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# Gauge coupling unification and the top mass in string models with $SU(4) \times O(4)$ symmetry

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## Abstract

We discuss the low energy implications of gauge coupling unification at the string scale, taking into account string threshold corrections in the  $SU(4) \times O(4)$  model. We express  $\sin^2 \theta_W$  and  $a_3$  as functions of the calculable string threshold differences and discuss simple examples of spectra which retain the successful predictions of the supersymmetric unification. Using further the low energy data and reasonable values of the common gauge coupling at the string scale, we obtain the range of the threshold corrections. Finally, we study the top Yukawa coupling ( $h_t$ ) evolution whose initial value is determined in terms of the common gauge coupling at the string scale. We find that  $h_t$  reaches its (quasi) infra-red fixed point at the weak scale and discuss the implications on the top mass.

## 1. Introduction

Recent experimental evidence indicates that the desired unification of all fundamental forces can take place (within a single gauge group) at a scale  $M_G \sim 10^{16}$  GeV, where all the couplings attain a common value, provided supersymmetry exists above a scale of the order 1 TeV. Within the context of supersymmetry however, the origin and magnitude of Yukawa couplings and other parameters are not explained. Among the present candidates, string theories can in principle give answers to the above questions. In most of the string derived models however, this simple unification scenario based on a single non-Abelian gauge group

is lost. String unification has been shown to occur at a scale some 20 times larger than the  $M_G$  scale predicted by the minimal supersymmetric standard model (MSSM).

$$M_{\text{str}} = g_{\text{str}} \frac{e^{(1-\gamma)3-\frac{3}{4}}}{4\pi} M_{\text{Pl}} \approx 5.2 g_{\text{str}} \times 10^{17} \text{ GeV} \quad (1)$$

In the above,  $g_{\text{str}}$  is the universal string coupling which is fixed by the vacuum expectation value of the dilaton field  $S$ ,  $g_{\text{str}}^2 = 2/(S + \bar{S})$ .

The gauge symmetry of the resulting theory below  $M_{\text{str}}$  is usually a product of groups  $G = \prod_{\alpha} G_{\alpha}$  rather than a single gauge group. The corresponding field theory describing the low energy phenomena is achieved by integrating out the massive string states. As a result, the evolution of the gauge couplings  $g_{\alpha}$  of the effective theory should take into account threshold

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corrections  $\hat{\Delta}_\alpha$  due to the infinite tower of the massive string modes. Thus, they are given by

$$\frac{4\pi}{g_\alpha^2(\mu)} = k_\alpha \frac{4\pi}{g_{\text{str}}^2} + \frac{1}{4\pi} \left( b_\alpha \ln\left(\frac{M_{\text{str}}^2}{\mu^2}\right) + \hat{\Delta}_\alpha \right) \quad (2)$$

where  $b_\alpha$  is the beta-function and  $k_\alpha$  characterizes the Kac-Moody level of the corresponding coupling  $g_\alpha$  ( $k_\alpha = 1$  in what follows).

As it is obvious from the above formula, string thresholds affect decisively the boundary conditions of the effective field theory gauge couplings. Therefore, the low energy predictions of a particular string model are also sensitive on  $\hat{\Delta}_\alpha$ .

String threshold corrections have been extensively studied in the literature [1,2]. Recently there was a revived interest from the point of view of the effective field theory [3] and their implications in the low energy phenomenology [4,5].

A class of string derived models [6–10], which offer a suitable ground to study the low energy implications of these thresholds, is based on the free fermionic formulation of the four dimensional superstring [11]. In the present work, we examine some related aspects of the string derived  $SU(4) \times O(4)$  model. We explore the possibility of reconciling the low energy data with the existence of the string unification point being twenty times larger than the conventional unification scale. We take into account the string threshold corrections and determine the low energy gauge couplings in terms of their differences and the spectrum of the model. We extend previous analysis on the top mass calculations and include the effects of the theory above the ‘‘GUT’’ scale including the string threshold corrections.

## 2. The model

We briefly start with the basic features of the minimal supersymmetric version of the  $SU(4) \times O(4) \sim SU(4) \times SU(2)_L \times SU(2)_R$  model. The field content is summarized in the following table

$$\begin{aligned} F &= (4, 2, 1); & \bar{F} &= (\bar{4}, 1, 2); \\ H &= (4, 1, 2); & \bar{H} &= (\bar{4}, 1, 2); \\ h &= (1, 2, 2); & D &= (6, 1, 1); \end{aligned}$$

$$\phi_{m,0} = (1, 1, 1), \quad m = 1, 2, 3;$$

$$\not{H} = (4, 1, 1); \quad \bar{\not{H}} = (\bar{4}, 1, 1);$$

$$a_R = (1, 1, 2); \quad a_L = (1, 2, 1)$$

Left and right handed fermions (including the right handed neutrinos) are accommodated in the  $(4, 2, 1)$ ,  $(\bar{4}, 1, 2)$  representations respectively. Both pieces form up the complete  $16^{\text{th}}$  representation of  $SO(10)$ . The  $SU(4) \times SU(2)_R \rightarrow SU(3) \times U(1)$  symmetry breaking is realized at a scale  $\sim 10^{15-16}$  GeV, with the introduction of a higgs pair belonging to  $H + \bar{H} = (4, 1, 2) + (\bar{4}, 1, 2)$  representations. The symmetry breaking of the standard model occurs in the presence of the two standard doublet higgses which are found in the  $(1, 2, 2)$  representation of the original symmetry of the model. (The decomposition of the latter under the  $SU(3) \times SU(2)_L \times U(1)_Y$  gauge group results to the two higgs doublets  $(1, 2, 2) \rightarrow (1, 2, \frac{1}{2}) + (1, 2, -\frac{1}{2})$ .) The three singlets  $\phi_m$ 's are engaged in the see-saw type mechanism providing  $M_G$ -order masses to right handed neutrinos, while  $\phi_0$  is responsible for the appearance of the Higgs mixing term. Finally, note the existence of the ‘exotic’ representations  $\not{H}, \bar{\not{H}}, a_R$  and  $a_L$ . Although they do not arise in the ‘ordinary’ decomposition of an  $SO(10)$  GUT symmetry, they do appear in string derived models constructed at the level  $k = 1$  of the Kac-Moody algebra. These states possess fractional electric charges [7] and are expected to transform non-trivially under a hidden gauge group [12] which becomes strong at an intermediate scale confining them into bound states. In our present analysis we are not going to discuss such complications.

The trilinear superpotential is

$$\begin{aligned} \mathcal{W} &= \lambda_1 F \bar{F} h + \lambda'_1 F \bar{H} h + \lambda_2 \bar{F} H \phi_m + \lambda_3 H H D \\ &+ \lambda_4 \bar{H} \bar{H} D + \lambda_5 F F D + \lambda_6 \bar{F} \bar{F} D + \lambda_7 D D \phi_{m,0} \\ &+ \lambda_8 h h \phi_{m,0} + \lambda_9 \phi_n \phi_m \phi_l + \lambda_{10} \phi_n \phi_m \phi_0 \\ &+ \lambda_{11} \not{H} \not{H} \phi_{m,0} + \lambda_{12} \not{H} \not{H} D + \lambda_{13} \bar{\not{H}} \bar{\not{H}} D \\ &+ \lambda_{14} h a_L a_R + \lambda_{15} a_L a_L \phi_{m,0} + \lambda_{16} a_R a_R \phi_{m,0} \\ &+ \lambda_{17} \not{H} \bar{H} a_R + \lambda_{18} \bar{\not{H}} H a_R + \lambda_{19} \not{H} F a_L \end{aligned} \quad (3)$$

The phenomenological implications of (3) have been discussed elsewhere [13–15]. Here we will concentrate on the renormalisation of the gauge and Yukawa

couplings from the string scale to low energies. From the spectrum in (3) we observe first that in the minimal case there is an excess of right doublet over left doublet fields. In fact the asymmetric form of the higgs fourplets with respect to the two  $SU(2)$  symmetries of the model, causes a different running for the  $g_{L,R}$  gauge couplings from the string scale down to  $M_G$ . The possible existence of a new pair of fourplets with  $SU(2)_L$  – transformation properties (as suggested in Ref. [14]), namely  $H_L = (4, 2, 1)$  and  $\bar{H}_L = (\bar{4}, 2, 1)$ , could adjust their running so as to have  $g_L = g_R$  at  $M_G$ . This case corresponds to family – antifamily representations which can become massive close to  $M_G$ , with a trilinear or higher order term of the form  $\langle \Phi \rangle (\bar{4}, 2, 1) (4, 2, 1)$ . Moreover, a relatively large number ( $n_D$ ) of sextet fields ( $n_D \sim 7$ ) remaining in the massless spectrum down to  $M_G$ , would also result to an approximate equality of the above with  $g_4$  coupling. Other cases of string spectra with the desired properties are also possible.

Obviously, the equality of the three gauge couplings  $g_{4,L,R}$  at the  $SU(4)$  breaking scale  $M_G$ , is of great importance. In practice, this means that the three standard gauge couplings  $g_{1,2,3}$  start running from  $M_G$  down to low energies, with the same initial condition. The only possible splitting would arise only from string and GUT threshold corrections [13]. Thus, choosing  $M_G \sim 10^{16}$  GeV, we are able to obtain the correct predictions for  $\sin^2 \theta_W$  and  $a_3(m_Z)$ . As a matter of fact, the intermediate gauge breaking step gives us one more free parameter (namely  $M_G$ ). Having obtained the desired string spectrum, we are free to choose its value in order to reconcile the high string scale  $M_{str}$  with the low energy data. Examples of string models with such properties have been proposed [12].

The renormalisation group equations of the string version have been derived and studied in previous works [13,16,17]. At the one loop level, taking into account the string threshold corrections we can obtain the following equations:

$$\frac{1}{a_3} - \frac{s^2}{\alpha} = (b_4 - b_L)Q_{UG} + (b_2 - b_3) \cdot Q + \Delta_4 - \Delta_L \tag{4}$$

$$\frac{1}{a_3} - \frac{3c^2}{5\alpha} = \frac{3}{5}(b_4 - b_R)Q_{UG} + (b_3 - b_1) \cdot Q + \Delta_4 - \Delta_R \tag{5}$$

In the above, we have denoted  $Q_{UG} = \frac{1}{2\pi} \log(M_{str}/M_G)$ , with  $s, c$  the sin and cos of the weak mixing angle, while  $b_i \cdot Q = \sum_n b_i^n Q_{n,n-1}$  takes into account all possible intermediate scales. Finally,  $\Delta_i = \hat{\Delta}_i/(4\pi)$ . The weak mixing angle is given

$$\sin^2 \theta_W = \frac{3}{8} + \frac{5}{8} \alpha \{ b_{LR4} Q_{UG} + b_{21} \cdot Q + \Delta_{LY} \} \tag{6}$$

with

$$b_{LR4} = \frac{5b_L - 3b_R - 2b_4}{5}, \quad b_{ij} = b_i - b_j \tag{7}$$

$$\Delta_{LY} = \frac{3(\Delta_L - \Delta_R) + 2(\Delta_L - \Delta_4)}{5} \tag{8}$$

If we assume the minimal supersymmetric spectrum bellow  $M_G$ , where only the supersymmetry breaking scale  $M_S$  enters, we can eliminate  $M_S$  and determine the scale  $M_G$  in terms of  $\alpha_3, \alpha, \sin^2 \theta_W$  and the differences of the string thresholds. Equivalently,  $\sin^2 \theta_W$  can be expressed as follows:

$$\begin{aligned} \sin^2 \theta_W = & \frac{5}{8 - 5\kappa} \left\{ \frac{3}{5} + \kappa \frac{\alpha}{\alpha_3} \right. \\ & + \alpha \{ (b_{LR4} - \kappa b_{4L}) Q_{UG} + (b_{21} - \kappa b_{23}) Q_G \\ & \left. - (b_{2Y}^0 - \kappa b_{23}^0) Q_Z + \Delta_{LY} - \kappa \Delta_{L4} \} \right\} \tag{9} \end{aligned}$$

where  $\Delta_{L4} = \Delta_L - \Delta_4, \kappa = \frac{b_{21}^0 - b_{21}}{b_{23}^0 - b_{23}}$  and the superscript 0 in the beta functions refers to the non-supersymmetric ones. In particular, if the beta functions  $b_4, b_L, b_R$  above the GUT scale are equal, then  $b_{4L} = b_{4R} = 0$  and the above expression for  $\sin^2 \theta_W$  becomes very simple. In this case the  $g_{4,L,R}$  gauge coupling splittings at  $M_G$  are determined only in terms of the differences  $\Delta_{ij}$ .

Before we proceed to the calculations, let us describe briefly the spectrum in two main energy regions. In the  $M_{str} - M_G$  region, in addition to the three generations of  $(4, 2, 1)$  and  $(\bar{4}, 1, 2)$ , we choose the following content:

$$\begin{aligned} n_H = 2, \quad n_h = 1, \quad n_D = 8, \quad n_{\bar{H}} = 4, \quad n_{a_L} = 4, \\ n_{a_R} = 4, \quad n_{H_L} = 2 \end{aligned} \tag{10}$$

The above content has the property of giving  $b_4 = b_L = b_R = 7$  (note the existence of the  $H_L$ 's that were mentioned before). Therefore, in the  $M_{str} - M_G$  region the three gauge couplings  $\alpha_4, \alpha_L, \alpha_R$  run in parallel, their initial points at the  $M_{str}$  scale differing only due to the string threshold corrections  $\Delta_L, \Delta_R, \Delta_4$ . In the

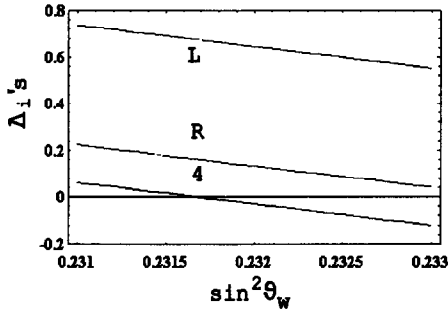


Fig. 1. The  $\Delta_L$ ,  $\Delta_R$  and  $\Delta_4$  thresholds as a function of  $\sin^2 \theta_W$ , for  $M_{\text{str}} = 0.4 \times 10^{18}$  GeV and  $M_S = 200$  GeV.

$M_G - M_Z$  region, the  $SU(4) \times SU(2)_L \times SU(2)_R$  model breaks down, at  $M_G$ , to the MSSM, which at the scale  $M_S$  turns to the non-supersymmetric standard model SM.

The procedure we are adopting is as follows: Using as input parameters the  $M_{\text{str}}$  scale and the string threshold differences  $\Delta_{L4} = \Delta_L - \Delta_4$ ,  $\Delta_{R4} = \Delta_R - \Delta_4$ , we determine  $M_G$  in order to have acceptable low energy parameters  $\sin^2 \theta_W$  and  $\alpha_3$  (we fix  $\alpha = 1/127.9$ ). As an example, we used the  $\Delta_{ij} = \Delta_i - \Delta_j$  values obtained in Ref. [4] for the specific  $SU(4) \times O(4)$  model [7]. In particular we take  $\hat{\Delta}_{L4} \sim 8.47$  and  $\hat{\Delta}_{R4} \sim 2.05$ . We can then run the gauge couplings from down to up and evaluate the quantities

$$\frac{1}{\alpha_{\text{str}}} + \Delta_4, \quad \frac{1}{\alpha_{\text{str}}} + \Delta_R, \quad \frac{1}{\alpha_{\text{str}}} + \Delta_L$$

The relation between  $M_{\text{str}}$  and  $\alpha_{\text{str}}$  can be used now to determine the absolute values of the string thresholds.

Let us now put our results into figures. In Fig. 1 we show the absolute values of the three string thresholds as a function of  $\sin^2 \theta_W$  for  $M_{\text{str}} = 0.4 \times 10^{18}$  GeV and  $M_S = 200$  GeV. We see a weak dependence on  $\sin^2 \theta_W$ . In Fig. 2 we plot the threshold  $\Delta_L$  as a function of both  $\sin^2 \theta_W$  and  $M_{\text{str}}$ . The dependence on the latter is strong. In fact, a change of  $M_{\text{str}}$  from  $0.3 \times 10^{18}$  GeV to  $0.5 \times 10^{18}$  GeV results a change in  $\Delta_L$  from  $\sim -200$  to  $\sim 100$  (in Fig. 1, we have chosen the value of  $M_{\text{str}}$  which corresponds to  $\Delta_i$ 's of the same order as their differences). In other words, even large threshold corrections demand only a small change in the value of  $M_{\text{str}}$ . If  $M_{\text{str}}$  is “pushed” towards  $M_G$ , then large negative threshold corrections are required. For completeness, in Fig. 3 we plot contours of constant  $M_G$  in the plane of  $(\sin^2 \theta_W, \alpha_3)$ , for  $M_S = 200$  GeV and  $M_{\text{str}} =$

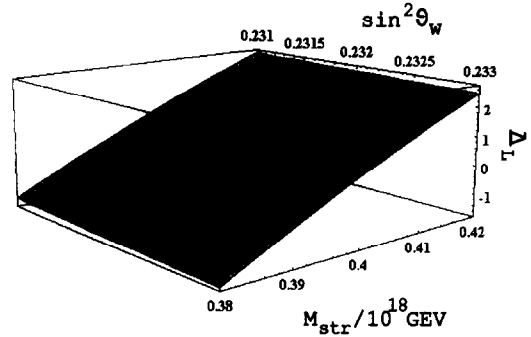


Fig. 2. The  $\Delta_L$  threshold as a function of  $\sin^2 \theta_W$  and  $M_{\text{str}}$ . The supersymmetry breaking scale is  $M_S = 200$  GeV.

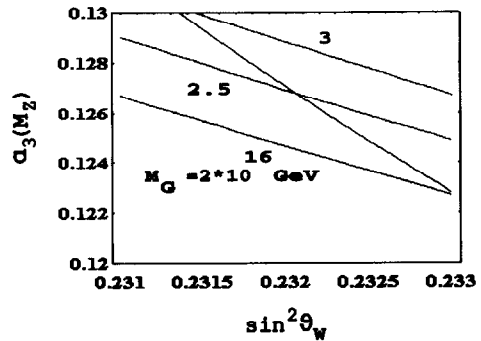


Fig. 3. Contours of constant  $M_G = (2, 2.5, 3) \times 10^{16}$  GeV in the  $(\sin^2 \theta_W, \alpha_3(M_Z))$  plane. The line crossing these contours gives the acceptable  $(\sin^2 \theta_W, \alpha_3(M_Z))$  pairs which correspond to the chosen values of  $M_{\text{str}} = 0.4 \times 10^{18}$  GeV and  $M_S = 200$  GeV ( $\alpha = 1/127.9$ ).

$0.4 \times 10^{18}$  GeV. The line that crosses these contours gives the acceptable pairs of  $(\sin^2 \theta_W, \alpha_3)$  that correspond to the chosen values of  $M_{\text{str}}$  and  $M_S$  (as we have mentioned before we keep  $\alpha = 1/127.9$ ).

### 3. Top Yukawa coupling fixed point

In the case of the minimal supersymmetric standard model, it is well known that the top–Yukawa coupling evolution from the unification scale down to the low energies, exhibits a quasi-fixed point structure<sup>3</sup>. In

<sup>3</sup> In Ref. [18] it has been shown that in MSSM the infrared fixed point is never reached. On the contrary, in theories with a stage of compactification the top coupling reaches its infrared fixed point since the evolution of couplings is much faster, following a power law rather than a logarithmic evolution.

fact, starting at the GUT-scale with  $h_{t,G} > 1$  the top mass is almost insensitive to the  $h_{t,G}$  value.

Large  $h_{t,G}$  values are in perfect agreement with the recent experimental evidence for a top mass of the  $\mathcal{O}(170\text{--}180)$  GeV. Moreover, the idea of the  $SU(2) \times U(1)$  symmetry breaking by radiative corrections in supersymmetric theories is realized by large negative top–Yukawa corrections to the Higgs mass, which need a sufficiently large top coupling. The MSSM theory with the unification assumption of the three gauge couplings at the scale  $\sim 10^{16}$  GeV does not provide a convincing reason why the initial value of the Yukawa coupling is large. It thus appears that the infra-red structure of the top coupling has its origin in a fundamental theory beyond the MSSM. An interesting possibility is that there is additional structure above the supersymmetric ‘unification’ at  $M_G \sim 10^{16}$  GeV which determines Yukawas and other parameters at  $M_G$ . The present string derived model provides such an example. The top Yukawa coupling is related to the gauge coupling at the string scale  $M_{\text{str}}$ . The  $SU(4)$ -breaking takes place at the intermediate scale  $M_G \sim 10^{16}$  GeV, which effectively corresponds to the SUSY ‘unification’ scale. Knowing the evolution equations of the gauge and Yukawas between  $(M_{\text{str}} - M_G)$ , it is rather easy to determine the  $h_t$  value at the GUT scale, which will serve as initial condition for the  $(M_G - M_Z)$  running. In particular, if the spectrum bellow the GUT  $SU(4)$  breaking scale is that of the minimal supersymmetric standard model, then one can make a definite prediction about the top mass.

In the present model, all charged fermions of the third generation receive masses from the superpotential term  $\lambda_1 \bar{F} F h$ . Therefore the  $SO(10)$  Yukawa unification condition  $h_{(t,G)} = h_{(b,G)} = h_{(\tau,G)} \equiv \lambda_1(M_G)$  is also retained in the  $SU(4) \times SU(2)_L \times SU(2)_R$  symmetry. In the range  $M_{\text{str}} - M_G$  the evolution of the Yukawa coupling  $\lambda_1$  is given by

$$16\pi^2 \frac{d\lambda_1}{dt} = \lambda_1 (8\lambda_1^2 - \sum_{\alpha} c_{\alpha} g_{\alpha}^2) \quad (11)$$

where  $t = \log(Q)$  is the logarithm of the scale, and the index  $\alpha$  refers to the three gauge group factors  $\alpha = 4, L, R$  in the range  $Q = (M_{\text{str}} - M_G)$ . For the sake of simplicity, in the above differential equation we have ignored terms proportional to the other Yukawa couplings of the superpotential. If all couplings were

included, only a numerical solution would be possible [20], however our results concerning the top-mass prediction would not be essentially affected. The coefficients  $c_{\alpha}$  are given by

$$\{c_{\alpha}\}_{\alpha=4,L,R} = \left\{ \frac{15}{2}, 3, 3 \right\}$$

Thus, in the range  $M_{\text{str}} - M_G$  the solution for  $\lambda_1$  is given by

$$\lambda_1(t) = \lambda_1(t_{\text{str}}) \gamma_U \zeta(t) \quad (12)$$

$$\zeta(t) = \frac{1}{(1 + \frac{8}{8\pi^2} \lambda_1^2(t_{\text{str}}) I_U(t))^{1/2}} \quad (13)$$

with

$$\gamma_U = \prod_{j=4,L,R} \left( \frac{\alpha_{\text{str}}}{\alpha_j} \right)^{\frac{c_j}{2b_j}}, \quad I_U(t) = \int_{t_{\text{str}}}^t \gamma_U^2(t') dt' \quad (14)$$

At the  $SU(4)$  breaking scale  $M_G$  the original symmetry breaks down to the standard gauge group. As pointed out previously the top Yukawa coupling has the same initial value at  $M_G$  with the  $b - \tau$ 's, i.e., we are in the case of  $\tan \beta \gg 1$ . In the case of the large  $\tan \beta$ , ignoring the  $\tau$ -Yukawa, for equal  $h_t, h_b$  couplings we can obtain the following expression [19] for the top-Yukawa evolution below  $M_G$

$$h_t(t) = \lambda_1(t_G) \gamma_Q(t) \xi(t) \quad (15)$$

$$\xi(t) = \frac{1}{(1 + \frac{7}{8\pi^2} \lambda_1(t_G)^2 I(t))^{1/2}} \quad (16)$$

where the relation  $h_{t,G} \equiv \lambda_1(t_G)$  has been taken into account. The expressions  $\gamma_Q(t), I_Q(t)$  are similar to those of  $\gamma_U, I_U$  in (14) respectively. Therefore, combining the above two equations, we determine the top Yukawa coupling and its mass at low energies directly from the initial value of the coupling  $\lambda_1$  at  $M_{\text{str}}$ . In particular, imposing the initial condition  $\lambda_1(t_{\text{str}}) = \sqrt{2} g_{\text{str}}$  predicted in this particular model, we obtain the following formula for the top mass:

$$\frac{m_t(t)}{\sin \beta} = \sqrt{2} g_{\text{str}} \gamma_U(t_G) \zeta(t_G) \gamma_Q(t) \xi(t) \frac{v}{\sqrt{2}} \quad (17)$$

with  $v = 246$  GeV.

Finally, in Table 1 we present the (physical) top mass predictions and the range of  $\tan \beta$  in order to

Table 1

$$M_{\text{str}} = 0.4 \times 10^{18} \text{ GeV}, \quad g_{\text{str}} = 0.77, \quad \sin^2 \theta_W = .232$$

$m_t^{\text{phys}}$ (GeV)	$\tan \beta$	$M_S$ (GeV)	$M_G$ (GeV)	$\alpha_3(M_Z)$
190	60–63	1000	$1.6 \times 10^{16}$	0.122
187	58–61	500	$2.0 \times 10^{16}$	0.124
183	56–59	200	$2.5 \times 10^{16}$	0.127

have the running bottom mass  $m_b(m_b) = (4.15\text{--}4.35)$  GeV, for three representative cases of the supersymmetry breaking scale  $M_S$ . We also show, for each case, the corresponding “GUT” scale  $M_G$  and  $\alpha_3(M_Z)$ . The string scale value is  $M_{\text{str}} = 0.4 \times 10^{18}$  GeV (which gives  $g_{\text{str}} = 0.77$ ) and  $\sin^2 \theta_W = .232$ . We have checked that the effect of string thresholds on  $m_t$  is of the order of (4–6)%. Thus for given  $m_b$  and  $\sin^2 \theta_W$  (or  $\alpha_3$ ) values,  $m_t^{\text{phys}}$  is well determined in terms of the infrared fixed property of the Yukawa coupling.

Note that in our actual calculations we have taken into account the  $M_S$  scale, thus running the (non-supersymmetric) SM beta-functions for gauge as well as Yukawa couplings. At the  $M_S$  scale, the initial conditions for the  $h_t$  and  $h_b$  running are of course  $h_t^{\text{NS}}(t_G) = h_t(t_G) \sin \beta$  and  $h_b^{\text{NS}}(t_G) = h_b(t_G) \cos \beta$ . We have also checked that, running (numerically) the coupled differential equations for  $h_t$  and  $h_b$  on the one hand and using the Eqs. (13)–(16) on the other, the differences between these procedures are negligible. Note that the RGEs for  $h_t$  and  $h_b$  in the range  $M_G\text{--}M_S$  differ only in the small  $U(1)$ -gauge coefficient.

#### 4. Conclusions

In the present work we have analysed the possibility to obtain low energy predictions compatible with the experimental data in string derived models with  $SU(4) \times O(4)$  symmetry. Generally, large string thresholds are required to reconcile the experimental data with the existence of the large gap between the string ( $M_{\text{str}} \sim 5 \times 10^{17}$  GeV) and supersymmetric ( $M_G \sim 1.5 \times 10^{16}$  GeV) unification scales. It is argued here that, a simple and viable scenario – compatible with the low energy phenomenological expectations – is to obtain a massless spectrum which allows approximately a parallel evolution of the gauge cou-

plings between  $M_{\text{str}}\text{--}M_G$ . Given the rich spectrum of such models [6–8], one could choose carefully the vacuum expectation values of the singlet fields associated with the large breaking scale of the possible surplus  $U(1)$  symmetries and make massive those states which allow equal beta function coefficients in most of the range above  $M_G$ . This would simply correspond to a judicious choice of a specific flat direction of the effective field theory superpotential.

In the above context, we have considered the evolution of the top Yukawa coupling from the string scale down to low energies. We have found that it exhibits a fixed point structure thus leading to definite predictions for the top mass compatible with its present experimentally determined range.

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