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Gauge coupling flux thresholds, exotic matter and the unification scale in F-$SU(5)$ GUT

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Abstract We explore the gauge coupling relations and the unification scale in F-theory $SU(5)$ GUT broken down to the Standard Model by an internal $U(1)_Y$ gauge flux. We consider variants with exotic matter representations which may appear in these constructions and investigate their rôle in the effective field theory model. We make a detailed investigation on the conditions imposed on the extraneous matter to raise the unification scale and make the color triplets heavy in order to avoid fast proton decay. We also discuss in brief the implications on the gaugino masses.

1 Introduction

Recent activity on model building in the context of F-theory has received considerable attention [1–5].[1] In this set up, gauge symmetries accommodating successful grand unified theories (GUTs) are naturally realized on seven-branes wrapping appropriate compact surfaces. As in the case of the heterotic string, there are no Higgs fields in the adjoint, and thus ordinary GUT symmetries cannot break with the usual Higgs mechanism. In the context of heterotic string, this inconvenience spawned new ideas [15, 16] of replacing unified GUTs with modified alternative groups which dispense with the use of the adjoint representation to break down to the Standard Model (SM) gauge symmetry. The advantage of F-constructions compared to the heterotic case, is that the gauge group can break by turning on suitable field configurations on the compact surface S in a subgroup of the GUT symmetry supported by the seven-brane. Suppose for example that the seven-brane supports an $SU(5)$ gauge symmetry which, as is well known, is the smallest unified gauge group accommodating the three gauge groups of the Standard Model. In this case, as described for example in [2] we can turn on a $U(1)_Y$ internal flux which breaks the $SU(5)$ gauge group down to the SM gauge symmetry. There are various potential problems in this approach. One important issue that arises in this scenario concerns the gauge coupling unification at the string scale. We know that the renormalization group (RG) running of the minimal supersymmetric SM gauge couplings is consistent with the embedding of the three gauge factors in a unified gauge group at around $M_U \sim 10^{16}$ GeV. This value of M_U is considerably smaller than the scale of the heterotic string and the Planck mass M_{Pl}. Thus, if one wishes to maintain the idea of unification of SM gauge couplings in a single gauge group, it is desirable to work out cases where these two scales decouple and M_U/M_{Pl} is a small number. It has been argued [2], that this can happen in cases where the seven-brane realizing the GUT gauge symmetry wraps a del Pezzo surface.

Another issue in these constructions related to the mechanism employed to break the GUT symmetry is the splitting [3, 17] of the values of the three gauge coupling constants when turning on a flux along the $U(1)_Y$. This splitting cannot be compatible with the unification scenario at a scale $M_U \sim 10^{16}$ GeV, at least not in the context of the MSSM. Nevertheless, the accommodation of the SM gauge symmetry into a unified group usually comes at the cost of extra matter at scales below M_U. For example, even in the simplest $SU(5)$ embedding of the SM gauge symmetry, the Higgs doublets are incorporated into complete $SU(5)$ fiveplets together with color triplets which mediate proton decay. Additional bulk matter may also be present at scales below the string scale. The problem of unification in F-$SU(5)$ is therefore more complicated than in the minimal GUT scenario [18–22] and a fresh look into the rôle of the extra-matter representations in conjunction with the new gauge coupling relations implied by the fluxes is needed. It

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[1]For related work see also [6–14].

is the purpose of this letter to clarify some of these issues. In Sect. 2 we give a short description of the F-$SU(5)$ GUT and its representation content. We discuss the proton decay and other potential problems caused by the possible appearance of exotic states. In Sect. 3 we perform a detailed RG analysis and discuss the correlation of the exotic matter states and the unification scale. In Sect. 4 we present our conclusions.

2 F-*SU*(5)

We start with a short description of the salient features of F-theory model building following the work of [2, 5]. In F-theory a gauge symmetry G_S is supported on seven-branes wrapping a del Pezzo surface S on the internal manifold. In this set-up, massless spectrum arises when a non-trivial background field configuration on S obtains a value along some subgroup H_S of $G_S \supset \Gamma_S \times H_S$. The spectrum is found in representations which arise from the decomposition of the adjoint of G_S under $\Gamma_S \times H_S$

$$\mathrm{adj}(G_S) = \bigoplus \tau_j \otimes T_j. \tag{1}$$

The net number of chiral minus anti-chiral states is given in terms of a topological index formula [2, 3],

$$n_\tau - n_{\tau^*} = -\int_S c_1(\mathcal{T}) c_1(S) = \chi\left(S, \mathcal{T}_j^*\right) - \chi(S, \mathcal{T}_j)$$

where τ^* is the dual representation of τ, $\mathcal{T}$ is the bundle transforming in the representation T and χ is the Euler character.

In a more general background containing intersecting branes, chiral matter appears on Riemann surfaces Σ_i which are located at the intersection loci of the compact surface S with other in general non-compact surfaces $S_1, S_2, \ldots$. These chiral states appear in bifundamental representations in close analogy to the case of intersecting D-brane models. Along the intersections the rank of the singularity increases. Designating G_S the gauge group on the surface S and G_{S_i} that associated with S_i, the gauge group on Σ_i is enhanced to $G_{\Sigma_i} \supset G_S \times G_{S_i}$ whose the adjoint in general decomposes as

$$\mathrm{adj}(G_{\Sigma_i}) = \mathrm{ad}(G_S) \oplus \mathrm{ad}(G_{S_i}) \oplus \left(\oplus_j U_j \otimes (U_i)_j\right) \tag{2}$$

with $U_j, (U_i)_j$ being the irreducible representations of G_S, G_{S_i}. In the simple case of $G_S = SU(n)$, $G_{S_i} = SU(m)$, and $G_{\Sigma_i} = SU(n+m)$ for example, the chiral $\mathcal{N} = 1$ multiplet is the bifundamental $(n, \bar{m})$.

We assume that a non-trivial background gauge field configuration acquires a value in a subgroup $H_S \subset G_S$ and similarly in $H_{S_i} \subset G_{S_i}$. If $G_S \supset \Gamma_S \times H_S$ and $G_{S_i} \supset \Gamma_{S_1} \times H_{S_i}$, with Γ_{S,S_i} being the corresponding maximal G_{S,S_i} subgroups, the $G_S \times G_{S_i}$ symmetry breaks to the commutant group $\Gamma = \Gamma_S \times \Gamma_{S_i}$. Denoting also $H = H_S \times H_{S_i}$, the decomposition of $U \otimes U_i$ into irreducible representations of $\Gamma \times H$ give

$$U \otimes U_i = \bigoplus_j (r_j, R_j). \tag{3}$$

The net number of chiral fermions $n_{r_j} - n_{r_j^*}$ in a specific representation is

$$n_{r_j} - n_{r_j^*} = \chi\left(\Sigma_i, K_{\Sigma_i}^{1/2} \otimes V_j\right) \tag{4}$$

$K_{\Sigma_i}^{1/2}$ being the spin bundle over Σ_i and V_j that of R_j. In the case of an algebraic curve Σ_i the Euler character is written as a function of the genus of the Riemann surface Σ_i and the first Chern class [2].

Having described the basic features of the F-theory constructions, we discuss now the minimal unified gauge group that this scenario can be realized [2–8]. In order to obtain a viable effective low-energy model, the seven-brane wrapping the del Pezzo surface S must support at least an $SU(5)$ gauge group which is the minimal GUT containing the SM gauge symmetry. The symmetry breaking down to the SM cannot occur via the adjoint Higgs field since the GUT surface does not admit adjoint scalars. The advantage of constructing $SU(5)$ in F-theory is that it is possible to turn on a non-trivial $U(1)_Y$ flux which breaks $SU(5)$ to the Standard Model gauge group. This flux is considered as a connection on a line bundle $\mathcal{L}_Y$ and taken to be localized, so that it can be non-trivial on the seven-brane wrapping S but trivial in the base of the F-theory compactification, thus the final group can be $SU(3) \times SU(2) \times U(1)_Y$. The flux also determines the matter context of the low-energy effective field theory model. In particular, in F-$SU(5)$ the possible representations with their decompositions under $SU(3) \times SU(2) \times U(1)$ are as follows

$$5 \to (3,1)_{-2} + (1,2)_3, \tag{5}$$

$$\bar{5} \to (\bar{3},1)_2 + (1,2)_{-3}, \tag{6}$$

$$10 \to (\bar{3},1)_{-4} + (3,2)_1 + (1,1)_6, \tag{7}$$

$$\overline{10} \to (3,1)_4 + (\bar{3},2)_{-1} + (1,1)_{-6}, \tag{8}$$

$$24 \to (8,1)_0 + (1,3)_0 + (1,1)_0 + (3,2)_{-5} + (\bar{3},2)_5. \tag{9}$$

The $U(1)$ normalization to obtain the correct charges is $Y = \frac{1}{6} Q_{U(1)}$, with the electric charge given by $Q = T_3 + Y$. The fermion families belong to 10_F and $\bar{5}_f$,

$$10_F \to u^c + Q + e^c, \tag{10}$$

$$\bar{5}_f \to d^c + \ell \tag{11}$$

and the SM Higgs fields to $5_h + \bar{5}_{\bar{h}}$

$$5_h \to D_h + h_u, \tag{12}$$

$$\bar{5}_{\bar{h}} \to \bar{D}_h + h_d. \tag{13}$$

The masses arise from the couplings

$$10_F 10_F 5_h \to Q u^c h_u + u^c e^c D_h + Q Q D_h, \tag{14}$$

$$10_F \bar{5}_f \bar{5}_h \to Q d^c h_d + e^c \ell h_d + u^c d^c \bar{D}_h + Q \ell \bar{D}_h. \tag{15}$$

Clearly, the necessary fermion mass terms are accompanied by dangerous trilinear terms which combine to the well known dimension five operators causing proton decay. We will deal with this issue in the context of F-$SU(5)$ in the next subsection. We note in passing that there could be also R-parity violating terms

$$10_F \bar{5}_f \bar{5}_f \to Q d^c \ell + e^c \ell \ell + u^c d^c d^c \tag{16}$$

which must be absent due to some ($U(1)_{PQ}$-type) symmetry, or they have to be highly suppressed to avoid rapid proton decay.

2.1 Exotics and proton decay

From the decomposition of the available $SU(5)$ representations in F-$SU(5)$, we see that in addition to the minimal SM spectrum there are also extra states which in principle could appear in the light spectrum. Below we discuss in brief their consequences and how they can be removed from the low-energy effective theory.

Decomposing the adjoint of $SU(5)$ under the SM gauge symmetry, one finds the representations $Q' = (3, \bar{2})_{-5}$ and $\bar{Q}' = (\bar{3}, 2)_5$ which carry non-trivial $U(1)$ charges. Usually, these bulk exotic states appear in pairs because of well defined transformation properties under discrete symmetries. The Q' field has the exotic charges

$$Q' \equiv (3, \bar{2})_{-5} = \begin{pmatrix} \xi_{-\frac{1}{3}} \\ \zeta_{-\frac{4}{3}} \end{pmatrix} \tag{17}$$

whilst $\bar{Q}'$ has their conjugates.

The appearance of Q', $\bar{Q}'$ in the spectrum of the effective theory has two undesired consequences. First, as we will see in detail in the next section, they modify the beta functions of the SM gauge couplings and as a result they lower the unification scale. Second, they can form couplings of the type $S\Sigma\Sigma$ with Standard Model matter fields. The following couplings originate involving bulk fields

$$\begin{aligned} &24_S \cdot 5_\Sigma \cdot \bar{5}_\Sigma \\ &\quad \to (3, \bar{2})_{-5} (1, 2)_3 (\bar{3}, 1)_2 \\ &\quad \to Q' h_u \bar{D}_h,\ Q' h_u d^c + (\bar{3}, 2)_5 (1, 2)_{-3} (3, 1)_{-2} \\ &\quad \to \bar{Q}' h_d D_h,\ \bar{Q}' \ell D_h. \end{aligned}$$

These include terms of the form $\zeta h_u^+ d^c + \xi h_u^0 d^c + \cdots$, thus the ζ-field decays to d^c and ξ-field couples through a mass term to d^c. The following mass terms can exist

$$h_d^0 d d^c + h_u^0 \xi d^c + M_{KK} \bar{\xi} \xi \tag{18}$$

which imply an unacceptable mixing between the ordinary down quark mass 3×3 matrix m_d and the exotic(s) state ξ:

$$m_{\text{down}} \sim \begin{pmatrix} m_d & 0 \\ m_u & M_{KK} \end{pmatrix}. \tag{19}$$

This problem suggests that either the extra fields Q', $\bar{Q}'$ should be absent, or that the couplings of type $(S\Sigma\Sigma)$ should be zero.[2]

A second drawback in $SU(5)$ models comes from the unsolicited color triplets D_h, $\bar{D}_h$ which are always present in the GUT spectrum. They are constituents of the same $5, \bar{5}$ multiplets where the Higgs doublets—needed to break the SM gauge symmetry—are found. It is well known that in the minimal $SU(5)$ there are dimension five operators generated by D_h, $\bar{D}_h$ as well as dimension six operators from diagrams mediated by gauge bosons, which both induce proton decay. Among them, the most dangerous ones are those which are constructed by the exchange of the color scalar triplets discussed above. A relevant graph generates in the superpotential the effective operator [23–27],

$$\mathcal{W}_5 \sim \frac{\lambda_1^i \lambda_2^k}{M_{\text{eff}}} V_{jk}^* Q_i Q_i Q_j \ell_k \tag{20}$$

where the coupling λ_1 is related to the corresponding up-quark Yukawa coupling (see (14)), while λ_2 has a common origin with the Yukawa coupling for the down quarks (see (15)). V_{jk}^* is the CKM mixing element and M_{eff} is an effective scale related to the mass of the color triplet and their exact relation is determined when the entire color triplet mass matrix is specified [25–27].

The operator (20), when dressed with charged-wino and/or higgsino leads to the most dominant decay $p \to K\bar{\nu}$. Therefore these triplets must be heavy enough in order to imply a proton decay rate consistent with the present bounds. According to experimental bounds put by KamioKande ($\tau_p \geq 6.7 \times 10^{32}$ years), even for relatively small Yukawa couplings $\lambda_{1,2} < 1$ the triplet mass must be at least heavier than $\sim 7 \times 10^{16}$ GeV. If the GUT breaking scale is that of the minimal supersymmetric $SU(5)$ model, $M_G \sim 2 \times 10^{16}$ GeV, the problem is clear, since the triplet mass is of the order of M_G. On way to evade this shortcoming in F-theory, is to assume that the two Higgs fields are localized at different curves and generating heavy mass terms

[2] We note in passing that in the presence of additional states originating from $10, \overline{10}$ vector fields more couplings of the following form might be present

$$24 \cdot 10 \cdot \overline{10} \to Q' Q \bar{u}_H^c + Q' \bar{Q}_H e^c + \bar{Q}' \bar{Q}_H u^c + \bar{Q}' Q \bar{e}_H^c.$$

with another triplet pair so that the effective operator (20) can be avoided [5]. We notice however that this color triplet arrangement may also impose restrictions or possible zero entries (texture-zeros) on the mass matrices.

In the following we wish to elaborate on a complementary solution to this problem. In particular we will examine the possibility of raising the $SU(5)$ GUT scale at least at the level of the heterotic string scale[3] $M_U \sim 2\times 10^{17}$ GeV so that the mass of the triplets can be heavy enough. We may consider this possibility in conjunction with the fact that in F-theory constructions it is possible that additional matter representations arise as vector-like pairs in the bulk or on some matter curves Σ_i. In this case one can take advantage of these extra matter and proceed to suitable modifications [28] of the doublet-triplet splitting problem in order to avoid rapid proton decay.

The possible vector-like multiplets available in these constructions descend from the representations $10+\overline{10}$ and $5+\bar{5}$ and thus have the quantum numbers of ordinary SM states. When these latter representations form complete $SU(5)$ multiplets they do not modify the value of the GUT scale. However, individual pairs of them do have nonvanishing effects on the GUT scale. In a viable model is expected of course that at some scale below the unification they will receive masses and decouple from the light spectrum. Depending on the particular combinations that appear in the spectrum, these states can decrease or increase the unification scale. In a bottom-up approach, one may determine the appropriate spectrum to obtain a 'unification' point at energies high enough to avoid rapid proton decay and then determine the admissible line bundle configurations that lead to the desired zero mode context of the effective theory.

3 Gauge coupling relations and the GUT scale

The issue of the Standard Model gauge couplings running and their integration into an $\mathcal{N}=1$ Supersymmetric $SU(5)$ GUT in the context of F-theory was considered in [17]. In this work it was observed that the $U(1)_Y$ flux turned on to break the $SU(5)$ symmetry leads to a splitting of gauge couplings at the unification scale. In the presence of the line bundles discussed above, the following gauge coupling relations are derived at the string scale[17]

$$\begin{aligned} \frac{1}{a_3(M_G)} &= \frac{1}{a_G} - y, \\ \frac{1}{a_2(M_G)} &= \frac{1}{a_G} - y + x, \\ \frac{1}{a_1(M_G)} &= \frac{1}{a_G} - y + \frac{3}{5}x \end{aligned} \tag{21}$$

[3]In the cases of M_U scales comparable to M_{Pl} one may relax the assumption of decoupling discussed in the introduction.

where $x = -\frac{1}{2}S\int c_1^2(\mathcal{L}_Y)$ and $y = \frac{1}{2}S\int c_1^2(\mathcal{L}_a)$ associated with a non-trivial line bundle $\mathcal{L}_a$ and $S = e^{-\phi} + \imath C_0$ the axion-dilaton field as discussed in [17]. Then

$$\frac{1}{a_2(M_G)} - \frac{1}{a_3(M_G)} = x, \tag{22}$$

$$\frac{1}{a_1(M_G)} - \frac{1}{a_3(M_G)} = \frac{3}{5}x \tag{23}$$

so that the following relation is found at the unification scale[17]

$$\frac{5}{3}\frac{1}{a_1(M_G)} = \frac{1}{a_2(M_G)} + \frac{2}{3}\frac{1}{a_3(M_G)}. \tag{24}$$

In addition, taking into account (21) and the fact that $-\frac{1}{2}\int c_1^2(\mathcal{L}_Y) = 1 > 0$, the following hierarchy of the couplings holds at M_G:

$$a_3(M_G) \geq a_1(M_G) \geq a_2(M_G) \tag{25}$$

where the equalities hold in the case of $x=0$. This limiting case can be reached by twisting $\mathcal{L}_a = \imath^*(L_a)$ appropriately by a trivial line bundle [3, 12, 17].

In view of (24) and (25) non-standard GUT relations, in what follows, we will investigate in some detail the issue of unification in the presence of extra-matter threshold effects.

3.1 The case of the color triplets $D_h, \bar{D}_h$

We consider first the simple case where only the triplet pairs appear in the spectrum below the $SU(5)$-GUT breaking scale M_G. We assume that at some scale $M_X < M_G$ the extra triplet pairs $D_h, \bar{D}_h$ decouple and only the MSSM spectrum remains massless for scales $\mu < M_X$. The low-energy values of the gauge couplings are then given by the evolution equations

$$\frac{1}{a_i(M_Z)} = \frac{1}{a_i(M_G)} + \frac{b_i^x}{2\pi}\ln\frac{M_G}{M_X} + \frac{b_i}{2\pi}\ln\frac{M_X}{M_Z} \tag{26}$$

where b_i^x, (b_i) are the beta functions above (below) the scale M_X and $a_i(M_G)$ given by (21).

Using the GUT relation (24) one arrives at the following equation[17]

$$\left[5\left(b_1^x - b_1\right) - 2\left(b_3^x - b_3\right)\right]\ln\left(\frac{M_G}{M_X}\right) = 0. \tag{27}$$

For n triplet pairs the differences $b_1^x - b_1 = \frac{1}{5}\cdot 2n$ and $b_3^x - b_3 = \frac{1}{2}\cdot 2n$. Therefore, as was observed[17], the expression with the β-functions in (27) vanishes and the equation is satisfied for any number of triplets irrespectively of the scale M_X they become massive. Thus, for a given value of x-shift we can choose the scale M_X where the triplets decouple to modify appropriately the gauge coupling running so that (24) and (25) are fulfilled.

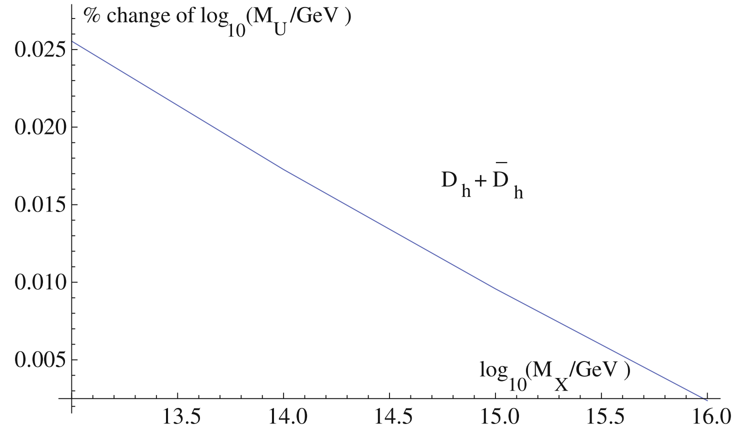

Fig. 1 Percentage correction, between 2- and 1-loop running, on the $SU(5)$ GUT scale M_U as a function of the decoupling scale of a color triplet pair $D_h + \bar{D}_h$

This exact M_X-scale independence at the 1-loop level, is of course spoiled when 2-loop contributions to the beta functions are taken into account. To estimate the effect, we run the renormalization group equations for the gauge couplings at 2 loop, including 1-loop contributions from the top quark coupling. In Fig. 1 we plot the percentage change of the unification scale as a function of the scale where the triplets acquire a mass. We notice only a marginal change from the "no-extra-matter" scenario. We also mention that in 1-loop running the scale where (24) is satisfied, with no extra matter at all, is 2.15×10^{16} GeV while in 2-loop running this scale is slightly higher 2.21×10^{16} GeV.

3.2 The general case

We assume next the more general situation where various types of extra states remain down to a scale $M_X < M_U$ after the breaking of the GUT symmetry M_U. When we take into account the contributions of complete $SU(5)$ $(10/\overline{10}, 5/\bar{5})$ vector-like multiplets, the unification point does not change, irrespectively of the scale these multiplets become massive. However, incomplete multiplets may originate from the bulk surface S as vector-like pairs. In fact, for the $SU(5)$ case, the exotic superfield pair $(3, \bar{2})_{-5} + (\bar{3}, 2)_5$ descends from the decomposition of the adjoint representation. It is also possible that after the $SU(5)$ breaking another type of extra vector-like states appear in the spectrum along the various matter curves Σ_i. In Table 1 we summarize the contributions of the various SM representations to the beta functions. To investigate their rôle in the determination of the string scale, we proceed to a more general analysis of the RGEs at the 1- and 2-loop level. To get an intuition of the specific contribution of each of the states presented in Table (1) we start first with the exploration at the 1-loop level where the analytic formulae are easy to handle. Thus, combining (24) and (26), we obtain

$$M_U = e^{\frac{2\pi}{\beta\mathcal{A}}\rho}\left(\frac{M_X}{M_Z}\right)^{1-\rho} M_Z \tag{28}$$

Table 1 The contributions of the various extra states to the beta functions and the combination $\delta\beta = \delta(b_Y - b_2 - (2/3)b_3)$

	b_Y	b_2	b_3	$\delta\beta$
Q	1/6	3/2	1	−2
u	4/3		1/2	1
d	1/3		1/2	0
L	1/2	1/2		0
e	1			1
H	3/10	1/2		0
Q'	1/2	1/2	0	0

with

$$\rho = \frac{\beta}{\beta_x} \tag{29}$$

and β, β_x are the beta-function combinations

$$\beta_x = b_Y^x - b_2^x - \frac{2}{3}b_3^x, \tag{30}$$

$$\beta = b_Y - b_2 - \frac{2}{3}b_3 \tag{31}$$

while $\mathcal{A}$ is a function of the experimentally known low-energy values of the SM gauge coupling constants

$$\frac{1}{\mathcal{A}} = \frac{5}{3}\frac{1}{a_1(M_Z)} - \frac{1}{a_2(M_Z)} - \frac{2}{3}\frac{1}{a_3(M_Z)}. \tag{32}$$

For $\rho = 1$, i.e., when there are no extra contributions in the beta functions or—more precisely—when the extra content contribution to the specific combination of the beta functions vanishes, we obtain the previous value of the GUT scale $M_U = M_G = e^{\frac{2\pi}{\beta\mathcal{A}}} M_Z \approx 2.1 \times 10^{16}$ GeV. We can substitute this into (28) to obtain a more illuminating relation between the 'old' M_G and 'new' unification scale M_U

$$\frac{M_U}{M_G} = \left(\frac{M_X}{M_G}\right)^{1-\rho}. \tag{33}$$

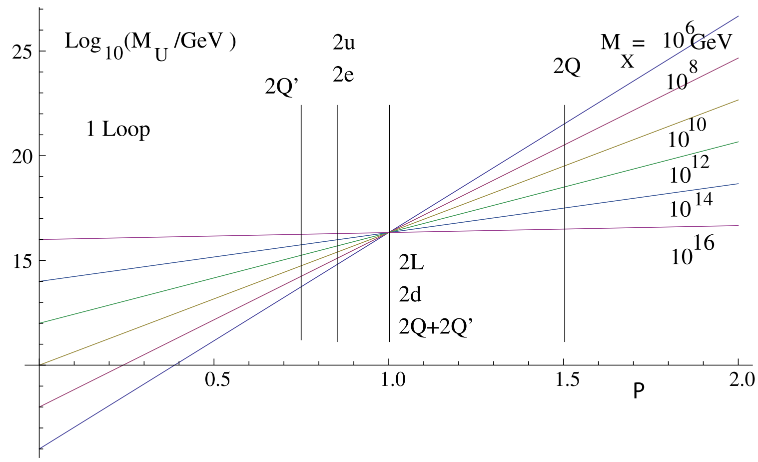

Fig. 2 The unification scale M_U as a function of $\rho = \beta/\beta_x$ for several energy scales M_X where the extra matter is assumed to contribute the β-functions. We also indicate the ρ value for several specific cases of (extra) matter content. We note that in practice ρ takes only discrete values and that only a subset of the above cases is compatible with the gauge coupling inequality (25). See text for details

Furthermore, as can be seen from Table 1 the contributions of the triplets to 1-loop β_x beta-function combination are zero. Therefore the scale M_D at which they become massive does not affect the scale M_U at which (24) is fulfilled and of course, does not necessarily coincide with M_X where the remaining extra states decouple. Now we can easily classify all the cases.

- If the contribution of extraneous matter in the beta-function combination β_x is zero, then $\beta_x = \beta$, or $\rho = 1$ and we have no dependence on the M_X scale. In this case the exponent in (33) is zero and then we obtain the old GUT scale $M_U = M_G$.
- If $0 < \rho < 1$ (i.e., $\beta_x > \beta$), consistency of the hierarchy of scales requires that we take the extra matter to be massive at scales[4] $M_X < M_G$, the scale M_U is suppressed.
- Finally, for $\rho > 1$ the power $1 - \rho$ of the same factor is negative and we can take only $M_X < M_G$, the M_U scale is enhanced.

To get an insight of the impact of the extra matter on the determination of the scale M_U, in Fig. 2 we plot the scale M_U vs. a sufficiently wide range of ρ values assuming that the extra matter states receive masses at a scale $M_X < M_G$. In this graph each line corresponds to a certain common (average) scale M_X where the extra matter becomes massive. When $\rho = 1$, i.e., when there is no extra matter, or more precisely when there is no contribution of the extra matter to the combination (31), we obtain the standard $SU(5)$ GUT scale $M_U \equiv M_G \sim 2 \times 10^{16}$ GeV. As expected all lines of the graph meet at the same point which is in accordance with the observation above. For values $\rho < 1$, the M_U scale decreases while the effect is obviously enhanced for lower M_X scales. This happens when the extra matter is composed by states which assign positive contributions to (31). Several vertical lines have been drawn on the graph to show the effect of various individual pairs on the unification scale. It is observed that the addition of an exotic $Q' + \bar{Q}'$ bulk pair descending from the adjoint reduces the unification scale to unacceptable values. For this reason as well as the unacceptable mixing with ordinary matter induced by the couplings (20) these exotic states must be eliminated from the spectrum. To this end, if L is assumed to be the line bundle on S associated with the breaking of $SU(5)$, we must impose the condition $\chi(S, L^{\pm 5}) = 0$.[5] In the presence of sufficiently large number of bulk states the parameter ρ can also attain values beyond the range discussed above and we will see specific examples in the subsequent analysis.

We have already pointed out that the admissible energy scales with their inextricable exotic matter spectra are restricted by the inequality (25) that holds among the gauge couplings' values at the unification scale. We can rewrite these inequalities in terms of the beta functions

$$\delta\left(b_3 - \frac{3}{5}b_Y\right)\ln\frac{M_U}{M_X} > +\frac{48}{5}\ln\frac{M_U}{M_Z} - 2\pi\left(\frac{3}{5}\frac{1}{a_Y} - \frac{1}{a_3}\right), \tag{34}$$

$$\delta\left(\frac{3}{5}b_Y - b_2\right)\ln\frac{M_U}{M_X} > -\frac{28}{5}\ln\frac{M_U}{M_Z} + 2\pi\left(\frac{3}{5}\frac{1}{a_Y} - \frac{1}{a_2}\right) \tag{35}$$

where in the right-hand side we have substituted the numerical values of the MSSM beta functions. On the left-hand side of these constraints we can express the differences in terms of the integer numbers n_Q, n_u, n_d, n_L, n_e representing the multiplicities of the extraneous matter introduced in

[4] If we take $M_X > M_G$, we see from $\frac{M_U}{M_X} = (\frac{M_G}{M_X})^\rho$ that this would imply $M_U < M_X$ which is unacceptable.

[5] Of course, if L is a line bundle, L^5 cannot also correspond to a root of a Lie algebra; however, it was shown that fractional powers of line bundles are also consistent [5].

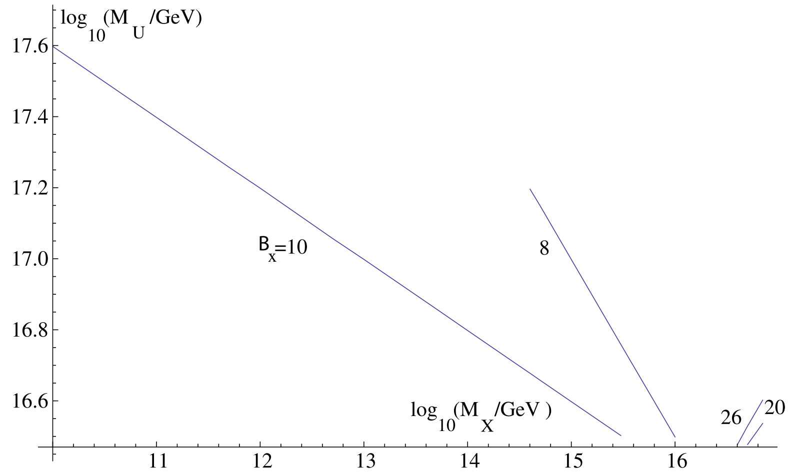

Fig. 3 The unification scale M_U as a function of the scale M_X (1-loop running) for extra-matter context $\beta_x = 10, 8$, and $\beta_x = 20, 26$

the spectrum above M_X. The $a_{Y,2,3}$ couplings on the right-hand side assume values at M_Z, thus for a desired unification scale $M_U \sim 10^{17}$ GeV, we can turn these inequalities to constraints on the extra-matter representations. Clearly, the above inequalities imply strong restrictions on the exotic matter needed to raise the unification scale while being compatible with the flux-modified conditions on the gauge coupling hierarchies at M_U. Expressing the beta functions in terms of the extra-matter multiplicities the above inequalities can be written

$$(n_d - 2n_e - n_u - n_L + 3n_Q)\ln\frac{M_U}{M_X} > 32\ln\frac{M_U}{M_Z} - 4\pi\left(\frac{\cos^2\theta_W}{a_{\text{em}}} - \frac{5}{3a_3}\right),$$

$$(n_d + 3n_e + 4n_u - n_L - 7n_Q)\ln\frac{M_U}{M_X} > -28\ln\frac{M_U}{M_Z} + 2\pi\frac{3 - 8\sin^2\theta_W}{a_{\text{em}}}.$$

Undoubtedly, there are various combinations of extra-matter representations satisfying the above conditions. In the following, we restrict our analysis to cases with minimal numbers of extra matter, while in our calculations we incorporate corrections implied by 2-loop contributions to the beta functions. We will also assume that the triplets become massive at the string scale M_U, thus they do not affect in any way the inequalities or the 2-loop RG running.

We start with the 1-loop analysis and depict our results in Fig. 3. We choose to plot the unification scale M_U as a function of the extra-matter decoupling scale M_X, for various values of the beta-function combination β_x given in (30).

For $\beta_x = 10$ we see that for M_X values in the range $[10^{10}–4 \times 10^{14}]$ GeV, the corresponding unification scale ranges

$$M_U \sim 3 \times 10^{16}–3.9 \times 10^{17} \text{ GeV}.$$

Table 2 The predictions for the $SU(5)$ breaking scale and the corresponding decoupling scale M_X for two values of the combination $\beta_x = b_Y^x - b_2^x - (2/3)b_3^x$. Various types of additional matter result to the same β_x and therefore to the same M_U prediction

β_x	Extra matter	min M_X (GeV)	max M_U (GeV)
10	$4Q, 6u$	10^{10}	3.96×10^{17}
	$6Q, 8u, 2e$		
	$2Q, 2u$	2×10^{15}	3.18×10^{16}
	$4Q, 4u, 2e$		
	$4Q, 6u, 4L$		
	$6Q, 6u, 4e$		
	$6Q, 8u, 2L, 2e$		
	$8Q, 8u, 6e$		
8	$6Q, 8u$	4×10^{14}	1.57×10^{17}
	$4Q, 4u$	10^{16}	3.15×10^{16}
	$6Q, 6u, 2e$		
	$6Q, 8u, 2L$		
	$8Q, 8u, 4e$		

As far as we keep the extra matter below five pairs, the only allowed combinations of extra matter consistent with the inequalities, is 2 pairs of Q-like and 3 pairs of u-like particles or 3 pairs of Q-, 4 pairs of u- and 1 pair of e-like matter (see also Table 2). Note that since we are interested in high values of the unification scale, we draw the lines only for scales bigger than $M_U > 3 \times 10^{16}$ GeV. For the $\beta_x = 10$ case in particular, the line is drawn down to a reasonable lower decoupling scale $M_X \sim 10^{10}$ GeV, although the conditions (34, 35) would allow even for smaller M_X scales.

As a second possibility, we assume the case of extra-matter content which gives $\beta_x = 8$ corresponding to the combination of 3 pairs of Q and 4 pairs of u. Now consistency with (34, 35) requires that the extra matter decouples at scales $M_X > 4 \times 10^{14}$ GeV. We observe that this case corresponds to a string scale as high as $M_U \sim 1.6 \times 10^{17}$ GeV.

Table 3 Two-loop results for the $SU(5)$ GUT scale M_U and the 'shifted' gauge coupling values $a_i(M_U)$ in the case of two vector-like $Q+\bar{Q}$ quark pairs and three $u^c+\bar{u}^c$ pairs. The corresponding decoupling scale M_X is shown in the first column

M_X (GeV)	M_U (GeV)	$\alpha_3(M_U)$	$(5/3)\alpha_1(M_U)$	$\alpha_2(M_U)$	x
10^{11}	6.23×10^{17}	0.15680	0.15097	0.14730	0.41035
10^{12}	2.90×10^{17}	0.09927	0.09734	0.09609	0.33346
10^{13}	1.51×10^{17}	0.07429	0.07354	0.07304	0.23075
10^{14}	8.22×10^{16}	0.05988	0.05963	0.5947	0.11540

For higher M_X values more extra-matter combinations respect the requirements, however the conditions (34, 35) restrict the string scale to lower and lower values. Because of the $\ln(M_U)$ dependence on the factor $1-\beta/\beta_x=1-\rho$, we observe that as long as $0<\beta_x<\beta=12$ the slope of the lines are negative thus the lower the M_X decoupling scale, the higher the unification mass M_U. For $\beta_x>\beta=12$ the slope of the lines changes and the opposite is true.

When 2-loop corrections are taken into account, the $SU(5)$ GUT scale may attain even higher values. To show the effect, we pick up the case $n_Q=4$ and $n_u=6$ of extra-matter representations. We run numerically the coupled RG equations using the SM beta functions from M_Z to m_{top}, and the MSSM spectrum from m_{top} to M_X, while we take into account the extra-matter contributions in the range M_X-M_U. In Table 3 we present the results for the unification scale and the values of the gauge couplings $a_i(M_U)$ as they are shifted by the fluxes' threshold corrections. From the findings presented in this table we conclude that in the presence of a rather moderate number of additional matter states it is possible to obtain a unification scale M_U sufficiently larger than the ordinary GUT breaking scale of the minimal $SU(5)$. This scale is of the order 10^{17} GeV and therefore allows the possibility to make the colored triplets incorporated in the fiveplets heavy enough in order to avoid fast proton decay. In the last column we present also the corresponding values of the 'flux' parameter x which is intimately related to the dilaton field. These values imply dilaton vevs leading to a strong coupling regime which is the appropriate description for F-theory.

We close our analysis with a remark concerning the gaugino masses. The above GUT relations and the threshold corrections induced by the fluxes discussed above are expected to have also significant implications on the gaugino masses M_i whose magnitude at low energies is determined by the renormalization group equations

$$\frac{dM_i}{dt}=\frac{b_i}{2\pi}a_iM_i. \tag{36}$$

Combining with the RG equations for the gauge couplings a_i we find that at any scale $t=\ln\mu$ the following equation

Table 4 The gaugino masses for three cases of $m_{1/2}$ and four possible M_X scales. The extra matter consists of two pairs of Q's and three pairs of u's. All values are in GeV units

M_X	$m_{1/2}=300$			$m_{1/2}=350$			$m_{1/2}=400$		
	M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3
10^{11}	48	100	300	56	117	347	64	134	394
10^{12}	62	128	392	72	149	453	83	171	515
10^{13}	76	156	482	89	182	559	102	208	634
10^{14}	90	183	572	106	214	663	121	245	752

holds

$$\frac{M_i(\mu)}{a_i(\mu)}=\frac{M_i(M_U)}{a_i(M_U)} \tag{37}$$

where $M_i(M_U), a_i(M_U)$ are the corresponding values at the unification scale. Since in a unified gauge group the gaugino masses belong to the same multiplet, we expect $M_i(M_U)=m_{1/2}$. This implies the following relation between the gaugino masses irrespectively of the unification scale and the mass spectrum of the theory

$$2\frac{M_3}{a_3}+3\frac{M_2}{a_2}-5\frac{M_1}{a_1}=0. \tag{38}$$

In Table 4 we give the predictions of the three masses for the cases discussed previously. Since the gaugino masses play a decisive role in the calculation of the various scalar masses, we note that these definite mass relations would have also a significant impact on the scalar spectrum of the model.

Up to now we have worked out in detail the effects of the fluxes on the string scale and the gaugino masses, however we have ignored threshold corrections which arise when integrating out Kaluza–Klein (KK) modes. Since the KK-scale may differ substantially from the decoupling scale in a given model [3, 12], it is important to give an estimate of the effect these thresholds can have on the various RG dependent quantities. In the F-version of the $SU(5)$ model that we discuss here, we have assumed the existence of chiral matter residing only on the curves defined by the seven-brane intersections. Therefore, we may assume threshold corrections arising only from KK-modes of gauge fields propagating in the bulk and those of chiral matter of the Σ_{10} and $\Sigma_{\bar{5}}$ matter curves. Denoting with $\delta_i = \delta_i^g + \delta_i^c + \delta_i^h$ the gauge, chiral matter and Higgs KK-threshold contributions to the three gauge couplings ($i = 1, 2, 3$) respectively, we find that the modified unification scale M'_U is given by

$$M'_U = e^{-\frac{2\pi}{\beta_x}\delta} M_U \tag{39}$$

where M_U is given by (28) and δ is the combination

$$\delta = \frac{5}{3}\delta_1 - \delta_2 - \frac{2}{3}\delta_3. \tag{40}$$

To estimate the effects of these thresholds on the unification scale, we follow closely the analysis of [3]. The massive modes contributing to these thresholds constitute the spectrum of the Dirac operator in the eight-dimensional theory which in the compactified four-dimensional theory decomposes to Dolbeault operators of the corresponding holomorphic bundle V with representation R_V, i.e., $\bar\partial : \Omega_S^{0,k} \otimes R_V \to \Omega_S^{0,k+1} \otimes R_V$, $k = 0, 1$. The quantities involved in the thresholds are related to the eigenvalues of the corresponding Laplacian $\Delta_k = \bar\partial\bar\partial^\dagger + \bar\partial^\dagger\bar\partial$ acting on $\Omega_S^{0,k} \otimes R_V$ and can be expressed in terms of the logarithm of the determinant $\log\det'\Delta = -\zeta'_\Delta(0)$ [31, 32], where the prime in $\det'$ means that the zero modes are excluded.

For the bulk gauge fields we may consider the decomposition (9) and denote each representation R_Y with respect to its hypercharge $Y = 0, \pm 5/6$, while for each group factor the corresponding contribution is

$$\delta_i^g = \frac{1}{4\pi}\sum_Y 2\,\mathrm{Tr}_{R_Y}\left(Q_i^2\right) K_Y \tag{41}$$

where

$$K_Y = 2\log\det'\frac{\Delta_{0,R_V}}{M^2} - \log\det'\frac{\Delta_{1,R_V}}{M^2}. \tag{42}$$

It has been argued [3] that this type of thresholds can be expressed in terms of the Ray–Singer torsion T_{R_V} [33] modulo the M^2 dependence. The corrections may finally be written

$$\delta_i^g = \frac{b_i^g}{4\pi}\log\frac{M^2}{\mu^2} + \tilde\delta_i^g \tag{43}$$

with $\tilde\delta_i^g = \frac{b_i^{(5/6)}}{2\pi}(T_{5/6} - T_0)$. To get an estimate of the order of corrections, following [3] we choose a special case of line bundle $\mathcal{O}(n, -n)$ on $P^1 \times P^1$ and apply for the case $n = 1$, so that $L^5 = \mathcal{O}(1, -1)$. Using the fact that $T_{\mathcal{O}(k)} = -\frac{1}{2}\zeta'_k(0)$ and the results of [31, 32] we find

$$\tilde\delta^g = \frac{4}{\pi}(T_{\mathcal{O}(-1)} - T_{\mathcal{O}(0)}) = -\frac{1}{\pi} \tag{44}$$

which implies a correction of the order $M'_U \sim 1.22 \times M_U$ on the scale M_U.

To estimate the remaining threshold effects we first note that all chiral and vector-like matter as well as the Higgs fields, are localized on the $\Sigma_{\bar 5}$ and Σ_{10} curves. Using the analogue of formula (41) we find that only the Σ_{10}-thresholds contribute to M_U since for the $\Sigma_{\bar 5}$ curve the threshold combination (40) is found to be zero. Following the same reasoning with [3], we consider the simplest non-trivial case of Σ_{10} being of genus one and express the corrections in terms of the torsion of flat line bundles L_z, as follows

$$\delta^{10} = \frac{1}{4\pi}(4T_{\mathcal{O}} - 4T_{L_z}) = \frac{1}{\pi}(T_{\mathcal{O}} - T_{L_z}). \tag{45}$$

Substituting in the exponent of (39) and noting that the difference $T_{\mathcal{O}} - T_{L_z}$ is positive for a large z-range [3], we infer that contributions from Σ_{10} tend to abate the effect of the thresholds on M_U since as we can easily observe this correction works in the opposite direction of the gauge threshold discussed above. Although there is no a priori reason that these two contributions are equal in magnitude, we may work out cases where the total effect is rather small compared to our previous calculations.

4 Conclusions

In the present work, we have discussed several phenomenological issues of the low-energy effective theory derived in the context of an F-theory $SU(5)$ GUT. We have investigated the rôle of the Yukawa couplings generated from exotic representations and other matter states—beyond those of the minimal supersymmetric spectrum—which are usually present in F-theory constructions. We have shown that the exotic color pairs $(\bar 3, 2)_5 + (3, \bar 2)_{-5}$ descending from the $SU(5)$ adjoint induce unacceptable mixing mass terms while they lower dangerously the unification scale and they should be diminished from the matter spectrum, in the way already described in [2]. Furthermore, we have suggested that a sensible way to evade the problem of fast proton decay caused by the triplets found in the $5/\bar 5$ Higgs fields, is to obtain a high enough GUT breaking scale $M_U \gg 10^{16}$ GeV. Indeed, taking advantage of the splitting of gauge couplings [17] at M_U—induced by the $SU(5)$-breaking $U(1)_Y$ flux—and the presence of appropriate types of exotic matter, we have shown that the GUT scale can take values of the order $M_U \gtrsim 10^{17}$ GeV. This way, if the triplets decouple at the GUT scale and acquire a mass of the same order, they could suppress adequately the relevant proton decay operators. We finally presented in brief the implications of the gauge coupling splittings at M_U, on the gaugino masses.

Note added

While this paper was reaching its final form, we noticed some recent work [29, 30] where several similar issues are also discussed.

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