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New results in models with reduced couplings

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The reduction of couplings concept consists in searching for renormalization group invariant relations among parameters that hold to all orders in perturbation theory. This technique has been applied to $N = 1$ supersymmetric Grand Unified Theories, some of

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which can become finite to all loops. We review the basic idea and tools, as well as two theories in which reduction of couplings has been applied: (i) an all-loop finite $N = 1$ $SU(5)$ model and (ii) a reduced version of the Minimal Supersymmetric Standard Model. The finite model exhibits high relic abundance of cold dark matter, while, on the contrary, the second model has underproduction in the early universe. For each model we select three representative benchmark scenarios. The heavy Higgs and supersymmetric spectrum of the finite $SU(5)$ model lies beyond the reach of the 14 TeV HL-LHC, while large parts of the predicted spectrum can be tested in the 100 TeV FCC-hh, although the higher mass regions are beyond its reach. On the other hand, the Reduced Minimal Supersymmetric Standard Model (Reduced MSSM) is found to be ruled out by LHC searches for heavy neutral MSSM Higgs bosons.

Keywords: Reduction of couplings; finiteness; supersymmetry; Higgs; dark matter.

1. Introduction

The reduction of couplings scheme[1–4] (see also Refs. 5–7) is a promising idea which relates parameters of a renormalizable theory to a single coupling. The method requires the resulting relation among the parameters to be valid at all energy scales, i.e. Renormalization Group Invariant (RGI). In order to achieve a reduction of the number of free parameters of the Standard Model (SM) the introduction of an extra symmetry was proposed, and in particular a Grand Unified Theory (GUT).[8–13] Then the next step was the unification of the gauge and Yukawa sectors [Gauge Yukawa Unification (GYU)]. This was the main characteristic of the *reduction of couplings* early stage application in $N = 1$ GUTs,[14–27] where RGI relations are set between the GUT scale and the Planck scale. Moreover, RGI relations which guarantee all-loop finiteness can be found. The method predicted the top quark mass in the finite $N = 1$ supersymmetric $SU(5)$ model,[14,15] as well as in the Reduced Minimal $N = 1$ supersymmetric $SU(5)$,[16] 1 year before its experimental discovery.[28]

Supersymmetry (SUSY) is an essential ingredient of the reduction of couplings idea, and thus a soft supersymmetry breaking (SSB) sector has to be included, which involves couplings with nonzero mass dimension. It has been shown that one can achieve complete all-loop finite models, i.e. including the SSB sector. The all-loop finite $N = 1$ supersymmetric $SU(5)$ model[29,30] has given a prediction for the light Higgs-boson mass in agreement with the experimental results[31–33] and a heavy SUSY mass spectrum, consistent with the experimental nonobservation of these particles. In the past two decades the reduction of couplings technique has been applied to many cases, including a reduced version of the minimal $N = 1$ supersymmetric $SU(5)$[16] and a reduced version of the $N = 1$ supersymmetric $SU(3)^3$ model.[34–36] The full analyses of the most successful models that include predictions in agreement with the experimental measurements of the top and bottom quark masses for each model can be found in Ref. 37.

In this work, after a brief review of the reduction of couplings and finiteness ideas in Sec. 2, we present the main features of the two models, namely the all-loop finite $N = 1$ $SU(5)$ model and the Reduced Minimal Supersymmetric SM

(Reduced MSSM), in Sec. 3. In Sec. 4, we list the phenomenological constraints used in our analyses and in Secs. 5 and 6 we review the examination of these two models. We briefly present some earlier results of our phenomenological analysis. In this context, the new version of the `FeynHiggs`[38–41] code plays a crucial role, as it was used to calculate the Higgs-boson predictions, in particular the mass of the lightest CP-even Higgs boson. The Cold Dark Matter (CDM) relic density is calculated using the `MicrOMEGAs 5.0` code[42–44] (for a more extensive discussion, see Ref. 45). Concerning the finite model we address the question to what extent the reduction of couplings idea can be experimentally tested at the HL-LHC and the future FCC hadron colliders. To this end we propose three benchmark points. We present the SUSY breaking parameters used as input in each benchmark to calculate the corresponding Higgs and SUSY particle masses using `SPheno`.[46,47] Then, having computed the expected production cross-sections at the 14 TeV (HL-)LHC and the 100 TeV FCC-hh, we investigate which production channels can be observed. The complete analyses for both models (and two more) are included in our recent work.[48] The final section contains a few conclusive remarks.

2. Theoretical Basis

We start with briefly reviewing the core idea of the reduction of couplings method. The target is to single out a basic parameter (which we will call the primary coupling), where all other parameters can be expressed in terms of this one through RGI relations. Such a relation has, in general, the form $\Phi(g_1, \ldots, g_A) = \text{const}$ which should satisfy the following partial differential equation (PDE):

$$\mu \frac{d\Phi}{d\mu} = \boldsymbol{\nabla}\Phi \cdot \boldsymbol{\beta} = \sum_{a=1}^{A} \beta_a \frac{\partial \Phi}{\partial g_a} = 0, \tag{1}$$

where β_a is the β-functions of g_a. The above PDE is equivalent to the following set of ordinary differential equations (ODEs), which are called Reduction Equations (REs),[2–4]

$$\beta_g \frac{dg_a}{dg} = \beta_a, \quad a = 1, \ldots, A-1, \tag{2}$$

where now g and β_g are the primary coupling and its corresponding β-function. There are obviously $A-1$ relations in the form of $\Phi(g_1, \ldots, g_A) = \text{const}$ in order to express all other couplings in terms of the primary one.

The crucial demand is that the above REs admit power series solutions

$$g_a = \sum_n \rho_a^{(n)} g^{2n+1}, \tag{3}$$

which preserve perturbative renormalizability. Without this requirement, we just trade each "dependent" coupling for an integration constant. The power series, which are a set of special solutions, fix that constant. It is very important to point out that the uniqueness of such a solution can be already decided at the one-loop level.[2–4] In supersymmetric theories, where the asymptotic behavior of several parameters are similar, the use of power series as solutions of the REs are justified. But, usually, the reduction is not "complete", which means that not all of the couplings can be reduced in favor of the primary one, leading to the so-called "partial reduction".[49,50]

We proceed to the reduction scheme for massive parameters, which is far from being straightforward. A number of conditions are required (see, for example, Ref. 51). Nevertheless, progress has been achieved, starting from Ref. 52, and finally we can introduce mass parameters and couplings carrying mass dimension[53,54] in the same way as dimensionless couplings. Consider the superpotential

$$W = \frac{1}{2}\,\mu^{ij}\,\Phi_i\,\Phi_j + \frac{1}{6}\,C^{ijk}\,\Phi_i\,\Phi_j\,\Phi_k, \tag{4}$$

and the SSB sector Lagrangian

$$-\mathcal{L}_{\rm SSB} = \frac{1}{6}\,h^{ijk}\,\phi_i\phi_j\phi_k + \frac{1}{2}\,b^{ij}\,\phi_i\phi_j + \frac{1}{2}\,(m^2)^j_i\,\phi^{*\,i}\phi_j + \frac{1}{2}\,M\,\lambda_i\lambda_i + \text{h.c.}, \tag{5}$$

where ϕ_i's are the scalar fields of the corresponding superfields Φ_i's and λ_i are the gauginos. Let us write down some well-known relations:

(i) The β-function of the gauge coupling at one-loop level is given by[55–59]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i T(R_i) - 3\,C_2(G)\right], \tag{6}$$

where $T(R_i)$ is the Dynkin index of the rep R_i where the matter fields belong and $C_2(G)$ is the quadratic Casimir operator of the adjoint rep G.

(ii) The anomalous dimension $\gamma^{(1)}{}^i_j$, at a one-loop level, of a chiral superfield is

$$\gamma^{(1)}{}^i_j = \frac{1}{32\pi^2}[C^{ikl}\,C_{jkl} - 2\,g^2\,C_2(R_i)\delta^i_j], \tag{7}$$

where C^{ijk} are the trilinear (Yukawa) couplings of the corresponding fields that are accommodated in the rep R_i.

(iii) The β-functions of C_{ijk}'s, at one-loop level, following the $N = 1$ nonrenormalization theorem,[60–62] are expressed in terms of the anomalous dimensions of the fields involved

$$\beta_C^{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl}\,\gamma^l_k + C_{ikl}\,\gamma^l_j + C_{jkl}\,\gamma^l_i. \tag{8}$$

We proceed by assuming that the REs admit power series solutions:

$$C^{ijk} = g\sum_{n=0}\rho^{ijk}_{(n)}g^{2n}. \tag{9}$$

Trying to obtain all-loop results we turn to relations among β-functions. The spurion technique[62–66] gives all-loop relations among SSB β-functions.[67–73] Then, assuming that the reduction of C^{ijk} is possible to all orders

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g}, \tag{10}$$

as well as for h^{ijk}

$$h^{ijk} = -M \frac{dC^{ijk}}{d\ln g}, \tag{11}$$

it can be proven[74,75] that the following relations are all-loop RGI:

$$M = M_0 \frac{\beta_g}{g}, \tag{12}$$

$$h^{ijk} = -M_0 \ \beta_C^{ijk}, \tag{13}$$

$$b^{ij} = -M_0 \ \beta_\mu^{ij}, \tag{14}$$

$$(m^2)^i_j = \frac{1}{2} \ |M_0|^2 \ \mu \frac{d\gamma^i{}_j}{d\mu}, \tag{15}$$

where M_0 is an arbitrary reference mass scale to be specified and Eq. (12) is the Hisano–Shifman relation[70] (note that in both assumptions we do not rely on specific solutions of these equations).

As a next step we substitute the last equation, Eq. (15), by a more general RGI sum rule that holds to all orders[76]

$$\begin{aligned} m_i^2 + m_j^2 + m_k^2 = |M|^2 &\left\{ \frac{1}{1 - g^2 C_2(G)/(8\pi^2)} \frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2} \frac{d^2 \ln C^{ijk}}{d(\ln g)^2} \right\} \\ &+ \sum_l \frac{m_l^2 T(R_l)}{C_2(G) - 8\pi^2/g^2} \frac{d\ln C^{ijk}}{d\ln g}, \end{aligned} \tag{16}$$

which leads to the following one-loop relation:

$$m_i^2 + m_j^2 + m_k^2 = |M|^2. \tag{17}$$

Finally, note that in the case of product gauge groups, Eq. (12) takes the form

$$M_i = \frac{\beta_{g_i}}{g_i} M_0, \tag{18}$$

where i denotes the group of the product. This will be used in the Reduced MSSM case.

Consider an $N = 1$ globally supersymmetric gauge theory, which is chiral and anomaly free, where G is the gauge group and g the associated gauge coupling. The theory has the superpotential of Eq. (4), while the one-loop gauge and C_{ijk}s

β-functions are given by Eqs. (6) and (8), respectively, and the one-loop anomalous dimensions of the chiral superfields by Eq. (7).

Demanding the vanishing of all one-loop β-functions, Eqs. (6) and (7) lead to the relations

$$\sum_i T(R_i) = 3C_2(G), \tag{19}$$

$$C^{ikl}C_{jkl} = 2\delta^i_j g^2 C_2(R_i). \tag{20}$$

The finiteness conditions for an $N = 1$ supersymmetric theory with $SU(N)$ associated group are found in Ref. 77, while discussion of the no-charge renormalization and anomaly free requirements can be found in Ref. 78. It should be noted that conditions (19) and (20) are necessary and sufficient to ensure finiteness at the two-loop level.[55–59]

The requirement of finiteness, at the one-loop level, in softly broken SUSY theories demands additional constraints among the soft terms of the SSB sector,[79] while, once more, these one-loop requirements assure two-loop finiteness, too.[80] These conditions impose restrictions on the irreducible representations R_i of the gauge group G as well as on the Yukawa couplings. For example, since $U(1)$ is not compatible with condition (19), the MSSM is excluded. Therefore, a GUT is considered, with the MSSM being its low-energy theory. Also, since condition (20) forbids the appearance of gauge singlets ($C_2(1) = 0$), F-type spontaneous symmetry breaking[81] is not compatible with finiteness. Finally, D-type spontaneous breaking[82] is also incompatible since it requires a $U(1)$ group.

The nontrivial point is that the relations among couplings (gauge and Yukawa) which are imposed by the conditions (19) and (20) should hold at any energy scale. The necessary and sufficient condition is to require that such relations are solutions to the REs (see Eq. (10))

$$\beta_g \frac{dC_{ijk}}{dg} = \beta_{ijk} \tag{21}$$

holding at all orders. We note, once more, that the existence of one-loop level power series solution guarantees the all-order series.

There exists the following theorem[83,84] which points down which are the necessary and sufficient conditions in order for an $N = 1$ SUSY theory to be all-loop finite. In Refs. 83–89, it was shown that for an $N = 1$ SUSY Yang–Mills theory, based on a simple gauge group, we need the following four conditions to be fulfilled:

(i) No gauge anomaly is present.
(ii) The β-function of the gauge coupling is zero at one-loop level

$$\beta_g^{(1)} = 0 = \sum_i T(R_i) - 3\, C_2(G). \tag{22}$$

(iii) The condition of vanishing for the one-loop anomalous dimensions of matter fields,

$$\gamma^{(1)}{}_j^i = 0 = \frac{1}{32\pi^2}[C^{ikl}\, C_{jkl} - 2\, g^2\, C_2(R)\delta_j^i], \tag{23}$$

admits solution of the form

$$C_{ijk} = \rho_{ijk} g, \quad \rho_{ijk} \in \mathbb{C}. \tag{24}$$

(iv) When considered as solutions of vanishing Yukawa β-functions (at one-loop order), i.e. $\beta_{ijk} = 0$, the above solutions are isolated and nondegenerate.

Then, each of the solutions in Eq. (24) can be extended uniquely to a formal power series in g, and the associated super Yang–Mills models depend on the single coupling constant g with a vanishing, at all orders, β-function.

While the validity of the above cannot be extended to non-SUSY theories, it should be noted that reduction of couplings and finiteness are intimately related.

3. Phenomenologically Interesting Models with Reduced Couplings

In this section, we briefly review the basic properties of two phenomenologically interesting supersymmetric models with reduced couplings. Their predictions for the heavy SM particles, their supersymmetric spectra and the CDM relic density (since the lightest neutralino is a CDM candidate) are discussed in Secs. 5 and 6. All experimental constraints considered are listed in Sec. 4. Other models with reduced couplings that were analyzed in Refs. 37 and 48 are the Reduced Minimal $N = 1$ $SU(5)$[16] and the two-loop Finite $N = 1$ $SU(3)^3$.[34–36]

3.1. *The finite $N = 1$ supersymmetric $SU(5)$ model*

The first model is the finite to all-orders $SU(5)$, where we restrict the application of the reduction of couplings method to the third generation. An older analysis of this Finite Unified Theory (FUT) was in agreement with the experimental constraints at the time[29] and predicted the light Higgs mass in the correct range almost 5 years before its discovery. As reviewed below, improved Higgs calculations predict a somewhat different interval that is still within current experimental limits.

The particle content of the model consists of three $(\overline{\mathbf{5}} + \mathbf{10})$ supermultiplets for the three generations of leptons and quarks, while the Higgs sector is accommodated in four supermultiplets $(\overline{\mathbf{5}} + \mathbf{5})$ and one $\mathbf{24}$. The one-loop anomalous dimensions are diagonal, fermions do not couple to $\mathbf{24}$ and the MSSM Higgs doublets are mostly composed from the 5 and $\bar{5}$ that couple to the third generation. The finite $SU(5)$ group is broken to the MSSM, which is no longer a finite theory, as expected.[14–17,21,24] When the GUT breaks to the MSSM, a suitable rotation in the Higgs sector,[14,15,90–93] allows only two Higgs doublets (coupled mostly to the third family) to remain light and acquire vacuum expectation values (vevs). Fast proton decay is avoided with the usual doublet–triplet splitting.

The superpotential (with an enhanced symmetry due to the reduction of couplings) is given by[25, 27]

$$W = \sum_{i=1}^{3}\left[\frac{1}{2}g_i^u\,\mathbf{10}_i\mathbf{10}_i H_i + g_i^d\,\mathbf{10}_i\overline{\mathbf{5}}_i\,\overline{H}_i\right] + g_{23}^u\,\mathbf{10}_2\mathbf{10}_3 H_4 + g_{23}^d\,\mathbf{10}_2\overline{\mathbf{5}}_3\,\overline{H}_4$$

$$+ g_{32}^d\,\mathbf{10}_3\overline{\mathbf{5}}_2\,\overline{H}_4 + g_2^f\,H_2\,\mathbf{24}\,\overline{H}_2 + g_3^f\,H_3\,\mathbf{24}\,\overline{H}_3 + \frac{g^\lambda}{3}\,(\mathbf{24})^3, \qquad (25)$$

where $\overline{\mathbf{5}}_i$, $\mathbf{10}_i$, H_i and $\overline{H}_i$ are the lepton/quark and Higgs chiral supermultiplets, respectively ($i = 1, 2, 3$ denotes the generation) and $\mathbf{24}$ is the vector supermultiplet that accommodates the gauge bosons.

The *nondegenerate* and *isolated* solutions to the vanishing of $\gamma_i^{(1)}$ are

$$(g_1^u)^2 = \frac{8}{5}g^2, \quad (g_1^d)^2 = \frac{6}{5}g^2, \quad (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5}g^2,$$

$$(g_2^d)^2 = (g_3^d)^2 = \frac{3}{5}g^2, \quad (g_{23}^u)^2 = \frac{4}{5}g^2, \quad (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5}g^2, \qquad (26)$$

$$(g^\lambda)^2 = \frac{15}{7}g^2, \quad (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2}g^2, \quad (g_1^f)^2 = 0, \quad (g_4^f)^2 = 0.$$

Regarding the parameters of nonzero dimension, we have the relation $h = -MC$, while the sum rules lead to

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2, \quad m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}, \quad m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}. \qquad (27)$$

We therefore result in just two free dimensionful parameters, $m_{\mathbf{10}}$ and M. The model is discussed in more detail in Refs. 14–16.

3.2. *The reduced minimal supersymmetric standard model*

The second case is the application of the method of coupling reduction to a version of the MSSM, where a covering GUT is assumed. The original partial reduction can be found in Refs. 94 and 95 where only the third fermionic generation is considered. Following this restriction, the superpotential reads

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2, \qquad (28)$$

where $Y_{t,b,\tau}$ refer only to the third family, and the SSB Lagrangian is given by (with the trilinear couplings $h_{t,b,\tau}$ for the third family)

$$-\mathcal{L}_{\rm SSB} = \sum_\phi m_\phi^2 \hat{\phi}^*\hat{\phi} + \left[m_3^2\hat{H}_1\hat{H}_2 + \sum_{i=1}^{3}\frac{1}{2}M_i\lambda_i\lambda_i + \text{h.c.}\right]$$

$$+ [h_t\hat{H}_2\hat{Q}\hat{t^c} + h_b\hat{H}_1\hat{Q}\hat{b^c} + h_\tau\hat{H}_1\hat{L}\hat{\tau^c} + \text{h.c.}]. \qquad (29)$$

We start with the dimensionless sector and consider initially the top and bottom Yukawa couplings and the strong gauge coupling. The rest of the couplings will be treated as corrections. If $Y^2_{(t,b)}/(4\pi) \equiv \alpha_{(t,b)}$, the REs and the Yukawa RGEs give

$$\alpha_i = G_i^2 \alpha_3, \quad \text{where } G_i^2 = \frac{1}{3}, \;\; i = t, b.$$

If the tau Yukawa is included in the reduction, the corresponding G^2 coefficient for tau turns negative,[96] explaining why this coupling is treated also as a correction (i.e. it cannot be reduced).

We assume that the ratios of the top and bottom Yukawa to the strong coupling are constant at the GUT scale, i.e. they have negligible scale dependence,

$$\frac{d}{dg_3}\left(\frac{Y^2_{t,b}}{g_3^2}\right) = 0.$$

Then, including the corrections from the $SU(2)$, $U(1)$ and tau couplings, at the GUT scale, the coefficients $G^2_{t,b}$ become

$$G_t^2 = \frac{1}{3} + \frac{71}{525}\rho_1 + \frac{3}{7}\rho_2 + \frac{1}{35}\rho_\tau, \quad G_b^2 = \frac{1}{3} + \frac{29}{525}\rho_1 + \frac{3}{7}\rho_2 - \frac{6}{35}\rho_\tau, \tag{30}$$

where

$$\rho_{1,2} = \frac{g^2_{1,2}}{g_3^2} = \frac{\alpha_{1,2}}{\alpha_3}, \quad \rho_\tau = \frac{g_\tau^2}{g_3^2} = \frac{\frac{Y_\tau^2}{4\pi}}{\alpha_3}. \tag{31}$$

Going to the two-loop level, we assume that the corrections take the following form:

$$\alpha_i = G_i^2 \alpha_3 + J_i^2 \alpha_3^2, \quad i = t, b, \tag{32}$$

where the two-loop coefficients, J_i, including the corrections from the gauge and the tau Yukawa couplings, are known quantities which can be found in Ref. 97. We shall treat Eq. (32) as boundary conditions at the GUT scale.

Proceeding to the SSB Lagrangian, Eq. (29), and the dimension one parameters, i.e. the trilinear couplings $h_{t,b,\tau}$, we first reduce $h_{t,b}$ and we get

$$h_i = c_i Y_i M_3 = c_i G_i M_3 g_3, \quad \text{where } c_i = -1, \;\; i = t, b,$$

where M_3 is the gluino mass. Adding the corrections from the gauge and the tau couplings we have

$$c_t = -\frac{A_A A_{bb} + A_{tb} B_B}{A_{bt} A_{tb} - A_{bb} A_{tt}}, \quad c_b = -\frac{A_A A_{bt} + A_{tt} B_B}{A_{bt} A_{tb} - A_{bb} A_{tt}}.$$

Again, A_{tt}, A_{bb} and A_{tb} can be found in Ref. 97.

We end up with the soft scalar masses m_ϕ^2 of the SSB Lagrangian. Assuming the relations $m_i^2 = c_i M_3^2$ ($i = Q, u, d, H_u, H_d$), and adding the corrections from the gauge, the tau couplings and h_τ, we get

$$\begin{gathered} c_Q = -\frac{c_{Q\mathrm{Num}}}{D_m}, \quad c_u = -\frac{1}{3}\frac{c_{u\mathrm{Num}}}{D_m}, \quad c_d = -\frac{c_{d\mathrm{Num}}}{D_m}, \\ c_{H_u} = -\frac{2}{3}\frac{c_{H_u\mathrm{Num}}}{D_m}, \quad c_{H_d} = -\frac{c_{H_d\mathrm{Num}}}{D_m}, \end{gathered} \tag{33}$$

where D_m, $c_{Q\mathrm{Num}}$, $c_{u\mathrm{Num}}$, $c_{d\mathrm{Num}}$, $c_{H_u\mathrm{Num}}$, $c_{H_d\mathrm{Num}}$ and the complete analysis are again given in Ref. 97. These values do not obey any soft scalar mass sum rule.

If only the reduced system was used, i.e. the strong, top and bottom Yukawa couplings as well as the h_t and h_b, the coefficients turn to be

$$c_Q = c_u = c_d = \frac{2}{3}, \quad c_{H_u} = c_{H_d} = -1/3,$$

which clearly obey the sum rules

$$\frac{m_Q^2 + m_u^2 + m_{H_u}^2}{M_3^2} = c_Q + c_u + c_{H_u} = 1, \quad \frac{m_Q^2 + m_d^2 + m_{H_d}^2}{M_3^2} = c_Q + c_d + c_{H_d} = 1. \tag{34}$$

There is an essential point for the gaugino masses that should be mentioned. The application of the Hisano–Shifman relation, Eq. (12), is made for each gaugino mass as a boundary condition with unified gauge coupling at M_{GUT}. Then, at one-loop level, the gaugino masses depend on the one-loop coefficient of the corresponding β-function and an arbitrary mass M_0, $M_i = b_i M_0$. This fact permits, with a suitable choice of M_0, to have the gluino mass equal to the unified gaugino mass, while the gauginos masses of the other two gauge groups are given by the gluino mass multiplied by the ratio of the appropriate one-loop β coefficient.

4. Phenomenological Constraints

In this section, we briefly review several experimental constraints that were applied in our phenomenological analysis. The used values do not correspond to the latest experimental results. This fact, however, has a negligible impact on our analysis.

In our models we have evaluated the pole mass of the top quark, while the bottom quark mass is evaluated at the M_Z scale (to avoid uncertainties to its pole mass). The experimental values, taken from Ref. 98 are

$$m_t^{\mathrm{exp}} = 173.1 \pm 0.9~\mathrm{GeV}, \quad m_b(M_Z) = 2.83 \pm 0.10~\mathrm{GeV}. \tag{35}$$

We interpret the Higgs-like particle discovered in July 2012 by ATLAS[31] and CMS[32] as the light CP-even Higgs boson of the MSSM.[99–101] The Higgs-boson experimental average mass is[98][a]

$$M_h^{\mathrm{exp}} = 125.10 \pm 0.14~\mathrm{GeV}. \tag{36}$$

The theoretical uncertainty,[38,39] however, for the prediction of M_h in the MSSM dominates the total uncertainty, since it is much larger than the experimental one. In our analysis, we used version 2.16.0 of the `FeynHiggs` code[38–40] to predict the Higgs mass.[b] This version gives a downward shift on the Higgs mass M_h of

[a]This is the latest available LHC combination. More recent measurements confirm this value.
[b]An analysis of the impact of the improved M_h calculation in various SUSY models can be found in Ref. 102.

$\mathcal{O}(2\text{ GeV})$ for large SUSY masses and in particular gives a reliable point-by-point evaluation of the Higgs-boson mass uncertainty.[41] The theoretical uncertainty calculated is added linearly to the experimental error in Eq. (36).

Furthermore, recent results from the ATLAS experiment[103] set limits to the mass of the pseudoscalar Higgs boson, M_A, in comparison with $\tan\beta$. For models with $\tan\beta \sim 45 - 55$, as the ones examined here, the lowest limit for the physical pseudoscalar Higgs mass is

$$M_A \gtrsim 1900\text{ GeV}. \tag{37}$$

For the production of the heavy Higgs sector and the full supersymmetric spectrum of each model a `SARAH`[104] generated, custom MSSM module for `SPheno`[46,47] was used. The cross-sections for their particle productions at the HL-LHC and FCC-hh were calculated with `MadGraph5_aMC@NLO`.[105]

We also considered the following four flavor observables where SUSY has non-negligible impact. For the branching ratio $\text{BR}(b \to s\gamma)$ we take a value from Refs. 106 and 107, while for the branching ratio $\text{BR}(B_s \to \mu^+\mu^-)$ we use a combination of Refs. 108–112:

$$\frac{\text{BR}(b \to s\gamma)^{\text{exp}}}{\text{BR}(b \to s\gamma)^{\text{SM}}} = 1.089 \pm 0.27, \quad \text{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 1.4) \times 10^{-9}. \tag{38}$$

For the B_u decay to $\tau\nu$ we use[107,113,114] and for ΔM_{B_s} we use[115,116]

$$\frac{\text{BR}(B_u \to \tau\nu)^{\text{exp}}}{\text{BR}(B_u \to \tau\nu)^{\text{SM}}} = 1.39 \pm 0.69, \quad \frac{\Delta M_{B_s}^{\text{exp}}}{\Delta M_{B_s}^{\text{SM}}} = 0.97 \pm 0.2. \tag{39}$$

Finally, we consider CDM constraints. Since the Lightest SUSY Particle (LSP), which in our case is the lightest neutralino, is a promising CDM candidate,[117] we examine if each model is within the CDM relic density experimental limits. The current bound on the CDM relic density at $2\,\sigma$ level is given by[118]

$$\Omega_{\text{CDM}} h^2 = 0.1120 \pm 0.0112. \tag{40}$$

For the calculation of the CDM relic density the `MicrOMEGAs 5.0` code[42–44] was used.

In the following sections, we review the way these constraints were applied to each model and discuss the corresponding phenomenology.

5. Numerical Analysis of the Finite $SU(5)$

We start with the analysis of the predicted spectrum of the Finite $N = 1$ $SU(5)$ that was discussed in Subsec. 3.1. Below the GUT scale we get the MSSM, where the third generation is given by the finiteness conditions (the first two remain unrestricted). However, these conditions do not restrict the low-energy renormalization properties, so the above relations between gauge, Yukawa and the various dimensionful parameters serve as boundary conditions at M_{GUT}. The third generation quark masses $m_b(M_Z)$ and m_t are predicted within 3σ and 2σ uncertainties, respectively, of their experimental values (see the complete analysis in Ref. 37). $\mu < 0$ is the only phenomenologically viable option, as shown in Refs. 37 and 119–126. The plot of the light Higgs mass satisfies all experimental constraints considered in Sec. 4 (including B-physics constraints) for a unified gaugino mass $M \sim 4500 - 7500$ GeV, while its point-by-point theoretical uncertainty[41] drops significantly (with respect to the previous analysis) to 0.65–0.70 GeV.

The improved evaluation of M_h and its uncertainty prefer a heavier (Higgs) spectrum (compared to previous analyses[37,119–125,127–131]), and thus allows only a heavy supersymmetric spectrum, which is in agreement with all existing experimental data. Very heavy colored supersymmetric particles are favored, in agreement with the nonobservation of such particles at the LHC.[132]

At this point there is an important remark. No point fulfills the strict bound of Eq. (40), since we have overproduction of CDM in the early universe, as it can be seen in Fig. 1 (for the original analysis, see Ref. 45). The LSP, which in our case is the lightest neutralino, is strongly Bino-like. Combined with the heavy mass it acquires (1–2 TeV), it cannot account for a relic density low enough to agree with experimental observation. Thus, we need a mechanism that reduces this CDM abundance. This could be related to the problem of neutrino masses, which cannot be generated naturally in this particular model. However, one could extend the model by considering bilinear R-parity violating terms (that preserve finiteness) and thus introduce neutrino masses.[133,134] R-parity violation[135] would have a small impact on the masses and production cross-sections, but remove the CDM bound of Eq. (40) completely. Other mechanisms, not involving R-parity violation, that could be invoked if the amount of CDM appears to be too large, concern the cosmology of the early universe. For example, "thermal inflation"[136] or "late time entropy injection"[137] can bring the CDM density into agreement with Planck measurements. For the original discussion, see Ref. 37.

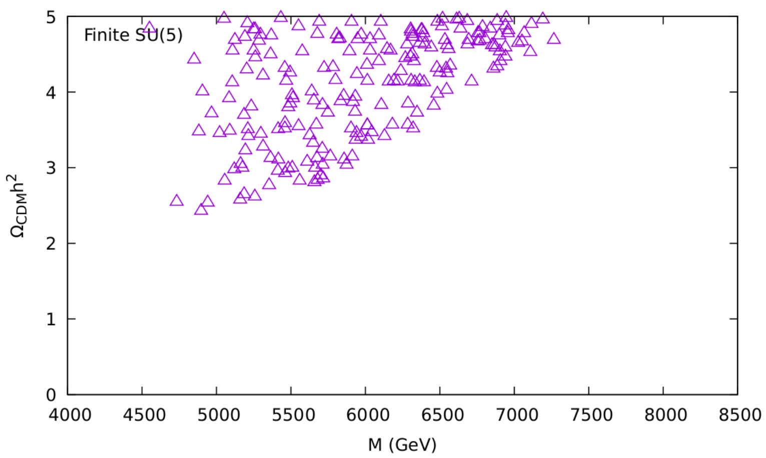

Fig. 1. The CDM relic density of the Finite $SU(5)$ as a function of the unified gaugino mass, M, for points with light Higgs mass within its calculated uncertainty. All points are well above the experimental value, $\Omega_{\rm CDM}h^2 = 0.1120 \pm 0.0112$.

Table 1. Masses for each of the three benchmarks of the Finite $N = 1$ $SU(5)$ (in TeV).[48]

	$\tan\beta$	$M_{A,H}$	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}_1^0}$	$M_{\tilde{\chi}_2^0}$	$M_{\tilde{\chi}_3^0}$	$M_{\tilde{\chi}_4^0}$	$M_{\tilde{\chi}_1^\pm}$	$M_{\tilde{\chi}_2^\pm}$
FUTSU5-1	49.9	5.688	5.688	8.966	2.103	3.917	4.829	4.832	3.917	4.833
FUTSU5-2	50.1	7.039	7.086	10.380	2.476	4.592	5.515	5.518	4.592	5.519
FUTSU5-3	49.9	16.382	16.401	12.210	2.972	5.484	6.688	6.691	5.484	6.691
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
FUTSU5-1	3.102	3.907	2.205	3.137	7.839	7.888	6.102	6.817	6.099	6.821
FUTSU5-2	3.623	4.566	2.517	3.768	9.059	9.119	7.113	7.877	7.032	7.881
FUTSU5-3	4.334	5.418	3.426	3.834	10.635	10.699	8.000	9.387	8.401	9.390

As explained in more detail in Ref. 48, the three benchmarks chosen (for the purposes of collider phenomenology) feature the LSP above 2100, 2400 and 2900 GeV, respectively. The resulting masses that are relevant to our analysis were generated by `SPheno` 4.0.4[46,47] and are listed in Table 1 for each benchmark (with the corresponding $\tan\beta$). The two first masses refer to the heavy Higgs bosons. The gluino mass is $M_{\tilde{g}}$, the neutralinos and the charginos are denoted as $M_{\tilde{\chi}_i^0}$ and $M_{\tilde{\chi}_i^\pm}$, while the slepton and sneutrino masses for all three generations are given as $M_{\tilde{e}_{1,2,3}}$, $M_{\tilde{\nu}_{1,2,3}}$. Similarly, the squarks are denoted as $M_{\tilde{d}_{1,2}}$ and $M_{\tilde{u}_{1,2}}$ for the first two generations. The third generation masses are given by $M_{\tilde{t}_{1,2}}$ for stops and $M_{\tilde{b}_{1,2}}$ for sbottoms.

As discussed in detail in Ref. 48, at 14 TeV HL-LHC none of the Finite $SU(5)$ scenarios listed above has a SUSY production cross-section above 0.01 fb, and thus will most probably remain unobservable.[138] The discovery prospects for the heavy Higgs-boson spectrum is significantly better at the FCC-hh.[139] Theoretical analyses[140,141] have shown that for large $\tan\beta$ heavy Higgs mass scales upto $\sim$8 TeV could be accessible. Since in this model we have $\tan\beta \sim 50$, the first two benchmark points are well within the reach of the FCC-hh (as explained in Ref. 48). The third point, however, where $M_A \sim 16$ TeV, will be far outside the reach of the collider.

Although 100 TeV are enough to produce SUSY particles in pairs in principle, prospects for detecting squark pairs and squark–gluino pairs are very dim, since their production cross-section is at the few fb level. This is a result of the heavy spectrum of the model. Comparing our benchmark predictions with the simplified model limits of Ref. 142, we have found that the lighter stop might be accessible in FUTSU5-1 (see Ref. 48). For the squarks of the first two generations there are somewhat better prospects of testing the model. All benchmarks could possibly be excluded at the $2\,\sigma$ level, but no discovery at the $5\,\sigma$ can be expected and the same holds for the gluino. The heavy LSP will keep charginos and neutralinos unobservable. We have to conclude that large parts of the possible mass spectra will not be observable at the FCC-hh.

6. Numerical Analysis of the Reduced MSSM

We finish our phenomenological analysis with the reduced version of the MSSM, as described in Subsec. 3.2. We choose the GUT scale to apply the corrections to all these RGI relations in our analysis. A detailed discussion on the free parameters selection of the model can be found in Ref. 37. In total, we vary ρ_τ, ρ_{h_τ}, M and μ. The predictions for the bottom and the top quark masses are within 2σ of Eq. (35). The light Higgs mass M_h is predicted within the experimental measured range and satisfies B-physics constraints for a unified gaugino mass $M \sim 2500 - 4000$ GeV, while its theoretical uncertainty[41] now drops below 1 GeV.

The lightest neutralino (LSP) is Wino-like, as imposed by the Hisano–Shifman relation, Eq. (12), and thus the CDM relic density is below the boundaries of Eq. (40), as demonstrated in Fig. 2 (see Ref. 45). This renders this model viable if Eq. (40) is applied only as an upper limit and additional sources of CDM are allowed. An additional DM component could be, e.g. a SUSY axion,[143] which would then bring the total DM density into agreement with the Planck measurement of $\Omega_{\text{CDM}}h^2$.

As demonstrated in Ref. 48, M_h sets a limit on the low-energy supersymmetric masses, which we briefly discuss. The three benchmarks selected correspond to $\overline{\text{DR}}$ pseudoscalar Higgs masses above 1900, 1950 and 2000 GeV, respectively. Table 2 shows the resulting masses of Higgs bosons and some of the LSPs (generated with `SPheno` 4.0.4[46,47]). In particular, there is one point that should be stressed. We find $M_A \lesssim 1.5$ TeV (for large values of $\tan\beta$ as in the other models), values substantially lower than in the previously considered model. This means that in this model, because of the large $\tan\beta \sim 45$, the physical mass of the pseudoscalar Higgs boson, M_A, is excluded by the searches $H/A \to \tau\tau$ at ATLAS with 139/fb[103] for all three benchmarks, and, as it was shown in Ref. 48, this holds for

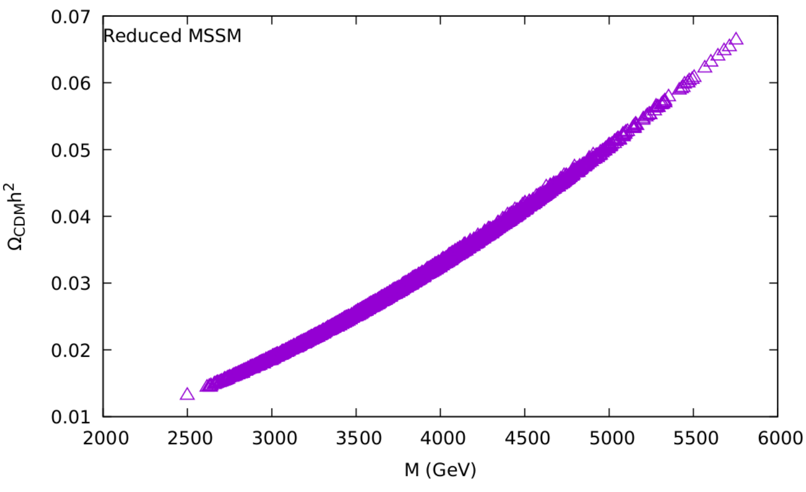

Fig. 2. The CDM relic density of the Reduced MSSM as a function of the unified gaugino mass, for points with light Higgs mass within its calculated uncertainty. All points are below the experimental value, $\Omega_{\text{CDM}}h^2 = 0.1120 \pm 0.0112$.

Table 2. Masses for each of the three benchmarks of the Reduced MSSM (in TeV). Original analysis in Ref. 48.

	$\tan\beta$	$M_{A,H}$	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}_1^0}$	$M_{\tilde{\chi}_2^0}$	$M_{\tilde{\chi}_3^0}$	$M_{\tilde{\chi}_4^0}$	$M_{\tilde{\chi}_1^\pm}$	$M_{\tilde{\chi}_2^\pm}$
RMSSM-1	44.9	1.393	1.387	7.253	1.075	3.662	4.889	4.891	1.075	4.890
RMSSM-2	44.6	1.417	1.414	7.394	1.098	3.741	4.975	4.976	1.098	4.976
RMSSM-3	45.3	1.491	1.492	7.459	1.109	3.776	5.003	5.004	1.108	5.004
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
RMSSM-1	2.124	2.123	2.078	2.079	6.189	6.202	5.307	5.715	5.509	5.731
RMSSM-2	2.297	2.139	2.140	2.139	6.314	6.324	5.414	5.828	5.602	5.842
RMSSM-3	2.280	2.123	2.125	2.123	6.376	6.382	5.465	5.881	5.635	5.894

the entire allowed parameter space. If we considered a heavier spectrum instead (in which we would have $M_A \gtrsim 1900$ GeV) the light Higgs boson mass would be above its acceptable region. Thus, this version of the model is ruled out experimentally.

7. Conclusions

In this work, after a review of the reduction of couplings idea, which is realized in certain $N = 1$ theories, rendering them more predictive, we turn to the question of testing this class of models experimentally. Two specific models, namely the all-loop Finite $N = 1$ $SU(5)$ and the Reduced MSSM, have been considered and updated results have been obtained for both, using the Higgs-boson mass calculation of `FeynHiggs`.The CDM relic density was calculated with the `MicrOMEGAs` code. In each case low-mass region benchmark points have been chosen, for which the `SPheno` code was used to calculate the spectrum of supersymmetric particles and their respective decay modes. Finally, the `MadGraph` event generator has been used (in the case of the Finite $SU(5)$) for the computation of the production cross-sections of relevant final states at the 14 TeV (HL-)LHC and 100 TeV FCC-hh colliders.

The finite model was found to be in agreement with LHC measurements. Both models predict relatively heavy spectra, which evade largely the detection of the SUSY particles at the HL-LHC. The finite model has an overproduction of CDM in the early universe, while the Reduced MSSM has a relic density below the experimental limit. Ways to tackle this problem are discussed. However, the Reduced MSSM features a relatively light spectrum of heavy Higgses. Combined with its relatively high $\tan\beta$, this spectrum is excluded by current searches at ATLAS. Concerning the finite model, we examined the accessibility of the SUSY and heavy Higgs spectrum at the FCC-hh with $\sqrt{s} = 100$ TeV. The lower parts of the parameter space will be testable, while the heavier parts of the possible SUSY spectra will remain elusive even at the FCC-hh.

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