Coupling Reduction and the Higgs Mass

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Abstract

Assuming the existence of a functional relation among the Standard Model (SM) couplings gauge α_1 and quartic λ , we determine the mass of the Higgs particle. Similar considerations for the top and bottom Yukawa couplings in the minimal supersymmetric SM lead to the prediction of a narrow window for tan β , one of the main parameters that determine the light Higgs mass.

1. Introduction. Copious theoretical efforts to establish a deeper understanding of Nature, led to very interesting constructions such as Superstring Theories that aim to unify consistently all interactions. The main goal expected from a unified description of interactions by the Particle Physics community is to understand the present day large number of free parameters of the Standard Model (SM) in terms of a few fundamental ones. Realistically, one expects to achieve at least a partial *reduction of couplings*. Indeed, the celebrated SM had so far outstanding successes in all its confrontations with experimental results. However, its apparent success is spoiled by the presence of a plethora of free parameters mostly related to the ad-hoc introduction of the Higgs and Yukawa sectors in the theory.

Towards reducing the independent parameters of a theory, a method has been developed which looks for renormalization group invariant (RGI) relations [1–9, 11] holding below the Planck scale, which in their turn are preserved down to Grand Unified (GUT) or lower scales. This program applied to dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, had already noticeable successes by predicting correctly, among other things, the top quark mass in the finite and in the minimal N =1 supersymmetric SU(5) GUTs [1,2]. An interesting prediction of the lightest Higgs mass in a N=1 Finite SU(5) GUT [1] will be confronted with the experiment soon. An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the couplings reduction program [5]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [9] (see also [10]). Here, we would like to examine to which extent the above method can be applied to minimal schemes such as the SM and its minimal supersymmetric extension, the MSSM. In fact, the former, was one of the first applications of the above reduction scheme [6, 8, 11] assuming a perturbative ansatz. The implications of a stronger condition have been examined in ref [12].

Let us first recall some basic issues concerning the reduction of couplings. A RGI relation $\Phi(g_1, ..., g_N) = 0$, has to satisfy the partial differential equation $\mu d\Phi/d\mu = \sum_{i=1}^{N} \beta_i \partial\Phi/\partial g_i = 0$, where β_i is the β -function of g_i . There exist (N-1) independent Φ 's, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations (REs), $\beta_g(dg_i/dg) = \beta_1$, i = 1, ..., N, where g and β_g are the primary coupling and its β -function correspondingly. Using all the $(N-1)\Phi$'s to impose RGI relations, one can, in principle, express all the couplings in terms of a single coupling g. The complete reduction, which formally preserves perturbative renormalizability, can be achieved by demanding a power series solution, where its uniqueness can be investigated at the one-loop level. The completely reduced theory contains only one independent coupling with the corresponding β -function. This possibility of coupling unification is attractive, but it can be too restrictive and hence unrealistic. To overcome this problem, one may use fewer Φ 's as RGI constraints.

After investigating specific examples, it becomes clear that the various couplings in supersymmetric theories have easily the same asymptotic behavior. Therefore, searching for a power series solution to the REs is justified. This is not the case in non-supersymmetric theories. Still in the SM α_3 and α_2 have the same behavior but one cannot be reduced in favor of the other [11]. Here, we will examine in some detail the possibility to reduce the couplings α_1 and the scalar quartic coupling λ of the SM, which have the same asymptotic behavior too.

As already mentioned, the method of reduction was applied in the couplings of the SM in refs [6, 8, 11]. The predictions for the Higgs boson mass in ref [6, 8] and for the Higgs and the top quark masses in ref [11] did not survive confrontation with experiment. In the present work, after studying the evolution of the SM couplings under the renormalization group flow, we look for solutions of the reduction equations following ref [1-5, 8, 9, 11] by generalizing their perturbative ansatz. Eventually, we are led to the updated solutions of ref [6] and a Higgs mass prediction in a region that is currently under experimental investigation, which we do not consider as totally conclusive yet. If the experimental results persist as in [13] when better statistics are available, then we will consider the SM case as an educative example and a motivation for applying our method in MSSM, which is examined here too.

2. Studies of the behavior of the couplings under RGEs. In the following, we will investigate the behavior of the SM and MSSM couplings under the renormalization group equations in order to establish a possible realisation of the reduction scenario. The most promising case appears to connect the scalar quartic coupling λ and the U(1) gauge coupling α_1 . We expect that such a relation leads to a prediction of the Higgs mass. Let us start with the 1-loop contributions. At this level, the RGE's for the gauge and



Figure 1: Plotting the derivative $d\eta_{\lambda}/dt$ as a function of t

the (top) Yukawa 1 can be solved analytically. The running of the quartic coupling is governed by the equation

$$\frac{d\lambda}{dt} = \beta_{\lambda} = \frac{1}{2\pi} \left[L_2 \tilde{\lambda}^2 + (A_{1L}\alpha_1 + A_{2L}\alpha_2) \tilde{\lambda} + A_{11}\alpha_1^2 + A_{12}\alpha_1\alpha_2 + A_{22}\alpha_2^2 + H_L\alpha_t \tilde{\lambda} + H_2\alpha_t^2 \right],$$
(1)

where

$$\tilde{\lambda} = \frac{\lambda}{4\pi}, \quad \alpha_t = \frac{h_t^2}{4\pi}, \quad t = \ln(E),$$

$$L_2 = 6, \quad A_{1L} = -\frac{3}{2}, \quad A_{2L} = -\frac{9}{2},$$

$$A_{11} = \frac{3}{8}, \quad A_{12} = \frac{3}{4}, \quad A_{22} = \frac{9}{8}, \quad H_L = 6, \quad H_2 = -6,$$

and α_i , i = 1, 2, 3 are the gauge couplings.

To check that the ratio λ over α_1 indeed tends to a constant value at high scales, we plot the derivative of the ratio $\eta_{\lambda} \equiv \tilde{\lambda}/\alpha_1$ as a function of t, for several initial values of the $\tilde{\lambda}$ coupling, which we trade for the (running) Higgs mass. In Fig.(1) we show such a plot. Starting from $m_H = 165$ GeV, we see that the derivative is positive for high energies. Upon lowering the Higgs

¹Only the top Yukawa coupling is taken into account in the running.

mass, the derivative decreases and, for $m_H \sim 162$ GeV, it goes asymptotically to zero. Further lowering the Higgs mass the derivative becomes negative but again for $m_H \sim 151.5$ GeV goes once more asymptotically to zero. For even smaller values of the Higgs mass the derivative becomes positive but now $\tilde{\lambda}$ passes through negative values². Notice that the η_{λ} becomes constant at energies well above the Planck scale, however at the 2-loop order the situation improves appreciably. Let us explore the above situation a bit further. We can easily express the running of the ratio η_{λ} in the form

$$\frac{d\eta_{\lambda}}{dt} = \frac{1}{\alpha_1} \frac{d\tilde{\lambda}}{dt} - \frac{\tilde{\lambda}}{\alpha_1^2} \frac{d\alpha_1}{dt} = \frac{1}{\alpha_1} \beta_{\lambda}(\alpha_1, \alpha_2, \alpha_t, \tilde{\lambda}) - \frac{\tilde{\lambda}}{\alpha_1^2} \beta_1(\alpha_1), \quad (2)$$

where β_1 is the 1-loop β -function for the α_1 coupling. This expression can be easily cast in the following form

$$\frac{d\eta_{\lambda}}{dt} = \alpha_1 \beta_{\lambda} (1, \alpha_2 / \alpha_1, \alpha_t / \alpha_1, \eta_{\lambda}) - \alpha_1 \eta_{\lambda} b_1 \tag{3}$$

where $\beta_1 = b_1 \alpha_1^2$. Since at the 1-loop level the differential equations for the gauge and Yukawa couplings can be solved independently of the $\tilde{\lambda}$ coupling, we can express α_1 , α_2 and α_t as functions of t and recast the above equation in the form

$$\frac{d\eta_{\lambda}}{dt} = \alpha_1(t)\beta_{\lambda}(t,\eta_{\lambda}) - \alpha_1(t)\eta_{\lambda}\beta_1(1) \equiv \alpha_1(t)F_{\eta_{\lambda}}(t,\eta_{\lambda}), \qquad (4)$$

using the same symbol β_{λ} for the new function of t and η_{λ} . In Fig.2 we plot contours of constant value (-0.01, 0 and 0.01) for $\alpha_1(t)F_{\eta_{\lambda}}(t,\eta_{\lambda})$ in the (t,η_{λ}) plane. We clearly see that the zero value contour tends, for albeit very high energies, to a constant value for the ratio η_{λ} (~ 1.3 and ~ 0.05).

Let us explore this situation from even another point of view and treat $\alpha_1, \alpha_2/\alpha_1 \equiv \eta_2, \alpha_t/\alpha_1 \equiv \eta_t$ and η_{λ} as independent variables. Then we rewrite Eq.3 in the form

$$\frac{d\eta_{\lambda}}{dt} = \alpha_1 F_{\eta_{\lambda}}(\eta_2, \eta_t, \eta_{\lambda}) \tag{5}$$

using again the same symbol $F_{\eta_{\lambda}}$. The derivative of η_{λ} with respect to α_1 is given by

$$\frac{d\eta_{\lambda}}{d\alpha_1} = \frac{\frac{d\eta_{\lambda}}{dt}}{\frac{d\alpha_1}{dt}} = \frac{\alpha_1 F_{\eta_{\lambda}}(\eta_2, \eta_t, \eta_{\lambda})}{b_1 \alpha_1^2} = \frac{F_{\eta_{\lambda}}(\eta_2, \eta_t, \eta_{\lambda})}{b_1 \alpha_1}.$$
(6)

²Recall that the assumption the λ stays always positive, for the whole energy scale, gives a lower bound to the Higgs mass ~ 149 GeV.



Figure 2: Contours of constant value of the derivative $d\eta_{\lambda}/dt$ in the (t, η_{λ}) plane for three values: -0.01, 0 and 0.01

If η_{λ} tends to a constant value, then the above derivative should tend to zero. This is of course true when α_1 becomes very large but also when the numerator, $F_{\eta_{\lambda}}(\eta_2, \eta_t, \eta_{\lambda})$ is equal to zero. Just to have a first impression, we put $\eta_2 = \eta_t = 0$ (both ratios tend to zero for very high energies). Then $F_{\eta_{\lambda}}(0, 0, \eta_{\lambda})$ is just a second order polynomial in η_{λ} with zeros at ~ 1.34 and ~ 0.047, which are the two fixed points observed many years ago [6]. We can plot, in the space of $(\eta_2, \eta_t, \eta_{\lambda})$, the surface where $F_{\eta_{\lambda}}(\eta_2, \eta_t, \eta_{\lambda}) = 0$. We can also numerically solve the differential equation and express η_{λ} as a function of t. Then we can make a parametric plot of the curve $(\eta_2(t), \eta_t(t), \eta_{\lambda}(t))$. We expect that for high energies, i.e. low values of η_2 and η_t , the curve will lie on the surface $F_{\eta_{\lambda}} = 0$. This is shown in Fig.3. There are two surfaces corresponding to $F_{\eta_{\lambda}} = 0$ and we have plotted the parametric curves for three Higgs masses. We clearly see that for the values $m_H \sim 162.5$ and 151.6 GeV, the parametric curves lie on the surfaces for low values of η_2 and η_t .

3. The Reduction Equations. The observations made in section 2 suggest that at least the couplings $\tilde{\lambda}$ and α_1 are not independent in the SM and there may exist a functional relation among them at high scales. It is therefore justified to look for solutions of the reduction equation

$$\frac{d\lambda}{d\alpha_1} = \frac{\beta_\lambda}{\beta_1}.\tag{7}$$



Figure 3: Surfaces of constant η_{λ} and parametric curves of $(\eta_2(t), \eta_t(t), \eta_{\lambda}(t))$ for three values of the Higgs mass (1-loop).

Let us first look for solutions of Eq. 7 at 1-loop

$$\lambda(t) = c_1 \alpha_1(t), \tag{8}$$

where c_1 would be a constant in the perturbative ansatz of ref [1-5, 8, 9, 11], but here we are searching for more general solutions. From the 1-loop $\alpha_1(t)$ we can solve for t and express $\alpha_2(t)$ and $\alpha_t(t)$ (which are present in β_{λ}) as functions of α_1 . Using the ansatz given in Eq.8, Eq.7 becomes a second order polynomial in c_1 , where, of course, the coefficients depend on α_1 . In Fig.4 we plot the two solutions of the polynomial as a function of α_1 . We clearly see that for large values of α_1 (i.e. large energies), the two solutions tend to constant values. This is easily understood, since for high energies, we can neglect all the couplings but α_1 itself, and Eq.7 reduces to

$$c_1 = \frac{L_2 \alpha_1^2 c_1^2 + c_1 A_{1L} \alpha_1^2 + A_{11} \alpha_1^2}{b_1 \alpha_1^2} = \frac{L_2 c_1^2 + c_1 A_{1L} + A_{11}}{b_1} \tag{9}$$

with the two solutions being independent of α_1 (1.34233 and 0.0465609). We have already encountered this behavior when examining Eq.6. The second order polynomial above is the $F_{\eta_{\lambda}}(0, 0, \eta_{\lambda})$. It is worth noting that the values of α_1 , when c_1 approaches one of its fixed points correspond to energies well above the Planck scale. (At the Planck scale $\alpha_1 \sim 0.017$!).

We can go one step further and postulate that

$$\eta_{\lambda} = \frac{\tilde{\lambda}}{\alpha_1} = c_1 + c_2(\eta_2). \tag{10}$$

For high energies, the ratio η_2 tends to zero and in order to obtain our first ansatz, we should require that $c_2(\eta_2 \to 0) = 0$. From Eqs.10 and 3 we easily get

$$\frac{dc_2}{dt} = \frac{d\eta_\lambda}{dt} = \alpha_1 \beta_\lambda (1, \eta_2, \eta_t, \eta_\lambda) - \alpha_1 \eta_\lambda b_1.$$
(11)

Writing

$$\frac{d\eta_2}{dt} = \frac{1}{\alpha_1} \frac{d\alpha_2}{dt} - \frac{\alpha_2}{\alpha_1^2} \frac{d\alpha_1}{dt} = \alpha_1 \left(b_2 \eta_2^2 - \eta_2 b_1 \right), \tag{12}$$

where b_2 is the one loop β -function coefficient for α_2 , and dividing the last two equations we get the derivative of c_2 with respect to η_2 . All that remains to be done is to express the ratio η_t as a function of η_2 . Having the 1-loop analytical expressions for α_t and α_1 as functions of t, we can substitute tfrom the relation

$$\eta_{2} = \frac{\alpha_{2}}{\alpha_{1}} = \frac{\frac{\alpha_{20}}{1 - \frac{b_{2}}{2\pi}\alpha_{20}(t - t_{0})}}{\frac{\alpha_{10}}{1 - \frac{b_{1}}{2\pi}\alpha_{10}(t - t_{0})}} \rightarrow (13)$$
$$t = t_{0} + \frac{\eta_{20} - \eta_{2}}{\frac{1}{2\pi} [\eta_{0}b_{1}\alpha_{10} - \eta_{2}b_{2}\alpha_{20}]},$$

where $\eta_{20} = \alpha_{20}/\alpha_{10}$ and α_{10} and α_{20} are the corresponding values at the scale t_0 . Substituting $\eta_{\lambda} = c_1 + c_2(\eta_2)$ and solving the differential equation



Figure 4: The "constant" c_1 as a function of α_1



Figure 5: Plotting of c_2 as a function of η_2 for the two values of c_1 (1-loop)



Figure 6: Plotting of $c_1 + c_2 = \eta_{\lambda}$ as a function of $\log_{10}(E)$ for the two values of c_1 (1-loop)

for $c_2(\eta_2)$, we get $c_2(\eta_2)$. In Fig.5 we show the solutions for the two choices of c_1 using the initial condition $c_2(\eta_2 = 0.2) = 0$ (see Fig.4). In Fig.6 we show c_1+c_2 (i.e. η_{λ}) as a function of the energy scale. The curve which corresponds to the higher c_1 value has almost reached that value at the Planck scale, while the one that corresponds to the lower c_1 value, apart from passing through unacceptable negative values, is still far away from that value. In Fig.7 we plot the function $(c_1 + c_2(t))\alpha_1(t)$, i.e. $\tilde{\lambda}(t)$ itself, for the higher c_1 value curve. The corresponding running (pole³) Higgs mass is ~ 162(154) GeV.

Going to 2-loop order, we should first determine the value(s) of the constant c_1 in Eq.8. In this order, the procedure of keeping only the large terms

³For the known value of the top mass and the specific region of the Higgs mass, the Higgs pole mass is lower than the running mass by an amount of ~ 4.6 - 4.7%. The relation between running and pole mass can be found in the references [14].

in the high-energy regime, does not lead to an independent from α_1 value(s) c_1 . Nevertheless, for a big range of α_1 , c_1 varies by less than 5% from the 1loop case: 0.0448 - 0.0465 for the lower value and 1.342 - 1.395 for the higher one. We solve now the 2-loop differential equation for c_2 using as an initial value of η_{λ} at (very) high energies (i.e. low value of η_2) the value $c_1 = 1.395$. The new value drives the ratio η_{λ} to its constant value early on the energy scale (see Fig.8). To be more specific, we see that $\eta_{\lambda}(M_{Planck}) = 1.459$ and it remains pretty stable for higher energies. The Higgs running (pole) mass is ~ 163(155) GeV. At the 2-loop level, the problem with the lower c_1 value persists: η_{λ} passes through negative values.

4. The MSSM case. If we assume that the top and bottom Yukawa couplings are related, the reduction equation is

$$\frac{d\alpha_t}{d\alpha_b} = \frac{\beta_t}{\beta_b} = \frac{\alpha_t \left(6\alpha_t + \alpha_b - c_i^{(t)} \alpha_i \right)}{\alpha_b \left(6\alpha_b + \alpha_t + \alpha_\tau - c_i^{(b)} \alpha_i \right)}$$

where $c_i^{(t)} = (13/30, 3/2, 8/3)$ and $c_i^{(b)} = (7/30, 3/2, 8/3)$. Let us ignore, for simplicity, the contribution of α_{τ} and the small difference between $c_1^{(t)}$ and $c_1^{(b)}$. Then, it is straightforward to deduce that if the ratio α_t/α_b is constant, then this ratio is equal to the corresponding ratio of the β -functions and is



Figure 7: Plotting of $(c_1 + c_2)\alpha_1 = \tilde{\lambda}$ as a function of $\log_{10}(E)$ for the higher value of c_1 . The corresponding running Higgs mass is ~ 162 GeV (1-loop).



Figure 8: Plotting of η_{λ} as a function of $\log_{10}(E)$ for $c_1 = 1.395$. The corresponding running (pole) Higgs mas is ~ 163(155) GeV (2-loop running).

equal to 1.

$$\frac{d}{dt}\left(\frac{\alpha_t}{\alpha_b}\right) = 0 \to \frac{1}{\alpha_b^2}\left(\alpha_b\beta_t - \alpha_t\beta_b\right) = 0 \to \frac{\alpha_t}{\alpha_b} = \frac{\beta_t}{\beta_b}$$

This result combined with the previous equation leads to

$$6\alpha_t + \alpha_b - c_i^{(t)}\alpha_i = 6\alpha_b + \alpha_t + \alpha_\tau - c_i^{(b)}\alpha_i \to \alpha_t = \alpha_b.$$

That is, if we start with equal α_t and α_b at an energy scale, equality will remain for all energies. Putting back the τ Yukawa coupling and the difference between the $c_1^{(t)}$ and $c_1^{(b)}$ constants, we expect a small deviation from that behavior.

Therefore, the procedure is the following: we start the running (with the SM RGEs) from the known values of the top-, bottom- and tau-mass. At M_{SUSY} , we choose the appropriate $\tan \beta$ value that keeps the ratio α_t/α_b constant for all energies. Of course, we expect⁴ this constant to be near 1.

In the MSSM scenario, at the scale M_{SUSY} , we have the relations

$$\alpha_t(SM) = \alpha_t(MSSM) \sin^2 \beta$$

$$\alpha_b(SM) = \alpha_b(MSSM) \cos^2 \beta$$

$$\alpha_\tau(SM) = \alpha_\tau(MSSM) \cos^2 \beta.$$
(14)

⁴ The fact that $\tan \beta$ could be predicted using reduction of couplings was suggested in [4] in a discussion with a different focus.



Figure 9: (a) The ratio h_t/h_b and (b) the derivative of the ratio as a function of energy for several values of tan β and $M_{SUSY} = 1$ TeV, $m_t = 172$ GeV and $m_b(M_Z) = 2.82$ GeV.

Above the M_{SUSY} scale, the running of all the parameters obeys the MSSM renormalization group equations, while below that scale, the SM regime is active.

In Fig.9 we plot the ratio h_t/h_b (a) and the derivative of the ratio (b) as a function of energy, for several values of $\tan \beta$ and $M_{SUSY} = 1$ TeV, $m_t = 172$ GeV and $m_b(M_Z) = 2.82$ GeV. We clearly see that for the range $\tan \beta = 52.25 - 58.55$, the derivative of the ratio stays almost zero (actually less than $6 \cdot 10^{-3}$). The two values of $\tan \beta$: 52.25 and 58.55, are the limiting cases. For values below the first one, the derivative stays positive, while above the second one the derivative stays negative for the whole energy range.

In Fig.10 we plot the ratio h_t/h_b (in (a) and (c)) as well as the derivative of the ratio (in (b) and (d)) as a function of energy for the central value of $\tan \beta = 56$. In (a) and (b) we show three curves corresponding to $M_{SUSY} = 1$, 5 and 10 TeV, keeping the masses of top and bottom at their central values. In (c) and (d) we vary the bottom mass $m_b(M_Z) = 2.75$, 2.82 and 2.89 GeV, keeping the top mass at its central value and $M_{SUSY} = 1$ TeV. The differences upon varying the top mass are negligible.

Now, using the program SUSPECT $[15]^5$, we can plot in the plane of $(m_0, m_{1/2})$ contours of constant (pole) mass values for the lightest supersym-

⁵We run the programm using the mSUGRA model, 2-loop running and evaluation of pole masses. In all cases $sign(\mu) = +1$.



Figure 10: Plots of the ratio h_t/h_b ((a) and (c)) as well as the derivative of the ratio ((b) and (d)) as a function of energy for $M_{SUSY} = 1, 5$ and 10 TeV ((a) and (b)) and varying the bottom mass in the experimental error region ((c) and (d)).

metric Higgs m_h ⁶ for tan $\beta = 56$. In Fig.11 we show these contours for $m_h = 114, 116, 118, 120$ GeV for initial A = 0 GeV and tan $\beta = 56$. The dotted-dashed contour corresponds to a gluino mass of 1 TeV, while the dashed contour to (the lightest) squark mass of 1.2 TeV. According to recent data from ATLAS/LHC and CMS/LHC [16], the two values represent the lower bounds for detection of the corresponding particle. Finally, in Fig.12 we plot the same contours for the two limiting tan β cases: 58.55 and 52.25. **5.** Conclusions. The idea of couplings reduction in a theory is very appealing since it increases its predictive power. Successful reduction led to all-loop finite theories and a prediction of the top-quark mass. The latter property was used as a selection criterion for a successful GUT. In the present

⁶We keep m_H for the SM Higgs and denote by m_h the lightest Higgs in the MSSM.



Figure 11: Contours of constant m_h (pole) mass in the plane of $(m_0, m_{1/2})$ for initial value A = 0 GeV and for $\tan \beta = 56$. The dashed and the dotteddashed contours correspond to (lightest) squark and gluino masses of 1.2 TeV and 1 TeV correspondingly.

work, we have studied the reduction of certain couplings within the SM, and have obtained a prediction for the Higgs particle mass. Previous studies either overlooked this possibility, or did not include the heavy top-quark contribution. We have also started an analogous analysis in the MSSM, which we plan to extend in a forthcoming publication.

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Figure 12: The same as in Fig.11 for the two limiting tan β cases: 58.55 and 52.25

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