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Low energy thresholds and the scalar mass spectrum in minimal supersymmetry

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Abstract

We discuss low energy threshold effects and calculate the sparticle masses in the context of the Minimal Supersymmetric Standard Model. We pay particular attention to the top squark and the Higgs mass parameters, and calculate the top Yukawa corrections, taking into account the successive decoupling of each particle at its threshold. We discuss the phenomenological implications in the context of the radiative symmetry breaking scenario.

The Minimal Supersymmetric Standard Model [2] (MSSM) has by now been accepted as the most natural extension of the Standard Model of Strong and Electroweak Interactions. As the recent experiments [2] approach closer to the energies where some of the superpartners seems to acquire their masses, it is very important to obtain higher precision in the theoretical predictions of the scalar mass parameters, Yukawa coupling corrections, threshold effects, etc. In recent analyses [4-8], it has been shown that the semi-analytic procedure in the calculations of the above quantities can offer the possibility of investigating reliably the above effects. Moreover, the advantage of analytic expressions for the low energy parameters is more than obvious: one can extract easily information about the role of the input parameters at the GUT scale (m_0, $m_{1/2}$, h_{t_G}), since in the analytic procedure the low energy measurable quantities can be expressed in terms of calculable functions of the former with the boundary conditions incorporated into these expressions. Nevertheless, detailed theoretical predictions may be still pushed further by estimating higher order effects, threshold corrections, etc.

In recent works it has been shown that the unification scenario [3] survives even when various uncertainties arising from several sources (GUT and low energy SUSY thresholds, input values for coupling constants, experimental uncertainties, etc.) are taken into account [4,6,9,10]. For example, in Ref. [4], it has been shown that an effective low energy SUSY scale can be defined which, for a realistic mass spectrum can account for low energy threshold effects. More recently [11] a more accurate way to estimate the uncertainties of such effects, which may also take into account two-loop corrections, has been explored.

Predictions of low energy parameters turn out to be very sensitive in all the above mentioned threshold effects. In estimating these effects, it has been

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shown [12] that it is adequate to use the "step" approximation in the definition of the beta function coefficients. Thus the values of the weak mixing angle ($\sin^2\theta_W$), the electromagnetic (α_{em}) and the strong (α_3) couplings, and other low energy parameters can be given by their one-loop formula with the addition of two small correction terms arising from two-loop and threshold effects in a semi-analytic procedure [4]. In calculating however the scalar masses themselves, one should be careful in particular for those affected from the top-Yukawa coupling h_t. In this case due to the non-negligible contribution of h_t, the evolution of $m_{\tilde{t}}^2$-squark and m_{H_2} mass parameters are determined by a coupled differential equation system. Thus in addition to the successive changes of the gauge coefficients as the sparticles decouple, the boundary conditions at each sparticle's threshold should be treated carefully.

In the present analysis, we wish to investigate the above effects in the context of the MSSM assuming that the gauge couplings unify in a simple non-abelian gauge group at an energy scale close to 10^{16} GeV. We find it useful to adopt a semi-analytic procedure and provide specific formulae for all the involved parameters, and compare them with those of previous estimates where such effects were not taken into account.

In particular the following issues will be discussed. We will start assuming that the radiative symmetry breaking (RSB) scenario [13] is an effective mechanism operating in the usual sense at low energy, i.e. by driving one of the Higgs mass-squared parameters negative at energies close to m_Z. We are going to use one-loop corrections to the effective potential and estimate the effects in the $|\mu|$-parameter which plays an essential role. Next we calculate the exact contributions of the trilinear parameter A and compare the results with previous estimates where these corrections were not included. Finally, we are going to calculate the scalar masses for various choices of the initial values $m_0, m_{1/2}$ taking into account the afore-mentioned threshold effects.

Starting at the GUT scale with a particular gauge group, one chooses specific values for five independent parameters, namely m_0, $m_{1/2}$, μ, A and B. In the simplest case, all the scalars have a universal [3] mass m_0. The masses evolve down to low energies where one expects that one Higgs mass-squared parameter becomes negative. This triggers the $SU(2)\times U(1)$ symmetry breaking. The calculation of the mass-squared parameters needed to check if this scenario is valid, requires the solution of the coupled differential R.G. equations obeyed by these parameters.

In the case of the small $\tan\beta \sim \mathcal{O}(1)$ scenario ($\tan\beta$ is the ratio of the two v.e.v.'s), one may approximate the relevant differential equations as follows

$$\frac{dm_{\tilde{t}_L}^2}{dt} = \frac{1}{8\pi^2}\left(h_t^2\left(m_{H_2}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A^2\right) - \sum_{i=1}^{3} c_i^Q g_i^2 M_i^2\right) \tag{1}$$

$$\frac{dm_{\tilde{t}_R}^2}{dt} = \frac{1}{8\pi^2}\left(2h_t^2\left(m_{H_2}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A^2\right) - \sum_{i=1}^{3} c_i^U g_i^2 M_i^2\right) \tag{2}$$

$$\frac{dm_{H_2}^2}{dt} = \frac{1}{8\pi^2}\left(3h_t^2\left(m_{H_2}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A^2\right) - \sum_{i=1}^{3} c_i^H g_i^2 M_i^2\right) \tag{3}$$

$$\frac{dm_{H_1}^2}{dt} = \frac{1}{8\pi^2}\left(-\sum_{i=1}^{3} c_i^H g_i^2 M_i^2\right) \tag{4}$$

$$\frac{dA}{dt} = \frac{1}{8\pi^2}\left(6h_t^2 A - \sum_{i=1}^{3} c_i^A g_i^2 M_i\right) \tag{5}$$

where only the top quark Yukawa coupling h_t has been kept. The coefficients c_i are given by

$$c_i^Q = \{\tfrac{1}{15}, 3, \tfrac{16}{3}\}, \quad c_i^U = \{\tfrac{16}{15}, 0, \tfrac{16}{3}\}$$
$$c_i^H = \{\tfrac{3}{5}, 3, 0\}, \quad c_i^A = \{\tfrac{13}{15}, 3, \tfrac{16}{3}\}$$

while M_i's are the gaugino masses and $t = \ln Q$.

The differential equation for H_1 can be solved straightforwardly, since it is independent from the others. The remaining four differential equations define a coupled system which depends strongly on the top Yukawa coupling. Making the identifications

$$m_{\tilde{t}_L} \equiv \tilde{m}_1, \quad m_{\tilde{t}_R} \equiv \tilde{m}_2, \quad m_{H_2} \equiv \tilde{m}_3$$

[3] For recent discussions where non-universal conditions at the GUT scale are assumed see [14].

the solution of the system is found to be

$$m_{H_1}^2(t) = m_0^2 + C_3(t)m_{1/2}^2 \tag{6}$$

$$\tilde{m}_n^2(t) = m_0^2 + C_n(t)m_{1/2}^2 - n\delta_m^2(t) - n\delta_A^2(t) \tag{7}$$

$$A(t) = q^{-1}(t)\left(A_G - m_{1/2}I_A(t)\right) \tag{8}$$

where all the relevant functions are presented in the Appendix.

The scale dependence of the Yukawa coupling $h_t(t)$ can be found by solving the RGE for that coupling

$$\frac{dh_t}{dt} = \frac{1}{8\pi^2}\left(6h_t^2 - \sum_{i=1}^{3} c_i g_i^2\right) h_t, \quad \text{where } c_i = \{\tfrac{13}{15}, 3, \tfrac{16}{3}\} \tag{9}$$

with the well known solution

$$h_t(t) = h_{t_G}\gamma_U(t)q^{-1/2}(t) \tag{10}$$

In all previous equations, the subscript G denotes the corresponding value at the GUT scale.

There are four arbitrary parameters entering the above formulae, namely m_0, $m_{1/2}$, A_G and h_{t_G}. The first three of them, as was already pointed out, are the soft input mass parameters at the unification scale M_G. Since the above solutions enter the minimization of the Higgs potential $\mathcal{V}(H_1, H_2)$, their range can be phenomenologically constrained by the requirement of generating a stable minimum for this potential. Of course, a crucial role is played also by the fourth parameter, h_{t_G}, which should be large enough to drive the H_2-Higgs mass-squared parameter negative and give a phenomenologically acceptable vacuum.

Experimental evidence [15] as well as theoretical expectations [16] treating the Yukawa couplings as dynamical variables, indicate that the top mass requires a large top-Yukawa coupling, close to its infrared fixed point, i.e. $(m_t/\sin\beta) \sim 190$ GeV. Therefore, since only the ratio $m_t/\sin\beta$ enters in the relevant running scalar mass parameters, one may safely conclude that in the fixed point solution for the top-mass, their values depend mainly on the initial values m_0 and $m_{1/2}$. The above argument may be more transparent if one writes the above mass-formulae in the limit of the infrared fixed point of h_t. One then obtains [8]

$$\tilde{m}_n^2(t) = \left(1 - \frac{n}{2}\right) m_0^2 + \left[C_n(t) - \frac{n}{6}\frac{J(t)}{I(t)}\right] m_{1/2}^2 \tag{11}$$

The contribution of the trilinear parameter A introduces one more input parameter at the GUT scale, but its role is less significant as long as A_G is of the order of m_0 (as expected). Indeed, by writing $A_G = A_0 m_0$, after some algebraic manipulations one can show that δ_A^2 is written as follows

$$\delta_A^2(t) = \frac{1}{q(t)}\int_{t_G}^{t} q(t')d\left(\Delta_A^2\right) = -\frac{1}{q(t)}\int_{t_G}^{t} q(t')\frac{h_t^2(t')}{8\pi^2}A^2(t')dt' \tag{12}$$

Expanding $A^2(t)$, using Eq. (8), we write formally

$$\delta_A^2 = \delta_{A_1}^2 A_0^2 m_0^2 + \delta_{A_2}^2 A_0 m_0 m_{1/2} + \delta_{A_3}^2 m_{1/2}^2 \tag{13}$$

where

$$\delta_{A_1}^2(t) = -\frac{1}{6}\frac{1}{q(t)}\left(\frac{1}{q(t)} - 1\right) \tag{14}$$

$$\delta_{A_2}^2(t) = -\frac{1}{6}\frac{1}{q(t)}\left(\frac{1}{q(t)}I_A(t) - \gamma_A(t)\right) \tag{15}$$

$$\delta_{A_3}^2(t) = -\frac{1}{6}\frac{1}{q(t)}\left(\frac{1}{q(t)}I_A^2(t) - 2\gamma_A(t)I_A(t) + 2I_A'(t)\right) \tag{16}$$

$$I_A'(t) = \int_{t_G}^{t} q(t')C_A(t')\gamma_A(t')dt' \tag{17}$$

$$\gamma_A(t) = \int_{t_G}^{t} C_A(t')dt' = \sum_{i=1}^{3}\frac{c_i^A}{b_i}\left(\frac{\alpha_i(t)}{\alpha_{i_G}} - 1\right) \tag{18}$$

The coefficients δ_{A_i} depend on simple integrals of scale dependent parameters. Evaluation of the relevant integrals give the following results (for $m_t \sim 175$ GeV and SUSY breaking $M_S = (500 - 1500)$ GeV)

$$\delta_A^2 = 0.005A_0^2 m_0^2 - 0.020A_0 m_0 m_{1/2} + (0.146 - 0.155)m_{1/2}^2 \tag{19}$$

which add too small corrections to the solutions where A was ignored. Moreover, these corrections become even smaller [5,7] as m_t approaches its infrared fixed point value.

In the above equation, the simplified assumption was made that all scalar masses decouple at the same scale, namely M_S. In the running of the RGE's it is assumed that there is a great "desert" between the GUT and the weak scale while a low energy SUSY scale M_S is assumed so that for energies lower than M_S one uses the Standard Model beta function and c_i coefficients. However, as has been already pointed out, a more careful treatment should also take into account threshold effects due to the successive decoupling of these scalar masses from the spectrum at different scales. In the semi-analytic approach of Ref. [4] an effective scale $M_S^{\rm eff}$ was assumed which can be roughly estimated to be

$$M_S^{\rm eff} = \left(\frac{\alpha_2(m_{\tilde{W}})}{a_3(m_{\tilde{g}})}\right)^{\frac{28}{19}} |\mu| \approx \tfrac{1}{5}|\mu| \tag{20}$$

to account for the SUSY scalar mass effects. The $|\mu|$ parameter can also be given in terms of known parameters by solving the minimization conditions of the neutral Higgs potential. Taking also into account one-loop contributions to the superpotential, we may obtain the following approximate formula for $|\mu|$ [18]

$$|\mu| = \sqrt{(\mu_0^2 + \eta^2)/(1 - \Omega^2)} \tag{21}$$

In deriving the above, the approximation $\ln m_{\tilde{t}_1}^2 \sim \ln m_{\tilde{t}_2}^2 \sim \ln\langle m_{\tilde{t}}^2\rangle$ has been used ($m_{\tilde{t}_1}^2$ and $m_{\tilde{t}_2}^2$ are the eigenvalues of the stop mass matrix). $|\mu_0|$ is the tree level contribution while η and Ω are defined in the Appendix. For $\tan\beta \geqslant 1.1$, $|\mu|$ is less than 1.5 GeV. One therefore, for sensible values of the parameter $|\mu| \leq (1-2)$ TeV, could define a reasonable scale $M_S^{\rm eff}$; below that scale the beta function coefficients should turn to their non-supersymmetric form.

The precise effects, however, are found by the successive change of all the beta function dependent coefficients at each particle's threshold. Assuming only one-loop corrections, in the Minimal Supersymmetry with three families and two Higgses, one can write the $b_i's$ in the following form [4,17,19]

$$\begin{aligned} b_1 = {}& \tfrac{4}{3}n_g + \tfrac{1}{10}n_H^{SM} + \tfrac{2}{5}\theta_{\tilde{H}} + \tfrac{1}{10}\theta_{H_2} \\ & + \tfrac{1}{5}\sum_{i=1}^{3}\Big[\tfrac{1}{12}(\theta_{\tilde{u}_{L_i}} + \theta_{\tilde{d}_{L_i}}) + \tfrac{4}{3}\theta_{\tilde{u}_{R_i}} + \tfrac{1}{3}\theta_{\tilde{d}_{R_i}} \\ & + \tfrac{1}{4}(\theta_{\tilde{e}_{L_i}} + \theta_{\tilde{\nu}_{L_i}}) + \theta_{\tilde{e}_{R_i}}\Big] \end{aligned} \tag{22}$$

$$\begin{aligned} b_2 = {}& -\tfrac{22}{3} + \tfrac{4}{3}n_g + \tfrac{1}{6}n_H^{SM} + \tfrac{4}{3}\theta_{\tilde{W}} \\ & + \tfrac{2}{3}\theta_{\tilde{H}} + \tfrac{1}{6}\theta_{H_2} + \tfrac{1}{2}\sum_{i=1}^{3}(\theta_{\tilde{u}_{L_i}}\theta_{\tilde{d}_{L_i}} + \tfrac{1}{3}\theta_{\tilde{e}_{L_i}}\theta_{\tilde{\nu}_{L_i}}) \end{aligned} \tag{23}$$

$$\begin{aligned} b_3 = {}& -11 + \tfrac{4}{3}n_g \\ & + 2\theta_{\tilde{g}} + \tfrac{1}{6}\sum_{i}^{3}\left[\theta_{\tilde{u}_{L_i}} + \theta_{\tilde{u}_{R_i}} + \theta_{\tilde{d}_{L_i}} + \theta_{\tilde{d}_{R_i}}\right] \end{aligned} \tag{24}$$

In the above formulae for b_i's, $\tilde{H}$ stands for the higgsino contribution $\tilde{W}$ for the winos, etc., while for any particle's threshold with mass $m_{s_i}^2$, we have denoted $\theta_{s_i} \equiv \theta(Q^2 - m_{s_i}^2)$.

In our semi-analytic approach, when evolving the gauge and Yukawa couplings as well as the scalar mass parameters down to low energies, we find it sufficient to define the following b_i-changing scales: We assume a common scale Q_L for the decoupling of $\tilde{u}_{L_{1,2}}, \tilde{d}_{L_{1,2}}$ sparticles, while we assume that they are not very different from their mass eigenstates. The next scale is the one defined as an average scale Q_R of their right-handed partners. In the case of the universal scalar masses at the GUT scale, however, these two scales could not differ substantially unless $m_{1/2}$ is very large ($\geqslant$ 1 TeV). Therefore, the thresholds arising between these two scales are not expected to have a significant effect.

We define as a third scale (subsequently denoted with $t_1 = \ln\tilde{m}_1$) where we change the b_i and c_i coefficients, the scale where the t_L-squark acquires its mass. As it is expected, due to the large negative contributions from the top-quark Yukawa coupling, this mass should be substantially smaller than those of the L, R squarks of the first two generations. Finally, we define two more new scales above the weak and top mass scale, namely the scale were the t_R-squark gets its mass (subsequently denoted with $t_2 = \ln\tilde{m}_2$) and an average scale for all other contributions (sleptons etc.). The hierarchy of these two latter scales depends strongly on the point of the parameters space $(m_0, m_{1/2})$ one has chosen. A simple inspection of the obtained evolution equations for their masses shows that slepton masses are larger than $m_{\tilde{t}_R}$ for m_0 values much bigger than $m_{1/2}$ while the opposite is true when $m_{1/2} > m_0$. Of course all the above scales should be carefully incorporated in the analytic formulae presented above. All the relevant integrals (see Appendix) should split into sums over the various scales.

Our next step is the determination of the $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ mass parameters which also define the aforementioned scales where the beta function coefficients should also change. For a given $(m_0, m_{1/2})$ pair, one can compute the left and right squark masses of the first two generations. The negative corrections $\delta_m^2(t_1)$ may then determine the mass of the $\tilde{t}_L$-squark. Below this scale, top-Yukawa negative corrections should not include contributions from diagrams involving $\tilde{t}_L$. Therefore in the range defined by $m_{\tilde{t}_L} \geqslant Q \geqslant m_{\tilde{t}_R}$ (i.e. for the range (t_1, t_2)) the evolution equations for $m_{\tilde{t}_R}^2 \equiv \tilde{m}_2^2$ and $m_{H_2}^2 \equiv \tilde{m}_3^2$ can be written as follows

$$\tilde{m}_2^2 = m_2^2(t_1) + C_2(t_1, t)\tilde{m}_{1/2}^2 - 2\delta(t_1, t) \qquad (25)$$

$$\tilde{m}_3^2 = m_3^2(t_1) + C_3(t_1, t)\tilde{m}_{1/2}^2 - 3\delta(t_1, t) \qquad (26)$$

where $\tilde{m}_n^2(t_1)$ are the mass parameters calculated at the scale $t_1 = ln(m_{\tilde{t}_L})$. Top-Yukawa corrections $\delta(t_1, t)$ contain now the sum only of t_R-squark and the H_2 Higgs mass parameters, and possibly (depending on the specific values of the $(m_0, m_{1/2})$ pair) the $A(t)$ trilinear parameter,

$$\delta(t_1, t) = \int_{t_1}^{t} \frac{h_t^2(t')}{8\pi^2}\left(\sum_{n=2}^{3} m_n^2(t') + \theta(t' - t_A)A^2(t')\right)dt' \qquad (27)$$

where t_A defines the logarithm of the scale at which the trilinear mass parameter stops running. The above corrections can be calculated easily, by solving (25), (26). The result is

$$\delta(t_1, t) = -q^{-\frac{5}{6}}(t_1, t)\int_{t_1}^{t} q^{\frac{5}{6}}(t_1, t')u_0(t')\frac{h_t^2(t')}{8\pi^2}dt' \qquad (28)$$

where

$$u_0(t) = 2m_0^2 + [C_2(t_1, t) + C_3(t_1, t)]\, m_{1/2}^2 - 5[\delta_m^2(t_1) - \delta_A^2(t_1)] \qquad (29)$$

After the decoupling from the massless spectrum of the t_R-squark at the scale $t_2 = \ln(m_{\tilde{t}_R})$, one ends up with only the Higgs mass parameter whose evolution from t_2 until the transmutation scale is given by the formula

$$\tilde{m}_3^2(t) = \tilde{m}_3^2(t_2) + C_3(t_2, t)m_{1/2}^2 - 3\,q^{-\frac{1}{2}}(t_2, t)\int_{t_2}^{t} q^{tfrac12}(t_2, t')v_0(t')\frac{h_t^2(t')}{8\pi^2}dt' \qquad (30)$$

where

$$v_0(t) = \tilde{m}_0^2(t_2) + C_3(t_2, t)m_{1/2}^2 \qquad (31)$$

In Tables 1 and 2 we present the calculated scalar mass spectrum for two initial values of the h_{t_G} (case 2, very close to its infrared fixed point value), and representative choices of $m_0, m_{1/2}$ pairs. The last column of these tables shows the prediction for α_s low energy parameter, for each sparticle spectrum. In obtaining our results, we have worked in the low $\tan\beta$ regime while we have allowed 10% deviations from the GUT relation $h_b(t_G) = h_\tau(t_G)$. On the other hand, the obtained values for $\sin^2\theta_W$ and m_t are consistent with the relation $\sin^2\theta_W(m_Z) = 0.2324 - 10^{-7} \times \{(m_t/\text{GeV})^2 - 143^2\} \pm 0.0003$.

As it can be inferred from the tables, the $t_{L,R}$-squarks and the average sparticle spectrum is lighter for larger top couplings and top mass. When m_0 is relatively small, slepton masses are the lighter sparticles while $m_{\tilde{t}_R}$ becomes light when $m_{1/2} \ll m_0$. Of course, right sleptons (not shown in the tables) have slightly smaller masses than their left partners.

Now let us discuss the effects of the decoupling of the heavier quarks in the rest of the sparticle spectrum. The successive decoupling of each scalar mass term from the relevant differential equation, has modified the negative corrections induced by the top-coupling below the t_L-squark mass. In the low $\tan\beta$ scenario this treatment has a direct effect only on the t_R-squark and the $m_{H_2}^2$ mass parameters. In particular, in our semi-analytic treatment we observe that the $\tilde{t}_R$ mass has increased by $(1-5)\%$, relative to a naive treatment of the boundary conditions at each particle's threshold [5], depending on the specific choice of the $(m_0, m_{1/2})$ point. Such corrections are therefore of the same order but with the opposite sign of the A-trilinear

Table 1
The supersymmetric sparticle spectrum together with the corresponding α_s prediction in the low $\tan\beta$ scenario ($\tan\beta \sim 1.5$) and $m_t \leq 165$ GeV, fixing $\sin^2\theta_W$ around its central value $\sim .232$

m_0	$m_{1/2}$	m_{Q_L}	m_{Q_R}	$m_{\tilde{t}_L}$	$m_{\tilde{t}_R}$	$m_{\tilde{l}}$	α_s
338	423	950	915	853	697	450	.118
250	359	800	770	722	596	357	.117
255	306	700	675	627	513	335	.116
110	324	700	672	640	539	256	.115
182	189	450	435	400	325	225	.114
322	100	400	395	316	192	340	.114
311	100	380	375	301	187	319	.114
280	95	350	345	280	178	288	.114
246	113	350	343	290	204	340	.114

Table 2
The supersymmetric sparticle spectrum together with the corresponding α_s prediction for h_{t_G} Yukawa coupling close to its non-perturbative value, and $m_t \sim 170$ GeV. Again $\sin^2\theta_W$ has been fixed to its central value $\sim .232$

m_0	$m_{1/2}$	m_{Q_L}	m_{Q_R}	$m_{\tilde{t}_L}$	$m_{\tilde{t}_R}$	$m_{\tilde{l}}$	α_s
251	359	800	770	718	588	356	.117
256	306	700	675	624	505	335	.116
488	236	700	685	576	395	515	.118
638	135	700	695	525	241	645	.119
73	277	600	577	547	451	450	.114
53	254	551	529	503	424	185	.114
182	188	450	435	399	320	226	.116
322	102	400	395	313	180	340	.114

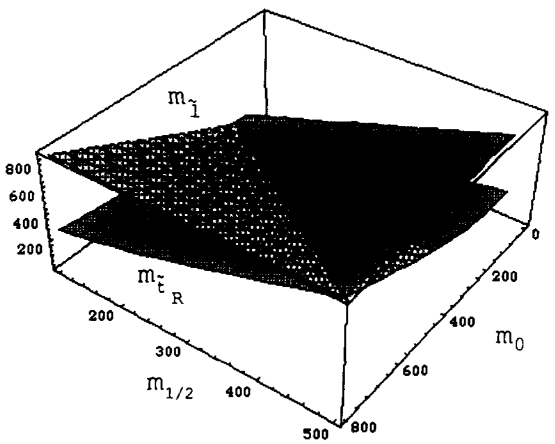

Fig. 1. Surfaces of $m_{\tilde{l}}$ and $m_{\tilde{t}_R}$ showing their variation as a function of m_0 and $m_{1/2}$.

term corrections given by the formula (8). The maximal change occurs when the top Yukawa coupling gets closer to its fixed point value (Table 2). The effect however is still small, since the modified equation (25) runs only on a small region, namely between the $\tilde{m}_{t_L}$ and $\tilde{m}_{t_R}$ scales. A larger effect is found in the m_{H_2} running mass whenever the transmutation scale is substantially different from $\tilde{m}_{t_L}$.

Finally, as it has been pointed out above, the $\tilde{t}_R$ mass is smaller than the slepton masses in specific regions of the $(m_0, m_{1/2})$ parameter space. A qualitative picture of the $m_{\tilde{t}_R}$ and $m_{\tilde{l}}$ variation in terms of m_0 and $m_{1/2}$ is given in the figure.

We can see that there is a considerable fraction of the parameter space $(m_0, m_{1/2})$ which allows solutions of relatively small $m_{\tilde{t}_R}$. For example, in the last entry of Table 2, $m_{\tilde{t}_R}$ is of the order of the top-quark mass. This would imply that, after the diagonalisation of the squark mass matrix, the light physical mass eigenstate $m_{\tilde{t}_1}$ could be as small as 150 GeV. This gives hope that future experiments may discover supersymmetric signatures.

To summarize our results, we have used a semi-analytic approach to calculate the supersymmetric spectrum in the small $\tan\beta$ regime, taking into account low energy threshold effects. We have given special emphasis to top squark and Higgs mass parameter calculation, which in the presence of a heavy top quark receive large negative contributions. We have examined in detail the effects of the "decoupling" of the heavier sparticles from the renormalization group equations of the lighter ones, and we have found that our treatment of the boundary conditions, results in an increase $(1-5)\%$ of the $\tilde{t}_R$ mass parameter, compared to a naive treatment. Corrections on the scalar masses from the trilinear parameter A are treated also analytically and found to be of the same order for moderate initial values ($A_G \sim -\sqrt{3}m_0$) Furthermore, we have examined general properties of the sparticle spectrum and observed interesting correlations. Thus, large values of m_0 compared to that of $m_{1/2}$ imply that $m_{\tilde{t}_R}$ is lighter than the left slepton masses, while the opposite is true for $m_{1/2} > m_0$. Moreover, in the small $\tan\beta$ regime that we are examining here, for a considerable fraction of the $(m_0, m_{1/2})$ space, a light t-squark (~ 150 GeV) can be obtained, which might be found in accessible energies by experiment in the near future.

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Appendix

The scale dependent coefficients in the scalar mass solutions (6,7,8) are given by the following general formula

$$C_n(t_1,t) = \sum_{i=1}^{3} \frac{c_i^n}{2b_i\alpha_{i_G}^2}\left(\alpha_i^2(t) - \alpha_i^2(t_1)\right),\quad n = 1,2,3 \qquad (32)$$

with the identifications $C_1 \equiv C_Q$, $C_2 \equiv C_D$ and $C_3 \equiv C_{H_2}$. We define $C_n(t) \equiv C_n(t_G,t)$.

The gauge dependent functions $\gamma_U(t)$, and $q(t)$ which arise from the top-Yukawa differential equation have the form

$$\gamma_U(t) = \prod_n \prod_{i=1}^{3}\left(\frac{\alpha_i(t_n)}{\alpha_i(t_{n-1})}\right)^{c_i^n/2b_i^n} \qquad (33)$$

$$q(t_1,t) = 1 + \frac{3h_{t_G}^2}{4\pi^2} I(t) = 1 - \frac{3h_{t_G}^2}{4\pi^2}\int_{t_1}^{t} \gamma_U^2(t')dt' \qquad (34)$$

where the index n runs over all the intermediate scales. Again we define $q(t) \equiv q(t_G,t)$.

The negative Yukawa contributions of Eq. (7) are found to be [5]

$$\delta_m^2(t) = \left(\frac{m_{\rm top}(t)}{2\pi v\gamma_U\sin\beta}\right)^2\left(3m_0^2 I(t) + m_{1/2}^2 J(t)\right), \qquad (35)$$

$$\delta_A^2(t) = \Delta_A^2(t) - \frac{3}{2}\left(\frac{m_{\rm top}(t)}{\pi v\gamma_U\sin\beta}\right)^2 E_A^2 \qquad (36)$$

where v = 246 GeV, while the quantities I, J, I_A are functions of scale dependent integrals given by

$$I(t) = -\int_{t_G}^{t} \gamma_U^2(t')dt',$$

$$J(t) = -\sum_{i=1}^{3}\int_{t_G}^{t} \gamma_U^2(t')C_i(t')dt' \qquad (37)$$

$$\Delta_A^2(t) = \int_{t}^{t_G} \frac{h_t^2(t')}{8\pi^2}A^2(t')dt',$$

$$E_A^2(t) = \int_{t}^{t_G} \gamma_U^2(t')\Delta_A^2(t')dt' \qquad (38)$$

$$I_A(t) = \int_{t_G}^{t} q(t')C_A(t')dt' = \frac{1}{2\pi}\sum_{i=1}^{3} c_i^A\alpha_{i_G}\int_{t_G}^{t} q(t')\frac{\alpha_i^2(t')}{\alpha_{i_G}^2}dt' \qquad (39)$$

The minimization conditions of the tree-level neutral Higgs potential give the following solution for the $|\mu_0|$-parameter [8]

$$|\mu_0| = \frac{1}{\sqrt{2}}\left\{\frac{k^2+2}{k^2-1}m_0^2 + \left(\frac{k^2}{k^2-1}\frac{J}{I} - 1\right)m_{1/2}^2 - M_Z^2\right\}^{1/2} \qquad (40)$$

with $k = \tan\beta$.

The parameters η, Ω entering the one-loop formula are given by

$$\eta^2 = \frac{\alpha_2}{8\pi\cos^2\theta_W}\left\{\left[\left(\tfrac{1}{4} - \rho^2\right)\left(M_{LL}^2 + M_{RR}^2\right) + \left(M_{LL}^2 - M_{RR}^2\right)\left(\tfrac{1}{4} - \tfrac{2}{3}\sin^2\theta_W\right) - \rho^2A^2\right] \times \left(\ln\tilde{\rho}^2 - 1\right) - 2m_t^2\left(\ln\rho^2 - 1\right)\frac{\rho^2}{k^2}\right\}\frac{k^2+1}{k^2-1} \qquad (41)$$

$$\Omega^2 = \frac{\alpha_2}{8\pi\cos^2\theta_W}\left\{\frac{\rho^2(k^2+1)}{k^2(k^2-1)}\right\}\left(\ln\tilde{\rho}^2 - 1\right) \qquad (42)$$

with $\rho = m_t/M_Z$, $\tilde{\rho} = \langle m_{\tilde{t}}\rangle/M_Z$ and μ_0 the tree level parameter defined in (40). Finally the t-squark mass-combinations $M_{LL}^2 \pm M_{RR}^2$ are given by

$$M_{LL}^2 + M_{RR}^2 = \tfrac{1}{2}m_0^2 + \left(C_1 + C_2 - \frac{J}{2I}\right)m_{1/2}^2 + 2m_t^2 + \tfrac{1}{2}m_Z^2\cos 2\beta$$

$$M_{LL}^2 - M_{RR}^2 = \tfrac{1}{2}m_0^2 + \left(C_1 - C_2 + \frac{J}{6I}\right)m_{1/2}^2 + \left(\tfrac{4}{3}M_W^2 - \tfrac{5}{6}M_Z^2\right)\cos 2\beta$$

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