Gauge unification and quark masses in a Pati–Salam model from branes

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Abstract. We investigate the phase space of parameters in the Pati–Salam model derived in the context of D-brane scenarios, requiring a low energy string scale. We find that a non-supersymmetric version complies with a string scale as low as ~ 10 TeV, while in the supersymmetric version the string scale rises to $\sim 2 \times 10^7$ TeV. The limited energy region for RGE running demands a large $\tan \beta$ in order to have experimentally acceptable masses for the top and bottom quarks.

Contents

1. Introduction

Over the last few years, there has been considerable work in trying to derive a low energy theory of fundamental interactions through a D-brane construction [1]–[13]. Recent investigations have shown that there is a variety of possibilities, concerning the group structure of the theory as well as the magnitude of the string scale and the nature of the particle spectrum.

A particularly interesting possibility in this context is the case of models with low scale unification of gauge and gravitational interactions. This is indeed a very appealing framework for solving the hierarchy problem, as one dispenses with the use of supersymmetry. There are a number of phenomenological questions, however, that should be answered in this case, including the smallness of neutrino mass[1].

Another interesting possibility which could solve a number of puzzles (as the neutrino mass problem mentioned previously) is the intermediate scale scenario. After some early attempts [14], a variety of models admit an intermediate unification scale; however supersymmetry is needed in this case to solve the hierarchy problem.

In this paper we concentrate on phenomenogical issues of the Pati–Salam (PS) [15] gauge symmetry proposed as a D-brane alternative [11] to the traditional grand unified version. In particular we investigate the gauge coupling relations in two cases: for a non-supersymmetric version and for a supersymmetric one. In both cases, in order to achieve a low string scale, we relax the idea of strict gauge coupling unification. However, this should not be considered as a drawback. Indeed, the various gauge group factors are associated with different stacks of branes and therefore it is natural that gauge couplings may differ at the string scale. In the non-supersymmetric case the string scale could be as small as a few TeV. On the other hand, the absence of a large mass scale puts the see-saw type mechanism (usually responsible for giving neutrino masses in the experimentally acceptable region) in trouble. In the supersymmetric case, the string scale is of the order of 10^3 TeV and a sufficiently suppressed neutrino mass may be obtained.

2. The model

We assume here a class of models which incorporate the PS symmetry [15], having representations that can be derived within a D-brane construction. In these models, gauge interactions are described by open strings with ends attached on various stacks of D-brane configurations and therefore fermions are constrained to be in representations smaller than the adjoint. A novelty of these constructions is the appearance of additional anomalous $U(1)$ factors. At most, one linear combination of these $U(1)$ is anomaly free and may remain unbroken at low energies. As we will see, the role of this extra $U(1)$ is important since when it is included in the hypercharge definition it allows the possibility of a low string scale.

We start with a brief review of the model [11]. The embedding of the PS model in the brane context leads to a $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_L \times U(1)_R$ gauge symmetry. Open strings with ends on two different branes carry quantum numbers of the corresponding groups. The standard model particles appear under the following multiplets of the PS group:

$$F_L = (\mathbf{4}, \mathbf{2}, \mathbf{1}; 1, 1, 0) \rightarrow Q(\mathbf{3}, \mathbf{2}, \tfrac{1}{6}) + L(\mathbf{1}, \mathbf{2}, -\tfrac{1}{2})$$
$$\bar{F}_R = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}; -1, 0, 1) \rightarrow u^c(\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}) + d^c(\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}) + e^c(\mathbf{1}, \mathbf{1}, 1) + \nu^c(\mathbf{1}, \mathbf{1}, 0) \qquad (1)$$
$$h = (\mathbf{1}, \mathbf{2}, \mathbf{2}; 0, -1, -1) \rightarrow H_u(\mathbf{1}, \mathbf{2}, \tfrac{1}{2}) + H_d(\mathbf{1}, \mathbf{2} - \tfrac{1}{2})$$

where we have also shown the quantum numbers under the three $U(1)$ and the breaking to the SM group. Some comments on the $U(1)$ charges are in order[2]. The $U(1)_L$ and $U(1)_R$ charges

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[1] For a recent proposal in the context of SM and the D-brane scenario see [10].
[2] For more details on the assignments of the $U(1)$ charges see [11].

of F_L and $\bar{F}_R$ can be chosen (without loss of generality) to be $+1$. Then, in order for the term $F_L \bar{F}_R h$ (which will provide masses to the quarks and the leptons) to be allowed, the $U(1)_L$ and $U(1)_R$ charges of the h should be -1. The Higgs which breaks the PS down to the SM is

$$\bar{H} = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}; -1, 0, \delta) \rightarrow u_H^c(\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}) + d_H^c(\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}) + e_H^c(\mathbf{1}, \mathbf{1}, 1) + v_H^c(\mathbf{1}, \mathbf{1}, 0). \tag{2}$$

The $U(1)$-charge parameter δ can take two values $\delta = \pm 1$. Each one of them is associated with a different symmetry breaking pattern. The down-quark-like triplets are the only remnants after the PS breaking while one Higgs $\bar{H}$ (and its complex conjugate) is enough to achieve this breaking. Additional states, such as

$$D(\mathbf{6}, \mathbf{1}, \mathbf{1}; 2, 0, 0) \rightarrow \tilde{d}^c(\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}) + \tilde{d}(\mathbf{3}, \mathbf{1}, -\tfrac{1}{3})$$
$$\eta(\mathbf{1}, \mathbf{1}, \mathbf{1}; 0, 0, 2) \tag{3}$$
$$h_R(\mathbf{1}, \mathbf{1}, \mathbf{2}; 0, 0, 1)$$

can arise which could provide masses to the PS breaking remnants (coloured triplets with down-type quark charges d_H, d_H^c) or break an additional Abelian symmetry (by a non-vanishing vacuum expectation value (VEV) of η and/or h_R).

While all three of the $U(1)$ that come with the PS group are anomalous, there exists only one combination which is anomaly free (even from gravitational anomalies):

$$Y_{\mathcal{H}} = Y_C - Y_L + Y_R \tag{4}$$

where $Y_X, X = C, L, R$ corresponds to the quantum number under the $U(1)_X$. None of the SM fermions and Higgs bidoublet (providing the SM Higgses) are charged under this $U(1)_{\mathcal{H}}$. To this end, we assume that all anomalous Abelian combinations break and we are left with a gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_{\mathcal{H}}$. The SM hypercharge[3] is given by the usual PS generators plus a contribution from the $U(1)_{\mathcal{H}}$:

$$Y = \tfrac{1}{2}Y_{B-L} + \tfrac{1}{2}T_{3R} + cY_{\mathcal{H}}. \tag{5}$$

The value of c depends on the symmetry breaking pattern and is related to the value of δ. In particular, when δ, determining the $\bar{H}$ charge under the $U(1)_{\mathcal{H}}$ (namely $\delta - 1$), takes the value $\delta = -1$ and e_H^c develops a VEV to break the $U(1)$ symmetries, the only possible value of c in the above equation is $1/2$. In the case where $\delta = 1$ and the breaking of the Abelian factors is achieved giving VEV to v_H^c, the value of c is zero (or better, can be chosen to be zero) leaving the standard assignment of PS model [11]. We are interested in the former case and we shall develop the RGE for gauge and Yukawa coupling running.

3. Setting the RGEs

Three different scales appear in our approach: the string scale M_U, the PS breaking scale M_R and the low energy scale M_Z. In principle, since the various groups leave in different stacks of branes, the corresponding gauge couplings may differ as well. However, in order not to lose predictability at the unification scale M_U, we require a 'petit' unification, namely $\alpha_4 = \alpha_R \neq \alpha_L$.[4] For further convenience we introduce the parameter $\xi = \alpha_L(M_U)/\alpha_4(M_U)$ (α_4, α_L and α_R correspond to

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[3] Note that the association of SM hypercharge operator with B-L and $SU(2)$ was first proposed in [16].
[4] The other two possibilities give either negative values of α_R at $\sim 10^{10}$ GeV (when $\alpha_4 = \alpha_L$) or a low unification scale ~ 7 GeV (when $\alpha_R = \alpha_L$) (see [11] for discussion).

the three groups of the model: $SU(4)$, $SU(2)_L$ and $SU(2)_R$). The value of $\alpha_{\mathcal{H}}$ at M_U is given by the following relation:

$$\frac{1}{\alpha_{\mathcal{H}}} = \frac{8}{\alpha_4} + \frac{4}{\alpha_R} + \frac{4}{\alpha_L}. \tag{6}$$

At M_R we have the following relations due to the PS group breaking:

$$\alpha_3 = \alpha_2, \qquad \alpha_2 = \alpha_L, \qquad \frac{1}{\alpha_Y} = \frac{2/3}{\alpha_4} + \frac{1}{\alpha_R} + \frac{c^2}{\alpha_{\mathcal{H}}} \tag{7}$$

where α_3, α_2 and α_Y correspond to the three groups of the SM.

As has been mentioned above, the parameter c can take two acceptable values. The value $c = 0$ corresponds to the standard definition of the hypercharge. Assuming *petit unification*, we find [11] $M_U \geq 10^{10}$ GeV. The $c = 1/2$ introduces a component of the extra $U(1)_{\mathcal{H}}$ in Y without affecting the SM charge assignment. This case allows the possibility of low unification in the TeV range. For the rest of the paper we will work with $c = 1/2$. Now for completeness we give the β-functions for all groups:

$M_U > M > M_R$

$$\beta_4 = -\frac{44}{3} + \frac{4}{3}n_g + \frac{1}{3}n_H + \frac{1}{3}n_D$$
$$\beta_L = -\frac{22}{3} + \frac{4}{3}n_g + \frac{1}{3}n_h$$
$$\beta_R = -\frac{22}{3} + \frac{4}{3}n_g + \frac{1}{3}n_h + \frac{2}{3}n_H + \frac{1}{6}n_{h_R}$$
$$\beta_{\mathcal{H}} = \frac{32}{3}n_H + 8n_D + \frac{4}{3}n_\eta + \frac{2}{3}n_{h_R} \tag{8}$$

$M_R > M > M_Z$

$$\beta_3 = -11 + \frac{4}{3}n_g + \frac{1}{6}n_{d_H^c} + \frac{1}{6}(n_{\tilde{d}^c} + n_{\tilde{d}})$$
$$\beta_2 = -\frac{22}{3} + \frac{4}{3}n_g + \frac{1}{6}(n_{H_u} + n_{H_d})$$
$$\beta_Y = \frac{20}{9}n_g + \frac{1}{9}n_{d_H^c} + \frac{1}{6}(n_{H_u} + n_{H_d}) + \frac{1}{9}(n_{\tilde{d}^c} + n_{\tilde{d}})$$

where n_g is the number of families ($n_g = 3$) while all other notation is in accordance with that of equations (1)–(3).

First we would like to set the range for the parameter $\xi = \alpha_L/\alpha_2$ in order to achieve a low energy M_U, while keeping $M_R < M_U$ as an upper limit and $M_R > 1$ TeV as a lower limit. We use the following low energy (M_Z) experimental values: $\sin^2 \theta_W = 0.231\,51$, $\alpha_{em} = 1/128.9$ and $\alpha_3 = 0.119 \pm 0.003$. Our particle content is the following:

$$n_g = 3, \qquad n_H = 1, \qquad n_D = 0, \qquad n_h = 1, \qquad n_\eta = 1, \qquad n_{h_R} = 0$$
$$n_{H_u} = n_{H_d} = 1, \qquad n_{d_H^c} = 0 \text{ or } 1, \qquad n_{\tilde{d}} = n_{\tilde{d}^c} = 0$$

and we use one-loop RGE equations.

In figure 1 we plot M_U and M_R versus ξ. The upper line for M_U and the lower line for M_R correspond to the highest acceptable value for α_3 (with the other lines corresponding to the lowest value). The maximum range for the gauge coupling ratio ξ at M_U is $\xi \sim (0.413, 0.445)$. At the lowest value both scales are of the order of 9.3 TeV while at the highest $M_U \sim 8$ TeV. In the case of absence of non-standard particles, the region of ξ is $(0.415, 0.445)$ and the corresponding values for the scales are 8.7 and 7.8 TeV. We have also checked that the gauge couplings stay well within the perturbative region.
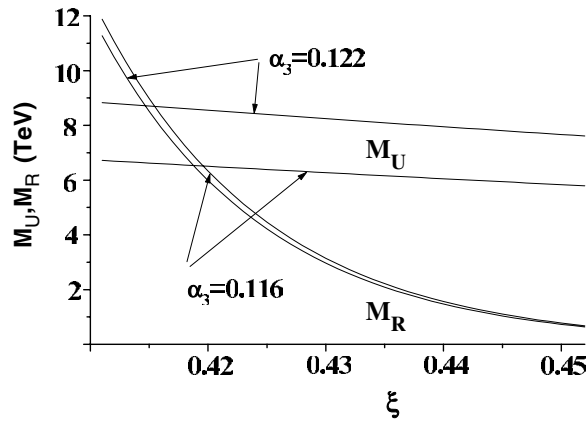
Figure 1. The scales M_U and M_R versus the parameter ξ. The requirements 1 TeV $< M_R < M_U$ sets the range for ξ. The particle content has $n_{d_H^c} = 0$ (see text).

We further observe that the M_R and M_U scales merge for the lower ξ values. Since consistency of the scale hierarchy demands $M_R \leq M_U$, this implies that there is a lower acceptable value of ξ or a higher M_U scale as figure 1 shows. On the other hand, experimental bounds on right-handed bosons imply $M_R \gtrsim 1$ TeV, this sets the upper bound on ξ or equivalently, the lower bound on M_U.

4. The supersymmetric model

In this section we repeat the above analysis for the supersymmetric version of the model, where we need the extra Higgs representation

$$H = (\mathbf{4}, \mathbf{1}, \mathbf{2}; 1, 0, \gamma) \to u_H(\mathbf{3}, \mathbf{1}, \tfrac{2}{3}) + d_H(\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{1}{3}) + e_H(\mathbf{1}, \mathbf{1}, -1) + \nu_H(\mathbf{1}, \mathbf{1}, 0). \tag{9}$$

The charge γ is not fully constrained (as opposed to the case of $\bar{H}$) and, in principle, can take two values $\gamma = \pm 1$. However, if supersymmetry is assumed, as the corresponding charge of the field $\bar{H}$ has been determined to $\delta = -1$, the value of γ should be fixed to $\gamma = 1$. Further, the following exotic representations could appear:

$$\begin{aligned}
&\bar{D}(\mathbf{6}, \mathbf{1}, \mathbf{1}; -2, 0, 0)\\
&h_L(\mathbf{1}, \mathbf{1}, \mathbf{2}; 0, 1, 0)\\
&\bar{h}_L(\mathbf{1}, \mathbf{1}, \mathbf{2}; 0, -1, 0)\\
&\bar{h}_R(\mathbf{1}, \mathbf{1}, \mathbf{2}; 0, 0, -1).
\end{aligned} \tag{10}$$

Keeping the same conditions as in the non-supersymmetric case, equations (6), (7) and fixing again the value of c to $1/2$, we plot M_U and M_R versus ξ in figure 2. The content is the minimum possible, i.e.

$$\begin{aligned}
&n_g = 3, & &n_H = 1, & &n_{\bar{H}} = 1, & &n_h = 1, & &n_D = 0,\\
&n_\eta = 0, & &n_{h_L} = 0 & &n_{h_R} = 0\\
&n_{H_u} = n_{H_d} = 1, & &n_{d_H^c} = 0, & &n_{\tilde{d}} = n_{\tilde{d}^c} = 0.
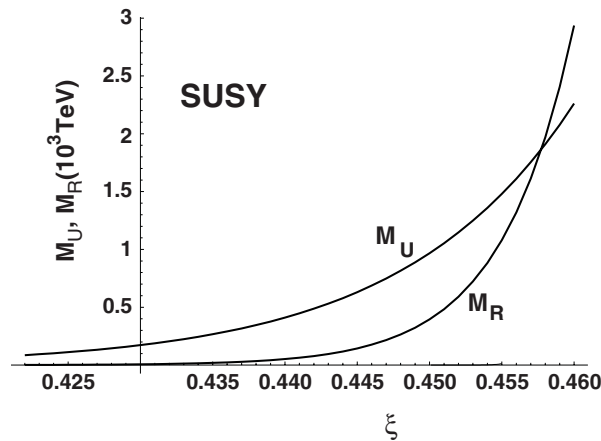\end{aligned}$$

Figure 2. The scales M_U and M_R versus the parameter ξ for the supersymmetric model, ($\gamma = 1$, $\delta = -1$). The requirements for the scales are as in figure 1. The particle content is the minimum one (see text).
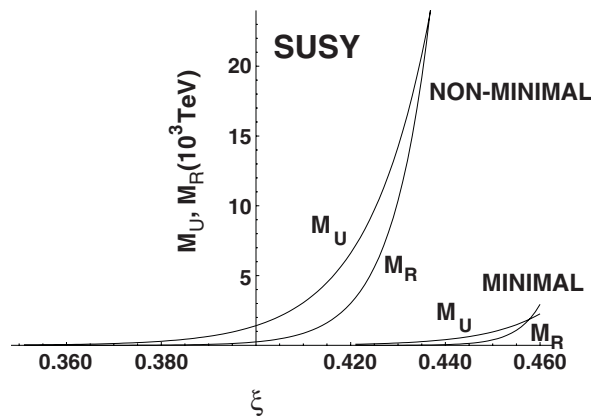

Figure 3. The scales M_U and M_R versus the parameter ξ for the supersymmetric model for the minimal and a non-minimal content.

We observe that, in contrast to the non-supersymmetric case examined in the previous section, here the limiting case $M_R = M_U$ is realized at the highest ξ value, while the lower ξ is correlated to the lower acceptable M_R value ($\sim$1 TeV). The energy scale of M_U and M_R now is three orders of magnitude higher than the corresponding non-supersymmetric case.

In figure 3 we show the same graph for the minimal and a non-minimal content for the supersymmetric case ($\gamma = 1$ and $\delta = -1$). The non-minimal content drives the ξ parameter to lower values but expands the acceptable region of the scales by almost one order of magnitude.

5. Yukawa coupling running for top and bottom

In the PS model with the minimal Higgs content, the Yukawa couplings for the top and the bottom quarks are equal at M_R, i.e. $h_t = h_b$. In this section we check whether such a constraint is compatible with the bottom and top quark masses as they are measured by the experiments.
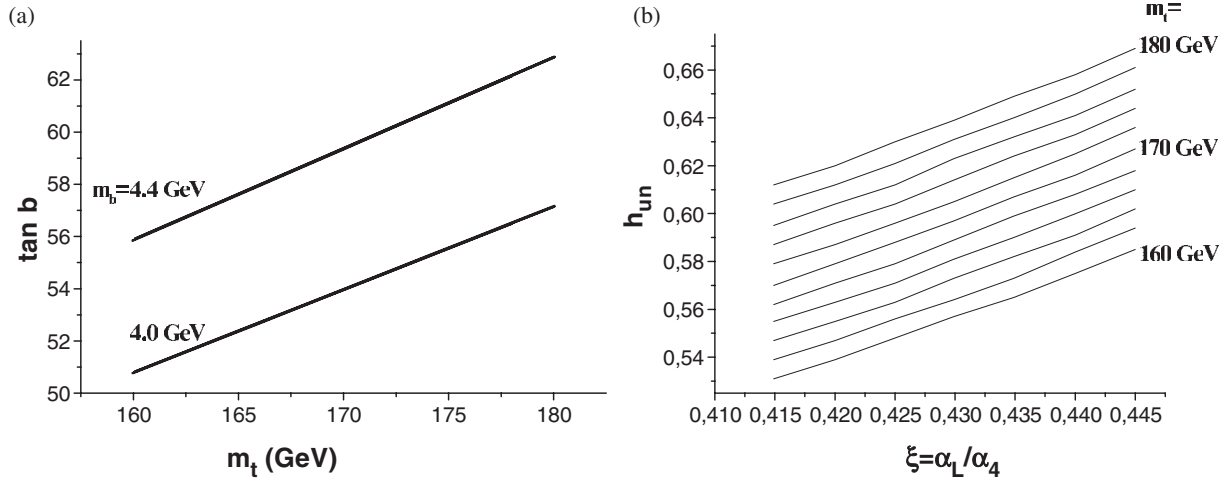
(a)

(b)

Figure 4. (a) The top mass versus $\tan b$ giving m_b in the experimental range (4.0–4.4) GeV and (b) the parameter $\xi = \alpha_L/\alpha_4$ versus $h(M_R)$ for several values of m_t.

If v_1 and v_2 are the two VEVs that correspond to H_d and H_u, we have of course

$$m_t(m_t) = h_t(m_t)v_2, \qquad m_b(m_b) = h_b(m_t)v_1\eta$$

where the factor $\eta = 1.4$ takes care for the QCD renormalization effects from the scale m_t down to the mass of the bottom quark. Since we have two VEVs (although we do not have supersymmetry), the relation with M_Z is

$$M_Z = \tfrac{1}{2}\sqrt{g_2^2 + g_Y^2}\sqrt{v_1^2 + v_2^2} = \tfrac{1}{2}\sqrt{g_2^2 + g_Y^2}\,v$$

while we insert, as usual, the parameter $\tan \beta = v_2/v_1$. The RGE for the two couplings are

$$
\begin{aligned}
16\pi^2\frac{\mathrm{d}h_t}{\mathrm{d}t} &= h_t\left[\frac{3}{2}h_t^2 - \frac{3}{2}h_b^2 - 4\pi\left(\frac{17}{12}\alpha_Y + \frac{9}{4}\alpha_2 + 8\alpha_3\right)\right] \\
16\pi^2\frac{\mathrm{d}h_b}{\mathrm{d}t} &= h_b\left[\frac{3}{2}h_b^2 - \frac{3}{2}h_t^2 - 4\pi\left(\frac{5}{12}\alpha_Y + \frac{9}{4}\alpha_2 + 8\alpha_3\right)\right]
\end{aligned}
\tag{11}
$$

where we have ignored all other Yukawa couplings. We run the equations from M_R down to scale M where $h_t(M)v_2 = M$, which is the top mass m_t.

In figure 4(a) we plot m_t versus $\tan \beta$ in order to have m_b in the acceptable experimental region (4.0–4.4) GeV. The choice of ξ (in the acceptable region defined above) makes a very small effect which shows itself in the thickness of the lines. Since we require unification of the two Yukawa couplings at M_R, the large difference in the mass of the two quarks can only be provided by a large angle, therefore the large values of $\tan \beta$ were expected. Moreover, being in the large $\tan \beta$ regime, m_t changes by a negligible amount as $\tan \beta$ changes to comply with the upper and lower limits of the bottom mass (remember that $v_2 = v \sin b$ while $v_1 = v \cos b$).

The form of equation (11) also shows that the two couplings run almost 'parallel' to each other and actually the main contribution to the running comes from the gauge couplings (as we can see in the next figure, the value of the Yukawas at M_R are small). The corresponding figure with $n_{d_H^c} = 2$ does not show any significant difference.
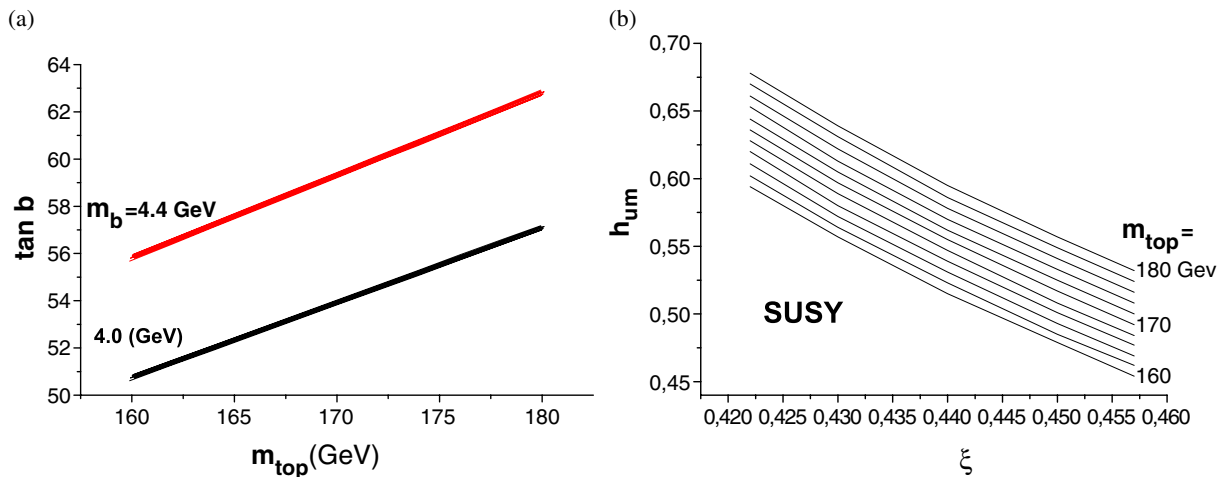
Figure 5. The SUSY case: (a) the top mass versus $\tan\beta$ giving m_b in the experimental range (4.0–4.4) GeV and (b) the parameter $\xi = \alpha_L/\alpha_4$ versus $h(M_R)$.

In figure 4(b) we plot the parameter ξ versus the unified value of the Yukawa coupling at M_R, for different values of m_t. The dependence is almost linear with higher value of m_t requiring higher values of the unified Yukawa coupling h. The absolute value of the Yukawa coupling justifies our previous claim that the running of h_b and h_t is governed by the gauge coupling contributions to the RGE equations.

The last figure, figure 5, corresponds to the supersymmetric case. The $\tan\beta$ versus m_t figure does not show any significant difference from the corresponding non-supersymmetric case. In contrast, the $h(M_R)$ versus ξ is different. Lower ξ values correspond to higher $h(M_R)$ ones while the range of the acceptable $h(M_R)$ values is a bit broader.

6. Conclusions

In this work we have examined the gauge and bottom–top Yukawa coupling evolution in models with PS symmetry obtained in the context of brane scenarios. In the case of 'petit' unification of gauge couplings, i.e. $a_4 = a_R \neq a_L$, it turns out that in the non-supersymmetric version of the above model one may have a string scale at a few TeV. Further, assuming $h_b - h_t$ Yukawa unification at the string scale, one finds that the correct $m_{b,t}$ quark masses are obtained for a a_4 approximately twice as big as a_L. A similar analysis for the supersymmetric case shows that the string scale raises up to 10^7 TeV while h_b–h_t unification also reproduces the right mass relations m_b, m_t.

Acknowledgments

References

[1] Antoniadis I 1990 *Phys. Lett.* B **246** 377
[2] Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 *Phys. Lett.* B **429** 263 (*Preprint* hep-ph/9803315)

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 [3] Antoniadis I, Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 *Phys. Lett.* B **436** 257 (*Preprint* hep-ph/9804398)
 [4] Lykken J D 1996 *Phys. Rev.* D **54** 3693 (*Preprint* hep-th/9603133)
 [5] Berkooz M, Douglas M R and Leigh R G 1996 *Nucl. Phys.* B **480** 265 (*Preprint* hep-th/9606139)
 [6] Balasubramanian V and Leigh R G 1997 *Phys. Rev.* D **55** 6415 (*Preprint* hep-th/9611165)
 [7] Aldazabal G, Ibanez L E and Quevedo F 2000 *J. High Energy Phys.* JHEP02(2000)015 (*Preprint* hep-ph/0001083)
 [8] Antoniadis I, Kiritsis E and Tomaras T 2001 *Fortschr. Phys.* **49** 573 (*Preprint* hep-th/0111269)
 [9] Antoniadis I, Kiritsis E and Tomaras T N 2000 *Phys. Lett.* B **486** 186 (*Preprint* hep-ph/0004214)
[10] Antoniadis I, Kiritsis E, Rizos J and Tomaras T N 2002 *Preprint* hep-th/0210263
[11] Leontaris G K and Rizos J 2001 *Phys. Lett.* B **510** 295 (*Preprint* hep-ph/0012255)
[12] Kokorelis C 2002 *J. High Energy Phys.* JHEP08(2002)036 (*Preprint* hep-th/0206108)
[13] Antoniadis I, Kiritsis E and Rizos J 2002 *Nucl. Phys.* B **637** 92 (*Preprint* hep-th/0204153)
[14] Chang D, Mohapatra R N and Parida M K 1984 *Phys. Rev. Lett.* **52** 1072
     Chang D, Mohapatra R N, Gipson J, Marshak R E and Parida M K 1985 *Phys. Rev.* D **31** 1718
[15] Pati J C and Salam A 1974 *Phys. Rev.* D **10** 275
[16] Mohapatra R N and Marshak R E 1980 *Phys. Lett.* B **91** 222