Probing Unified Theories with Reduced Couplings at Future Hadron Colliders

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Abstract

The search for renormalization group invariant relations among parameters to all orders in perturbation theory constitutes the basis of the reduction of couplings concept. Reduction of couplings can be achieved in certain $N = 1$ supersymmetric Grand Unified Theories and few of them can become even finite at all loops. We review the basic idea, the tools that have been developed as well as the resulting theories in which successful reduction of couplings has been achieved so far. These include: (i) a reduced version of the minimal $N = 1$ $SU(5)$ model, (ii) an all-loop finite $N = 1$ $SU(5)$ model, (iii) a two-loop finite $N = 1$ $SU(3)^3$ model and finally (vi) a reduced version of the Minimal Supersymmetric Standard Model. In this paper we present a number of benchmark scenarios for each model and investigate their observability at existing and future hadron colliders. The heavy supersymmetric spectra featured by each of the above models are found to be beyond the reach of the 14 TeV HL-LHC. It is also found that the reduced version of the MSSM is already ruled out by the LHC searches for heavy neutral MSSM Higgs bosons. In turn the discovery potential of the 100 TeV FCC-hh is investigated and found that large parts of the predicted spectrum of these models can be tested, but the higher mass regions are beyond the reach even of the FCC-hh.

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1 Introduction

The *reduction of couplings* method [1–4] (see also [5–7]) is a promising method which relates originally seemingly independent parameters to a single, "primary" coupling. The method requires the original theory to which it is applied to be a renormalizable one, and the resulting relation among the parameters to be valid at all energy scales, i.e. Renormalization Group Invariant (RGI).

A next (natural) step, after the introduction of a novel symmetry through a Grand Unified Theory (GUT) [8–13]), in order to achieve reduction of free parameters of the SM is the relation of the gauge sector to the Yukawa sector (Gauge Yukawa Unification, GYU). This was the central characteristic of the *reduction of couplings* approach in the first period of searches, applied in $N = 1$ GUTs [14–27]. According to that approach, being in a GUT environment, RGI relations are set between the unification scale and the Planck scale. One-loop consideration can guarantee the all-loop validity of those relations. Moreover, RGI relations can be found which guarantee all order finiteness of a theory. The method has predicted the top quark mass in the finite $N = 1$ $SU(5)$ model [14, 15] as well as in the minimal $N = 1$ $SU(5)$ one [16] before its experimental measurement [28].

Since SuperSymmetry (SUSY) seems an essential ingredient for the *reduction of couplings* method, we have to include a supersymmetry breaking sector (SSB), which involves dimension-1 and -2 couplings. The supergraph method and the spurion superfield technique played an important role for the progress in that sector, leading to complete all-loop finite models, i.e. including the SSB sector. The all-loop finite $N = 1$ $SU(5)$ model [29] has given a prediction for the Higgs mass compatible with the experimental results [30–32] and a heavy SUSY mass spectrum, consistent with the experimental non-observation of these particles. The reduction of couplings method has been applied to several other cases. The full analysis of the most successful models, that includes predictions in agreement with the experimental measurements of the top and bottom quark masses for each model, can be found in a recent work [37].

In this paper we address the question to what extent the reduction of couplings idea, as applied in the so far phenomenologically successful models, can be experimentally tested at HL-LHC and future FCC hadron collider. To this end we propose a number of benchmark points for each model. We present the SUSY breaking parameters used as input in each benchmark to calculate the corresponding Higgs boson and supersymmetric particles masses. Then we compute the expected production cross sections at the 14 TeV (HL-)LHC and the 100 TeV FCC-hh and investigate which production channels can be observed.

The present work is organized as follows. In Section 2 we review the basic idea of the reduction of couplings. In Section 3 we list the phenomenological constraints used in our analyses, wile in Section 4 we explain the computational setup. In Sections 5, 6, 7 and 8 we review four interesting models, namely (i) the Minimal $N = 1$ Supersymmetric $SU(5)$, (ii) the Finite $N = 1$ Supersymmetric $SU(5)$, (iii) the Finite $SU(3)^3$ and (iv) the MSSM, in which the reduction of couplings has been successfully applied. We briefly review some earlier results of our phenomenological analysis. In this context the new version of the `FeynHiggs` [33–36] code plays a crucial role, which was used to calculate the Higgs-boson predictions, in particular the mass of the lightest CP-even Higgs boson. The improved predictions of `FeynHiggs` are compared with the LHC measurements and the Beyond Standard Model (BSM) Higgs boson searches. As a new part of the analysis we examine in each model the discovery potential of the Higgs and SUSY spectrum at approved future and hypothetical future hadron colliders. Finally, Section 8 is dedicated to brief conclusive remarks.

2 Theoretical Basis

Here we will briefly review the core idea of the *reduction of couplings* method. The target is to single out a basic parameter (which we will call the primary coupling), where all other parameters can be expressed in terms of this one through RGI relations. Such a relation has, in general, the form $\Phi(g_1, \cdots, g_A) = \text{const.}$

which should satisfy the following partial differential equation (PDE)

$$\mu \frac{d\Phi}{d\mu} = \vec{\nabla}\Phi \cdot \vec{\beta} \; = \; \sum_{a=1}^{A} \beta_a \frac{\partial \Phi}{\partial g_a} \; = \; 0 \; , \tag{1}$$

where β_a is the β-functions of g_a. The above PDE is equivalent to the following set of ordinary differential equations (ODEs), which are called Reduction Equations (REs) [2–4],

$$\beta_g \frac{dg_a}{dg} = \beta_a \; , \; a = 1, \cdots, A-1 \; , \tag{2}$$

where now g and β_g are the primary coupling and its corresponding β-function. There are obviously $A-1$ relations in the form of $\Phi(g_1, \cdots, g_A) \; = \;$ const. in order to express all other couplings in term of the primary one.

The crucial demand is that the above REs admit power series solutions

$$g_a = \sum_n \rho_a^{(n)} \, g^{2n+1} \; , \tag{3}$$

which preserve perturbative renormalizability. Without this requirement, we just trade each "dependent" coupling for an integration constant. The power series, which are a set of special solutions, fix that constant. It is very important to point out that the uniqueness of such a solution can be already decided at the one-loop level [2–4]. In supersymmetric theories, where the asymptotic behaviour of several parameters are similar, the use of power series as solutions of the REs are justified. But, usually, the reduction is not "complete", which means that not all of the couplings can be reduced in favor of the primary one, leading to the so called "partial reduction" [38, 39].

We proceed to the reduction scheme for massive parameters, which is far from being straightforward. A number of conditions is required (see for example [40]). Nevertheless, progress has been achieved, starting from [41], and finally we can introduce mass parameters and couplings carrying mass dimension [42, 43] in the same way as dimensionless couplings.

Consider the superpotential

$$W = \frac{1}{2} \, \mu^{ij} \, \Phi_i \, \Phi_j + \frac{1}{6} \, C^{ijk} \, \Phi_i \, \Phi_j \, \Phi_k \; , \tag{4}$$

and the SSB sector Lagrangian

$$-\mathcal{L}_{\rm SSB} = \frac{1}{6} \, h^{ijk} \, \phi_i \phi_j \phi_k + \frac{1}{2} \, b^{ij} \, \phi_i \phi_j + \frac{1}{2} \, (m^2)_i^j \, \phi^{*\,i} \phi_j + \frac{1}{2} \, M \, \lambda_i \lambda_i + \text{h.c.}, \tag{5}$$

where ϕ_i's are the scalar fields of the corresponding superfields Φ_i's and λ_i are the gauginos.

Let us write down some well known relations:

(i) The β-function of the gauge coupling at one-loop level is given by [44–48]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2} \left[\sum_i T(R_i) - 3 \, C_2(G) \right] \; , \tag{6}$$

where $T(R_i)$ is the Dynkin index of the rep R_i where the matter fields belong and $C_2(G)$ is the quadratic Casimir operator of the adjoint rep G.

(ii) The anomalous dimension $\gamma^{(1)\,i}_{\;\;j}$, at a one-loop level, of a chiral superfield is

$$\gamma^{(1)\,i}_{\;\;j} = \frac{1}{32\pi^2} \left[C^{ikl} \, C_{jkl} - 2 \, g^2 \, C_2(R_i) \delta^i_j \right] \; . \tag{7}$$

(iii) The β-functions of C_{ijk}'s, at one-loop level, following the $N=1$ non-renormalization theorem [49–51], are expressed in terms of the anomalous dimensions of the fields involved

$$\beta_C^{ijk} = \frac{dC_{ijk}}{dt} \;=\; C_{ijl}\,\gamma_k^l + C_{ikl}\,\gamma_j^l + C_{jkl}\,\gamma_i^l \;. \tag{8}$$

We proceed by assuming that the REs admit power series solutions:

$$C^{ijk} = g\sum_{n=0}\rho_{(n)}^{ijk} g^{2n} \;. \tag{9}$$

Trying to obtain all-loop results we turn to relations among β-functions. The spurion technique [51–55] gives all-loop relations among SSB β-functions [56–62]. Then, assuming that the reduction of C^{ijk} is possible to all orders

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} \;, \tag{10}$$

as well as for h^{ijk}

$$h^{ijk} = -M\frac{dC^{ijk}}{d\ln g} \;, \tag{11}$$

it can be proven [63, 64] that the following relations are all-loop RGI

$$M = M_0\;\frac{\beta_g}{g}, \tag{12}$$

$$h^{ijk} = -M_0\;\beta_C^{ijk}, \tag{13}$$

$$b^{ij} = -M_0\;\beta_\mu^{ij}, \tag{14}$$

$$(m^2)_j^i = \frac{1}{2}\;|M_0|^2\;\mu\frac{d\gamma^i{}_j}{d\mu}, \tag{15}$$

where M_0 is an arbitrary reference mass scale to be specified and Eq. (12) is the Hisano-Shifman relation [59] (note that in both assumptions we do not rely on specific solutions of these equations).
As a next step we substitute the last equation, Eq. (15), by a more general RGI sum rule that holds to all orders [65]

$$\begin{aligned} m_i^2 + m_j^2 + m_k^2 = |M|^2 \Bigg\{ & \frac{1}{1 - g^2 C_2(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2} \Bigg\} \\ & + \sum_l \frac{m_l^2 T(R_l)}{C_2(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g} \;, \end{aligned} \tag{16}$$

which leads to the following one-loop relation

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \;. \tag{17}$$

Finally, note that in the case of product gauge groups, Eq. (12) takes the form

$$M_i = \frac{\beta_{g_i}}{g_i} M_0 \;, \tag{18}$$

where i denotes the group of the product. This will be used in the Reduced MSSM case.

Consider an $N=1$ globally supersymmetric gauge theory, which is chiral and anomaly free, where G is the gauge group and g the associated gauge coupling. The theory has the superpotential of Eq. (4),

while the one-loop gauge and C_{ijk}s β-functions are given by Eq. (6) and Eq. (8) respectively and the one-loop anomalous dimensions of the chiral superfields by Eq. (7).
Demanding the vanishing of all one-loop β-functions, Eqs.(6,7) lead to the relations

$$\sum_i T(R_i) = 3C_2(G)\,, \tag{19}$$

$$C^{ikl}C_{jkl} = 2\delta^i_j g^2 C_2(R_i)\ . \tag{20}$$

The finiteness conditions for an $N = 1$ supersymmetric theory with $SU(N)$ associated group is found in [66] while discussion of the no-charge renormalization and anomaly free requirements can be found in [67]. It should be noted that conditions (19) and (20) are necessary and sufficient to ensure finiteness at the two-loop level [44–48].

The requirement of finiteness, at the one-loop level, in softly broken SUSY theories demands additional constraints among the soft terms of the SSB sector [68], while, once more, these one-loop requirements assure two-loop finiteness, too [69]. These conditions impose restrictions on the irreducible representations R_i of the gauge group G as well as on the Yukawa couplings. For example, since $U(1)$s are not compatible with condition (19), the MSSM is excluded. Therefore, a GUT is initially required with the MSSM being its low energy theory. Also, since condition (20) forbids the appearance of gauge singlets ($C_2(1) = 0$), F-type spontaneous symmetry breaking [70] are not compatible with finiteness. Finally, D-type spontaneous breaking [71] is also incompatible since it requires a $U(1)$ group.

The nontrivial point is that the relations among couplings (gauge and Yukawa) which are imposed by the conditions (19) and (20) should hold at any energy scale. The necessary and sufficient condition is to require that such relations are solutions to the REs (see Eq. (10))

$$\beta_g \frac{dC_{ijk}}{dg} = \beta_{ijk} \tag{21}$$

holding at all orders. We note, once more, that the existence of one-loop level power series solution guarantees the all-order series.

There exist the following theorem [72, 73] which points down which are the necessary and sufficient conditions in order for an $N = 1$ SUSY theory to be all-loop finite. In refs [72–78] it was shown that for an $N = 1$ SUSY Yang-Mills theory, based on a simple gauge group, if the following four conditions are fulfilled:
(i) No gauge anomaly is present.
(ii) The β-function of the gauge coupling is zero at one-loop level

$$\beta_g^{(1)} = 0 = \sum_i T(R_i) - 3\,C_2(G). \tag{22}$$

(iii) The condition of vanishing for the one-loop anomalous dimensions of matter fields,

$$\gamma^{(1)i}_{\ \ j} \ = \ 0 = \frac{1}{32\pi^2}\ [\ C^{ikl}\,C_{jkl} - 2\ g^2\ C_2(R)\delta^i_j], \tag{23}$$

admits solution of the form

$$C_{ijk} = \rho_{ijk} g, \qquad \rho_{ijk} \in \mathbb{C}\ . \tag{24}$$

(iv) When considered as solutions of vanishing Yukawa β-functions (at one-loop order), i.e. $\beta_{ijk} = 0$, the above solutions are isolated and non-degenerate;
then, each of the solutions in Eq. (24) can be extended uniquely to a formal power series in g, and the associated super Yang-Mills models depend on the single coupling constant g with a vanishing, at all orders, β-function.

While the validity of the above cannot be extended to non-SUSY theories, it should be noted that reduction of couplings and finiteness are intimately related.

3 Phenomenological Constraints

In this section we briefly review several experimental constraints that were applied in our phenomenological analysis. The used values do not correspond to the latest experimental results, which, however, has a negligible impact on our analysis.

In our models we evaluate the pole mass of the top quark while the bottom quark mass is evaluated at the M_Z scale (to avoid uncertainties to its pole mass). The experimental values, taken from ref. [79] are:

$$m_t^{\rm exp} = 173.1 \pm 0.9 \text{ GeV} \quad , \quad m_b(M_Z) = 2.83 \pm 0.10 \text{ GeV} \, . \tag{25}$$

We interpret the Higgs-like particle discovered in July 2012 by ATLAS and CMS [30,31] as the light CP-even Higgs boson of the MSSM [80–82]. The Higgs boson experimental average mass is [79] [a]

$$M_h^{\rm exp} = 125.10 \pm 0.14 \text{ GeV} \, . \tag{26}$$

The theoretical uncertainty [33, 34], however, for the prediction of M_h in the MSSM dominates the total uncertainty, since it is much larger than the experimental one. In our following analyses we shall use the new `FeynHiggs` code [33–35] (Version 2.16.0) to predict the Higgs mass.[b] `FeynHiggs` evaluates the Higgs masses based on a combination of fixed order diagrammatic calculations and resummation of the (sub)leading logarithmic contributions at all orders. This provides a reliable M_h even for a large SUSY scale. This new version gives a downward shift on the Higgs mass M_h of $\mathcal{O}(2 \text{ GeV})$ for large SUSY masses and in particular gives a reliable point-by-point evaluation of the Higgs-boson mass uncertainty [36]. The theoretical uncertainty calculated is added linearly to the experimental error in Eq. (26).

Furthermore, recent results from the ATLAS experiment [84] set limits to the mass of the pseudoscalar Higgs boson, M_A, in comparison with $\tan\beta$. For models with $\tan\beta \sim 45 - 55$, as the ones examined here, the lowest limit for the physical pseudoscalar Higgs mass is

$$M_A \gtrsim 1900 \text{ GeV}. \tag{27}$$

We also consider the following four flavor observables where SUSY has non-negligible impact. For the branching ratio $\text{BR}(b \to s\gamma)$ we take a value from [85,86], while for the branching ratio $\text{BR}(B_s \to \mu^+\mu^-)$ we use a combination of [87–91]:

$$\frac{\text{BR}(b \to s\gamma)^{\rm exp}}{\text{BR}(b \to s\gamma)^{\rm SM}} = 1.089 \pm 0.27 \quad , \quad \text{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 1.4) \times 10^{-9} \, . \tag{28}$$

For the B_u decay to $\tau\nu$ we use [86, 92, 93] and for ΔM_{B_s} we use [94, 95]:

$$\frac{\text{BR}(B_u \to \tau\nu)^{\rm exp}}{\text{BR}(B_u \to \tau\nu)^{\rm SM}} = 1.39 \pm 0.69 \quad , \quad \frac{\Delta M_{B_s}^{\rm exp}}{\Delta M_{B_s}^{\rm SM}} = 0.97 \pm 0.2 \, . \tag{29}$$

In the following sections we will apply these constraints to each model and discuss the corresponding collider phenomenology.

4 Computational setup

The setup for our phenomenological analysis is as follows. Starting from an appropriate set of MSSM boundary conditions at the GUT scale, parameters are run down to the SUSY scale using a private code. Two-loop RGEs are used throughout, with the exception of the soft sector, in which one-loop RGEs are used.

[a] This is the latest available LHC combination. More recent measurements confirm this value.

[b] An analysis of the impact of the improved M_h calculation in various SUSY models can be found in [83].

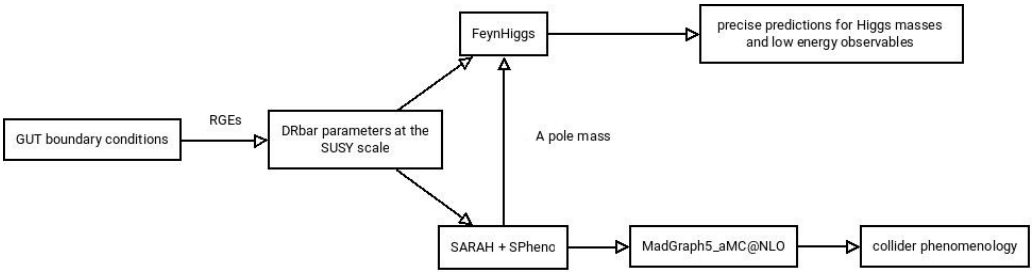

Figure 1: Flow of information between used computer codes (see text for details).

The running parameters are then used as inputs for both `FeynHiggs` [33–36] and a `SARAH` [96] generated, custom MSSM module for `SPheno` [97, 98]. It should be noted that `FeynHiggs` requires the $m_b(m_b)$ scale, the *physical* top quark mass m_t as well as *the physical* pseudoscalar boson mass M_A as input. The first two values are calculated by the private code while M_A is calculated only in $\overline{\rm DR}$ scheme. This single value is obtained from the `SPheno` output where it is calculated at the two-loop level in the gaugeless limit [99, 100]. The flow of information between codes in our analysis is summarised in Fig. 1.

At this point both codes contain a consistent set of all required parameters. SM-like Higgs boson mass as well as low energy observables mentioned in Sec. 3 are evaluated using `FeynHiggs`. To obtain collider predictions we use `SARAH` to generate `UFO` [101, 102] model for `MadGraph` event generator. Based on SLHA spectrum files generated by `SPheno`, we use `MadGraph5_aMC@NLO` [103] to calculate cross sections for Higgs boson and SUSY particle production at the HL-LHC and a 100 TeV FCC-hh. Processes are generated at the leading order, using `NNPDF31_lo_as_0130` [104] structure functions interfaced through `LHAPDF6` [105]. Cross sections are computed using dynamic scale choice, where the scale is set equal to the transverse mass of an event, in 4 or 5-flavor scheme depending on the presence or not of b-quarks in the final state. The results are given in Sec. 5, 6 and 7.

5 The Minimal $N = 1$ Supersymmetric $SU(5)$ Model

We start with the partial reduction of the $N = 1$ SUSY $SU(5)$ model [16, 41]. Our notation is as follows: $\Psi^I(\mathbf{10})$ and $\Phi^I(\overline{\mathbf{5}})$ refer to the three generations of leptons and quarks ($I = 1, 2, 3$), $\Sigma(\mathbf{24})$ is the adjoint which breaks $SU(5)$ to $SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}$ and $\overline{H}(\overline{\mathbf{5}})$ represent the two Higgs superfields for the electroweak symmetry breaking (ESB) [106, 107]. The choice of using only one set of $(\mathbf{5}+\overline{\mathbf{5}})$ for the ESB renders the model asymptotically free (i.e. $\beta_g < 0$). The superpotential of the model is described by

$$\begin{aligned} W = & \frac{g_t}{4}\,\epsilon^{\alpha\beta\gamma\delta\tau}\,\Psi^{(3)}_{\alpha\beta}\Psi^{(3)}_{\gamma\delta}H_\tau + \sqrt{2}g_b\,\Phi^{(3)\alpha}\Psi^{(3)}_{\alpha\beta}\overline{H}^\beta + \frac{g_\lambda}{3}\,\Sigma^\beta_\alpha\Sigma^\gamma_\beta\Sigma^\alpha_\gamma + g_f\,\overline{H}^\alpha\Sigma^\beta_\alpha H_\beta \\ & + \frac{\mu_\Sigma}{2}\,\Sigma^\gamma_\alpha\Sigma^\alpha_\gamma + \mu_H\,\overline{H}^\alpha H_\alpha\ , \end{aligned} \tag{30}$$

where only the third generation Yukawa couplings are taken into account. The indices $\alpha, \beta, \gamma, \delta, \tau$ are $SU(5)$ ones. A detailed presentation of the model can be found in [108] as well as in [109, 110].

Our primary coupling is the gauge coupling g. In this model the gauge-Yukawa unification can be achieved through two sets of solutions which are asymptotically free [108]:

$$\begin{aligned} a: g_t &= \sqrt{\frac{2533}{2605}}g + \mathcal{O}(g^3)\ ,\ g_b = \sqrt{\frac{1491}{2605}}g + \mathcal{O}(g^3)\ ,\ g_\lambda = 0\ ,\ g_f = \sqrt{\frac{560}{521}}g + \mathcal{O}(g^3)\ , \\ b: g_t &= \sqrt{\frac{89}{65}}g + \mathcal{O}(g^3)\ ,\ g_b = \sqrt{\frac{63}{65}}g + \mathcal{O}(g^3)\ ,\ g_\lambda = 0\ ,\ g_f = 0\ , \end{aligned} \tag{31}$$

where the higher order terms denote uniquely computable power series in g. Let us note that the reduction of the dimensionless sector is independent of the dimensionful one. These solutions describe the boundaries of a RGI surface in the parameter space which is AF and where g_f and g_λ could be different from zero. Therefore, a partial reduction is possible where g_λ and g_f are independent (non-vanishing) parameters without endangering asymptotic freedom (AF). The proton decay constraints favor solution a, therefore we choose this one for our discussion. [c]

The SSB Lagrangian is

$$\begin{aligned}-\mathcal{L}_{\rm soft} &= m_{H_u}^2 \hat{H}^{*\alpha}\hat{H}_\alpha + m_{H_d}^2 \hat{\overline{H}}^*_\alpha \hat{\overline{H}}^\alpha + m_\Sigma^2 \hat{\Sigma}^{\dagger\ \alpha}_\beta \hat{\Sigma}^\beta_\alpha + \sum_{I=1,2,3} [\, m_{\Phi I}^2 \hat{\Phi}^{*\ (I)}_\alpha \hat{\Phi}^{(I)\alpha} \\ &+ m_{\Psi I}^2 \hat{\Psi}^{\dagger\ (I)\alpha\beta} \hat{\Psi}^{(I)}_{\beta\alpha}\,] + \{\, \frac{1}{2} M\lambda\lambda + B_H \hat{\overline{H}}^\alpha \hat{H}_\alpha + B_\Sigma \hat{\Sigma}^\alpha_\beta \hat{\Sigma}^\beta_\alpha + h_f\, \hat{\overline{H}}^\alpha \hat{\Sigma}^\beta_\alpha \hat{H}_\beta \\ &+ \frac{h_\lambda}{3}\, \hat{\Sigma}^\beta_\alpha \hat{\Sigma}^\gamma_\beta \hat{\Sigma}^\alpha_\gamma + \frac{h_t}{4}\, \epsilon^{\alpha\beta\gamma\delta\tau}\, \hat{\Psi}^{(3)}_{\alpha\beta} \hat{\Psi}^{(3)}_{\gamma\delta} \hat{H}_\tau + \sqrt{2} h_b\, \hat{\Phi}^{(3)\alpha} \hat{\Psi}^{(3)}_{\alpha\beta} \hat{\overline{H}}^\beta + \text{h.c.}\,\}\ ,\end{aligned} \tag{32}$$

where the hat denotes the scalar components of the chiral superfields. The parameters M, μ_Σ and μ_H are treated as independent ones, since they cannot be reduced in a suitable form. The lowest-order reduction for the parameters of the SSB Lagrangian are given by:

$$B_H = \frac{1029}{521}\,\mu_H M\ ,\ B_\Sigma = -\frac{3100}{521}\,\mu_\Sigma M\ , \tag{33}$$

$$\begin{aligned} h_t = -g_t\, M\ ,\ h_b = -g_b\, M\ ,\ h_f = -g_f\, M\ ,\ h_\lambda = 0\ , \\ m_{H_u}^2 = -\frac{569}{521} M^2\ ,\ m_{H_d}^2 = -\frac{460}{521} M^2\ ,\ m_\Sigma^2 = \frac{1550}{521} M^2\ , \\ m_{\Phi 3}^2 = \frac{436}{521} M^2\ ,\ m_{\Phi 1,2}^2 = \frac{8}{5} M^2\ ,\ m_{\Psi 3}^2 = \frac{545}{521} M^2\ ,\ m_{\Psi 1,2}^2 = \frac{12}{5} M^2\ . \end{aligned} \tag{34}$$

We choose the gaugino mass M for characterizing the SUSY breaking scale. Finally, we note that (i) B_Σ and B_H are treated as independent parameters without spoiling the one-loop reduction solution of Eq. (34) and (ii) the soft scalar mass sum rule still holds despite the specific relations among the gaugino mass and the soft scalar masses.

We analyze the particle spectrum predicted for $\mu < 0$ as the only phenomenologically acceptable choice (in the $\mu > 0$ the quark masses do not match the experimental measurements). Below $M_{\rm GUT}$ all couplings and masses of the theory run according to the RGEs of the MSSM. Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones imposing the corresponding boundary conditions.

As presented in [37], the pole top mass m_t is predicted within 2σ of Eq. (25). Concerning the $m_b(M_Z)$ prediction (also in [37]), we take into account a theoretical uncertainty of $\sim 3\%$. But even taking theoretical and experimental uncertainties into account in combination, we find agreement with the experimental value only at the 4σ level. However, since there additional uncertainties of a few percent on the quark Yukawa couplings at the SUSY-breaking scale, that were not fully included (see [20]) into the evaluation of the bottom mass, we still consider the model as viable and proceed with its analysis.

The prediction for M_h as a function of the unified gaugino mass M with $\mu < 0$ is given in Fig. 2 (left). The ΔM_{B_s} channel is responsible for the gap at the B-physics allowed points (green points). The scattered points come from the fact that for each M we vary the free parameters μ_Σ and μ_H. Fig. 2 (right) gives the theoretical uncertainty of the Higgs mass for each point, calculated with `FeynHiggs` 2.16.0 [36]. There is substantial improvement to the Higgs mass uncertainty compared to past analyses, since it has dropped by more than 1 GeV.

[c] $g_\lambda = 0$ is inconsistent, but $g_\lambda \lesssim 0.005$ is necessary in order for the proton decay constraint [20] to be satisfied. A small g_λ is expected to not affect the prediction of unification of SSB parameters.

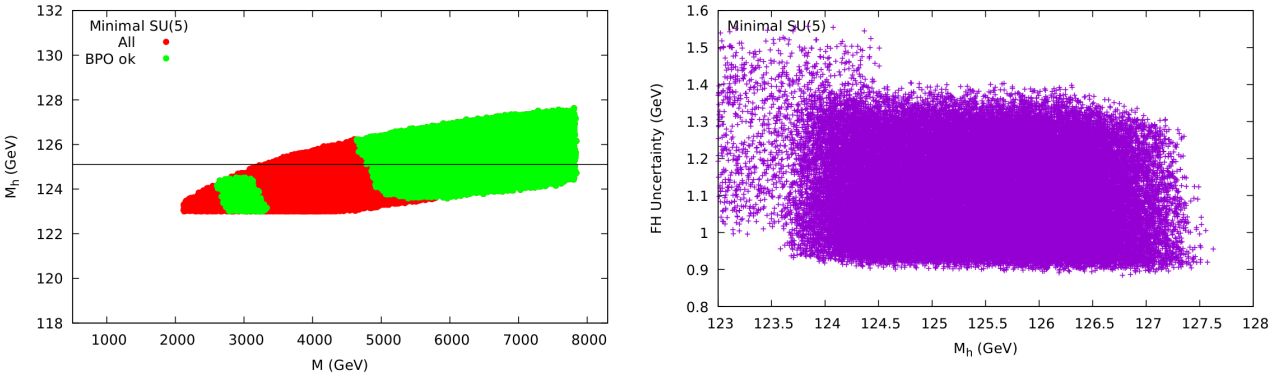

Figure 2: Scatter plots for the Minimal $N = 1$ $SU(5)$ model. Left: The lightest Higgs mass, M_h, as a function of M. The B-physics constraints allow (mostly) higher scale points (with green color). Right: The theoretical uncertainty of the light Higgs mass. [36].

Large parts of the predicted particle spectrum are in agreement with the B-physics observables and the lightest Higgs boson mass measurement and its theoretical uncertainty. We choose three benchmarks in the low-mass region, marking the points with the lightest SUSY particle (LSP) above 1200 GeV (MINI-1), 1500 GeV (MINI-2) and 2200 GeV (MINI-3), respectively. The mass of the LSP can go as high as ~ 3800 GeV, but the cross sections calculated below will then be negligible and we restrict ourselves here to the low-mass region. The values presented in Table 1 were used as input to get the full supersymmetric spectrum from `SPheno 4.0.4` [97,98]. M_i are the gaugino masses and the rest are squared soft sfermion masses which are diagonal ($\mathbf{m^2} = \mathrm{diag}(\mathrm{m}_1^2, \mathrm{m}_2^2, \mathrm{m}_3^2)$), and soft trilinear couplings (also diagonal $\mathbf{A_i} = \mathbb{1}_{3\times 3} A_i$).

	M_1	M_2	M_3	$\lvert\mu\rvert$	b	A_u	A_d	A_e	$\tan\beta$	$m^2_{Q_{1,2}}$
MINI-1	1227	2228	5310	4236	401^2	4325	4772	1732	50.3	6171^2
MINI-2	1507	2721	6376	5091	496^2	5245	5586	2005	52.0	7445^2
MINI-3	2249	4019	9138	7367	1246^2	7571	8317	3271	50.3	10762^2
	$m^2_{Q_3}$	$m^2_{L_{1,2}}$	$m^2_{L_3}$	$m^2_{\bar{u}_{1,2}}$	$m^2_{\bar{u}_3}$	$m^2_{\bar{d}_{1,2}}$	$m^2_{\bar{d}_3}$	$m^2_{\bar{e}_{1,2}}$	$m^2_{\bar{e}_3}$	
MINI-1	4548^2	3714^2	2767^2	5974^2	4181^2	5478^2	4177^2	4160^2	2491^2	
MINI-2	5469^2	4521^2	3358^2	7206^2	5039^2	5478^2	4994^2	5070^2	3019^2	
MINI-3	7890^2	6639^2	4934^2	10412^2	7233^2	9495^2	7211^2	7459^2	4464^2	

Table 1: Minimal $N = 1$ $SU(5)$ predictions that are used as input to `SPheno`. Mass parameters are in GeV and rounded to 1 GeV.

The resulting masses of all the particles that will be relevant for our analysis can be found in Table 2. The three first values are the heavy Higgs masses. The gluino mass is $M_{\tilde{g}}$, the neutralinos and the charginos are denoted as $M_{\tilde{\chi}_i^0}$ and $M_{\tilde{\chi}_i^\pm}$, while the slepton and sneutrino masses for all three generations are given as $M_{\tilde{e}_{1,2,3}}$, $M_{\tilde{\nu}_{1,2,3}}$. Similarly, the squarks are denoted as $M_{\tilde{d}_{1,2}}$ and $M_{\tilde{u}_{1,2}}$ for the first two generations. The third generation masses are given by $M_{\tilde{t}_{1,2}}$ for stops and $M_{\tilde{b}_{1,2}}$ for sbottoms.

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}_1^0}$	$M_{\tilde{\chi}_2^0}$	$M_{\tilde{\chi}_3^0}$	$M_{\tilde{\chi}_4^0}$	$M_{\tilde{\chi}_1^\pm}$	$M_{\tilde{\chi}_2^\pm}$
MINI-1	2.660	2.660	2.637	5.596	1.221	2.316	4.224	4.225	2.316	4.225
MINI-2	3.329	3.329	3.300	6.717	1.500	2.827	5.076	5.077	2.827	5.078
MINI-3	8.656	8.656	8.631	9.618	2.239	4.176	7.357	7.358	4.176	7.359
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
MINI-1	3.729	3.728	2.445	2.766	5.617	6.100	4.332	4.698	4.312	4.704
MINI-2	4.539	4.538	2.968	3.356	6.759	7.354	5.180	5.647	5.197	5.652
MINI-3	6.666	6.665	4.408	4.935	9.722	10.616	7.471	8.148	7.477	8.151

Table 2: Masses of Higgs bosons and some of the SUSY particles for each benchmark of the Minimal $N = 1$ $SU(5)$ (in TeV).

Table 3 shows the expected production cross section for selected channels at the 100 TeV future FCC-hh collider. We do not show any cross sections for $\sqrt{s} = 14$ TeV, since the prospects for discovery of MINI scenarios at the HL-LHC are very dim. SUSY particles are too heavy to be produced with cross sections greater that 0.01 fb. Concerning the heavy Higgs bosons, the main search channels will be $H/A \to \tau^+\tau^-$. Our heavy Higgs-boson mass scale shows values $\gtrsim 2500$ GeV with $\tan\beta \sim 50$. The corresponding reach of the HL-LHC has been estimated in [142]. In comparison with our benchmark points we conclude that they will not be accessible at the HL-LHC.[d]

The situation changes for the FCC-hh. Theory analyses [143, 144] have shown that for large $\tan\beta$ heavy Higgs-boson mass scales up to ~ 8 TeV may be accessible, both for neutral as well as for charged Higgs bosons. The relevant decay channels are $H/A \to \tau^+\tau^-$ and $H^\pm \to \tau\nu_\tau, tb$. This places our three benchmark points well within the covered region (MINI-1 and MINI-2) or at the border of the parameter space that can be probed (MINI-3).

The energy of 100 TeV is big enough to produce SUSY particles in pairs. However, the cross sections remain relatively small. Only for the MINI-1 scenario the squark pair and squark-gluino (summed over all squarks) production cross sections can reach tens of fb. For MINI-2 and MINI-3 scenarios the cross sections are significantly smaller. In these scenarios squarks decay preferentially into a quark+LSP (with BR ~ 0.95), gluino into $\tilde{t}\bar{t}$ and $\tilde{b}\bar{b}$ $+h.c$ with BR ~ 0.33 each.

The SUSY discovery reach at the FCC-hh with $3\,\text{ab}^{-1}$ was evaluated in [145] for a certain set of simplified models. In the following we will compare these simplified model limits with our benchmark points to get an idea, which part of the spectrum can be covered at the FCC-hh. A more detailed evaluation with the future limits implemented into proper recasting tools would be necessary to obtain a firmer statement. However, such a detailed analysis goes beyond the scope of our paper and we restrict ourselves to the simpler direct comparison of the simplified model limits with our benchmark predictions.

Concerning the scalar tops, the mass predictions of MINI-1 and MINI-2 are well within the anticipated reach of the FCC-hh, while MINI-3 predicts a too heavy stop mass. On the other hand, even for MINI-1 and MINI-2 no $5\,\sigma$ discovery can be expected. The situation looks more favorable for the first and second generation squarks. All the predicted masses can be excluded at the FCC-hh, whereas a $5\,\sigma$ discovery will be difficult, but potentially possible (see Fig. 19 in [145]). Even more favorable appear the prospects for gluino searches at the FCC-hh. All three benchmark points may lead to a $5\,\sigma$ discovery (see Fig. 13 in [145]). On the other hand, chances for chargino/neutralino

[d] The analysis presented in [142] only reaches $M_A \leq 2000$ GeV, where an exclusion down to $\tan\beta \sim 30$ is expected. An extrapolation to $\tan\beta \sim 50$ reaches Higgs-boson mass scales of ~ 2500 GeV.

searches are slim at the FCC-hh. The Next-to LSP (NLSP) can only be accessed for $M_{\tilde{\chi}_1^0} \lesssim 1$ TeV (see Fig. 21 in [145]), where all our benchmark points have $M_{\tilde{\chi}_1^0} > 1$ TeV. Taking into account that our three benchmark points represent only the lower part of the possible mass spectrum (with LSP masses of up to ~ 1.5 TeV higher), we conclude that even at the FCC-hh large parts of the possible SUSY spectrum will remain elusive.

scenarios $\sqrt{s}$	MINI-1 100 TeV	MINI-2 100 TeV	MINI-3 100 TeV	scenarios $\sqrt{s}$	MINI-1 100 TeV	MINI-2 100 TeV	MINI-3 100 TeV
$\tilde{\chi}_1^0\tilde{\chi}_1^0$	0.04	0.02		$\tilde{u}_i\tilde{\chi}_1^-, \tilde{d}_i\tilde{\chi}_1^+ + h.c.$	1.00	0.35	0.03
$\tilde{\chi}_1^0\tilde{\chi}_3^0$	0.02	0.01		$\tilde{u}_i\tilde{\chi}_2^-, \tilde{d}_i\tilde{\chi}_2^+ + h.c.$	0.07	0.02	
$\tilde{\chi}_2^0\tilde{\chi}_2^0$	0.06	0.02		$\tilde{q}_i\tilde{\chi}_1^0, \tilde{q}_i^*\tilde{\chi}_1^0$	0.38	0.14	0.02
$\tilde{\chi}_2^0\tilde{\chi}_3^0$	0.03	0.01		$\tilde{q}_i\tilde{\chi}_2^0, \tilde{q}_i^*\tilde{\chi}_2^0$	0.51	0.17	0.02
$\tilde{\chi}_2^0\tilde{\chi}_4^0$	0.02	0.01		$\tilde{\nu}_i\tilde{e}_j^*, \tilde{\nu}_i^*\tilde{e}_j$	0.06	0.02	
$\tilde{\chi}_3^0\tilde{\chi}_4^0$	0.05	0.02		$Hb\bar{b}$	84.04	30.10	0.17
$\tilde{\chi}_2^0\tilde{\chi}_1^+$	2.20	0.98	0.18	$Ab\bar{b}$	84.79	29.79	0.18
$\tilde{\chi}_3^0\tilde{\chi}_2^+$	0.10	0.04	0.01	$H^+b\bar{t} + H^-t\bar{b}$	33.24	12.76	0.1
$\tilde{\chi}_4^0\tilde{\chi}_2^+$	0.10	0.04	0.01	$H^-b\bar{b}$	0.04	0.02	
$\tilde{g}\tilde{g}$	7.76	2.02	0.11	$Ht\bar{t}$	0.03	0.01	
$\tilde{g}\tilde{\chi}_1^0$	0.28	0.11	0.01	$At\bar{t}$	0.02	0.01	
$\tilde{g}\tilde{\chi}_2^0$	0.34	0.12	0.01	Htb	0.01		
$\tilde{g}\tilde{\chi}_1^+$	0.70	0.27	0.03	HA	0.03	0.01	
$\tilde{q}_i\tilde{q}_j, \tilde{q}_i\tilde{q}_j^*$	21.15	7.44	0.74	HH^+	0.06	0.02	
$\tilde{\chi}_1^+\tilde{\chi}_1^-$	1.19	0.54	0.09	H^+W^-	6.50	2.96	0.03
$\tilde{\chi}_1^+\tilde{\chi}_2^-$	0.05	0.02		HW^+	0.02	0.01	
$\tilde{\chi}_2^+\tilde{\chi}_1^-$	0.05	0.02		H^+H^-	0.04	0.01	
$\tilde{\chi}_2^+\tilde{\chi}_2^-$	0.06	0.02		AH^+	0.06	0.02	
$\tilde{e}_i\tilde{e}_j^*$	0.16	0.08	0.01	AW^+	0.02	0.01	
$\tilde{q}_i\tilde{g}, \tilde{q}_i^*\tilde{g}$	30.57	9.33	0.66	HZ	1.38	0.58	0.01
$\tilde{\nu}_i\tilde{\nu}_j^*$	0.04	0.02		AZ	1.20	0.52	0.01

Table 3: Expected production cross sections (in fb) for SUSY particles in the MINI scenarios. There are no channels with cross sections exceeding 0.01 fb at $\sqrt{s} = 14$ TeV.

6 The Finite $N = 1$ Supersymmetric $SU(5)$ Model

We proceed now to the finite to all-orders $SU(5)$ gauge theory, where the reduction of couplings is restricted to the third generation. An older examination of this specific Finite Unified Theory (FUT) was shown to be in agreement with the experimental constraints at the time [29] and has predicted, almost five years before its discovery, the light Higgs mass in the correct range. As discussed below, improved Higgs calculations predict a somewhat different interval that is still in agreement with current experimental data. The particle content of the model has three $(\overline{\mathbf{5}} + \mathbf{10})$ supermultiplets for the three generations of leptons and quarks, while the Higgs sector consists of four supermultiplets $(\overline{\mathbf{5}} + \mathbf{5})$ and one **24**. The finite $SU(5)$ group is broken to the MSSM, which of course in no longer a finite theory [14–17, 21, 24].

In order for this finite to all-orders $SU(5)$ model to achieve Gauge Yukawa Unification (GYU), it should have the following characteristics:

(i) The one-loop anomalous dimensions are diagonal i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$.
(ii) The fermions of the $\overline{\mathbf{5}}_i$ and $\mathbf{10}_i$ $(i = 1, 2, 3)$ are not coupled to the $\mathbf{24}$.
(iii) The pair of the MSSM Higgs doublets are mostly composed from the 5 and $\bar{5}$ Higgs that couple to the third generation

The superpotential of the model, with an enhanced symmetry due to the reduction of couplings, is given by [25, 27]:

$$W = \sum_{i=1}^{3} \left[\frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \right] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4 \tag{35}$$
$$+ g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \, \overline{H}_4 + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \, \overline{H}_4 + g_2^f \, H_2 \, \mathbf{24} \, \overline{H}_2 + g_3^f \, H_3 \, \mathbf{24} \, \overline{H}_3 + \frac{g^\lambda}{3} (\mathbf{24})^3 \, .$$

Discussion of the model with a more detailed description can be found in [14–16]. The non-degenerate and isolated solutions to the vanishing of $\gamma_i^{(1)}$ are:

$$\begin{aligned} &(g_1^u)^2 = \frac{8}{5} \, g^2 \, , \; (g_1^d)^2 = \frac{6}{5} \, g^2 \, , \; (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5} \, g^2 \, , \\ &(g_2^d)^2 = (g_3^d)^2 = \frac{3}{5} \, g^2 \, , \; (g_{23}^u)^2 = \frac{4}{5} \, g^2 \, , \; (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5} \, g^2 \, , \\ &(g^\lambda)^2 = \frac{15}{7} g^2 \, , \; (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2} \, g^2 \, , \; (g_1^f)^2 = 0 \, , \; (g_4^f)^2 = 0 \, . \end{aligned} \tag{36}$$

We have also the relation $h = -MC$, while the sum rules lead to:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2 \, , \; m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3} \, , \; m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3} \, . \tag{37}$$

Therefore, we only have two free parameters, namely $m_{\mathbf{10}}$ and M in the dimensionful sector.

When $SU(5)$ breaks down to the MSSM, a suitable rotation in the Higgs sector [14, 15, 111–114], permits only a pair of Higgs doublets (coupled mostly to the third family) to remain light and acquire vev's. Avoiding fast proton decay is achieved with the usual doublet-triplet splitting, although different from the one applied to the minimal $SU(5)$ due to the extended Higgs sector of the finite model. Therefore, below the GUT scale we get the MSSM where the third generation is given by the finiteness conditions while the first two remain unrestricted.

Conditions set by finiteness do not restrict the renormalization properties at low energies, so we are left with boundary conditions on the gauge and Yukawa couplings (36), the $h = -MC$ relation and the soft scalar-mass sum rule at $M_{\rm GUT}$. The quark masses $m_b(M_Z)$ and m_t are predicted within 2σ and 3σ uncertainty, respectively of their experimental values (see [37] for details). The only phenomenologically viable option is to consider $\mu < 0$, as shown in [37, 115–121].

The scatter plot of the light Higgs boson mass is given in Fig. 3 (left), while its theory uncertainty [36] is given in Fig. 3 (right), with the same color coding as in Fig. 2. This point-by-point uncertainty (calculated with `FeynHiggs`) drops significantly (w.r.t. past analyses) to $0.65-0.70$ GeV. The scattered points come from the free parameter m_{10}.

Compared to our previous analyses [37, 115–124], the improved evaluation of M_h and its uncertainty prefer a heavier (Higgs) spectrum and thus allows only a heavy supersymmetric spectrum

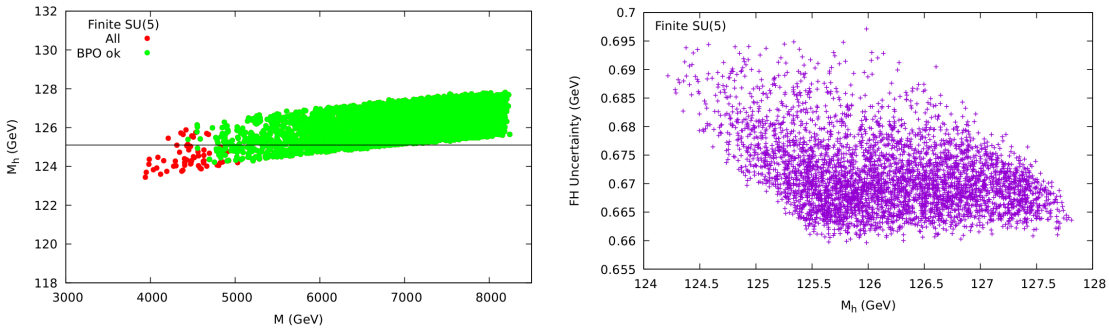

Figure 3: Scatter plot for the Finite $N = 1$ $SU(5)$ model. Left: M_h as a function of M. Green points comply with B-physics constraints. Right: The lightest Higgs mass theoretical uncertainty calculated with `FeynHiggs` 2.16.0 [36].

	M_1	M_2	M_3	$\lvert\mu\rvert$	b	A_u	A_d	A_e	$\tan\beta$	$m^2_{Q_{1,2}}$
FUTSU5-1	2124	3815	8804	4825	854^2	7282	7710	2961	49.9	8112^2
FUTSU5-2	2501	4473	10198	5508	1048^2	8493	9023	3536	50.1	9387^2
FUTSU5-3	3000	5340	11996	6673	2361^2	10086	10562	4243	49.9	11030^2
	$m^2_{Q_3}$	$m^2_{L_{1,2}}$	$m^2_{L_3}$	$m^2_{\bar{u}_{1,2}}$	$m^2_{\bar{u}_3}$	$m^2_{\bar{d}_{1,2}}$	$m^2_{\bar{d}_3}$	$m^2_{\bar{e}_{1,2}}$	$m^2_{\bar{e}_3}$	
FUTSU5-1	6634^2	3869^2	3120^2	7684^2	5053^2	7635^2	4177^2	3084^2	2241^2	
FUTSU5-2	7669^2	4521^2	3747^2	8887^2	6865^2	8826^2	6893^2	3602^2	2551^2	
FUTSU5-3	9116^2	5355^2	3745^2	10419^2	8170^2	10362^2	7708^2	4329^2	3403^2	

Table 4: Finite $N = 1$ $SU(5)$ predictions that are used as input to `SPheno`. Mass parameters are in GeV and rounded to 1 GeV.

(which is in agreement with all existing experimental constraints). In particular, very heavy colored SUSY particles are favored (nearly independent of the M_h uncertainty), in agreement with the non-observation of those particles at the LHC [126].
We choose three benchmarks, each featuring the LSP above 2100 GeV, 2400 GeV and 2900 GeV respectively. Again, they are chosen from the low-mass region. Although the LSP can be as heavy as ~ 4000 GeV, but in such cases the production cross sections even at the FCC-hh would be too small. The input and output of `SPheno` 4.0.4 [97,98] can be found in Table 4 and Table 5 (with the notation as in Sect. 5).

The expected production cross sections for various final states are listed in Table 6. At 14 TeV HL-LHC none of the Finite $N = 1$ $SU(5)$ scenarios listed in Table 4 has a SUSY production cross section above 0.01 fb, and thus will (likely) remain unobservable. All superpartners are too heavy to be produced in pairs. Also the heavy Higgs bosons are far outside the reach of the HL-LHC [142].

At the FCC-hh the discovery prospects for the heavy Higgs-boson spectrum is significantly better. With $\tan\beta \sim 50$ the first two benchmark points, FUTSU5-1 and FUTSU5-2, are well within the reach of the FCC-hh. The third point, FUTSU5-3, however, with $M_A \sim 16$ TeV will be far outside

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}_1^0}$	$M_{\tilde{\chi}_2^0}$	$M_{\tilde{\chi}_3^0}$	$M_{\tilde{\chi}_4^0}$	$M_{\tilde{\chi}_1^\pm}$	$M_{\tilde{\chi}_2^\pm}$
FUTSU5-1	5.688	5.688	5.688	8.966	2.103	3.917	4.829	4.832	3.917	4.833
FUTSU5-2	7.039	7.039	7.086	10.380	2.476	4.592	5.515	5.518	4.592	5.519
FUTSU5-3	16.382	16.382	16.401	12.210	2.972	5.484	6.688	6.691	5.484	6.691
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
FUTSU5-1	3.102	3.907	2.205	3.137	7.839	7.888	6.102	6.817	6.099	6.821
FUTSU5-2	3.623	4.566	2.517	3.768	9.059	9.119	7.113	7.877	7.032	7.881
FUTSU5-3	4.334	5.418	3.426	3.834	10.635	10.699	8.000	9.387	8.401	9.390

Table 5: Masses for each benchmark of the Finite $N = 1$ $SU(5)$ (in TeV).

the reach of the FCC-hh. Prospects for detecting production of squark pairs and squark-gluino pairs are also very dim since their production cross section is also at the level of a few fb. This is as a result of a heavy spectrum in this class of models (see [145] with the same Figures as discussed in Sec. 5). Concerning the stops, the lighter one might be accessible in FUTSU5-1. For the squarks of the first two generations the prospects of testing the model are somewhat better. All three benchmark models could possibly be excluded at the $2\,\sigma$ level, but no discovery at the $5\,\sigma$ can be expected. The same holds for the gluino. Charginos and neutralinos will remain unobservable due to the heavy LSP. As in the previous section, since only the lower part of the possible mass spectrum has been considered (with LSP masses higher by up to ~ 1 TeV), we have to conclude that again large parts of the possible mass spectra will not be observable at the FCC-hh.

scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV	scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV
$\tilde{\chi}_2^0\tilde{\chi}_3^0$	0.01	0.01		$\tilde{\nu}_i\tilde{\nu}_j^*$	0.02	0.01	0.01
$\tilde{\chi}_3^0\tilde{\chi}_4^0$	0.03	0.01		$\tilde{u}_i\tilde{\chi}_1^-, \tilde{d}_i\tilde{\chi}_1^+ + h.c.$	0.15	0.06	0.02
$\tilde{\chi}_2^0\tilde{\chi}_1^+$	0.17	0.08	0.03	$\tilde{q}_i\tilde{\chi}_1^0, \tilde{q}_i^*\tilde{\chi}_1^0$	0.08	0.03	0.01
$\tilde{\chi}_3^0\tilde{\chi}_2^+$	0.05	0.03	0.01	$\tilde{q}_i\tilde{\chi}_2^0, \tilde{q}_i^*\tilde{\chi}_2^0$	0.08	0.03	0.01
$\tilde{\chi}_4^0\tilde{\chi}_2^+$	0.05	0.03	0.01	$\tilde{\nu}_i\tilde{e}_j^*, \tilde{\nu}_i^*\tilde{e}_j$	0.09	0.04	0.01
$\tilde{g}\tilde{g}$	0.20	0.05	0.01	$Hb\bar{b}$	2.76	0.85	
$\tilde{g}\tilde{\chi}_1^0$	0.03	0.01		$Ab\bar{b}$	2.73	0.84	
$\tilde{g}\tilde{\chi}_2^0$	0.03	0.01		$H^+b\bar{t} + h.c.$	1.32	0.42	
$\tilde{g}\tilde{\chi}_1^+$	0.07	0.03	0.01	H^+W^-	0.38	0.12	
$\tilde{q}_i\tilde{q}_j, \tilde{q}_i\tilde{q}_j^*$	3.70	1.51	0.53	HZ	0.09	0.03	
$\tilde{\chi}_1^+\tilde{\chi}_1^-$	0.10	0.05	0.02	AZ	0.09	0.03	
$\tilde{\chi}_2^+\tilde{\chi}_2^-$	0.03	0.02	0.01				
$\tilde{e}_i\tilde{e}_j^*$	0.23	0.13	0.05				
$\tilde{q}_i\tilde{g}, \tilde{q}_i^*\tilde{g}$	2.26	0.75	0.20				

Table 6: Expected production cross sections (in fb) for SUSY particles in the FUTSU5 scenarios.

7 The Finite $SU(N)^3$ Model

We proceed now to a FUT based on a product gauge group. Consider an $N = 1$ SUSY theory with $SU(N)_1 \times SU(N)_2 \times \cdots \times SU(N)_k$ having n_f families transforming as $(N, N^*, 1, \ldots, 1) +$

$(1, N, N^*, \ldots, 1) + \cdots + (N^*, 1, 1, \ldots, N)$. Then, the first order coefficient of the β-function, for each $SU(N)$ group is:

$$b = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N \,. \tag{38}$$

Demanding the vanishing of the gauge one-loop β-function, i.e. $b = 0$, we are led to the choice $n_f = 3$. Phenomenological reasons lead to the choice of the $SU(3)_C \times SU(3)_L \times SU(3)_R$ model, discussed in Ref. [128], while a detailed discussion of the general well known example can be found in [129–132]. The leptons and quarks transform as:

$$q = \begin{pmatrix} d & u & D \\ d & u & D \\ d & u & D \end{pmatrix} \sim (3, 3^*, 1), \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ D^c & D^c & D^c \end{pmatrix} \sim (3^*, 1, 3), \quad \lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*) \tag{39}$$

where D are down-type quarks acquiring masses close to $M_{\rm GUT}$. A cyclic Z_3 symmetry is imposed on the multiplets to achieve equal gauge couplings at the GUT scale and in that case the vanishing of the first-order β-function is satisfied. Continuing to the vanishing of the anomalous dimension of all the fields (see Eq. (20)), we note that there are two trilinear invariant terms in the superpotential, namely:

$$f\ Tr(\lambda q^c q) + \frac{1}{6} f'\ \epsilon_{ijk}\epsilon_{abc}(\lambda_{ia}\lambda_{jb}\lambda_{kc} + q^c_{ia} q^c_{jb} q^c_{kc} + q_{ia} q_{jb} q_{kc}), \tag{40}$$

with f and f' the corresponding Yukawa couplings. The superfields $(\tilde{N}, \tilde{N}^c)$ obtain vev's and provide masses to leptons and quarks

$$m_d = f\langle\tilde{N}\rangle, \quad m_u = f\langle\tilde{N}^c\rangle, \quad m_e = f'\langle\tilde{N}\rangle, \quad m_\nu = f'\langle\tilde{N}^c\rangle. \tag{41}$$

Having three families, 11 f couplings and 10 f' couplings are present in the most general superpotential. Demanding the vanishing of all superfield anomalous dimensions, 9 conditions are imposed

$$\sum_{j,k} f_{ijk}(f_{ljk})^* + \frac{2}{3}\sum_{j,k} f'_{ijk}(f'_{ljk})^* = \frac{16}{9} g^2 \delta_{il} \,, \tag{42}$$

where

$$f_{ijk} = f_{jki} = f_{kij}, \qquad f'_{ijk} = f'_{jki} = f'_{kij} = f'_{ikj} = f'_{kji} = f'_{jik} \,. \tag{43}$$

The masses of leptons and quarks are acquired from the vev's of the scalar parts of the superfields $\tilde{N}_{1,2,3}$ and $\tilde{N}^c_{1,2,3}$.

At $M_{\rm GUT}$ the $SU(3)^3$ FUT breaks[e] to the MSSM, where as was already mentioned, both Higgs doublets couple mostly to the third generation. The FUT breaking leaves its mark in the form of Eq. (42), i.e. boundary conditions on the gauge and Yukawa couplings, the relation among the soft trilinear coupling, the corresponding Yukawa coupling and the unified gaugino mass and finally the soft scalar mass sum rule at $M_{\rm GUT}$. In this specific model the sum rule takes the form:

$$m^2_{H_u} + m^2_{\tilde{t}^c} + m^2_{\tilde{q}} = M^2 = m^2_{H_d} + m^2_{\tilde{b}^c} + m^2_{\tilde{q}} \,. \tag{44}$$

[e] [133, 134] and refs therein discuss in detail the spontaneous breaking of $SU(3)^3$.

The model is finite to all-orders if the solution of Eq. (42) is both *isolated* and unique. Then, $f' = 0$ and we have the relations

$$f^2 = f_{111}^2 = f_{222}^2 = f_{333}^2 = \frac{16}{9} g^2 \,. \tag{45}$$

Since all f' vanish, at one-loop order, the lepton masses vanish. Since these masses, even radiatively, cannot be produced because of the finiteness conditions, we are faced with a problem which needs further study. If the solution of Eq. (42) is unique but not isolated (i.e. parametric), we can have non zero f' leading to non-vanishing lepton masses and at the same time achieving two-loop finiteness. In that case the set of conditions restricting the Yukawa couplings read:

$$f^2 = r \left(\frac{16}{9}\right) g^2 \,, \quad f'^2 = (1-r) \left(\frac{8}{3}\right) g^2 \,, \tag{46}$$

where r parametrises the different solutions and as such is a free parameter. It should be noted that we use the sum rule as boundary condition for the soft scalar masses.

In our analysis we consider the two-loop finite version of the model, where again below $M_{\rm GUT}$ we get the MSSM. We take into account two new thresholds for the masses of the new particles at $\sim 10^{13}$ GeV and $\sim 10^{14}$ GeV resulting in a wider phenomenologically viable parameter space [125].

Looking for the values of the parameter r which comply with the experimental limits, we find that both the top and bottom masses are in the experimental range (within 2σ) for the same value of r between 0.65 and 0.80 (we singled out the $\mu < 0$ case as the most promising). The inclusion of the above-mentioned thresholds gives an important improvement on the top mass from past versions of the model [128, 135–137].

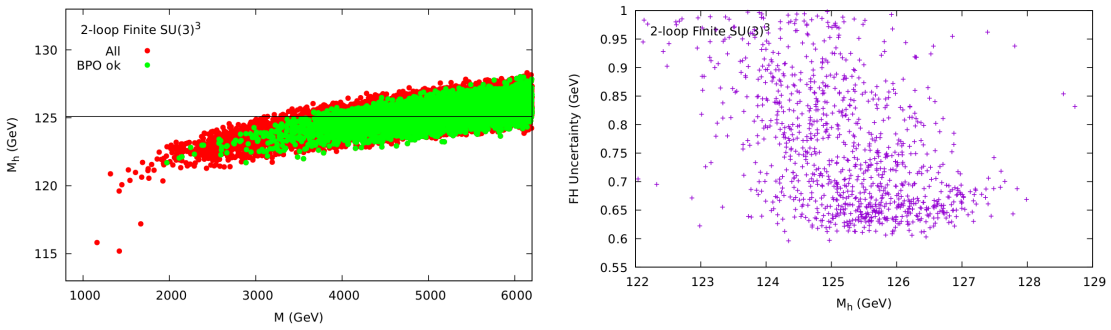

Figure 4: Scatter plot for the Finite $N = 1$ $SU(3)^3$ model. Left: M_h as a function of M. Right: The Higgs mass theoretical uncertainty [36].

Fig. 4 (left) shows the scatter plot of the light Higgs boson mass (green points satisfy the B-physics constraints), while the point-by-point calculated theoretical uncertainty is presented in Fig. 4 (right). The scattered points are due to the fact that we vary five parameters, namely r and four of the parameters that form the sum rule. The uncertainty is found in the range between 0.6 GeV and 1.0 GeV. All constraints regarding quark masses, the light Higgs boson mass and B-physics are

	M_1	M_2	M_3	$\lvert\mu\rvert$	b	A_u	A_d	A_e	$\tan\beta$	$m^2_{Q_{1,2}}$
FSU33-1	1522	2758	6369	6138	1002^2	4520	4413	1645	46.2	5574^2
FSU33-2	2070	3722	8330	7129	1083^2	5841	5734	2357	45.5	7255^2
FSU33-3	2500	4484	10016	6790	972^2	7205	7110	2674	49.7	8709
	$m^2_{Q_3}$	$m^2_{L_{1,2}}$	$m^2_{L_3}$	$m^2_{\overline{u}_{1,2}}$	$m^2_{\overline{u}_3}$	$m^2_{\overline{d}_{1,2}}$	$m^2_{\overline{d}_3}$	$m^2_{\overline{e}_{1,2}}$	$m^2_{\overline{e}_3}$	
FSU33-1	4705^2	2382^2	3754^2	5234^2	5548^2	5197^2	7043^2	1558^2	3095^2	
FSU33-2	7255^2	3136^2	4131^2	6749^2	7225^2	6745^2	8523^2	2238^2	3342^2	
FSU33-3	9074^2	3831^2	5483^2	8152^2	7207^2	2558^2	8600^2	2507^2	4000^2	

Table 7: Finite $N = 1$ $SU(3)^3$ predictions that are used as input to `SPheno`. Mass parameters are in GeV and rounded to 1 GeV.

satisfied, rendering the model very successful. The prediction of the SUSY spectrum results in relatively heavy particles, in full agreement with the current experimental searches.

Again, we choose three benchmarks, each featuring the LSP above 1500 GeV, 2000 GeV and 2400 GeV respectively (but the LSP can go as high as $\sim$ 4100 GeV, again with too small cross sections). The input and output of `SPheno` 4.0.4 [97, 98] can be found in Table 7 and Table 8 respectively (with the notation as in Sect. 5).

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}^0_1}$	$M_{\tilde{\chi}^0_2}$	$M_{\tilde{\chi}^0_3}$	$M_{\tilde{\chi}^0_4}$	$M_{\tilde{\chi}^\pm_1}$	$M_{\tilde{\chi}^\pm_2}$
FSU33-1	7.029	7.029	7.028	6.526	1.506	2.840	6.108	6.109	2.839	6.109
FSU33-2	6.484	6.484	6.431	8.561	2.041	3.817	7.092	7.093	3.817	7.093
FSU33-3	6.539	6.539	6.590	10.159	2.473	4.598	6.780	6.781	4.598	6.781
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
FSU33-1	2.416	2.415	1.578	2.414	5.375	5.411	4.913	5.375	4.912	5.411
FSU33-2	3.188	3.187	2.269	3.186	7.026	7.029	6.006	7.026	6.005	7.029
FSU33-3	3.883	3.882	2.540	3.882	8.334	8.397	7.227	8.334	7.214	7.409

Table 8: Masses for each benchmark of the Finite $N = 1$ $SU(3)^3$ (in TeV).

It should be noted that in this model the scale of the heavy Higgs bosons does not vary monotonously with $M_{\tilde{\chi}^0_1}$, as in the previously considered models. This can be understood as follows. The Higgs bosons masses are determined by a combination of the sum rule at the unification scale, and the requirement of successful electroweak symmetry breaking at the low scale. Like in the finite scenario of the previous section, there are no direct relations between the soft scalar masses and the unified gaugino mass, but they are related through the corresponding sum rule and thus vary correlatedly, a fact that makes the dependence on the boundary values more restrictive. Furthermore (and even more importantly), the fact that we took into account the two thresholds at $\sim 10^{13}$ GeV and $\sim 10^{14}$ GeV (as mentioned above), allows the new particles, mainly the Higgsinos of the two other families (that were considered decoupled at the unification scale in previous analyses) and the down-like exotic quarks (in a lower degree), to affect the running of the (soft) RGEs in a non-negligible way. Thus, since at low energies the heavy Higgs masses depend mainly on the values of $m^2_{H_u}$, $m^2_{H_d}$, $|\mu|$ and $\tan\beta$, they are substantially less connected to $M_{\tilde{\chi}^0_1}$ than in the other models, leading to a different exclusion potential, as will be discussed in the following.

Scenarios of Finite $SU(3)^3$ are beyond the reach of the HL-LHC. Not only superpartners are too heavy, but also heavy Higgs bosons with a mass scale of ~ 7 TeV cannot be detected at the HL-LHC. At 100 TeV collider (see Table 9), on the other hand, all three benchmark points are well within the reach of the $H/A \to \tau^+\tau^-$ as well as the $H^\pm \to \tau\nu_\tau, tb$ searches [143, 144], despite the slightly smaller values of $\tan\beta \sim 45$. This is particularly because of the different dependence of the heavy Higgs-boson mass scale on $M_{\tilde{\chi}_1^0}$, as discussed above. However, we have checked that M_A can go up to to ~ 11 TeV, and thus the heaviest part of the possible spectrum would escape the heavy Higgs-boson searches at the FCC-hh.

Interesting are also the prospects for production of squark pairs and squark-gluino, which can reach ~ 20 fb for the FSU33-1 case, going down to a few fb for FSU33-2 and FSU33-3 scenarios. The lightest squarks decay almost exclusively to the third generation quark and chargino/neutralino, while gluino enjoys many possible decay channels to quark-squark pairs each one with branching fraction of the order of a percent, with the biggest one $\sim 20\%$ to $t\tilde{t}_1 + h.c.$.

We briefly discuss the SUSY discovery potential at the FCC-hh, referring agian to [145] with the same Figures as discussed in Sec. 5. Stops in FSU33-1 and FSU33-2 can be tested at the FCC-hh, while the masses turn out to be too heavy in FSU33-3. The situation is better for scalar quarks, where all three scenarios can be tested, but will not allow for a $5\,\sigma$ discovery. Even more favorable are the prospects for gluino. Possibly all three scenarios can be tested at the $5\,\sigma$ level. As in the previous scenario, the charginos and neutralinos will not be accessible, due to the too heavy LSP. Keeping in mind that only the lower part of possible mass spectrum is represented by the three benchmarks (with the LSP up to ~ 1.5 TeV heavier), we conclude that as before large parts of the parameter space will not be testable at the FCC-hh. The only partial exception here is the Higgs-boson sector, where only the the part with the highest possible Higgs-boson mass spectra would escape the FCC-hh searches.

scenarios	FSU33-1	FSU33-2	FSU33-3	scenarios	FSU33-1	FSU33-2	FSU33-3
$\sqrt{s}$	100 TeV	100 TeV	100 TeV	$\sqrt{s}$	100 TeV	100 TeV	100 TeV
$\tilde{\chi}_1^0\tilde{\chi}_1^0$	0.04	0.01	0.01	$\tilde{q}_i\tilde{g}, \tilde{q}_i^*\tilde{g}$	22.12	3.71	1.05
$\tilde{\chi}_2^0\tilde{\chi}_2^0$	0.04	0.01		$\tilde{\nu}_i\tilde{\nu}_j^*$	0.10	0.03	0.01
$\tilde{\chi}_2^0\tilde{\chi}_1^+$	0.58	0.16	0.07	$\tilde{u}_i\tilde{\chi}_1^-, \tilde{d}_i\tilde{\chi}_1^+ + h.c.$	1.22	0.25	0.08
$\tilde{\chi}_3^0\tilde{\chi}_2^+$	0.02	0.01	0.01	$\tilde{q}_i\tilde{\chi}_1^0, \tilde{q}_i^*\tilde{\chi}_1^0$	0.55	0.13	0.05
$\tilde{\chi}_4^0\tilde{\chi}_2^+$	0.02	0.01	0.01	$\tilde{q}_i\tilde{\chi}_2^0, \tilde{q}_i^*\tilde{\chi}_2^0$	0.60	0.13	0.04
$\tilde{g}\tilde{g}$	2.61	0.30	0.07	$\tilde{\nu}_i\tilde{e}_j^*, \tilde{\nu}_i^*\tilde{e}_j$	0.36	0.12	0.04
$\tilde{g}\tilde{\chi}_1^0$	0.20	0.05	0.02	$Hb\bar{b}$	0.71	1.23	1.19
$\tilde{g}\tilde{\chi}_2^0$	0.20	0.04	0.01	$Ab\bar{b}$	0.72	1.23	1.18
$\tilde{g}\tilde{\chi}_1^+$	0.42	0.09	0.03	$H^+b\bar{t} + h.c.$	0.37	0.75	0.58
$\tilde{q}_i\tilde{q}_j, \tilde{q}_i\tilde{q}_j^*$	25.09	6.09	2.25	H^+W^-	0.10	0.25	0.19
$\tilde{\chi}_1^+\tilde{\chi}_1^-$	0.37	0.10	0.04	HZ	0.02	0.04	0.04
$\tilde{e}_i\tilde{e}_j^*$	0.39	0.12	0.06	AZ	0.02	0.04	0.04

Table 9: Expected production cross sections (in fb) for SUSY particles in the FSU33 scenarios.

8 The Reduced MSSM

We finish our phenomenological analyses with the application of the method of coupling reduction to a version of the MSSM, where a covering GUT is assumed. The original partial reduction can be found in refs. [138, 139] where only the third fermionic generation is considered. Following this restriction, the superpotential reads:

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2 \,, \tag{47}$$

where $Y_{t,b,\tau}$ refer only to the third family, and the SSB Lagrangian is given by by (with the trilinear couplings $h_{t,b,\tau}$ for the third family)

$$\begin{aligned} -\mathcal{L}_{\rm SSB} = \sum_\phi m_\phi^2 \hat{\phi}^* \hat{\phi} + \left[m_3^2 \hat{H}_1 \hat{H}_2 + \sum_{i=1}^{3} \frac{1}{2} M_i \lambda_i \lambda_i + \text{h.c} \right] \\ + \left[h_t \hat{H}_2 \hat{Q} \hat{t^c} + h_b \hat{H}_1 \hat{Q} \hat{b^c} + h_\tau \hat{H}_1 \hat{L} \hat{\tau^c} + \text{h.c.} \right] . \end{aligned} \tag{48}$$

We start with the dimensionless sector and consider initially the top and bottom Yuakwa couplings and the strong gauge coupling. The rest of the couplings will be treated as corrections. If $Y_{(t,b)}^2/(4\pi) \equiv \alpha_{(t,b)}$, the REs and the Yukawa RGEs give

$$\alpha_i = G_i^2 \alpha_3, \quad \text{where} \quad G_i^2 = \frac{1}{3}, \qquad i = t, b.$$

If the tau Yukawa is included in the reduction, the corresponding G^2 coefficient for tau turns negative [140], explaining why this coupling is treated also as a correction (i.e. it cannot be reduced).

We assume that the ratios of the top and bottom Yukawa to the strong coupling are constant at the GUT scale, i.e. they have negligible scale dependence,

$$\frac{d}{dg_3} \left(\frac{Y_{t,b}^2}{g_3^2} \right) = 0.$$

Then, including the corrections from the $SU(2)$, $U(1)$ and tau couplings, at the GUT scale, the coefficients $G_{t,b}^2$ become:

$$G_t^2 = \frac{1}{3} + \frac{71}{525}\rho_1 + \frac{3}{7}\rho_2 + \frac{1}{35}\rho_\tau, \qquad G_b^2 = \frac{1}{3} + \frac{29}{525}\rho_1 + \frac{3}{7}\rho_2 - \frac{6}{35}\rho_\tau \,, \tag{49}$$

where

$$\rho_{1,2} = \frac{g_{1,2}^2}{g_3^2} = \frac{\alpha_{1,2}}{\alpha_3}, \qquad \rho_\tau = \frac{g_\tau^2}{g_3^2} = \frac{\frac{Y_\tau^2}{4\pi}}{\alpha_3} \,. \tag{50}$$

We shall treat Eqs.(49) as boundary conditions at the GUT scale.

Going to the two-loop level, we assume that the corrections take the following form:

$$\alpha_i = G_i^2 \alpha_3 + J_i^2 \alpha_3^2, \qquad i = t, b \,.$$

Then, the two-loop coefficients, J_i, including the corrections from the gauge and the tau Yukawa couplings, are:

$$J_t^2 = \frac{1}{4\pi}\frac{N_t}{D}, \quad J_b^2 = \frac{1}{4\pi}\frac{N_b}{5D},$$

where D, N_t and N_b are known quantities which can be found in ref. [141].

Proceeding to the the SSB Lagrangian, Eq. (48), and the dimension-one parameters, i.e the trilinear couplings $h_{t,b,\tau}$, we first reduce $h_{t,b}$ and we get

$$h_i = c_i Y_i M_3 = c_i G_i M_3 g_3, \quad \text{where} \quad c_i = -1 \qquad i = t, b,$$

where M_3 is the gluino mass. Adding the corrections from the gauge and the tau couplings we have

$$c_t = -\frac{A_A A_{bb} + A_{tb} B_B}{A_{bt} A_{tb} - A_{bb} A_{tt}}, \qquad c_b = -\frac{A_A A_{bt} + A_{tt} B_B}{A_{bt} A_{tb} - A_{bb} A_{tt}}.$$

Again, A_{tt}, A_{bb} and A_{tb} can be found in ref. [141].

We end up with the soft scalar masses m_ϕ^2 of the SSB Lagrangian. Assuming the relations $m_i^2 = c_i M_3^2$ $(i = Q, u, d, H_u, H_d)$, and adding the corrections from the gauge, the tau couplings and h_τ, we get

$$c_Q = -\frac{c_{Q\text{Num}}}{D_m}, \quad c_u = -\frac{1}{3}\frac{c_{u\text{Num}}}{D_m}, \quad c_d = -\frac{c_{d\text{Num}}}{D_m}, \quad c_{H_u} = -\frac{2}{3}\frac{c_{H_u\text{Num}}}{D_m}, \quad c_{H_d} = -\frac{c_{H_d\text{Num}}}{D_m}, \quad (51)$$

where D_m, $c_{Q\text{Num}}$, $c_{u\text{Num}}$, $c_{d\text{Num}}$, $c_{H_u\text{Num}}$, $c_{H_d\text{Num}}$ and the complete analysis are again given in ref. [141]. These values do not obey any soft scalar mass sum rule.

If only the reduced system was used, i.e. the strong, top and bottom Yukawa couplings as well as the h_t and h_b, the coefficients turn to be

$$c_Q = c_u = c_d = \frac{2}{3}, \quad c_{H_u} = c_{H_d} = -1/3,$$

which clearly obey the sum rules

$$\frac{m_Q^2 + m_u^2 + m_{H_u}^2}{M_3^2} = c_Q + c_u + c_{H_u} = 1, \qquad \frac{m_Q^2 + m_d^2 + m_{H_d}^2}{M_3^2} = c_Q + c_d + c_{H_d} = 1. \quad (52)$$

There is an essential point for the gaugino masses that should be mentioned. The application of the Hisano-Shifman relation, Eq. (12), is made for each gaugino mass as a boundary condition with unified gauge coupling at M_{GUT}. Then, at one-loop level, the gaugino mass depends on the one-loop coefficient of the corresponding β-function and an arbitrary mass M_0, $M_i = b_i M_0$. This fact permits, with a suitable choice of M_0, to have the gluino mass equal to the unified gaugino mass, while the gauginos masses of the other two gauge groups are given by the gluino mass multiplied by the ratio of the appropriate one-loop β coefficient.

For our analysis we choose the unification scale to apply the corrections to all these RGI relations. The full discussion on the selection of the free parameters of the model can be found in [37]. In total, we vary ρ_τ, ρ_{h_τ}, M and μ. This results in the scattered points of the next figure.

The model's predictions for the bottom and top mass lie within 2σ of Eq. (25). The scatter plot of the light Higgs boson mass M_h is shown in Fig. 5 (left),while the theory uncertainty given

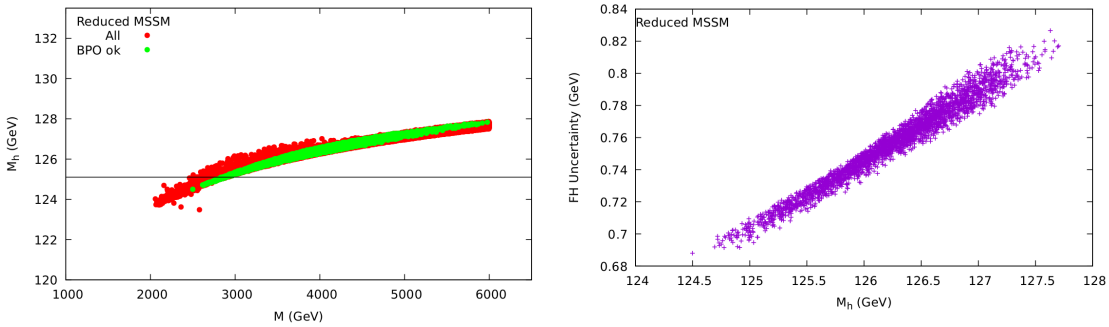

Figure 5: Left: The lightest Higgs boson mass, M_h in the Reduced MSSM. The green points is the full model prediction. Right: the lightest Higgs mass theoretical uncertainty [36].

in Fig. 5 (right) has dropped below 1 GeV. The Higgs mass predicted by the model lies perfectly in the experimentally measured range.

The M_h limits set a limit on the low-energy supersymmetric masses, which we briefly discuss. The three selected benchmarks correspond to $\overline{\text{DR}}$ pseudoscalar Higgs boson masses above 1900 GeV, 1950 GeV and 2000 GeV respectively. The input of `SPheno` 4.0.4 [97, 98] can be found in Table 10 (notation as in Sect. 5).

	M_1	M_2	M_3	$\lvert\mu\rvert$	b	A_u	A_d	A_e	$\tan\beta$	$m^2_{Q_{1,2}}$
RMSSM-1	3711	1014	7109	4897	284^2	5274	5750	20	44.9	5985^2
RMSSM-2	3792	1035	7249	4983	294^2	5381	5871	557	44.6	6103^2
RMSSM-3	3829	1045	7313	5012	298^2	5427	5942	420	45.3	6161^2
	$m^2_{Q_3}$	$m^2_{L_{1,2}}$	$m^2_{L_3}$	$m^2_{\bar{u}_{1,2}}$	$m^2_{\bar{u}_3}$	$m^2_{\bar{d}_{1,2}}$	$m^2_{\bar{d}_3}$	$m^2_{\bar{e}_{1,2}}$	$m^2_{\bar{e}_3}$	
RMSSM-1	5545^2	2106^2	2069^2	6277^2	5386^2	5989^2	5114^2	3051^2	4491^2	
RMSSM-2	5656^2	2122^2	2290^2	6385^2	5476^2	6110^2	5219^2	3153^2	4181^2	
RMSSM-3	5708^2	2106^2	2279^2	6427^2	5506^2	6172^2	5269^2	3229^2	3504^2	

Table 10: Reduced MSSM predictions that are used as input to `SPheno` Mass parameters are in GeV and rounded to 1 GeV.

Table 11 shows the resulting masses of Higgs bosons and some of the lightest SUSY particles. In particular, we find $M_A \lesssim 1.5$ TeV (for large values of $\tan\beta$ as in the other models), values substantially lower than in the previously considered models. This can be understood as follows.

In this model, we have direct relations between the soft scalar masses and the unified gaugino mass, which receive corrections from the two gauge couplings g_1 and g_2 and the Yukawa coupling of the τ lepton. As mentioned above, in the absence of these corrections the relations obey the soft scalar mass sum rule. However, unlike all the previous models, these corrections make the sum rule only approximate. Thus, these unique boundary conditions result in very low values for the masses of the heavy Higgs bosons (even compared to the minimal $SU(5)$ case presented above, which also exhibits direct relations which however obey the sum rule). A relatively light spectrum

is also favored by the prediction for the light CP-even Higgs boson mass, which turns out to be relatively high in this model and does not allow us to consider heavier spectra. Thus, in this model, contrary to the models analyzed before, because of the large $\tan\beta \sim 45$ found here, the *physical* mass of the pseudoscalar Higgs boson, M_A, is excluded by the searches $H/A \to \tau\tau$ at ATLAS with 139/fb [84] for all three benchmarks. One could try considering a heavier spectrum, in which we would have $M_A \gtrsim 1900$ GeV, but in that case the light Higgs mass would be well above its acceptable region. Particularly, it would be above 128 GeV, a value that is clearly excluded, especially given the improved (much smaller) uncertainty calculated by the new `FeynHiggs` code). Thus, the current version of this model has been ruled out experimentally. Consequently, we do not show any SUSY or Higgs production cross sections.

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}_1^0}$	$M_{\tilde{\chi}_2^0}$	$M_{\tilde{\chi}_3^0}$	$M_{\tilde{\chi}_4^0}$	$M_{\tilde{\chi}_1^\pm}$	$M_{\tilde{\chi}_2^\pm}$
RMSSM-1	1.393	1.393	1.387	7.253	1.075	3.662	4.889	4.891	1.075	4.890
RMSSM-2	1.417	1.417	1.414	7.394	1.098	3.741	4.975	4.976	1.098	4.976
RMSSM-3	1.491	1.491	1.492	7.459	1.109	3.776	5.003	5.004	1.108	5.004
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
RMSSM-1	2.124	2.123	2.078	2.079	6.189	6.202	5.307	5.715	5.509	5.731
RMSSM-2	2.297	2.139	2.140	2.139	6.314	6.324	5.414	5.828	5.602	5.842
RMSSM-3	2.280	2.123	2.125	2.123	6.376	6.382	5.465	5.881	5.635	5.894

Table 11: Masses for each benchmark of the Reduced MSSM (in TeV).

9 Conclusions

The reduction of couplings scheme consists in searching for RGE relations among parameters of a renormalizable theory that hold to all orders in perturbation theory. In certain $N = 1$ theories such a reduction of couplings indeed appears to be theoretically realised and therefore it developed to a powerful tool able to reduce the parameters and increase the predictivity of these theories. In the present paper first we briefly reviewed the ideas concerning the reduction of couplings of renormalizable theories and the theoretical methods which have been developed to confront the problem. Then we turned to the question of testing experimentally the idea of reduction of couplings. Four specific models, namely the Reduced Minimal $N = 1$ $SU(5)$, the all-loop Finite $N = 1$ $SU(5)$, the two-loop Finite $N = 1$ $SU(3)^3$ and the Reduced MSSM, have been considered for which new results have been obtained using the updated Higgs-boson mass calculation of `FeynHiggs`. In each case benchmark points in the low-mass regions have been chosen for which the `SPheno` code has been used to calculate the spectrum of SUSY particles and their decay modes. Finally the `MadGraph` event generator was used to compute the production cross sections of relevant final states at the 14 TeV (HL-)LHC and 100 TeV FCC-hh colliders.

The first three (unified) models were found to be in comfortable agreement with LHC measurements and searches, with the exception of the bottom quark mass in the Reduced Minimal $SU(5)$, for which agreement with measurements can be achieved only at the 4σ level. In addition it was found that all models predict relatively heavy spectra, which evade largely the detection in the HL-LHC. We found one noticeable exception. The reduced MSSM features a relatively light heavy Higgs-boson mass spectrum. Together with the relatively high value of $\tan\beta$ this spectrum is excluded already

by current searches at ATLAS and CMS for in the $pp \to H/A \to \tau^+\tau^-$ mode. We also analyzed the accessibility of the SUSY and heavy Higgs spectrum at the FCC-hh with $\sqrt{s} = 100$ TeV. We found that the lower parts of the parameter space will be testable at the $2\,\sigma$ level, with only an even smaller part discoverable at the $5\,\sigma$ level. However, the heavier parts of the possible SUSY spectra will remain elusive even at the FCC-hh. One exception here is the heavy Higgs-boson sector of the two-loop finite $N = 1$ $SU(3)^3$ model, which exhibits a spectrum where only the highest possible mass values could escape the searches at the FCC-hh.

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