LEADING LOG SUMMATION OF GLUONIC CORRECTIONS TO WEAK QUARK DECAYS

N.D. TRACAS
Theoretical Physics Group, National Technical University, Zografou, Athens 624, Greece

and

N.D. VLACHOS [1]
Department of Physics, University of Athens, Athens 621, Greece

Received 4 July 1983

We sum the leading gluon corrections to the $\Delta S = 1$ weak quark decays. The results are consistent with the operator product expansion approach.

Phenomena dominated by large invariants can in several cases be treated by means of the operator product expansion (OPE). Properties depending on these invariants can then be factorized into multiplicative constants (Wilson coefficients), which are calculable if the underlying theory posesses the property of asymptotic freedom.

There are cases however where OPE cannot be applied directly because of kinematic restrictions. For this reason, a diagrammatic approach, even in cases where OPE has been found to work, is useful not only for providing an independent test, but also for helping an intuitive understanding of the underlying physics. The $\Delta S = 1$ weak quark decays through charged W exchange is a process which can, qualitatively at least, be understood within the OPE framework, where the Wilson coefficients are found to be of the form $(\ln M_W^2/\mu^2)^d$ where M_W is the W mass and μ a typical QCD momentum scale [1].

It is the purpose of this work to show the equivalence of the two approaches. The process is being examined in perturbation theory. The leading logarithms in the heavy mass M_W, which are of the form $g^{2n}\ln^n M_W^2/p^2$, are identified and summed by means of a recursion equation of the Bethe–Salpeter type.

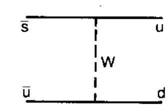


Fig. 1. Lowest order graph to $\Delta S = 1$ weak quark decays.

The method used is quite general and can be easily extended to include all similar processes.

The quark multiplets will be considered to be elements of an unbroken SU(4) flavour group where the underlying colour group will be SU(3) as usual.

The lowest order graph is shown in fig. 1 and corresponds to a colour singlet amplitude

$$M_0 = -M_W^{-2}\bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)d\,. \qquad (1)$$

Addition of one gluon will produce the graphs of fig. 2 which by means of standard methods are found to give the leading log approximation (LLA) result

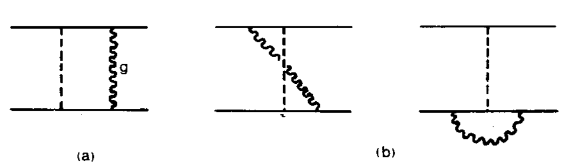


Fig. 2. g^2-order correction to the lowest order graph.

[1] Supported in part by the Hellenic National Research Foundation.

0.370-2693/84/$ 03.00 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

$$M_1 = -6M_W^{-2}\bar{s}\gamma_\mu(1-\gamma_5)t^a\bar{u}\gamma^\mu(1-\gamma_5)t^a d \times g^2 \ln(M_W^2/p^2)\,, \tag{2}$$

where p^2 is the quark virtuality and t^a the usual SU(3) colour matrices.

It is interesting to point out that in Landau gauge the graphs (b) in fig. 2 do not contribute and eq. (2) will come out from graph (a) alone.

The same result is obtained by shrinking the W line to a point. The resulting reduced graph is then superficially divergent and has to be calculated by means of an effective cut-off M_W.

Addition of extra gluons will not modify the chiral structure (V – A) X (V – A) of eq. (2) and the general form of the amplitude will be a linear combination of the available colour singlets

$$\Gamma_{O_1} = \bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)d\,, \tag{3}$$

$$\Gamma_{O_2} = \bar{s}\gamma_\mu(1-\gamma_5)t^a u\bar{u}\gamma^\mu(1-\gamma_5)t^a d\,, \tag{4}$$

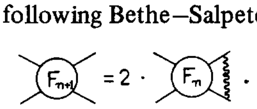
namely

$$M \sim M_W^{-2}\,[F^1(p^2,M_W^2,\mu^2)\Gamma_{O_1} + F^2(p^2,M_W^2,\mu^2)\Gamma_{O_2}]\,. \tag{5}$$

The trick of shrinking the W line can actually be used to all orders in the skeleton expansion. It is clear that internal momenta larger than M_W are not important and therefore we can use M_W as a UV cutoff for all graphs. In this region the W propagator can be shrunk to a point and all graphs will acquire a superficial logarithmic divergence by power counting. The dominant graphs are then identified as those which, order by order, acquire the highest degree of UV singularity [2].

The above statement will enforce the ladder structure of the dominant graphs. Moreover, the use of Landau gauge will force all gluon lines to be parallel to the W line.

Having established the ladder structure of the dominant skeleton graphs we now proceed to the summation of the ladders. This can be accomplished by means of the following Bethe–Salpeter equation



A short calculation leads to the following simple matrix equation

$$\begin{pmatrix} F_{n+1}^1 \\ F_{n+1}^2 \end{pmatrix} = g^2 \begin{pmatrix} 0 & -\frac{4}{3} \\ -6 & 2 \end{pmatrix} \begin{pmatrix} G(F_n^1) \\ G(F_n^2) \end{pmatrix}, \tag{6}$$

where

$$G(F_n^i) = \int_{p^2}^{M_W^2} \frac{dk^2}{k^2} F_n^i(k^2)\,, \quad i = 1,2\,.$$

This expression can be brought into canonical form by means of the new vector

$$\tilde{F}_n^1 = \tfrac{1}{3}F_n^1 - \tfrac{2}{9}F_n^2\,, \tag{7}$$

$$\tilde{F}_n^2 = \tfrac{1}{3}F_n^1 + \tfrac{1}{9}F_n^2\,, \tag{8}$$

and so we get

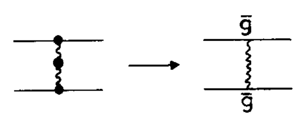
$$\tilde{F}_{n+1}^1 = 4g^2 G(\tilde{F}_n^1)\,, \tag{9}$$

$$\tilde{F}_{n+1}^2 = -2g^2 G(\tilde{F}_n^2)\,. \tag{10}$$

We now turn to the problem of "dressing" the ladders by adding renormalization parts.

Note that we do not have to dress the fermion propagators and the W vertex, because the use of the Landau gauge forbids the appearance of leading logarithms from these graphs.

The dressing is accomplished by the use of the running coupling constant at the vertices according to the well known recipe [3]



where the running coupling constant is given by

$$\bar{g}^2 = 1/\beta_0 \ln(k^2/\Lambda^2)\,,$$

and Λ is the well-known QCD invariant [4], $\beta_0 = \frac{25}{3} > 0$. Our recursion relations now become

$$\tilde{F}_{n+1}^1 = \frac{4}{\beta_0}\int_{p^2}^{M_W^2} dk^2\, \frac{\tilde{F}_n^1(k^2)}{k^2 \ln(k^2/\Lambda^2)}\,, \tag{11}$$

$$\tilde{F}_{n+1}^2 = -\frac{2}{\beta_0}\int_{p^2}^{M_W^2} dk^2\, \frac{\tilde{F}_n^2(k^2)}{k^2 \ln(k^2/\Lambda^2)}\,. \tag{12}$$

The above equations can be very easily solved by substituting as an input

$F_0^1 = 1\,, \quad F_0^2 = 0 \quad \text{or} \quad \tilde{F}_0^1 = \tilde{F}_0^2 = 1/3\,.$

We get the following results

$$\tilde{F}_n^1 = \frac{1}{3}\,\frac{(4/\beta_0)^n}{n!}\left(\int_{p^2}^{M_W^2} \frac{\mathrm{d}k^2}{k^2 \ln(k^2/\Lambda^2)}\right)^n\,, \tag{13}$$

$$\tilde{F}_n^2 = \frac{1}{3}\,\frac{(-2/\beta_0)^n}{n!}\left(\int_{p^2}^{M_W^2} \frac{\mathrm{d}k^2}{k^2 \ln(k^2/\Lambda^2)}\right)^n\,, \tag{14}$$

or

$$\tilde{F}_1 = \tfrac{1}{3}\,[\ln(p^2/\Lambda^2)]^{-4/\beta_0}\,[\ln(M_W^2/\Lambda^2)]^{4/\beta_0}\,, \tag{15}$$

$$\tilde{F}_2 = \tfrac{1}{3}\,[\ln(p^2/\Lambda^2)]^{2/\beta_0}\,[\ln(M_W^2/\Lambda^2)]^{-2/\beta_0}\,, \tag{16}$$

and the amplitude becomes

$$M = M_W^{-2}\,[\tfrac{1}{3}(\Gamma_{O_1} - 3\Gamma_{O_2})\tilde{F}_1 + \tfrac{2}{3}(\Gamma_{O_1} + \tfrac{3}{2}\Gamma_{O_2})\tilde{F}_2]\,. \tag{17}$$

The above expression represents exactly the results obtained by means of the OPE; i.e. the summation of the leading logarithms of perturbation theory reproduces correctly the well-known anomalous dimensions [1].

In a broken SU(4) flavour group, where $m_c \neq 0$, graphs of the "penguin" type should also be taken into account [5]. The problem now involves three momentum scales (p, M_W, m_c) and the resulting OPE effective hamiltonian is quite complicated [6]. The equivalence between leading log summation and OPE has, to the best of our knowledge, been checked so far only up to g^4 [7]. We are still working on the subject.

The final question to be posed, is whether the equivalence of the two approaches will persist at the next to leading logarithmic level. This remains to be seen.

References

[1] K. Wilson, Phys. Rev. 179 (1969) 1499;
M. Gaillard and B.W. Lee, Phys. Rev. Lett. 33 (1974) 108;
G. Altarelli and L. Maiani, Phys. Lett. 52B (1974) 3;
N.D. Tracas and N.D. Vlachos, Phys. Lett. 115B (1982) 419;
G. Altarelli et al., Nucl. Phys. B187 (1981) 461.

[2] N.D. Tracas and N.D. Vlachos, National Technical University, Athens, preprint (1983).

[3] A.B. Carter and C.H. Llewellyn Smith, Nucl. Phys. B162 (1980) 397.

[4] R. Cocqueraux, Ann. Phys. 125 (1980) 401.

[5] M.B. Wise and E. Witten, Phys. Rev. D20 (1979) 1216;
C.T. Hill and G.G. Ross, Phys. Lett. 94B (1980) 234.

[6] M.A. Shifman et al., Nucl. Phys. B120 (1977) 316; Sov. Phys. JETP 45 (1977) 670.

[7] N.D. Vlachos, Ph.D. Thesis, University of Sussex (1981), unpublished;
V.M. Khatsymovsky, Novosibirsk preprint 82-95.