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GUT PRECURSORS IN SU(3)^3-TYPE MODEL AND $N_{\text{colour}} > 3$

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We investigate the SU(3)^3 GUT model when signs of the model (precursors), due to low compactification scale, appear before the gauge couplings of the Standard Model get unified. The Kaluza–Klein state contribution seems to lead the gauge couplings to unification through a wide energy scale only in the case when the colour group is augmented to SU(4).

Keywords: GUT; Kaluza–Klein; precusors; gauge coupling unification.

1. Introduction

The unification of the SM gauge couplings to a common value at some high energy scale still plays an attractive role in our efforts to understand the fundamental interactions of nature. The supersymmetric extensions of the SM provides us with such a scenario. The energy scale of this unification is, however, of the order of 10^{16} GeV, rendering experimental evidence at best indirect. On the other hand, large extra dimension can lower the scale of gauge coupling unification due to the appearance of the Kaluza–Klein (KK) tower of states above the compactification scale.[1–6] Inclusion of KK states (either from the Higgs and the gauge bosons or from all the spectrum) in the MSSM could lead to lower energy scale unification.[2,7] The same idea was also applied[7] to successful GUT models that could be originated from strings, namely the $S(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and the SU(3)^3 models. In both models, the inclusion of several numbers of exotic states (i.e. states not appearing in the SM spectrum but present in models derived from strings) helps in providing the gauge coupling unification. We can use the standard Higgs mechanism to break the GUT model. It is obvious that the masses of the extra GUT fields are of the order of the vev used which in turn is of the order of the GUT breaking scale. On the other hand, using the orbifold method to break the larger symmetry, the extra fields acquire masses of the order of the compactification scale (the inverse of the circle S^1 for one extra dimension). Thus, in case that this scale is lower than the scale where the gauge coupling meet (M_{GUT}), we have the appearance of KK states of the extra GUT fields. These fields are the so-called "precursors" which of course influence the running of the β-functions for energy scales above the compactification one. In the present work, we will concentrate on the $\mathrm{SU}(3)^3$ GUT model. Assuming a low compactification scale we will try to accomplish the gauge coupling unification incorporating in the MSSM β-functions the contribution of the GUT precursors.

2. The Model

Let us briefly describe the $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ GUT model, which is one of the few that can be derived from strings. The MSSM content can be found in the $\underline{27}$ representation of the E_6:

$$27 \to (3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3}) ,$$

where

$$Q = (3, \bar{3}, 1) = \begin{pmatrix} u \\ d \\ D \end{pmatrix} , \qquad Q^c = (\bar{3}, 1, 3) = \begin{pmatrix} u^c \\ d^c \\ D^c \end{pmatrix} ,$$
$$L = (1, 3, \bar{3}) = \begin{pmatrix} h_0 & h^+ & e^c \\ h^- & \bar{h}_0 & \nu^c \\ e & \nu & N \end{pmatrix} . \tag{1}$$

The emergence of the SM comes as follows: The $\mathrm{SU}(3)_C$ is the colour group. The second $\mathrm{SU}(3)$ breaks to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_L$ while $\mathrm{SU}(3)_R$ breaks to $\mathrm{U}(1)_R$. The SM $\mathrm{U}(1)$ comes as a linear combination of the two $\mathrm{U}(1)_{L,R}$. The hypercharge Y is related to the X and Z charges of the $\mathrm{U}(1)_L$ and $\mathrm{U}(1)_R$ correspondingly, through the relation

$$Y = \frac{1}{\sqrt{5}} X + \frac{2}{\sqrt{5}} Z \tag{2}$$

while the corresponding relation between the couplings at the breaking scale is

$$\alpha_3 = \alpha_C , \qquad \alpha_2 = \alpha_L , \qquad \alpha_Y^{-1} = \frac{1}{5}\alpha_L^{-1} + \frac{4}{5}\alpha_R^{-1} .$$

The one-loop β-functions are given by

$$\begin{aligned} \beta_C &= -9 + \frac{1}{2}(3n_Q + 3n_{Q^c}) , \\ \beta_L &= -9 + \frac{1}{2}(3n_Q + 3n_L) , \\ \beta_R &= -9 + \frac{1}{2}(3n_{Q^c} + 3n_L) , \end{aligned} \tag{3}$$

where n shows the number of the corresponding representation.

3. The Precursor Contribution to the MSSM $\boldsymbol{\beta}$-Functions

The general form for the Kaluza–Klein state contribution to the β-function is:

$$\beta_{\text{KK}} = -2C_2(G) + \sum_i T(R_i)\,, \tag{4}$$

where the first term comes from the gauge multiplet (gauge bosons and gauginos), while the second comes from the chiral multiplets (quarks, leptons, Higgs and superpartners). We should find the contribution to the MSSM β-functions coming from the members of the gauge sector and chiral sector which do not appear in the corresponding MSSM sectors. Let us start from the former ones. In the $\text{SU}(3)_L \to \text{SU}(2)_L \times \text{U}(1)$ breaking, the adjoint of the SU(3) gives:

$$8 \to 3 + 2 + \bar{2} + 1$$

while the U(1) charges are zero for the 3 and the singlet and $\pm\sqrt{3}/2$ for the 2's. The 3 are the gauge bosons of the $\text{SU}(2)_L$ of the SM, while the singlet is completely blind in all SM interactions. The doublets appear as spin 1 (plus the SUSY partners) particles having $\text{SU}(2)_L$ gauge interactions.

Now, the $C_2(G)$, appearing in Eq. (4), for an $\text{SU}(N)$ group is equal to N which comes from the contraction of the structure constants of the group: $f_{ijk}f_{i'jk} = N\delta_{ii'}$. In our case we should find this summation when i, i' and j correspond to the doublet while k corresponds to the triplet. In the SU(3) case this summation gives 3/2 instead of 3.[a] Therefore the contribution of these two doublets to the β-function of the SU(2) is $-2 * (3/2) = -3$ for each doublet.

Let us now find the contribution of the doublets to the SM U(1) β-function. The charge under $\text{U}(1)_L$ is $\pm\sqrt{3}/2$. Their charges under $\text{U}(1)_R$, coming from the breaking of the $\text{SU}(3)_R$, is of course zero. Therefore, using Eq. (2), the contribution of each doublet to the U(1) β-function is: $-2[(1/\sqrt{5})(\sqrt{3}/2)]^2 * 2$.

Following the same procedure, the contribution of each doublet, coming from the breaking of the adjoint of the $\text{SU}(3)_R$, to the SM U(1) is: $-2[(2/\sqrt{5})(\sqrt{3}/2)]^2 * 2$.

We turn now to the contribution of the D and D^c appearing in the Q and Q^c representations. Both, D and D^c, being in the fundamental representation of the $\text{SU}(3)_C$, contribute a term $(1/2)N_g$ each, in the colour group β-function (N_g is the number of generations). In the breaking of $\text{SU}(3)_L$, the D's appear as the singlets in the breaking of the fundamental representation: $3 \to 2 + 1$, with charge under $\text{U}(1)_L$ equal to $-1/\sqrt{3}$. Therefore, they do not contribute to the $\text{SU}(2)_L$ β-function while their contribution to the U(1) β-function is $[(1/\sqrt{5})(-1/\sqrt{3})]^2 3N_g = (1/5)N_g$. Finally, the D^c do not contribute to the $\text{SU}(2)_L$ β-function and since $\text{SU}(3)_R$ breaks to $\text{U}(1)_R$, the charge under the last group could be arbitrary, fixed in such a way as to give the correct electrical charge. Indeed, the $\text{U}(1)_R$ charge of the D^c is $1/(2\sqrt{3})$ and the contribution to the U(1) β-function is $[(2/\sqrt{5})(1/(2\sqrt{3}))]^2 * 3N_g = (1/5)N_g$. The two states, N and ν^c, appearing in L are totally blind in the SM interactions.

[a]For the general case $\text{SU}(N+1) \to \text{SU}(N) + \text{U}(1)$, this summation gives $(N^2-1)/N$.

Gathering all the above we can write the contribution of the Kaluza–Klein states of the non-SM particles to the β-function as follows ($N_g = 3$):

$$\beta_3 = 3\,, \qquad \beta_2 = -6\,, \qquad \beta_Y = -24/5\,. \tag{5}$$

4. The Running of the One-Loop β-Functions

We assume that from M_Z to $M_{\rm SUSY} = 1$ TeV, we have the non-SUSY SM β-functions. We use the following experimental values as our starting point at M_Z:

$$\sin^2\theta_W = 0.23151 \pm 0.00017\,, \qquad \alpha_{\rm em} = 1/128.9\,, \qquad \alpha_s = 0.119 \pm 0.003\,.$$

From $M_{\rm SUSY}$ to the compactification scale M_C we have the MSSM β-functions. Above M_C, all Kaluza–Klein states start to appear. We assume the successful approximation of incorporating the massive KK-states with masses less than the running scale. The running of the couplings above M_C is given by

$$\alpha_i^{-1}(M') = \alpha_i^{-1}(M_C) - \frac{\beta_i}{2\pi}\left(2N\log\frac{M'}{M_C} - 2\log(N!)\right),$$

where N is an integer such that $(N+1)M_C > M' > NM_C$, which counts the KK-states that have masses below the running scale (we have assumed only one extra dimension and in that case the multiplicity of the states at each level is 2). In Fig. 1 we show such a running. It is obvious from the positivity of the KK-state contribution to the β-function that the three couplings could not converge to a point. And this fact, of course, persists whatever the choice of the scale M_C is, since we know that the MSSM couplings converge at the scale 10^{16} GeV which means that the strong coupling is always larger than the other two up to that scale.

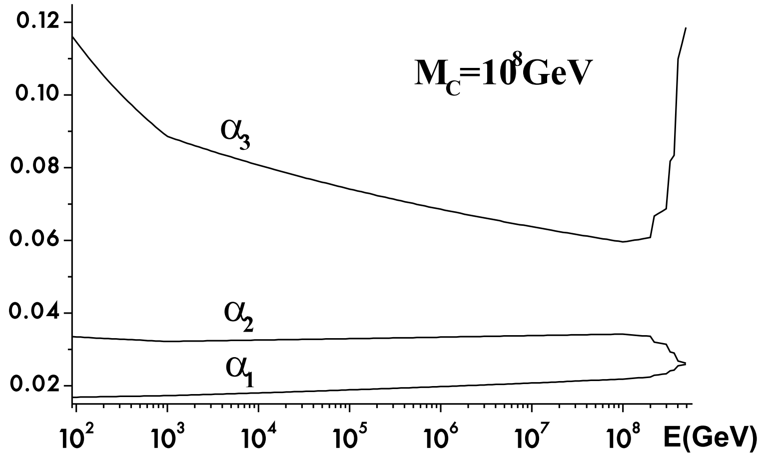

Fig. 1. Running of the SM couplings. $M_C = 10^8$ GeV.

5. Upgrading the Colour Group to SU(N), $N > 3$

The idea that at high energies the colour group is SU($N > 3$) has a long history[8] and was considered as requirement for "asymptotic convergence" on top of asymptotic freedom. Recently, this idea was applied to Grand Unification.[9] In our case, therefore, it is more than tempting to investigate the case where the colour group of our model is upgraded to SU(N) with $N > 3$. We assume that the breaking from $\mathrm{SU}(N)_C$ down to $\mathrm{SU}(3)_C$ is stepwise: $\mathrm{SU}(N)_C \to \mathrm{SU}(N-1)_C + \mathrm{U}(1)$. The conjugate and the fundamental representation breaking are:

$$\begin{aligned} (N^2-1) &\to ((N-1)^2-1) + (N-1) + \overline{(N-1)} + 1\,, \\ N &\to (N-1) + 1\,. \end{aligned} \tag{6}$$

We further assume that all singlets produced by these breakings get masses and therefore they do not contribute to the colour group. At the end, we are left with the fundamental representation of $\mathrm{SU}(3)_C$ coming from the fundamental one of SU(N) (i.e. the coloured quarks of the SM) while from the conjugate representation of SU(N) we get the gluons plus a number of 3's and $\bar{3}$'s. The number of these states is proportional to $N-3$ (the number of times breaking occurs from $\mathrm{SU}(N)_C$ to $\mathrm{SU}(3)_C$). For example, if we have an $\mathrm{SU}(5)_C$ group, the breaking of the adjoint down to $\mathrm{SU}(3)_C$ is:

$$\begin{array}{ccccccc} 24 \to & 15 & + & 4 & + & \bar{4} & + 1 \\ & \downarrow & & \downarrow & & \downarrow & \\ & & & & & \bar{3}+1 & \\ & & & 3+1 & & & \\ & 8+3+\bar{3}+1 & & & & & \end{array}$$

The contribution of the 3's and $\bar{3}$'s (KK-) states to the β-function of $\mathrm{SU}(3)_C$ will be:

$$-2\frac{N(N-2)}{(N-1)}2(N-3)\,.$$

Now we are ready to run the coupling constants with the above new contribution. In Fig. 2 we show the running for two values of $N = 4$ and 5.

We clearly see that for $N = 4$ the three coupling constant runnings are suitable for unification, while for $N = 5$ the strong coupling decreases very rapidly and unification is missed. Of course, for higher values of N the situation will be even worst (the strong β-function will be even more negative).

In Fig. 3 we show the running for $N = 4$ and for several values of the compactification scale M_C. We see that for all chosen values the three couplings show the same tendency to unify to a common value. Of course, the unification scale is just above the compactification one and only a small number of the KK-states contribute to the running. Nevertheless, we can achieve a low energy unification and the value of the unified coupling is well in the perturbative region.

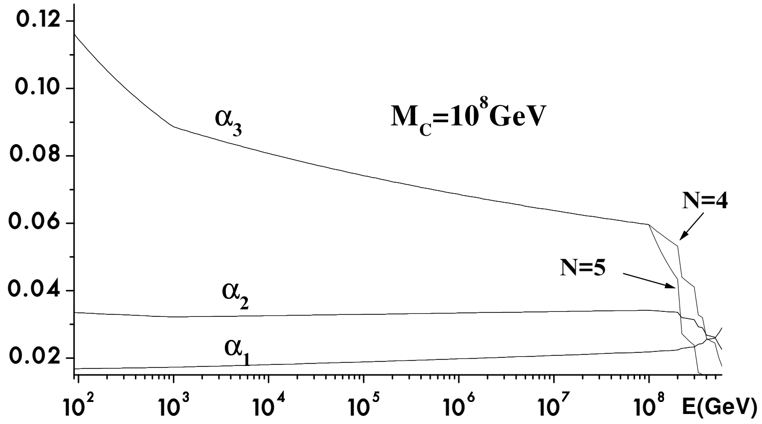

Fig. 2. Running of the SM couplings for $N = 4$ and $N = 5$. $M_C = 10^8$ GeV.

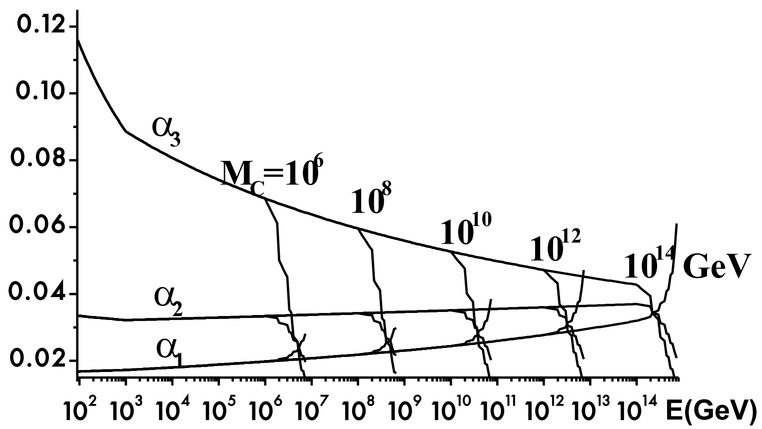

Fig. 3. Running of the SM couplings for $N = 4$ and several values of the compactification scale $M_C = 10^6$, 10^8, 10^{10}, 10^{12} and 10^{14} GeV.

We would like to stress that the above is of course a toy model. Nevertheless, it shows how the precursor idea can be applied to a successful grand unified group, namely $\mathrm{SU}(3)^3$, and that only in the case where the colour group is coming from an SU(4) the gauge coupling unification seems possible. Application of two-loop running in the RGE (in order to check how precise is the gauge coupling unification) will not have any meaning at this stage since there are theoretical simplifications (the breaking of $\mathrm{SU}(4) \to \mathrm{SU}(3) + \mathrm{U}(1)$, inclusion of KK states step-by-step as well as the detailed compactification scenario) which introduce uncertainties. On the other hand it is clear that for the specific group, i.e. $\mathrm{SU}(3)^3$, only the case where $\mathrm{SU}(3)_C \subset \mathrm{SU}(4)$ could lead to gauge coupling unification.

6. Conclusions

Using the precursor idea, i.e. the appearance of KK states of the SU(3)^3 GUT model before the SM couplings get unified, we have studied the gauge coupling running. The contribution of KK states, due to low compactification scale, accelerates the convergence of the couplings but unification seems possible only if the colour group is SU(4) at the GUT scale providing therefore extra precursors. The unification scale appears near the chosen compactification scale but the tendency of the gauge couplings to unify is independent of the latter scale.

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References

1. I. Antoniadis, *Phys. Lett.* **B246**, 377 (1990); I. Antoniadis, K. Benakli and M. Quiros, *ibid.* **B331**, 313 (1994).
2. K. Dienes, E. Dudas and T. Gherghetta, *Phys. Lett.* **B436**, 55 (1998); *Nucl. Phys.* **B537**, 47 (1999); hep-ph/9807522; hep-ph/0210294; F. P. Correia, M. G. Schmidt and Z. Tavartkiladze, *Phys. Lett.* **B566**, 226 (2003).
3. D. Ghilencea and G. G. Ross, *Phys. Lett.* **B442**, 165 (1998).
4. K. Benakli, *Phys. Rev.* **D60**, 104002 (1999).
5. C. P. Burges, L. E. Ibanẽz and F. Quevedo, *Phys. Lett.* **B447**, 257 (1999); L. E. Ibanẽz, C. Muñoz and S. Rigolin, *Nucl. Phys.* **B553**, 43 (1999); T. Li, hep-ph/9903371.
6. A. Delgado and M. Quiros, *Nucl. Phys.* **B559**, 235 (1999).
7. G. K. Leontaris and N. D. Tracas, *Phys. Lett.* **B470**, 84 (1999).
8. G. 't Hooft, *Phys. Lett.* **B109**, 1848 (1982).
9. G. C. Branco, J.-M. Gérard, R. González Felipe and B. M. Nobre, hep-ph/0305092.