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Modular weights, $U(1)$'s and mass matrices [1]

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Abstract

We derive the scalar mass matrices in effective supergravity models with the standard gauge group augmented by a $U(1)_F$ family symmetry. Simple relations between $U(1)_F$ charges and modular weights of the superfields are derived and used to express the matrices with a minimum number of parameters. The model predicts a branching ratio for the $\mu \to e\gamma$ process close to the present experimental limits. © 1998 Elsevier Science B.V.

The Minimal Supersymmetric Standard Model (MSSM) emerges as the most natural extension of the Standard Model (SM) in the context of the unification of all interactions. Although supersymmetric models solve the hierarchy problem, the plethora of arbitrary parameters requires a further step beyond the MSSM. The $N=1$ supergravity coupled to matter stands promising [1]. Yet, there are many essential parameters (Yukawa couplings, content of the chiral multiplets, etc.) to be chosen by the model builder. In this scene, string theory appears the only known candidate theory that can in principle predict all the required parameters. String theory puts rather strong constraints on many of the parameters of the resulting $N=1$ effective supergravity, which appears as its low-energy limit. Thus, the kinetic terms must have a certain structure, the Lagrangian should obey the string duality symmetries, while several constraints are imposed on the superpotential and the Yukawa couplings [2].

The subject of this letter is to reproduce the observed family hierarchy of the fermion masses and moreover to predict the corresponding mass matrices in the scalar supersymmetric sector. This is done in the context of residual stringy $U(1)$ symmetries [3] left from the large gauge group at a high scale. In particular, combining modular invariance constraints and $U(1)$ invariance of the superpotential, the scalar mass matrices are given in terms of powers of an expansion parameter $\langle\theta\rangle/M$, where $\langle\theta\rangle$ is the vacuum expectation value of a singlet field and M is a high (string) scale. These powers are written in terms of modular weight differences. Further, the consequences in the lepton flavour non-conserving reaction $\mu \to e\gamma$ are examined. Its branching ratio is found close to the present experimental limits.

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We start with a quick review of the $N=1$ supergravity, which introduces a real gauge-invariant Kähler function with the general form [4]

$$G(z,\bar{z}) = \mathscr{K}(z,\bar{z}) + \log|\mathscr{W}(z)|^2 \tag{1}$$

where $\mathscr{K}(z,\bar{z})$ is the Kähler potential and $\mathscr{W}(z)$ is the superpotential. Denoting (Φ,Q) by z, where Φ stands for the dilaton field S and other moduli T_i, while Q represents the chiral superfields, the Kähler potential at tree level can be written as follows

$$K\left(\Phi,\overline{\Phi},Q,\overline{Q}\right) = -\log(S+\bar{S}) - \Sigma_n h_n \log\left(T_n + \overline{T}_n\right) + Z_{\bar{i}j}(\Phi,\overline{\Phi})\overline{Q}_{\bar{i}} e^{2V} Q_j + \cdots \tag{2}$$

The superpotential $\mathscr{W}(z)$ is a holomorphic function of the chiral superfields Q_i and at the tree level is given by

$$\mathscr{W}(\Phi,Q) = \frac{1}{3}\lambda_{ijk}(\Phi)Q_iQ_jQ_k + \frac{1}{2}\mu_{ij}(\Phi)Q_iQ_j + \cdots \tag{3}$$

In both (2) and (3), $\cdots$ stand for possible non-renormalizable contributions. Terms bilinear in the fields Q_i refer in fact to an effective Higgs mixing term.

Now under the modular symmetries, the moduli transform as $T \to (aT - \imath b)/(\imath cT + d)$, where a,b,c,d constitute the entries of the $SL(2,\mathbf{Z})$ group elements with $a,b,c,d \in \mathbf{Z}$ and $ad-bc=1$. These imply the following transformation rules [5]

$$Q_i \to Q_i \prod_k t_k^{n_i^k}, \quad Z_{i\bar{j}} \to Z_{i\bar{j}} \prod_k (t_k)^{-n_i^k}(\bar{t}_k)^{-n_j^k},$$
$$\mathscr{W} \to \prod_k t_k^{n_W^k}\mathscr{W}, \tag{4}$$

where we have introduced the notation $t_k = \imath c_k T_k + d_k$. The exponent n_i^k is the modular weight of Q_i with respect to the modulus T_k.

Let us now introduce into the Kähler function non-renormalizable terms through two fields, θ and $\bar{\theta}$, which are singlets under the low energy standard gauge group, while they carry charges $q_\theta = -q_{\bar{\theta}}$ under the $U(1)_F$ family group. The lower order (in Q_i's) non-renormalizable terms can be written in the form

$$K_{r_{i\bar{j}}}^{i\bar{j}}\left(\frac{\langle\theta\rangle}{M_1}\right)^{r_{i\bar{j}}} Q_i\overline{Q}_{\bar{j}} + K_{\tilde{r}_{i\bar{j}}}^{i\bar{j}}\left(\frac{\langle\bar{\theta}\rangle}{M_2}\right)^{\tilde{r}_{i\bar{j}}} Q_i\overline{Q}_{\bar{j}}. \tag{5}$$

These terms should be invariant under the $U(1)_F$ symmetry. Assigning $U(1)_F$ charges q_i for the matter fields one gets

$$q_i + q_j + q_\theta r_{ij} = 0, \quad q_i + q_j + q_{\bar{\theta}}\tilde{r}_{ij} = 0. \tag{6}$$

Similar non-renormalizable terms could also appear in the superpotential.

After this short review we come to the mass matrix textures. The $SU(2)_L$ invariance, together with the requirement to have symmetric mass matrices, leads us to assign the same $U(1)_F$ charge to all quark members of the same family q_i, while the same should be applied to the leptons of the same family l_i. The full anomaly-free Abelian group involves an additional family-independent component and with this freedom we may make $U(1)_F$ traceless without any loss of generality. Thus $q_1+q_2+q_3=0$ and $l_1+l_2+l_3=0$.

If the light Higgs H_2, responsible for the masses of the up quarks, and H_1, responsible for the down quarks and leptons have $U(1)_F$ charge, so that only the (3,3) renormalizable Yukawa couplings to H_2 and H_1 are allowed, namely

$$2q_3 + h_2 = 0, \quad \text{and} \quad 2l_3 + h_1 = 0, \tag{7}$$

only the (3,3) element of the associated mass matrix will be non-zero. The remaining entries are generated when the $U(1)_F$ symmetry is broken. A straightforward consequence of this fact is the equality of the two Higgs $U(1)_F$ charges $(h_1 = h_2)$, since H_1 provides also the mass to the bottom quark while we have assumed equal $U(1)_F$ charges within a family.

A general non-renormalizable relevant term in the superpotential is of the form

$$Y_{ij}Q_iu_j^cH_2\left(\frac{\theta}{M}\right)^{x_{ij}}. \tag{8}$$

Owing to $U(1)_F$ invariance of the superpotential, we have the constraint

$$q_i + q_j + h_2 + x_{ij} q_\theta = 0 \tag{9}$$

and similarly for the parameter $\bar{\theta}$ terms. The allowed powers of non-renormalizable terms in each entry are determined by the charges $(q_\theta = -q_{\bar{\theta}})$

$$x_{ij} = \left|\frac{q_2 - q_3}{q_\theta}\right| \begin{pmatrix} 2|a+1| & |a| & |a+1| \\ |a| & 2 & 1 \\ |a+1| & 1 & 0 \end{pmatrix} \tag{10}$$

where $a = 3q_3/(q_2 - q_3)$, and we have used the condition (7). Suppressing unknown Yukawa couplings Y_{ij} and their phases, which are all expected to be of order 1, we arrive at the following mass matrices

$$m_U \approx \begin{pmatrix} \epsilon^{2|a+1|} & \epsilon^{|a|} & \epsilon^{|a+1|} \\ \epsilon^{|a|} & \epsilon^2 & \epsilon \\ \epsilon^{|a+1|} & \epsilon & 1 \end{pmatrix} m_t \tag{11}$$

$$m_D \approx \begin{pmatrix} \tilde{\epsilon}^{2|a+1|} & \tilde{\epsilon}^{|a|} & \tilde{\epsilon}^{|a+1|} \\ \tilde{\epsilon}^{|a|} & \tilde{\epsilon}^2 & \tilde{\epsilon} \\ \tilde{\epsilon}^{|a+1|} & \tilde{\epsilon} & 1 \end{pmatrix} m_b \tag{12}$$

where $\tilde{\epsilon} = (\frac{\langle\theta\rangle}{M_1})^{|(q_2 - q_3)/q_\theta|}$, $\epsilon = (\frac{\langle\theta\rangle}{M_2})^{|(q_2 - q_3)/q_\theta|}$ (M_1 and M_2 being two high scales). The charged lepton mass matrix may similarly be determined. The equality $h_1 = h_2$, together with (7) has also the consequence $q_3 = l_3$, which implies the successful relation $m_b = m_\tau$ at unification. We then get

$$m_L \approx \begin{pmatrix} \tilde{\epsilon}^{2|a+b|} & \tilde{\epsilon}^{|a|} & \tilde{\epsilon}^{|a+b|} \\ \tilde{\epsilon}^{|a|} & \tilde{\epsilon}^{2|b|} & \tilde{\epsilon}^{|b|} \\ \tilde{\epsilon}^{|a+b|} & \tilde{\epsilon}^{|b|} & 1 \end{pmatrix} m_\tau \tag{13}$$

where $b = (l_2 - q_3)/(q_2 - q_3)$.

The powers of the above matrices can be written in terms of the modular weights as follows. As we have already discussed in the introduction, the superpotential transforms covariantly under the modular symmetry. Let us denote by $n_{Q_i}, n_{u_i}, n_{d_i}, n_{h_2}, n_\theta$ the modular weights for the corresponding fields with respect to a certain modulus. For the non-renormalizable term of the form (8), the modular weights obey the equation $n_{Q_i} + n_{u_j} + n_{h_2} + x_{ij} n_\theta = n_{\mathscr{W}}$. Combining this relation with the $U(1)_F$ invariance and the fact that $\sum_{i=1}^3 q_i = 0$, we obtain the general formula

$$q_j = \frac{q_\theta}{3n_\theta} \sum_i n_{Q_{ji}} = \frac{q_\theta}{3n_\theta} \sum_i n_{u_{ji}} = \frac{q_\theta}{3n_\theta} \sum_i n_{d_{ji}} \tag{14}$$

where $n_{Q_{ji}} = n_{Q_j} - n_{Q_i}$ and correspondingly for $n_{u_{ji}}$ and $n_{d_{ji}}$. The third equality comes from the down-quark mass matrix non renormalizable contributions corresponding to a term like (8). Similar relations hold for the lepton modular weights. Using the above relation, we may obtain an elegant form of the matrix (10), which expresses the powers of the allowed non-renormalizable entries only in terms of modular weight differences [6]. We obtain

$$x_{ij} = \frac{1}{n_\theta} \begin{pmatrix} 2n_{Q_{31}} & n_{Q_{31}} + n_{Q_{32}} & n_{Q_{31}} \\ n_{Q_{31}} + n_{Q_{32}} & 2n_{Q_{32}} & n_{Q_{32}} \\ n_{Q31} & n_{Q32} & 0 \end{pmatrix} \tag{15}$$

The positivity of the entries requires the conditions $n_{Q_{31}} n_\theta > 0$ and $n_{Q_{32}} n_\theta > 0$. We can also express the powers of the matrix (12) in terms of modular weight difference. This is easily done by expressing the parameter a in the form

$$a = \frac{n_{Q_{13}} + n_{Q_{23}}}{n_{Q_{23}}}. \tag{16}$$

From (15) we conclude that the hierarchical fermion mass spectrum requires all three n_{Q_i}'s to be different. Models with equal n_{Q_i}'s, but different q_i's (necessary to create hierarchy), require $q_\theta = 0$. In this case the $U(1)_F$ charges are not related to the modular weights and the constraint (14) does not hold.

We next turn to the lepton fermion mass matrix. The phenomenological constraint $l_3 = q_3$ imposes the following relation on the modular weights of the quark and lepton generations

$$n_{L_{13}} - n_{Q_{13}} = n_{Q_{23}} - n_{L_{23}} \equiv \delta. \tag{17}$$

As a result, the $U(1)_F$ structure permits to express the powers y_{ij} of the lepton term

$$L_i e_j H_1 \left(\frac{\overset{(-)}{\theta}}{M} \right)^{y_{ij}} \tag{18}$$

by the following matrix

$$y_{ij} = \frac{1}{n_\theta} \begin{pmatrix} 2(n_{Q13} + \delta) & n_{Q_{13}} + n_{Q_{23}} & n_{Q_{13}} + \delta \\ n_{Q_{13}} + n_{Q_{23}} & 2(n_{Q_{23}} - \delta) & n_{Q_{23}} - \delta \\ n_{Q_{13}} + \delta & n_{Q_{23}} - \delta & 0 \end{pmatrix} \tag{19}$$

whilst the corresponding constraints for the positivity of the entries are $n_{L_{13}} n_\theta > 0$ and $n_{L_{23}} n_\theta > 0$. The powers of the matrix (13) can also be expressed in the same way by writing b in the form

$$b = \frac{n_{Q_{23}} - \delta}{n_{Q_{23}}}. \tag{20}$$

We now turn to the scalar part. At the tree level the scalar mass matrices receive contributions only along the diagonal, since terms of the form $\propto Q_i Q_i^*$ have zero $U(1)_F$ charge. Using powers of the fields $\theta, \bar{\theta}$ scaled by the M, we may fill in the remaining entries. It can easily be seen that the allowed $U(1)_F$ structure of the powers in the scalar mass term is

$$\frac{1}{n_\theta}\begin{pmatrix} 0 & n_{Q_{12}} & n_{Q_{13}} \\ n_{Q_{12}} & 0 & n_{Q_{23}} \\ n_{Q_{13}} & n_{Q_{23}} & 0 \end{pmatrix} \tag{21}$$

Thus, the powers of the parameters $\langle \theta \rangle, \langle \bar{\theta} \rangle$ are simply determined by the differences $n_{Q_{ij}}$ for the squark matrix and similarly for the sleptons (remember that since the $U(1)_F$ charge is the same within a family, (14) tells us that $n_{Q_{ij}} = n_{u_{ij}} = n_{d_{ij}}$).

Using again the parameters a and b entered in the fermion mass matrices, we can express the squark mass matrix in the form

$$m^2_{\tilde{Q}} \approx \begin{pmatrix} 1 & \epsilon^{|a+6|} & \epsilon^{|a+1|} \\ \epsilon^{|a+6|} & 1 & \epsilon \\ \epsilon^{|a+1|} & \epsilon & 1 \end{pmatrix} m^2_{3/2} \tag{22}$$

where $m_{3/2}$ is the gravitino mass. Similarly, for the sleptons we obtain

$$m^2_{\tilde{l}_{L,R}} \approx \begin{pmatrix} 1 & \tilde{\epsilon}^{|a+6b|} & \tilde{\epsilon}^{|a+b|} \\ \tilde{\epsilon}^{|a+6b|} & 1 & \tilde{\epsilon}^{|b|} \\ \tilde{\epsilon}^{|a+b|} & \tilde{\epsilon}^{|b|} & 1 \end{pmatrix} m^2_{3/2} \tag{23}$$

Obviously in the case of $b = 1$ the two matrices are identical since this case corresponds to equal $U(1)_F$ charges in the quark and the leptonic sector, $l_i = q_i$. In fact, it can be checked that the phenomenological analysis of the fermion mass spectrum allows two values of b, namely $b = 1$ or $1/2$ [7], while $\epsilon \approx 0.053$ and $\tilde{\epsilon} \approx 0.23$.

The above results show that $U(1)_F$ symmetries necessarily lead to low energy models where the Yukawa and its corresponding scalar mass matrices are not simultaneously diagonalized. As a result, flavour violation is possible and in general one should check whether such models can pass also the flavour violation tests. One of the most popular flavour non-conserving processes is the $\mu \to e\gamma$ decay. We have calculated the branching ratio for this process in order to compare it with the present experimental limits. This calculation requires the diagonalization of the 6×6 scalar mass matrix

$$\tilde{M}^2_l = \begin{pmatrix} m^2_{\tilde{l}_L} & A_l \langle H_1 \rangle + m_L \mu \tan\beta \\ (A_l \langle H_1 \rangle + m_L \mu \tan\beta)^\dagger & m^2_{\tilde{l}_R} \end{pmatrix} \tag{24}$$

Here, as usual, A_l is the trilinear parameter entering the scalar potential, μ is the Higgs mixing term and $\tan\beta$ is the Higgs vev ratio. Since lepton mass matrices are symmetric, left and right diagonalizing matrices coincide. Further, due to the properties of the $U(1)_F$ symmetry of the model, left ($m^2_{\tilde{l}_L}$) and right ($m^2_{\tilde{l}_R}$) scalar mass matrices are the same. Moreover, here we restrict on the case of small $\tan\beta$ regime, where the chirality changing diagrams are suppressed. In the general case, of course, and in the large $\tan\beta$ scenario, they become important. We have considered contributions from one loop graphs involving neutralino-charged slepton or chargino-sneutrino states in the loop. The diagrams of this process are shown in Fig. 1. We have diagonalized the lepton and the slepton mass matrices and found the corresponding amplitude for each neutralino/chargino graph. Then by diagonalizing

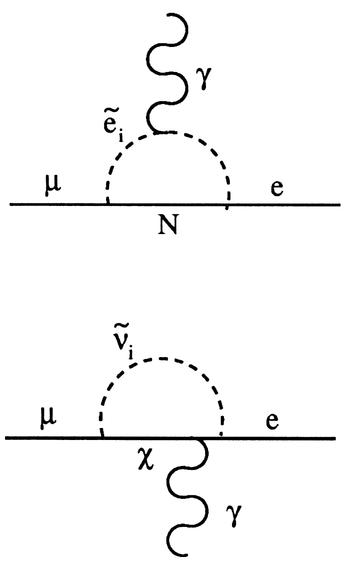

Fig. 1. The $\mu \to e\gamma$ decay via supersymmetric graphs.

the Wino mass matrices we evaluated the total amplitude and the branching ratio. For sensible values of $m_{3/2} \sim m_{1/2} \sim \mathscr{O}(m_W)$ (initial values for the scalar and gaugino masses respectively) and standard GUT initial conditions for gauge couplings, the value of the $\mathrm{BR}_{\mu \to e\gamma}$ can reach the order of 10^{-12}. Thus, this rare decay gives the opportunity to test the viability of the above $U(1)_F$-like model in future experiments.

In conclusion, we have considered the scalar mass matrices in supergravity models with the standard $SU(3) \times SU(2)_L \times U(1)_Y$ gauge group augmented by a $U(1)_F$ family symmetry. Using modular invariance of the Kähler potential and the superpotential, we have derived certain relations between $U(1)_F$ charges and the modular weights of the fields. As a result, the scalar mass matrix entries are found to depend only on certain powers, which are proportional to the difference of modular weights. We have calculated, as an example, the process $\mu \to e\gamma$, which is found, for a wide range of the parameter space ($\tan\beta$, $m_{3/2}$, $m_{1/2}$), to be very close to the present experimental limits. This fact makes it possible to test such theories in near future-experiments.

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