ELSEVIER

1 June 1995

PHYSICS LETTERS B

Physics Letters B 351 (1995) 487–496

# Soft SUSY masses and the dynamical determination of the gravitino mass

G.K. Leontaris [a,1,2], N.D. Tracas [b,3]

[a] *Centre de Physique Theorique, Ecole Polytechnique, F-91128 Palaiseau, France*

[b] *Physics Department, National Technical University, 157 80 Zografou, Athens, Greece*

Received 27 January 1995
Editor: R. Gatto

**Abstract**

We discuss in detail the possibility of determining dynamically the gravitino mass $m_{3/2}$, which is related to the supersymmetry breaking scale, within the minimal supersymmetric standard model (MSSM). Using the complete MSSM spectrum, we minimize the vacuum energy including one-loop corrections and a cosmological term of $\mathcal{O}(m_{3/2}^4)$ induced by the underlying fundamental theory. We find that both terms are necessary to determine dynamically the gravitino mass. Other useful constraints for the low energy phenomenology are also obtained.

It is widely believed that the only plausible solution to the gauge hierarchy problem is $N = 1$ local supersymmetry [1]. The gauge hierarchy problem arises from quadratically divergent one-loop corrections to the effective potential, those being of the form $(\Lambda^2 \mathrm{Str}\, \mathcal{M}^2/(32\pi^2))$, where $\Lambda$ is the momentum cut-off, while

$$\mathrm{Str}\, \mathcal{M}^2(z,\bar{z}) = \sum_n (-1)^{2s_n}(2s_n+1)m_n^2(z,\bar{z}) \quad (1)$$

The sum is over all particles with field-dependent masses squared $m_n^2$ and spin $s_n$. Since $\mathcal{M}^2$ contains also the Higgs mass-squared, this term induces a divergent contribution destabilising the hierarchy $M_W \ll M_{\mathrm{Pl}}$, where $M_{\mathrm{Pl}}$ is the Planck mass.

[1] On leave of absence from Physics Department, University of Ioannina, 451 10 Ioannina, Greece.

[2] Work supported in part by C.E.C. Project (CHRX-CT93-0132).

[3] Work supported in part by C.E.C. Projects (SC1-CT91-0729, CHRX-CT93-0319).

In the spontaneously broken $N = 1$ local supersymmetry the Str $\mathcal{M}^2$, which appears as a coefficient of the one-loop quadratically divergent contributions, is given in terms of the field dependent gravitino mass $m_{3/2}$ by the formula [2]

$$\mathrm{Str}\, \mathcal{M}^2 = 2Q(z,\bar{z})m_{3/2}^2(z,\bar{z}) \quad (2)$$

where the dimensionless function $Q(z,\bar{z})$ depends on the fields $z$ and $\bar{z}$ through the Riçi tensor of the Kähler manifold and the function $f_{ab}(z,\bar{z})$ which determines the kinetic terms of the vector supermultiplets as well as the gauge coupling constants.

In the fundamental theory of quantum gravity the non-vanishing of $Q(z,\bar{z})$ would imply corrections to the effective potential of the order $\mathcal{O}(m_{3/2}^2 M_{\mathrm{Pl}}^2)$ which cannot be cancelled by any contribution of low energy physics. The gravitino mass is given by

$$m_{3/2}^2(z,\bar{z}) = |\mathcal{W}(z)|^2 e^{k(z,\bar{z})} \quad (3)$$


SSDI 0370-2693(95)00433-5

where $\mathcal{W}(z)$ is the superpotential. The value of $m_{3/2}$ is related to the scale of supersymmetry breaking which should not be much larger than the electroweak breaking scale. Since $m^2_{3/2}(z,\bar{z})$ is field dependent, its vacuum expectation value (vev) should arise from the minimization of the potential. Then, quadratically divergent loop corrections proportional to Str $\mathcal{M}^2$ will induce either $m_{3/2} \to 0$ (unbroken supersymmetry) or $m_{3/2} \to M_{\rm Pl}$, therefore destabilizing again the hierarchy.

A possible solution to the hierarchy problem requires the vanishing of $Q(z,\bar{z})$ which motivated the no-scale supergravity models [3]. A further step towards this problem has been taken the last few years by going beyond the $N = 1$ local supersymmetry, the superstring theory. In the context of the latter, and in particular of their four-dimensional version [4], the effective supergravity theory is strongly restricted. It has been shown [2] that there exist examples in supergravity theories preserving the general features of the superstring underlying theory which predicts a vanishing $\mathcal{O}(m^2_{3/2}M^2_{\rm Pl})$ contribution. Such theories, however, will still leave a non-vanishing contribution to the vacuum of the order $\mathcal{O}(m^4_{3/2})$, which can be interpreted as a contribution to the cosmological constant

$$\Delta V_{\rm COSM} = \eta(Q)m^4_{3/2} \tag{4}$$

The energy scale dependent coefficient $\eta(Q)$ has a certain boundary condition on the unification scale. Its value there is dictated by the structure of the 'hidden' sector in the specific string model that has been chosen.

In Ref. [5], the gravitino mass has been treated as a dynamical variable. This would in turn imply that the low energy effective potential should be minimized not only with respect to the vevs of the Higgs fields but also with respect to $m_{3/2}$. It has been stressed that this term cannot be absent in low energies as far as the gravitino mass is not taken as an external parameter. On the contrary, its contribution is determined by the evolution of the coefficient $\eta(Q)$ from the GUT scale down to the low energies on the one hand, and the dynamical determination of $m_{3/2}$ on the other hand.

In what follows we wish to analyse the above procedure in a realistic low energy supersymmetric theory. We take as an example the minimal supersymmetric standard model (MSSM) which is endowed with all the salient features of an effective supergravity theory. We will show that under the very general characteristics of the above theories, the $\eta(Q)$ is non-zero and negative at $Q \sim M_Z$, as long as $m_{3/2}$ lies in the desirable range of 100 GeV to 1 TeV. Moreover the dynamical determination of the $m_{3/2}$ scale through the minimization of the effective potential puts constraints of the scalar mass spectrum of the theory.

We consider therefore the MSSM. Following the discussion above, the only terms relevant to the potential (including quantum corrections) are the following

$$V_1(Q) = V_0(Q) + \eta(Q)m^4_{3/2} + \frac{1}{64\pi^2}\text{Str}\,\mathcal{M}^4(\ln\frac{\mathcal{M}^2}{Q^2} - \frac{3}{2}) \tag{5}$$

$V_0(Q)$ is the (RGE improved) tree-level potential while the appearance of the last term is due to the radiative corrections (at one-loop level) and its inclusion is necessary in order to stabilize the minimization procedure of the potential against $Q$ [6].

The evolution of the parameter $\eta(Q)$ is determined by a RGE which can be derived by demanding that the potential $V_1(Q)$ is scale independent to the one-loop order, i.e.

$$\frac{dV_1(t)}{dt} = 0, \quad t = \ln Q \tag{6}$$

Since the above relation should hold for all values of the fields, in the case where $v_1 = v_2 = 0$, we have [5,7] $V_0|_{v_i=0} = dV_0/dt|_{v_i=0} = 0$, thus

$$m^4_{3/2}\frac{d\eta(t)}{dt} - \frac{2}{64\pi^2}\text{Str}\,\mathcal{M}^4|_{v_i=0} = 0 \tag{7}$$

The above differential equation determines the value of $\eta(t)$ in terms of Str $\mathcal{M}^4$ and the gravitino mass, once the initial values of $\eta$ and of the mass parameters entering Str $\mathcal{M}^4$, at the unification scale, are known.

The initial value $\eta_G$, for example, is related in some specific models to the difference $n_B - n_F$ where $n_{B(F)}$ are the bosonic (fermionic) degrees of freedom after supersymmetry breaking. An explicit derivation of the cosmological term, which can be identified with the contribution $\eta(Q)m^4_{3/2}$ of Eq. (5), is given in Ref. [8]. In this treatment, the supersymmetry breaking scale is related to the size of a large internal dimension $R$. It was found that after the SUSY breaking, the one-loop contribution to cosmological constant is of the order of $(\alpha_{\rm String}/4\pi R^4)(\eta_B - \eta_F)$. For $Z_2 \times Z_2$

orbifolds, the gravitino mass is $1/\sqrt{8R}$, thus for broken SUSY one estimate that $\eta_G \leq 0$.

The initial values of the scalar masses $\tilde{m}_i^2$, the gaugino mass $m_{1/2}$ and of the $\mu$ parameter at the unification scale $M_G$, can be parametrized in terms of $m_{3/2}$

$$\tilde{m}_i^2 = \xi_i m_{3/2}^2, \quad m_{1/2}^2 = \xi_{1/2} m_{3/2}^2, \quad \mu_G = \xi_\mu m_{3/2} \tag{8}$$

where the $\xi$ coefficients are of $\mathcal{O}(1)$ (calculable in specific models). Therefore, the value of $\eta(Q)$ at any scale $Q < M_G$ is given by

$$\eta(Q) = \eta_G + \frac{1}{32\pi^2} \int_{M_G}^{Q} \mathrm{Str}\, \hat{\mathcal{M}}^4(Q', \xi_i, \xi_{1/2})|_{v_i=0} d\ln Q' \tag{9}$$

where

$$\hat{\mathcal{M}}^4|_{v_i=0} = \sum_i (2s_i+1)(-1)^{2s_i} \frac{m_i^4(Q)}{m_{3/2}^4} = \sum_i (2s_i+1)(-1)^{2s_i} [f_i(\xi_i, Q)]^4$$

and $f_i(\xi_i, Q)$ can be calculated from the RGE running of the masses. Therefore, the parametrization of Eq. (8) renders the value of $\eta(Q)$, obtained from Eq. (9), independent of $m_{3/2}$.

For a given set of $(\xi_\alpha, \eta_G)$, the $m_{3/2}$ value will be given by the minimization condition of the low energy potential with respect to $m_{3/2}$. This condition results to the equation [5]

$$V_1 + \frac{1}{128\pi^2} \mathrm{Str}\, \mathcal{M}^4 = 0 \tag{10}$$

The latter has been interpreted as defining an infrared fixed point of the cosmological term, as it corresponds to the vanishing of the associated $\beta$-function. It is a significant constraint that should be satisfied by the $m_{3/2}$ and $\xi_\alpha$ parameters and the low energy values of the gauge couplings involved in $V_0(v_1, v_2)$.

In order to exploit the constraint of Eq. (10), in the case where the complete spectrum of the MSSM is taken into account, we need the detailed $Q$-dependence of all the relevant parameters. We start with the classical tree-level potential which is given by

$$\begin{aligned} V_0(Q) = {} & (m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2 \\ & + m_3^2(H_1 H_2 + \text{h.c.}) + \tfrac{1}{8} g^2 (H_2^\dagger \sigma H_2 + H_1^\dagger \sigma H_1)^2 \\ & + \tfrac{1}{8} g'^2 (|H_1|^2 - |H_2|^2)^2 \end{aligned} \tag{11}$$

The minimization of the $V_0$ potential with respect to $v_{1,2}$ leads to the well known conditions

$$m_3^2 = -\tfrac{1}{2}(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2) \sin 2\beta \tag{12}$$

$$\tfrac{1}{2} M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \tag{13}$$

These conditions, for given initial conditions of the Higgses and the angle $\beta$, can determine the low energy values of the $\mu$ and $m_3^2 \equiv B\mu$ parameters. In what follows, we will demand a top mass to the present experimentally determined region, thus for any chosen $h_{t_G}$ Yukawa coupling we can also determine $\tan\beta$. This in turn implies that in our procedure we also fix the low energy values of $\mu$ and $B$. Using their RGE evolution we finally determine their values at the GUT scale.

Furthermore, the conditions of Eqs. (12,13) allow us to write the tree level potential in a simple form, exhibiting its dependence on the $M_Z$ mass. Substituting Eq. (13) into Eq. (11) we get

$$V_0(v, \beta) = -\tfrac{1}{32}(g^2 + g'^2) v^4 \cos^2 2\beta = -\frac{1}{8\pi} \frac{M_Z^4 \cos^2 2\beta}{(\alpha + \alpha')} \tag{14}$$

where $v = 246$ GeV. Thus, the details of the supersymmetry breaking parameters do not essentially affect the $V_0(v, \beta)$ piece at the low energy scale. In, particular, in determining the low energy value of $\eta_Z$ from low energy physics, the main dependence of SUSY breaking parameters enters through the Str contributions. In fact, we can use the minimization condition Eq. (10) to determine the required low energy value of $\eta(Q)$ as a function only of the parameters $\xi_\alpha$ and $\xi_Z = (M_Z/m_{3/2})^2$ and $\tan\beta$, i.e. for $Q \sim M_Z$ we get

$$\eta(M_Z) = \frac{1}{8\pi} \left\{ \frac{\xi_Z^2 \cos^2 2\beta}{\alpha + \alpha'} - \frac{1}{8\pi} \mathrm{Str}\, \hat{\mathcal{M}}^4 (\ln \hat{\mathcal{M}}^2 - 1) \right\} \tag{15}$$

The above relation enables us to calculate the required low energy value of the cosmological coefficient $\eta(Q)$ for any set of the parameters $\xi_\alpha$ choosing a phenomenologically acceptable $m_{3/2}$ range. By solving then the corresponding RGE for $\eta(Q)$, Eq. (7), we can determine a consistent range of values of $\eta$ at the unification scale. Some general remarks concerning Eq. (15) are worth noting here.

First there is a positive contribution from the tree level potential which depends on $\xi_Z$ and the angle $\beta$. For very large $\tan\beta$, this term becomes almost independent of $\beta$ as $|\cos 2\beta| \to 1$. As $m_{3/2}$ shifts to values much larger than $M_Z$, then $\xi_Z^2 \ll 1$ and the positive contribution becomes negligible. There is a negative contribution, on the other hand, from the supertrace dependence which finally leads $\eta(M_Z)$ to negative values at $m_Z$. Scalar mass and gaugino contributions in the supertrace scaled by $m_{3/2}$ are independent of the latter, being functions only of the $\xi_\alpha$ parameters and the scale dependent gauge functions. Therefore, the main $m_{3/2}$ dependence enters through the logarithmic terms of the form $\ln(\tilde{m}_i^2(Q)/Q^2) - 1 \equiv \ln f(\xi_i, Q) + \ln(m_{3/2}^2/Q^2) - 1$. Therefore, the $m_{3/2}$ value at which the minimum of the potential occurs, is intimately related to these terms.

The calculation of Str $\hat{\mathcal{M}}^4$ requires the knowledge of the boundary conditions (b.c.) for the scalars at the GUT scale, i.e. the knowledge of the $\xi_\alpha$ parameters. In the case of universal b.c., for example, one has $\xi_i = \xi_0 = m_0/m_{3/2}$ and $\xi_{1/2} = m_{1/2}/m_{3/2}$, i.e. only two parameters in addition to $\xi_Z$. However, in the general case of supergravity theories $\xi_i$ are in general different (non-universality) and the parameter space becomes more complicated. In addition, the RGEs for the scalars should also contain the contribution of the U(1)-D terms which plays a significant role for large deviations from the universality condition $\xi_i = \xi_0$.

The important fact of the above described approach is that, for a specific supergravity or superstring model, up to an overall constant which can be identified with the gravitino mass, all the $\xi_\alpha$'s are known. If in addition the initial value of $\eta(Q)$ at $M_G$ is known, equations Eqs. (7,10) can determine exactly the gravitino mass.

In practice, it is not trivial to write down, at least for the moment, a detailed spectrum of a realistic string model. Therefore, in the present analysis we prefer to follow the above described procedure using the general features of a supergravity theory. In this procedure, we treat as free parameters the coefficients $\xi_\alpha$, varying them in a range close to unity, and use the complete spectrum of the MSSM to predict a consistent range of $\eta(Q)$ at the unification scale. This bottom-up approach has as a prerequisite the knowledge of the gravitino mass whose value is supposed to be determined dynamically. We know however that, since supersymmetry breaking is related closely to the $m_{3/2}$ scale, its value should be necessarily of the order of the electroweak scale. Our purpose is then to show that under realistic conditions and for a wide choice of the parameter space $\xi = (\xi_i, \xi_{1/2}, \xi_\mu)$ there are some stable and well defined predictions of the input value $\eta(M_G)$ which can be hopefully determined independently in specific string models. To put it in another way, using all the possible information of low energy physics, one can certainly support, or rule out, possible string constructions.

In the present work we stick in the low $\tan\beta$ regime and prefer to use semianalytic formulae to calculate the Str contributions. To start with, in the case of non-universal conditions at the GUT scale for the soft terms, we generalize our previous formulae [9] for the third generation of squarks which are the only one affected by the heavy top contribution.

As in Ref. [5], we prefer to restrict our analysis in the case of the universal condition in the Higgs sector, although it seems interesting to consider the more general case. However, working in the low $\tan\beta$ regime, the non-universality in the Higgs sector, $m_{H_1}^0 = m_{H_2}^0$, is not expected to play a significant role, contrary to the case of large $\tan\beta$ scenario. In the latter case, departure from universality [10] is sometimes necessary to avoid instabilities in the low energy effective potential due to large negative corrections to both Higgs mass parameters. We give now the specific formulae which we are going to use.

The RGEs for the scalars receiving large $h_t$ Yukawa contribution are

$$\frac{d\tilde{m}_{Q_L}^2}{dt} = \sum \frac{c_i^Q M_i^2 g_i^2}{8\pi^2} - \frac{h_t^2}{8\pi^2}\left(\tilde{m}_{Q_L}^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_t\right) - \frac{1}{6}\frac{\alpha_1}{2\pi} S \quad (16)$$

$$\frac{d\tilde{m}_U^2}{dt} = \sum \frac{c_i^U M_i^2 g_i^2}{8\pi^2} - \frac{2h_t^2}{8\pi^2}\left(\tilde{m}_{Q_L}^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_t\right) + \frac{2}{3}\frac{\alpha_1}{2\pi} S \quad (17)$$

$$\frac{dm_{H_2}^2}{dt} = \sum \frac{c_i^H M_i^2 g_i^2}{8\pi^2} - \frac{3h_t^2}{8\pi^2}\left(\tilde{m}_{Q_L}^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_t\right) - \frac{1}{2}\frac{\alpha_1}{2\pi}S \quad (18)$$

where $S$, in the case of MSSM, is given by

$$S = m_{H_2}^2 - m_{H_1}^2 + \sum_{\text{gen}}(\tilde{m}_Q^2 + \tilde{m}_D^2 + \tilde{m}_E^2 - \tilde{m}_L^2 - 2\tilde{m}_U^2)$$

The solution of the above system can be easily found through the solution of the differential equation obeyed by the sum of the three masses $u(t) = \sum \tilde{m}_i^2$, (where we have assigned $\tilde{m}_1 \to \tilde{m}_Q^2$, $\tilde{m}_2 \to \tilde{m}_U^2$ and $\tilde{m}_3 \to m_{H_2}^2$),

$$\frac{du(t)}{dt} = u_0(t) - \frac{6h_t^2}{8\pi^2}u(t) \quad (19)$$

where

$$u_0 = \sum_j \sum_i \frac{c_i^j M_i^2 g_i^2}{8\pi^2} - \frac{6h_t^2}{8\pi^2}A_t, \quad j = Q, U, H_2$$

It is worth noticing here that the Eq. (19) is independent of the $S$ contribution, since the sum of the U(1) charges should be zero in the term $QUH_2$ (invariance of the Yukawa Lagrangian under U(1)). Of course, each individual mass gets a contribution from the $S$ term. The solution of the differential equation is given by [9]

$$u(t) = \int_{t_0}^{t} u_0(t)dt - 6\delta_A^2(t) - 6\delta_m^2(t)$$

Following closely the formalism of Ref. [9] and taking into account that the $A_t$ contributions are small, we can write the solution of the Eqs. (16–18) in the form

$$\tilde{m}_n^2 = \xi_n m_{3/2}^2 + C_n^i(t)\xi_{1/2}^i m_{3/2}^2 + C_n^S(t)S_0 m_{3/2}^2 - n\delta_m^2(t) \quad (20)$$

where the coefficients $C_n^i(t)$ are defined in Ref. [9] and

$$C_n^S = \{-\tfrac{1}{6}, \tfrac{2}{3}, -\tfrac{1}{2}\}\frac{1}{b_1}\left(\frac{\alpha_1(t)}{\alpha_1(t_G)} - 1\right)$$

$$S_0 = \xi_3 - \xi_{H_1} + \sum_{\text{gen}}(\xi_1 + \xi_D + \xi_E - \xi_L - 2\xi_U)$$

where the differential equation obeyed by $S$, namely $dS/dt = \alpha_1 b_1 S/(2\pi)$, has been used. In the following we will stick in the case $\xi_3 = \xi_{H_1}$ (universality in the Higgs sector) and that all three $\xi_{1/2}$ are the same (universality in the gaugino sector).

The Str-term contributions can be calculated now easily. We should point out that this calculation involves the $\mu$ parameter of the superpotential, which is unknown at the $M_G$ scale. However, in the bottom-up approach we are using here, the minimization conditions at $Q \sim M_Z$ determine the value $\mu$ at this scale. Its value at any scale can be obtained by generalizing Eq. (26) of Ref. [9] for the case for non-universal b.c., and evolve it using the relevant renormalisation group equation. Similarly, the $B$ (or $m_3^2$) parameter is not left arbitrary once a particular value of the $\beta$ angle is chosen. As we have already commented, for given $\tan\beta$ and $\xi_{H_{1,2}}$ initial Higgs parameters, their values are fixed by the minimization procedure. Thus, the only parameter left arbitrary, is the value of the trilinear coupling $A$. Remarkably, its contribution in the supertrace is not important, provided that the initial value $A_0$ is not very large, $A_0 \sim \mathcal{O}(\sqrt{3}m_0)$, and the top coupling is large enough $h_{t_G} > 1$. In fact, its contribution enters mainly through the soft scalar mass terms $m_{H_2}, \tilde{m}_{t_R}, \tilde{m}_{t_L}$ whose expressions are given by Eq. (20). There, $\delta_A$ corrections where ignored since it has been found that under the above conditions which concern us here $\delta_A^2 \ll \delta_m^2$ [9].

A final issue we should discuss before we present our numerical results, is the scale at which the required parameters should be calculated. Indeed, as we shall show soon, $\eta(Q)$ varies substantially as the scale approaches $M_Z$ and its value in very sensitive to the chosen scale. For a gravitino mass close to the value $M_Z$ it seems sensible to calculate all the relevant parameters at $Q \sim M_Z$. If we seek however solutions for $m_{3/2} \gg M_Z$, it would be appropriate to calculate the relevant quantities at a scale close to this value of $m_{3/2}$. Then, according to our program we define as low energy value of $\eta(Q)$ that one obtained from the minimization condition at $Q = m_{3/2}$ and calculate the required initial condition $\eta_G$ at $M_G$.

We start our numerical investigations with the renormalisation group of the coefficient $\eta(Q)$. Using Eq. (9), in Fig. 1a we plot the coefficient $\eta(Q)$ using as initial value $\eta_G = 0$, for three characteristic choices of the coefficients $\xi_\alpha$,

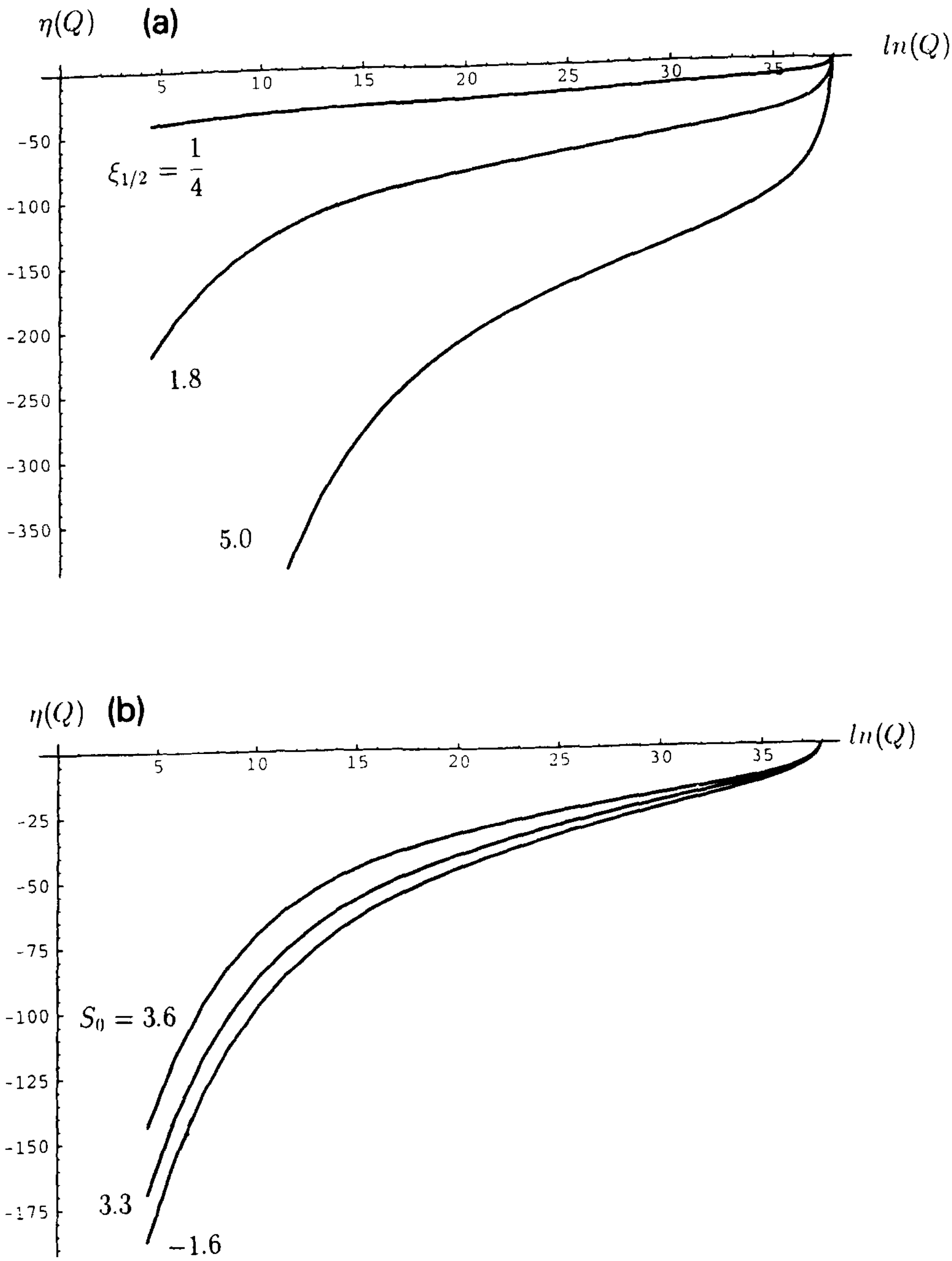


Fig. 1. The running of the parameter $\eta(Q)$ with initial value $\eta(M_G) = 0$. In (a) we plot $\eta$ for three different values of $\xi_{1/2} = \frac{1}{4}$, 1.8 and 5, with all other $\xi$'s fixed. In (b), keeping $\xi_{1/2} = 1.8$ we plot $\eta$ for three values of $S_0 = 3.6, 3.3$ and $-1.6$. In (c), we keep $\xi_{1/2} = 1.8$, all other $\xi$'s, but $\xi_\mu$, fixed and we vary the initial top Yukawa coupling $h_t(M_G) = 1.8, 2.6$ and 3.0.

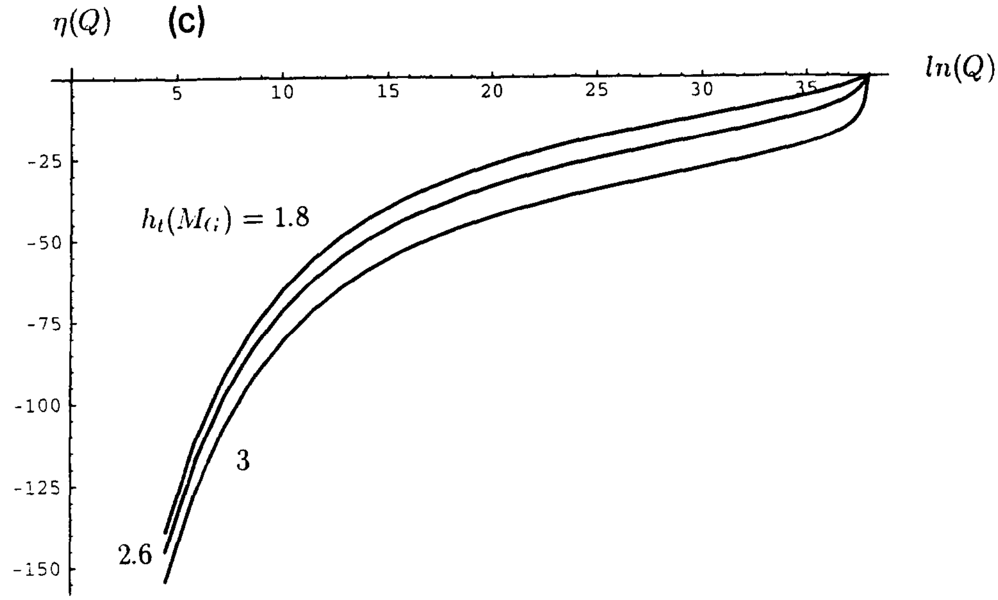


Fig. 1 — continued.

$\alpha = i, \frac{1}{2}, \mu$. By varying them in a reasonable range, we find that a crucial role is played by the choice of the coefficient $\xi_{1/2}$. For each particular choice of $\xi_i$'s, we choose the value of $\xi_\mu$ so as to ensure radiative breaking of the $SU(2) \times U(1)$ symmetry at the low energy scale. From the three curves shown in Fig. 1a, the upper one corresponds to the choice $\xi_{1/2} = \frac{1}{4}$, the middle to the case to $\xi_{1/2} = 1.8$, while the lower to the value $\xi_{1/2} = 5$. We observe that the bigger the coefficient $\xi_{1/2}$, the lower the value of $\eta(Q_Z)$ obtained for the same initial condition $\eta_G$. This is of course expected since larger contributions in the Str $\mathcal{M}^4$, result also to a bigger value of $\eta(Q)$ through Eq. (9). It is clear from Eq. (9) that a different initial condition $\eta_G$ will result to a parallel shift of the obtained curves by the same amount. In Fig. 1b we examine the sensitivity of the $\eta(Q)$ with respect to the $\xi_i$ parameters for given $\xi_{1/2} = 1.8$. We present three cases where the parameter $S_0$ takes the values $3.6, 3.3, -1.6$. Although we observe a significant variation of the $\eta(Q_Z)$ value for the above choices, this is smaller than the one obtained by varying $\xi_{1/2}$. On the other hand, there is no obvious interrelation between $\eta(Q)$ and $S_0$ values. The final $\eta(M_Z)$'s depend solely on the specific choice of $\xi_i$'s. On the contrary, we find a rather interesting correlation between $\eta(Q)$ curves and the top Yukawa coupling. In Fig. 1c we plot curves for $h_{t_G} = 1.8, 2.6$ and $3.0$, while fixing all $\xi_i$'s with $\xi_{1/2} = 1.8$. As can be read off from the the curves, the higher the top coupling the lower the $\eta(M_Z)$ value. This result seems to contradict the one obtained previously in Fig. 1 of Ref. [5]. There the curves appear in the opposite order with respect to $h_{t_G}$ values. This discrepancy is however apparent. The difference lies in the fact that in our case *not* all the initial conditions $\xi_\alpha$ are fixed. In fact we choose to fix only $\xi_i$'s at the GUT scale while we vary $\xi_\mu$ so as to obtain a reasonable transmutation scale at $Q_{\rm trans} \leq 1$ TeV and satisfy the minimization conditions for any chosen set of the parameters $\xi_i$, $\tan\beta$, etc. In particular, choosing a specific value for $h_{t_G}$, we are forced to change the initial value of $\mu$ to compensate for the corresponding negative corrections on $m^2_{H_2}$ Higgs mass and result to a reasonable scale $Q_{\rm trans}$. To make clear the above argument let us present a simple numeric example. Choosing for example $\xi_{Q,U} = 0.4$, $\xi_{1/2} = 1.2$ and universality for the rest of the scalars, for two $h_{t_G}$ values we obtain the results shown in Table 1.

Obviously, when the $Q_{\rm trans}$ are the same, then the bigger the top the lower the $\eta(M_Z)$ value, in accordance with our Fig. 1. If on the other hand one starts with the same initial condition $\mu$ at $M_{GUT}$, then larger $h_{t_G}$ coupling leads to larger $\eta(M_Z)$ values as has been shown in Fig. 1 of Ref. [5].

Table 1

| $h_{tG}$ | $\xi_\mu$ | $\mathrm{Log}_e\{Q_{\mathrm{trans}}\}$ | $\eta(M_Z)$ | $m_t$ [GeV] |
|---|---|---|---|---|
| 3.0 | 2.41 | 8.90 | −72.0 | ∼ 177.5 |
| 2.0 | 2.41 | 6.02 | −75.2 | ∼ 176.0 |
| 3.0 | 2.98 | 6.03 | −81.7 | ∼ 177.5 |

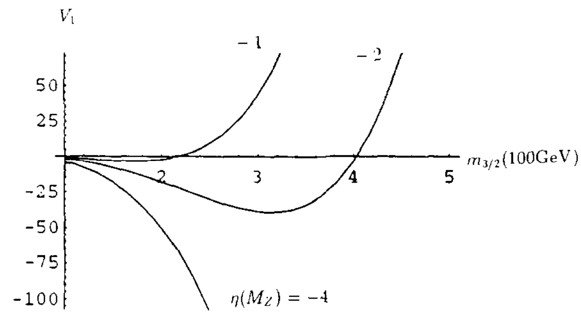


Fig. 2. The potential $V_1(M_Z)$ as a function of $m_{3/2}$ for a selected case where $\xi_{1/2} = 1/4$, $\xi_Q = \xi_U = 2.5$ and $S_0 = -3.5$. The three curves correspond to $\eta(M_Z) = -4, -2, -1$.

In Figs. 2–4 we present our results from the minimization procedure with respect to $m_{3/2}$, varying the value of $\eta(Q \sim M_Z)$ to a range close to the one obtained by the minimization condition of Eq. (10). In our calculations we use $M_G \approx 1.3 \times 10^{16}$ GeV, $\alpha_G \approx 1/24.6$ and a SUSY scale close to $m_{\mathrm{top}}$. The obtained top mass is $m_{\mathrm{top}} \approx 175$ GeV while we take $\tan\beta \approx 1.8$.

In Fig. 2 we plot the low energy effective potential $V_1(M_Z)$ versus $m_{3/2}$ for a selected case where

$$\xi_{1/2} = 0.25, \quad \xi_Q = \xi_U = 2.5 \quad \text{and} \quad S_0 = -3.5$$

and three choices of $\eta_Z = -4, -2, -1$. The electroweak breaking occurs at $Q \sim 450$ GeV. We notice in the graph that in the specific case mentioned above, for $\eta_Z$ in the range $(-1, -3)$, the minimum of $m_{3/2}$ is in the range $(150, 550)$ GeV. Of course, such a low $\xi_{1/2}$ will result low masses for the gauginos, in particular for the larger $\eta_Z$ values of the above range which give the lower $m_{3/2}$ minimum. In Table 2 we give the masses of the SUSY particles scaled with the $m_{3/2}$ mass.

In Figs. 3a,b we present the case where $\xi_{1/2} = 1.8$ while all other $\xi$'s are as before, for two different scales namely $Q \approx M_Z$ and $Q \approx 250$ GeV. The parameter $\eta$ takes the values $(-20, -30, -40, -50)$. Table 3 shows the obtained supersymmetric spectrum scaled again with the $m_{3/2}$. The scale of electroweak breaking is $Q \sim 280$ GeV. Since now $\xi_{1/2}$ is higher we expect the SUSY masses to be heavier than before.

In Fig. 3a, $V(Q, m_{3/2})$ develops a minimum for $n(M_Z) \approx (-30, -50)$ with a corresponding range of $m_{3/2} \approx$ (120–550) GeV. In Fig. 3b, the minimum is

Table 2
The masses of the three gauginos and the other SUSY particles, scaled with the $m_{3/2}$ mass, for the choices $\xi_{1/2} = .25$, $\xi_Q = \xi_U = 2.5$ and $S_0 = -3.5$

| $M_1$ | $M_2$ | $M_3$ | $\tilde{m}_Q$ | $\tilde{m}_U$ | $\tilde{m}_D$ | $\tilde{m}_{t_L}$ | $\tilde{m}_{t_R}$ | $\tilde{m}_L$ | $\tilde{m}_E$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.21 | 0.41 | 1.36 | 1.75 | 1.52 | 1.58 | 1.69 | 1.18 | 1.54 | 0.85 |

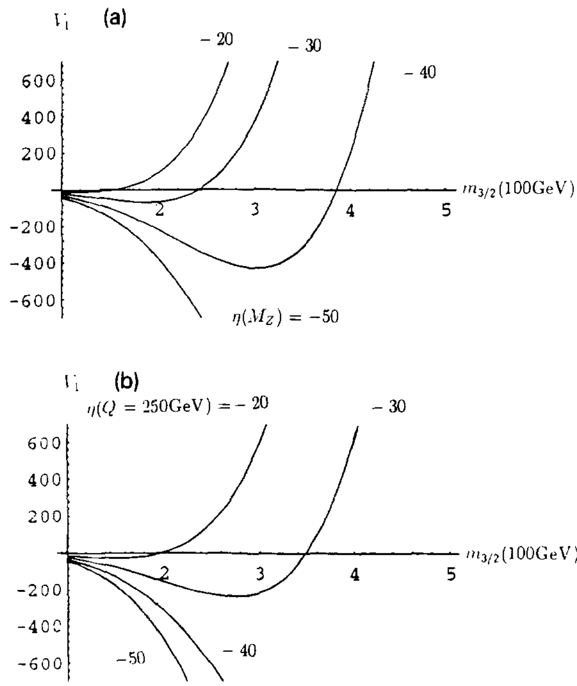


Fig. 3. As in Fig. 2, with $\xi_{1/2} = 1.8$. All other $\xi$'s are the same. In (a) we plot the potential for $Q = M_Z$, while in (b) we plot the potential for $Q = 250$ GeV.

Table 3
The same as in Table 2, for the choices $\xi_{1/2} = 1.8$, $\xi_Q = \xi_U = 2.5$ and $S_0 = -3.5$

| $M_1$ | $M_2$ | $M_3$ | $\tilde{m}_Q$ | $\tilde{m}_U$ | $\tilde{m}_D$ | $\tilde{m}_{t_L}$ | $\tilde{m}_{t_R}$ | $\tilde{m}_L$ | $\tilde{m}_E$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.56 | 1.11 | 3.65 | 3.55 | 3.35 | 3.37 | 3.32 | 2.73 | 2.33 | 0.98 |

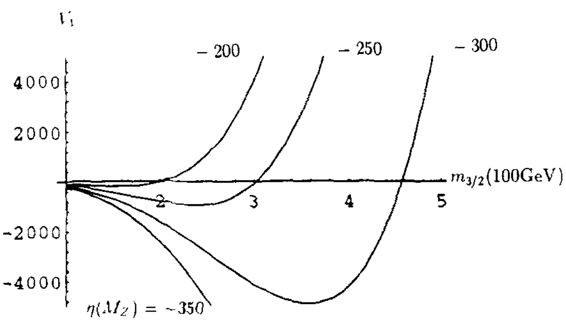


Fig. 4. As in Fig. 2, with $\xi_{1/2} = 5$, $\xi_Q = \xi_U = 0.8$ and $S_0 = 0.3$. The curves correspond to $\eta = -200, -250, -300, -350$.

obtained for larger $\eta(Q)$ values being now in the range $\eta(M_Z) \approx (-25, -35)$. The minimum at $m_{3/2} = 300$ GeV corresponds to $\eta(M_Z) \approx -40$ in the first case (Fig. 3a) and to $\eta(250\text{ GeV}) \approx -31$ for the second (Fig. 3b). Again as $\eta$ shifts to lower values, the minimum of $m_{3/2} \to \infty$. Notice that within the above range, $V_1(Q)$ is stable with respect to the scale $Q$ as expected.

Fig. 4 represents a case with relatively large value of $\xi_{1/2} = 5.0$ and $\xi_Q \approx \xi_D = 0.8$ and $S_0 = -0.3$. All the relevant parameters are calculated at $Q = M_Z$, while the curves correspond to $\eta = (-200, -250, -300, -350)$. Finally we wish to point out that the cosmological coefficient receives naturally small values close to zero only in the first case, namely when $m_{1/2} \leq m_{3/2}$. From this point of view, a vanishing cosmological constant at $Q \sim M_Z$ would require a considerable fine tuning of the various parameters.

The three cases chosen above are in correspondence with the curves obtained from the renormalisation group running of the $\eta$-coefficient. Comparing the results with Fig. 1, it can be seen that large positive $\eta_G \gtrsim \mathcal{O}(100)$ values are required in order for the $\eta(Q)$ value obtained from the RGE running to match with the low energy $\eta$'s consistent with the minimization condition. The larger the value of $\xi_{1/2}$, the higher the $n_G$ value required to obtain reasonable $m_{3/2}$ values dynamically. Remarkably enough, it is possible in specific supersymmetry breaking scenarios, to obtain such a large initial value for $\eta_G$ already at the classical level [11].

In conclusion, we have discussed in detail the implications of the minimization of the vacuum energy with respect to the gravitino mass. We have shown that the requirement of determining a hierarchically consistent gravitino mass dynamically, leads to useful constraints in low energy and unification scale physics. In particular, we have seen that the existence of a $V$-minimum with respect to $m_{3/2}$ necessitates the inclusion of the one loop corrections and of the cosmological term $\eta(Q)m_{3/2}^4$, remnant from the underlying supergravity or string theory. Furthermore the minimization of the vacuum energy can naturally lead to $m_{3/2}$ values at the order of the electroweak scale $m_{3/2} \sim (100\text{–}500)$ GeV and acceptable supersymmetric mass spectrum, in particular if $m_{1/2} > m_{3/2}$. Further constraints are also put on the $\eta_Z$ parameter which can be easily converted to constraints for the initial value of the cosmological coefficient $\eta_G \equiv \eta(Q = M_G)$. In particular, small $\eta_G$ values as required by specific string models are compatible with $m_{1/2} \leq m_{3/2}$ and small deviations from the universality condition for the scalars. In this case a sparticle spectrum compatible with the experimental bounds, requires $m_{3/2} \gtrsim (3\text{–}4) \times M_Z$.

It is interesting that the above minimization procedure may also apply to other undetermined parameters of the standard model, i.e. Yukawa couplings and fermion masses [5,12,13].

We have benefited from discussions with I. Antoniadis, S. Dimopoulos C. Kounnas and F. Zwirner. G.K.L would like to thank the CERN Theory Division for hospitality during the early stages of this work.

## References

[1] S. Dimopoulos and H. Georgi, Nucl. Phys. B 193 (1981) 150;
H.P. Nilles, Phys. Rep. 110 (1984) 1;
H.E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 75;
A.B. Lahanas and D.V. Nanopoulos, Phys. Rep. 145 (1987) 1;
S. Ferrara, ed., supersymmetry (North-Holland, Amsterdam, 1987);
F. Zwirner, in Proceedings of the 1991 Summer School in High Energy Physics and Cosmology, Trieste, 17 June, 9 August 1991 (E. Gava, K. Narain, S. Randjbar-Daemi, E. Sezgin and Q. Shafi, eds.), Vol. 1, p. 193.

[2] S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B 429 (1994) 589, and references therein.

[3] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, Phys. Lett. B 133 (1983) 61;
J. Ellis, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis,

Phys. Lett. B 134 (1984) 429;
J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B 241 (1984) 406; B 247 (1984) 373.
[4] K.S. Narain, Phys. Lett. B 169 (1986) 41;
W. Lerche, D. Lüst and A.N. Schellekens, Nucl. Phys. B 287 (1987) 477;
H. Kawai, D.C. Lewellen and S.-H.H. Tye, Nucl. Phys. B 288 (1987) 1;
I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. B 289 (1987) 87;
I. Antoniadis and C. Bachas, Nucl. Phys. B 298 (1988) 1.
[5] C. Kounnas, I. Pavel and F. Zwirner, Phys. Lett. B 335 (1994) 403.
[6] G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. B 331 (1990) 331;
C. Kounnas, in Properties of SUSY particles (World Scientific, Erice proceedings, 1994), p. 496.
[7] S. Kelley, J.L. Lopez, D.V. Nanopoulos and A. Zichichi, CERN preprint CERN-TH.7433.
[8] I. Antoniadis, Phys. Lett. B 246 (1990) 377.
[9] G.K. Leontaris, Phys. Lett. B 317 (1993) 569;
G.K. Leontaris and N.D. Tracas, Phys. Lett. B 342 (1995) 163.
[10] L.E. Ibanez and D. Lüst, Nucl. Phys. B 382 (1992) 305;
A. Lleyda and C. Muñoz, Phys. Lett. B 317 (1993) 82;
T. Kobayashi, D. Suematsu and Y. Yamagishi, Kanazawa report, 94-06;
D. Mataliotakis and H.P. Nilles, Munich preprint TUM-HEP-201/94;
M. Carena and C.E.M. Wagner, CERN preprint CERN-TH.7393/94;
N. Polonsky and A. Pomarol, Pennsylvania preprint UPR-0627T, 1994.
[11] C. Bachas, Ecole Polytechnique preprint, ph-th 9503030.
[12] C. Kounnas, I. Pavel, G. Ridolfi and F. Zwirner, CERN-TH. preprint, in preparation.
[13] P. Binetruy and E. Dudas, LPTHE Orsay 94/73.