2. QCD corrections to the graphs of fig. 1.

gauge, where the fermion wave-function renormalization constant, and consequently the anomalous dimension of the fermion field, is zero, we obtain

$$\gamma_R = \gamma_L = \frac{g^2}{16\pi^2} \begin{pmatrix} \frac{80}{9} & -\frac{8}{3} \\ -\frac{160}{27} & -\frac{92}{9} \end{pmatrix} \quad (16)$$

with eigenvalues $\gamma_1 = -(g^2/16\pi^2)9.68$ and $\gamma_2 = (g^2/16\pi^2)11.02$ and for the anomalous dimension of O_1^{LR}

$$\gamma_3 = -(g^2/16\pi^2)16. \quad (17)$$

Now the effective lagrangian is given by [10]

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{G_F}{\sqrt{2}} \sum_{a=1}^5 \frac{1}{M_a^2} \{ \xi_1^{a^2} C_R^T S^{-1} W S O_R \\ & + \xi_2^{a^2} C_L^T S^{-1} W S O_L + 2\xi_1^a \xi_2^a C_{LR} W_{LR} O_{LR} \} + \text{h.c.}, \end{aligned} \quad (18)$$

where $C_{R(L)}^T = (1, 0)$ and $C_{LR} = 1$ are the zeroth order C -coefficients, S is a matrix which diagonalizes γ_L ($S\gamma_L S^{-1} = \gamma_{\text{diagonal}}$), W is the diagonal matrix

$$W = \text{diag} \left(\exp \int \frac{dg'}{g'} \frac{\gamma_1}{b_0}, \exp \left(\int \frac{dg'}{g'} \frac{\gamma_2}{b_0} \right), \right) \quad (19)$$

W_{LR} is given by an analogous form

$$W_{LR} = \exp \left(\int \frac{dg'}{g'} \frac{\gamma_3}{b_0} \right), \quad (20)$$

and O_R^T, O_L^T are the row vectors of the operators

$$O_R^T = (O_1^R, O_2^R), \quad O_L^T = (O_1^L, O_2^L). \quad (21)$$

Finally \mathcal{L}_{eff} takes the form

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{G_F}{\sqrt{2}} \sum_{a=1}^5 \frac{1}{M_a^2} \\ & \times \{ \xi_1^{a^2} [(\bar{g}^2(M_a^2)/g^2(\mu^2))^{\gamma_1/2b_0} \hat{O}_1^R \\ & + (\bar{g}^2(M_a^2)/g^2(\mu^2))^{\gamma_2/2b_0} \hat{O}_2^R] \\ & + \xi_2^{a^2} [(\bar{g}^2(M_a^2)/g^2(\mu^2))^{\gamma_1/2b_0} \hat{O}_1^L \\ & + (\bar{g}^2(M_a^2)/g^2(\mu^2))^{\gamma_2/2b_0} \hat{O}_2^L] \\ & + 2\xi_1^a \xi_2^a (\bar{g}^2(M_a^2)/g^2(\mu^2))^{\gamma_3/2b_0} O_{LR} \} + \text{h.c.}, \end{aligned} \quad (22)$$

where $O_1^{R(L)} = O_1^{R(L)} - 0.13O_2^{R(L)}$, $O_2^{R(L)} = 0.30O_1^{R(L)} + O_2^{R(L)}$ are the eigenvectors of the anomalous dimension matrix.

4. Results and conclusions. In order to use the vacuum-insertion approximation [11] to the effective interactions (22) we recall that

$$\begin{aligned} \langle \bar{K}^0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_\mu \gamma_5 d | K^0 \rangle &= f_K^2 m_K^2 \equiv A, \\ \langle \bar{K}^0 | \bar{s} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | \bar{K}^0 \rangle &= -f_K^2 m_K^4 / (m_s + m_d)^2 \equiv B. \end{aligned} \quad (23)$$

In the vacuum-insertion approximation we find

$$\begin{aligned} \langle \bar{K}^0 | \hat{O}_1 | K^0 \rangle &= -0.431B, \quad \langle \bar{K}^0 | \hat{O}_2 | K^0 \rangle = -0.014B, \\ \langle \bar{K}^0 | O_{LR} | K^0 \rangle &= -2B + \frac{1}{3}A. \end{aligned} \quad (24)$$

The CP -noninvariant contribution to $K^0 - \bar{K}^0$ transition due to Higgs exchanges in tree order is

$$\begin{aligned} \langle \bar{K}^0 | \mathcal{L}^{CP-} | K^0 \rangle = & -\frac{G_F}{\sqrt{2}} \sum_{a=1}^5 \frac{1}{M_a^2} \text{Im} \{ [\frac{5}{12} (\xi_1^{a^2} + \xi_2^{a^2}) \\ & - \xi_1^a \xi_2^a] B + \frac{1}{6} \xi_1^a \xi_2^a A + \text{h.c.} \}. \end{aligned} \quad (25)$$

The dominant contribution to the CP -conserving part of $K^0 - \bar{K}^0$ transition is still expected to come from double- W exchange because $M_a \gg M_W$ [11]. An estimate of the Higgs boson mass has been obtained [9] by assuming that (25) is dominated by the contribution of the lightest Higgs boson. Then one obtains approximately

$$\langle \bar{K}^0 | \mathcal{L}^{CP-} | K^0 \rangle \simeq -(G_F/\sqrt{2}) B (m_s^2/M_H^2) r^2, \quad (26)$$

where r is a model-dependent suppression factor.

From (26), the known contribution of $W-W$ and the experimental value of $|\epsilon|$ one obtains

$$M_H \simeq (30r) \text{ TeV}. \quad (27)$$

Thus a suppression factor

$$r = 3 \times 10^{-2} \quad (28)$$

is needed in order for M_H to not exceed the critical value of ~ 1 TeV. Beyond this value the perturbation expansion and therefore all the analysis of spontaneous CP violation does not hold.

We find that the QCD corrected CP -noninvariant contribution to $K^0 - \bar{K}^0$ transition due to Higgs exchange is:

$$\begin{aligned}
\langle \bar{K}^0 | \mathcal{L}^{CP} | K^0 \rangle = & -\frac{G_F}{\sqrt{2}} \sum_{a=1}^5 \frac{1}{M_a^2} \text{Im} \{ (\xi_1^{a^2} + \xi_2^{a^2}) \\
& \times [0.431(\bar{g}^2(M_a^2)/g^2(\mu^2))^{0.69} \\
& + 0.014(\bar{g}^2(M_a^2)/g^2(\mu^2))^{-0.79}] \\
& - \xi_1^a \xi_2^a (\bar{g}^2(M_a^2)/g^2(\mu^2))^{1.14} \} B \\
& + \frac{1}{6} \xi_1^a \xi_2^a (\bar{g}^2(M_a^2)/g^2(\mu^2))^{1.14} A + \text{h.c.} \}, \quad (29)
\end{aligned}$$

where $\mu \sim 1$ GeV. Within the same approximations that eqs. (27), (28) have been obtained we find that when QCD corrections are taken into account

$$M_{\text{H QCD}} \simeq (500r) \text{ TeV}, \quad r \simeq 2 \times 10^{-3}. \quad (30)$$

This result being one order of magnitude different from the tree order one rules out the models proposed so far in which the only source of CP violation is the heavy Higgs exchange. Though we have used the framework of a particular model it is clear that the size of the corrections is model-independent in this class of models. Of course in eqs. (25), (29) various unknown mixing angles enter. Therefore it is in principle possible that somebody could construct a clever model which could take care of a suppression factor of order 10^{-3} . However, what our calculation suggests is that whenever, in a reasonable approximation, a heavy Higgs (1–100 TeV) is needed in tree order to account for CP violation the QCD corrected calculation requires a Higgs at least one order of magnitude (10–13 times) heavier.

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References

- [1] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* 49 (1973) 652.
- [2] A.J. Buras, *Proc. Workshop on the Future of intermediate energy physics in Europe* (1984) p. 53; C. Jarlskog, *Proc. Workshop on the Electroweak interactions and particle structure* (DESY, 1984) p. 98.
- [3] G.C. Branco, J.-M. Frère and J.-M. Gérard, *Nucl. Phys.* B221 (1983) 317, and references therein; D. Chang, *Nucl. Phys.* B214 (1983) 435; G. Ecker and W. Grimus, *Wien preprint UWTRPH-1984-47*.
- [4] T.D. Lee, *Phys. Rev.* D8 (1973) 1226; S. Weinberg, *Phys. Rev. Lett.* 37 (1976) 657; A.B. Lahanas and C.E. Vayonakis, *Phys. Rev.* D19 (1979) 1258; G.C. Branco, A.J. Buras and J.-M. Gérard, *Max-Planck-Institut preprint MPI-PAE/PTh 4/85*.
- [5] G.C. Branco, *Phys. Rev. Lett.* 44 (1980) 504.
- [6] T. Maehara and T. Yanagida, *Prog. Theor. Phys.* 60 (1978) 822; 61 (1979) 1434; A. Davidson, M. Koca and K.C. Wali, *Phys. Rev. Lett.* 43 (1979) 92; V.A. Monich, B.V. Struminsky and G.G. Volkov, *Phys. Lett.* 104B (1981) 382; G. Zoupanos, *Phys. Lett.* 115B (1982) 221; M.B. Gavela and H. Georgi, *Phys. Lett.* 119B (1982) 141; R. Decker, J.-M. Gérard and G. Zoupanos, *Phys. Lett.* 137B (1984) 83.
- [7] See e.g., J.-M. Gérard, W. Grimus, A. Raychaudhuri and G. Zoupanos, *Phys. Lett.* 140B (1984) 349; P. Langacker and B. Sathiapalan, *Phys. Lett.* 144B (1984) 401.
- [8] G.C. Branco, A.J. Buras and J.-M. Gérard, *Phys. Lett.* 155B (1985) 192.
- [9] G.C. Branco and A.I. Sanda, *Phys. Rev.* D26 (1982) 3176.
- [10] A.J. Buras, *Rev. Mod. Phys.* 52 (1980) 199; N.D. Vlachos, Ph.D. thesis (Sussex University, 1981); N.D. Tracas and N.D. Vlachos, *Phys. Lett.* 115 (1982) 41.
- [11] M.K. Gaillard and B.W. Lee, *Phys. Rev.* D10 (1974) 89.