Implications of a heavy top in supersymmetric theories

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Abstract

In the context of the radiative electroweak symmetry breaking scenario, we investigate the implications of a heavy top quark mass, close to its infrared fixed point, on the low energy parameters of the minimal supersymmetric standard model. We use analytic expressions to calculate the Higgs masses as well as the supersymmetric masses of the third generation. We further assume bottom-tau unification at the GUT scale and examine the constraints put by this condition on the parameter space ($\tan \beta$, α_3), using the renormalization group procedure at the two-loop level. For supersymmetry breaking scale varying in the range (250–1000) GeV we find only a small fraction of the parameter space where the above conditions can be satisfied, namely $1 \leq \tan \beta \leq 2$, while $0.114 \leq \alpha_3(M_Z) \leq 0.125$. We further analyse in brief the case where all three Yukawa couplings reach the perturbative limit just after the unification scale. In this case, due to the large corrections on the bottom mass from sparticle loops, the top-mass and $\tan \beta$ predictions are significantly lower than those estimated when such corrections were ignored.

1. Introduction

In the last years there has been a revived interest in the supergravity unified models and their low energy effective theories, mainly due to the fact that LEP measurements are in good agreement with a gauge coupling constant unification scenario with supersymmetric β-function coefficients down to the scale of $\sim$ 1 TeV. However, the existence of supersymmetry will only be confirmed when new particles – the superpartners of the standard model spectrum – will be observed in (near) future experiments. Thus, the study of supersymmetric grand unification is very important and should be seen in conjunction with the predictions for the new particles which may be observed soon. So far, the constraints put by the unification of the three gauge couplings require a superpartner mass spectrum in the range of (0.1–1) TeV which can be accessible in the near future.

There is another experimental fact the last few years which seems to be related with the fate of the electroweak symmetry breaking in an effective supersymmetric low energy theory. Experimental evidence shows that the mass of the top quark has a lower bound $m_t \gtrsim 110$ GeV. Although this result is disappointing from the experimental point of view, on the other hand, it fits perfectly with the idea of radiative symmetry breaking scenario suggested several years ago [1,3]. Indeed the renormalization group improved SUSY Higgs potential breaks the $[SU(2) \times U(1)]_{EW}$ symmetry when the top Yukawa coupling is large

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enough to drive one of the soft supersymmetry breaking parameters (namely $m_{H_2}^2$) negative.

Grand unification based on the most popular groups, with the minimal number of fermion and Higgs content, implies additional relations in the initial values of the parameters of the theory. Thus, for example in the Yukawa sector, one such well known constraint requires the bottom and tau lepton Yukawa couplings h_b and h_τ, to be equal at the unification scale E_G

$$h_b(E_G) = h_\tau(E_G) \qquad (1)$$

In certain cases, and particularly in string derived unified models, additional constraints on the Yukawa sector of the theory are often obtained, i.e.

$$h_b(E_G) \approx h_\tau(E_G) \approx h_t(E_G) \sim g_{String} \qquad (2)$$

where h_t is the top Yukawa coupling and g_{String} is the value of the unified gauge coupling at the string scale $E_{String} \geq E_G \sim 10^{16}$ GeV. In particular, a large top Yukawa coupling which is implied by the last equality in Eq. (2), motivates again the study of the fixed point solutions proposed several years ago in the context of non supersymmetric theories [2].

Motivated by the experimental fact that the top quark mass is rather high as well as from the aforementioned theoretical speculations, in the present work we wish to study the implications of the above considerations on the low energy theory. In order to minimize the arbitrary parameters and to avoid complications with flavour changing neutral currents, we assume universality of the scalar mass parameters at the GUT scale. Using renormalization group techniques, we derive the mass formulae of the scalar masses (in particular those affected by a large top quark mass) and examine their properties close to the infrared fixed point of the top mass. Furthermore we investigate the regions of the parameter $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$ which are compatible with the above constraints and the minimization conditions put by the renormalization group improved Higgs potential.

2. Radiative symmetry breaking in the presence of a heavy top quark

One of the most appealing features of supergravity theories is the radiative symmetry breaking mecha-

nism [3] which may occur in the presence of a heavy top quark mass. Indeed, the renormalization group improved Higgs potential breaks the electroweak symmetry if the top Yukawa coupling is large enough to drive the $m_{H_2}^2$ mass parameter negative below a certain scale Q_0.

At the tree level the supersymmetric Higgs potential can be written as follows

$$V_0(Q) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (\epsilon_{ij} H_1{}^i H_2{}^j + \text{h.c.})$$
$$+ \tfrac{1}{8}(g^2 + g'^2)\left(|H_1|^2 - |H_2|^2\right)^2 + \tfrac{1}{2} g^2 |H_1^{i*} H_2^i|^2 \qquad (3)$$

where $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$ are the standard Higgs superfields and ϵ_{ij} is the antisymmetric tensor in two dimensions. We have also introduced the two Higgs mass parameters

$$m_1^2 = m_{H_1}^2 + \mu^2 \qquad (4)$$

$$m_2^2 = m_{H_2}^2 + \mu^2 \qquad (5)$$

Finally $m_{H_{1,2}}$ and m_3 are the soft SUSY breaking mass terms and μ is the Higgs mixing mass parameter.

The above tree-level potential $V_0(Q)$ depends strongly on the energy scale Q. It has been shown [4] however, that a correct minimization procedure can be achieved (making the potential relatively stable), if one includes the one-loop corrections $\Delta V_1(Q)$

$$\Delta V_1(Q) = \frac{1}{64\pi^2} \text{Str} \left[\mathcal{M}^4 \left(\ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \right] \qquad (6)$$

where $\mathcal{M}^2$ is the field dependent tree level mass matrix squared. Thus finally

$$V_H(Q) = V_0(Q) + \Delta V_1(Q) \qquad (7)$$

The symbol Str stands for the supertrace which is defined as follows

$$\text{Str} f(\mathcal{M}^2) = \sum_i q_i (-1)^{2s_i} (2s_i + 1) f(m_i^2) \qquad (8)$$

with q_i being the color degrees of freedom while m_i and s_i are the mass and the spin of the corresponding particle.

Now, electroweak symmetry breaking occurs if the following two conditions are met:

– The supersymmetric Higgs potential should develop an asymmetric minimum below some scale $Q \leq Q_0$. This requirement is expressed by the condition

$$m_1^2(Q)m_2^2(Q) - m_3^4(Q) \leq 0 \qquad (9)$$

– The Higgs potential should be bounded from below. This requirement sets the second condition, which reads

$$m_1^2(Q) + m_2^2(Q) \geq 2|m_3(Q)|^2 \qquad (10)$$

The minimization conditions $\partial V_H/\partial v_i = 0$, where $v_i \equiv \langle H_i \rangle$, result the well known equations

$$\frac{1}{2}M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2 \beta}{\tan^2 \beta - 1} \qquad (11)$$

$$\frac{1}{2}\sin 2\beta = -\frac{m_3^2}{\mu_1^2 + \mu_2^2} \qquad (12)$$

where we have introduced the new mass parameters $\mu_i^2 = m_{H_i}^2 + \mu^2 + \sigma_i^2$, which take into account the corrections to the Higgs potential from the one-loop contributions σ_i^2

$$\sigma_i^2 \equiv \frac{\partial \Delta V_1}{\partial v_i} \qquad (13)$$

From the above equations one can conclude that the one-loop corrections to the Higgs potential appear in the minimization conditions through shifts of the Higgs mass parameters $m_i^2 \rightarrow m_i^2 + \sigma_i^2$. It has been shown [5] that, although 31 particles contribute to σ_i^2 corrections, there are finally large cancellations which reduce significantly their effect to the electroweak symmetry breaking. Moreover, the one-loop contribution of the t-squark-quark sector to the masses of the neutralinos, Higgsinos and gauginos seems to be well below the 10% [6] (except in the unfavorable case of a very light tree-level mass).

The most important contributions arize from the squarks of the third generation and the top quark mass. Therefore, it is obvious that the Higgs mass parameters m_{H_i} and the t-squarks play an important role in the minimization of the Higgs potential.

The scale dependence of m_{H_i} and t-squarks is given by the renormalization group equations [3] which can be integrated to give the following results. The m_{H_1} Higgs mass parameter is given by

$$m_{H_1}^2 = m_0^2 + C_{H_1}(t)m_{1/2}^2 \qquad (14)$$

where $t = \ln Q$, m_0 and $m_{1/2}$ are the universal scalar and gaugino mass parameters at E_G, and $C_{H_1} \sim 0.57$ for $t \sim \ln M_Z$. For the rest of the scalar masses, denoting for convenience $m_{\tilde{t}_L} \equiv \tilde{m}_1$, $m_{\tilde{t}_R} \equiv \tilde{m}_2$ and $m_{H_2} \equiv \tilde{m}_3$, we can write the general analytic form [7]

$$\tilde{m}_n^2 = \alpha_n m_0^2 + C_n(t)m_{1/2}^2 - n\delta_m^2(t) - n\delta_A^2(t) \qquad (15)$$

where α_n depends on the Käller manifold and hereafter we assume that $\alpha_n = 1$. The quantities $\delta_{m,A}^2(t)$ are given by

$$\delta_m^2(t) = \left(\frac{m_t(t)}{2\pi v \gamma_Q(t) \sin \beta}\right)^2 (3m_0^2 I(t) + m_{1/2}^2 J(t)) \qquad (16)$$

and,

$$\delta_A^2(t) = \Delta_A^2(t) - \frac{3}{2}\left(\frac{m_t(t)}{2\pi v \gamma_Q(t) \sin \beta}\right)^2 E_A^2(t) \qquad (17)$$

where $v = 246$ GeV and I, J, Δ_A^2 and E_A^2, are integrals containing functions of gauge couplings, i.e.

$$I = \int_t^{t_0} dt' \gamma_Q^2(t') \qquad (18)$$

$$J = \int_t^{t_0} dt' \gamma_Q^2(t') C(t') \qquad (19)$$

$$\Delta_A^2 = \int_t^{t_0} \frac{h_t^2(t')}{8\pi^2} A^2(t') \, dt' \qquad (20)$$

$$E_A^2 = \int_t^{t_0} dt' \gamma_Q^2(t') \Delta_A^2(t') \qquad (21)$$

with $t_0 = \ln E_G$, $C(t) \equiv \sum_{n=1}^3 C_n(t)$, while $\gamma_Q(t) = \prod_{j=1}^3 \left(\alpha_{j,0}/\alpha_j\right)^{c_Q^j/2b_j}$. Clearly, a large top mass implies also a large value of the top-Yukawa coupling, and therefore the negative contributions δ^2 will become also significant. It is possible then to have $\tilde{m}_3^2 \equiv m_{H_2}^2$ negative, and the radiative symmetry breaking scenario will take place.

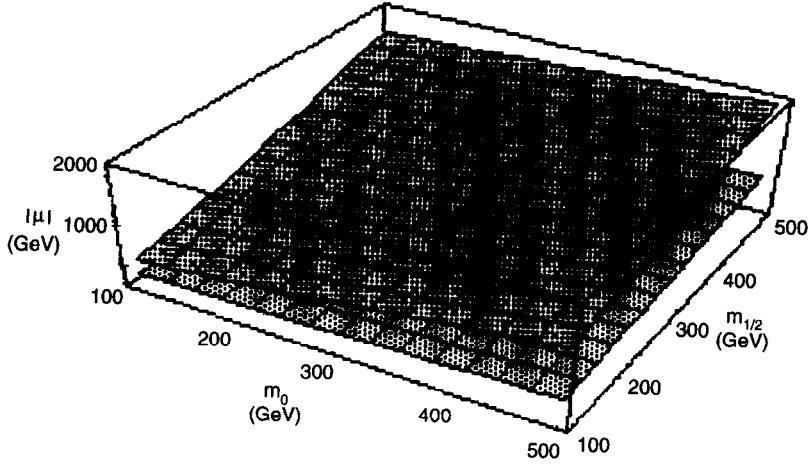A very interesting possibility arizes in the case where the top mass is close to its infrared quasi-fixed

Fig. 1. Surfaces of constant $|\mu|$ in the parameter space $(m_0, m_{1/2})$. The upper surface corresponds to $\tan\beta = 1.1$ while the lower one corresponds to $\tan\beta = 5$.

point. The evolution of the top quark coupling, assuming all the way down supersymmetry, is given by

$$h_t = \frac{h_t(t_0)\gamma_Q(t)}{(1 + \frac{3}{4\pi^2} h_t^2(t_0) I(t))^{1/2}} \tag{22}$$

In this case, i.e. for $h_t^2(t_o)/4\pi \sim 1$, since $I(t \sim \ln m_t) \gg 1$, we can approximate the above

$$h_t(\text{fixed}) \approx \frac{2\pi}{\sqrt{3I(t)}}\gamma_Q(t) \tag{23}$$

i.e. independent of the initial value $h_t(t_0)$. Thus $m_t(\text{fixed}) = m_t^0 \sin\beta \sim (190\text{-}200)\sin\beta$ GeV, depending on the precise values of α_3, E_G etc.

The scalar masses of $m_{\tilde{t}_L}$, $m_{\tilde{t}_R}$ and the Higgs which couples to the up-quarks, take a very simple m_t-independent form in this case. For $h_t = h_t(\text{fixed})$ Eq. (15) simplifies to

$$\tilde{m}_n^2 = (1 - \frac{n}{2})m_0^2 + \left[C_n(t) - \frac{n}{6}\frac{J}{I}\right]m_{1/2}^2 \tag{24}$$

As far as one assumes $A(E_G) \leq 3|m_0|$, corrections due to A-contributions to the above formula have been found to be very small and thus they have been totally ignored. Calculation of the various t-dependent quantities at $t \simeq \ln m_t$ gives [8,7]

$$C_1(t) \simeq 5.30, \quad C_2(t) \simeq 4.90, \quad C_3 \simeq .57,$$
$$I \simeq 113, \quad J \simeq 590 \tag{25}$$

There are some worth-noting properties of the above mass formulae. Indeed, first note that $m_{\tilde{t}_R}$ depends only

on $m_{1/2}^2$ up to A corrections which have been found negligible. A second property is that the dependence of the sum $m_{\tilde{t}_L}^2 + m_{H_2}^2$ on m_0^2, vanishes at the limit $m_t \to m_t(\text{fixed})$. The above properties have also been noticed in Ref. [9]. It is interesting to see the implications of the above simplified formulae in the case of the minimization conditions of the Higgs potential. We start first with the Higgs mixing parameter μ involved in the minimization conditions Eqs. (11), (12).

Ignoring one-loop effects for simplicity, the μ parameter can be given in terms of the known parameters I, J, the unknown Higgs-vev ratio and the initial values $(m_0, m_{1/2})$, by the following equation

$$|\mu| = \frac{1}{\sqrt{2}}\left\{\frac{k^2+2}{k^2-1}m_0^2 + \left(\frac{k^2}{k^2-1}\frac{J}{I} - 1\right)m_{1/2}^2 - M_Z^2\right\}^{1/2} \tag{26}$$

with $k = \tan\beta$. In Fig. 1, we plot the $|\mu|$-values in the parameter space $(m_0, m_{1/2})$, for $\tan\beta = 1.1$ and $\tan\beta = 5$. For the most of the parameter space, $|\mu| \leq 1.5$ TeV. Of course, as $\tan\beta \to 1$, μ grows larger, and a fine tuning problem may arize in Eq. (11), in order to obtain the correct experimental value of M_Z. Thus, to avoid fine tuning, we may put the condition on $\tan\beta \geq 1.1$, which finally translates to the bound $M_t \equiv m_t(\text{pole}) \geq (150\text{-}155)$ GeV.

The μ-parameter plays also important role in the squark mass matrices. In particular, the t-squark mass matrix is
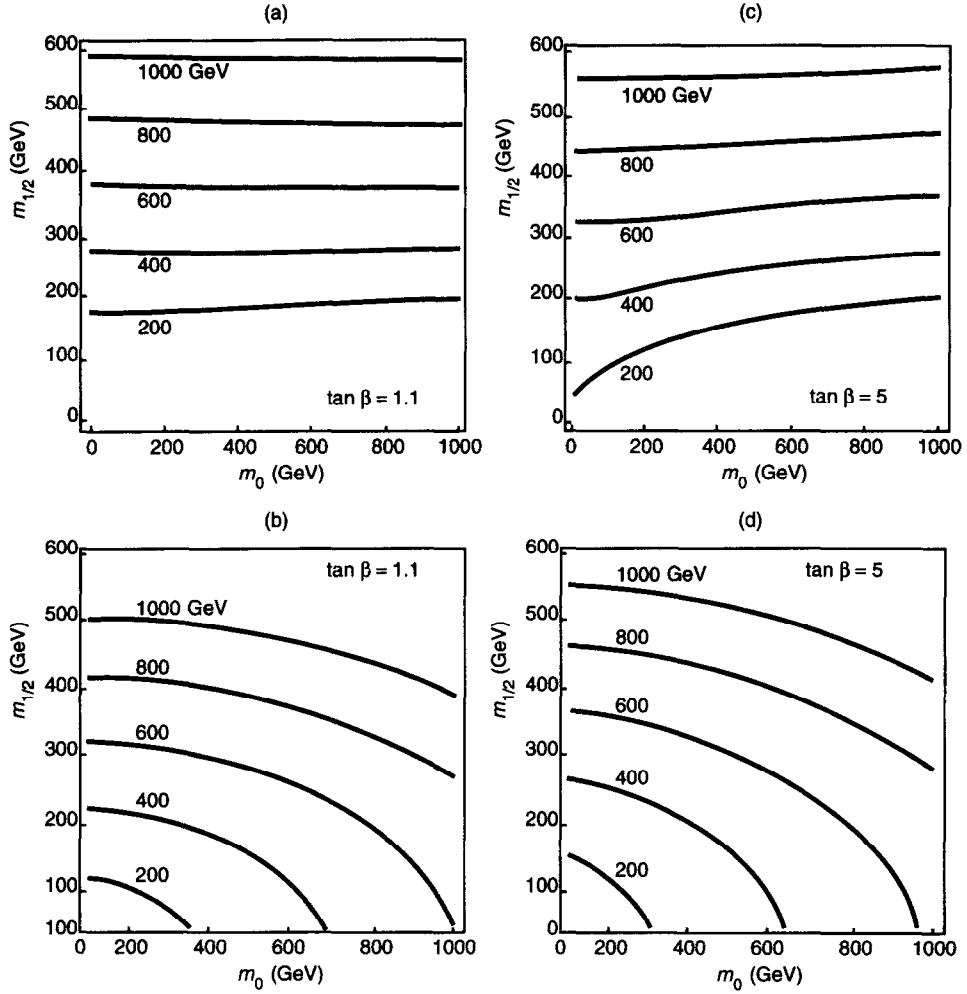
Fig. 2. Contours of constant $m_{\tilde{t}_{1,2}}$ in the parameter space $(m_0, m_{1/2})$, for two values of $\tan\beta = 1.1$ and 5. (a) and (c) corresponds to the lighter eigenstate while (b) and (d) to the heavier one.

$$M_Q^2 = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^2 & M_{RR}^2 \end{pmatrix} \qquad (27)$$

with eigenvalues given by

$$m_{\tilde{t}_{1,2}}^2 = \frac{M_{LL}^2 + M_{RR}^2 \pm \sqrt{\left(M_{LL}^2 - M_{RR}^2\right)^2 + 4M_{LR}^4}}{2} \qquad (28)$$

where

$$M_{LL}^2 + M_{RR}^2 = \tfrac{1}{2}m_0^2 + (C_1 + C_2 - \frac{J}{2I})m_{1/2}^2 + 2m_t^2$$

$$+ \tfrac{1}{2}m_Z^2 \cos 2\beta$$

$$M_{LL}^2 - M_{RR}^2 = \tfrac{1}{2}m_0^2 + (C_1 - C_2 + \frac{J}{6I})m_{1/2}^2$$

$$+ (\tfrac{4}{3}M_W^2 - \tfrac{5}{6}M_Z^2) \cos 2\beta$$

$$M_{LR}^2 = m_t^0 (A \sin\beta + \mu \cos\beta)$$

In Fig. 2 (assuming $\mu > 0$), we plot contours of the above eigenmasses in the parameter space $(m_0, m_{1/2})$ for the choice $A(E_G) = -\sqrt{3}m_0$, and two representative values of $\tan\beta$ in the low range (1.1–10), namely $\tan\beta = 1.1$ and $\tan\beta = 5$. In most of the parameter space the light eigenstate preserves the independence of m_0 mass parameter. For reasonable initial values of the parameters m_0 and $m_{1/2}$, the squark masses are

well bellow the 1 TeV, and therefore accessible to future experiment.

Notice finally that the one-loop contributions to the effective potential will also result to a shift in the $|\mu|$ parameter. Making use of the fact that in the limit $m_{1/2} \gg m_0$ we can approximate

$$\ln \frac{m_{\tilde{t}_1}^2}{M_Z^2} \sim \ln \frac{m_{\tilde{t}_2}^2}{M_Z^2} \sim \ln \frac{\langle m_{\tilde{t}}^2 \rangle}{M_Z^2}$$

we may obtain an analytic form for the one-loop corrected $|\mu|$ parameter, when m_t and $m_{\tilde{t}_{1,2}}$ corrections are taken into account [5]

$$|\mu| = \sqrt{(\mu_0^2 + \eta^2)/(1 - \Omega^2)} \qquad (29)$$

where

$$\eta^2 = \frac{\alpha_2}{8\pi \cos^2 \theta_W} \Big\{ [(M_{LL}^2 + M_{RR}^2)(\tfrac{1}{4} - \rho^2)$$
$$+ (M_{LL}^2 - M_{RR}^2)(\tfrac{1}{4} - \tfrac{2}{3}\sin^2 \theta_W) - \rho^2 A^2](\ln \bar{\rho}^2 - 1)$$
$$- 2m_t^2(\ln \rho^2 - 1)\frac{\rho^2}{k^2} \Big\} \frac{k^2 + 1}{k^2 - 1} \qquad (30)$$

$$\Omega^2 = \frac{\alpha_2}{8\pi \cos^2 \theta_W} \left\{ \frac{\rho^2(k^2 + 1)}{k^2(k^2 - 1)} \right\} (\ln \bar{\rho}^2 - 1) \qquad (31)$$

with $\rho = m_t/M_Z$, $\bar{\rho} = \langle m_{\tilde{t}} \rangle / M_Z$ and μ_0 the tree level parameter defined in (26). For moderate values of $m_{1/2}$ however, these corrections are not going to alter substantially our previous results.

3. Bottom–tau Yukawa unification and the IR fixed point

One of the great successes of the most popular GUTs is the equality of the bottom and tau Yukawa couplings at the GUT scale which lead to the correct prediction of the experimentally determined relation $m_b \approx 3\, m_\tau$ at low energies. Several groups [10] have examined the effects of h_b, h_τ relations implied by various unified theories, assuming minimal supersymmetry with grand unification at an energy scale close to 10^{16} GeV. It has been claimed that the GUT relation $h_b = h_\tau$ implies a heavy top quark with a value of the Yukawa coupling close to its infrared fixed point. In this section, we wish to present a detailed numerical analysis in the context of the GUT constraints mentioned above. We will mainly discuss the constraints

on the parameter space $(\tan \beta, \alpha_3)$ when bottom-tau Yukawa unification is assumed and examine the connection of this constraint in relation with the top-mass. We will further examine the case where the three Yukawa couplings reach the perturbative limit just after the unification scale. Our analysis will be done at the two-loop level, taking into account the contribution of the Yukawa couplings, and in particular that of the h_t into the running of the gauge coupling constants. The case where all three Yukawa couplings are equal at E_G occurs theoretically in (minimal) $SO(10)$ [11] and in $SU(4) \times SU(2)_L \times SU(2)_R$ [12].

We shall present now a detailed description of the procedure we are following. We adopt the so called bottom-up approach starting from M_Z. Which are the inputs at this energy level?

– the experimentally known values of α, $\sin^2 \theta_W = 0.2319 \pm 0.0004$ and α_3, or equivalently of the three gauge couplings α_i, $i = 1, 2, 3$. The relatively small experimental errors on α and $\sin^2 \theta_W$ permit us to talk about the "bands" of α_1 and α_2 in the running of those couplings while we treat α_3 as a "free" parameter, inside its experimental limits of course. On the other hand, the obtained values for $\sin^2 \theta_W$ and the physical top mass at the one loop level are correlated by the formula [13]

$$\sin^2 \theta_W(m_Z) = 0.2324$$
$$- 10^{-7}\{(m_t/\text{GeV})^2 - 138^2\} \pm 0.0003 \qquad (32)$$

while α should lie within the range obtained from low energy data

$$\alpha^{-1}(m_Z) = 127.9 \pm 0.1 \qquad (33)$$

– the value of the SUSY scale E_S: as we are going from the electroweak up to the GUT scale, at E_S the beta function coefficients change from the non-supersymmetric to the supersymmetric ones. In the usual approach this was considered as the average mass of the SUSY spectrum, or equivalently all sparticles were assumed to have degenerate masses. The correct procedure however would be to take in to account the decoupling of each sparticle at its mass and change the beta function coefficients successively starting from the heavier down to the lighter sparticle, until we reach the electroweak scale. The precise effects of these low energy thresholds have been discussed recently in the

literature [13–15]. It has been pointed out [13] that in the case where the squark and slepton spectra are assumed degenerate, taking into account only contributions with leading exponents, one can define an effective SUSY scale to account for the above mention effects which can be given approximately by the following formula

$$E_S^{eff} = \left(\frac{\alpha_2(m_{\tilde{W}})}{\alpha_3(m_{\tilde{g}})} \right)^{\frac{28}{19}} |\mu| \approx \frac{1}{5}|\mu| \qquad (34)$$

It is obvious from the above formula that E_S^{eff} depends strongly on the value of the $|\mu|$ parameter.

Since we are studying here the constraints on α_3, $\sin^2 \theta_W$ and m_t low energy values, it suffices for our purposes to assume two limiting cases of E_S, namely $E_S \approx 1$ TeV and $E_S \approx .25$ TeV. We should also note however, that GUT threshold effects may also affect our predictions, in particular that for the strong gauge coupling. Such corrections however are more or less model dependent [13,14,16] and will not be considered in our present analysis.

– the value of $\tan \beta$, starting its role when we reach the energy E_S where SUSY is valid.

– the value of the h_t Yukawa coupling of the top quark. Essentially it is a free parameter as long as it gives the mass of the top quark in the allowed experimental region (110–190) GeV while being compatible with Eq. (32). We use the 1 loop QCD corrections to define the pole mass M_t of the top quark

$$M_t = (h_t(M_t)v/\sqrt{2})(1 + \frac{4}{3\pi}\alpha_3(M_t))$$

– the values of h_b and h_τ, taken from the relations

$$m_b(m_b) = \frac{h_b(M_Z)v/\sqrt{2}}{\eta_b}, \quad m_\tau = h_\tau(M_Z)v/\sqrt{2}$$

We take the mass of the bottom quark $m_b(m_b) = (4.15–4.35)$ GeV while that of the τ lepton $m_\tau(m_\tau) = 1.7841$ GeV. The factor η_b, appearing in the mass of the bottom quark, includes the 1 loop QCD corrections from m_b to M_Z.

Between M_Z and E_S, which we take to be in the range (0.25–1) TeV, we run the couplings with the β-functions of the S.M. At E_S we apply the following boundary conditions for the Yukawa couplings

$$h_t^S = \frac{h_t}{\sin \beta}, \quad h_b^S = \frac{h_b}{\cos \beta}, \quad h_\tau^S = \frac{h_\tau}{\cos \beta}$$

Then onwards we run the couplings using the MSSM β-functions. At an energy E_G, around 10^{16} GeV, the "bands" of the couplings α_1 and α_2 meet and determine what we call "the unification band". The strong coupling $\alpha_3(M_Z)$ should be chosen so that it passes through this unification band in order to achieve gauge coupling unification. A short comment is in order at this point. Since we are using 2 loop β-functions the differential equations for all the couplings are coupled (this fact shows its presence even harder when we demand one or all the Yukawa couplings to grow large at E_G). Therefore the unification band is not uniquely determined but depends, though not strongly, on the particular choice for α_3 as well as on the Yukawa couplings at M_Z. We try to find the values of $\tan \beta$ that permit the growth of h_t to the perturbative limit ($h_t \sim 3.5$) at the energy scale E_G or later, checking always that α_3 passes through the unification band. At the same time we try to unify, again at E_G, the other two Yukawa couplings: $h_b(E_G) = h_\tau(E_G)$. This could be achieved by varying the mass of the bottom quark inside its experimentally allowed region. Finally we try to arrange the possibility that all three Yukawa couplings grow to the perturbative limit at E_G. This last step could be achieved by using large values of $\tan \beta$.

We approach, step by step, the above three points, constraining in each step the allowed region of the parameter space of our inputs. In Fig. 3 we plot, for several values of the mass of the top quark M_t, $\tan \beta$ versus $\alpha_3(M_Z)$ demanding gauge coupling unification and $h_t(E_G) \lesssim 3.5$. Let us explain the features of the graphs. The lower limits on $\alpha_3(M_Z)$ appear because the lower the gauge couplings the larger the slope dh_t/dt (recall that gauge and Yukawa couplings have opposite contributions to the β-functions). This fact permits h_t to grow very fast and reach the perturbative limit before gauge coupling unification is achieved. The same line of thought explains the slope of the "lines" in Fig. 3. Choosing a higher $\alpha_3(M_Z)$ we need a higher initial point $h_t^S(E_S)$ to reach the perturbative limit, therefore we need a smaller $\tan \beta$. The turning edges of each line is more intriguing. At the right end the value of $\alpha_3(M_Z)$ is so high that, although h_t permits gauge coupling unification, $\alpha_3(E_G)$ passes above the unification band of (α_1, α_2). Choosing a higher
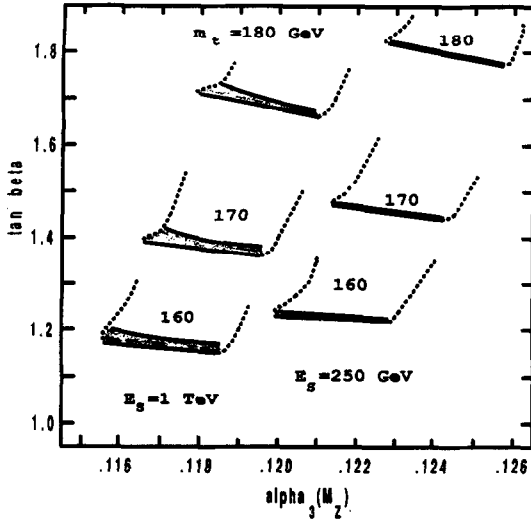
Fig. 3. Allowed regions in the space of $(\tan\beta, \alpha_3(M_Z))$, in order to achieve gauge coupling unification, for several values of the top mass M_t and E_S. Shaded regions correspond to $b - \tau$ unification.

value of $\tan\beta$ (therefore smaller h_t) $d\alpha_i/dt$ grows to larger values. The coupling α_2 receives the biggest contribution, the unification band shifts to higher values and allows α_3 to pass through it. Of course, in that case $h_t(E_G) < 3.5$. Similar arguments explain the left end of the lines. Now $\alpha_3(M_Z)$ is so small that very easily drops below the unification band. Choosing a somewhat higher $\alpha_3(M_Z)$ permits a higher $\tan\beta$. Again, in this case, $h_t(E_G) < 3.5$. Therefore the allowed region for each M_t is inside the envelope-like shape.

Our next step is to demand $h_b(E_G) = h_\tau(E_G)$. For each M_t, in Fig. 3 we plot shaded regions corresponding to the allowed range of $m_b(m_b) = (4.15\text{–}4.35)$ GeV. We notice that b–τ unification requires M_t to be near its fixed point, being closer to it for $E_S \sim 250$ GeV.

The last step is to require all three Yukawa couplings to grow large near E_G. To achieve that point we need a large value for $\tan\beta$ (for h_b and h_τ) and a large M_t (for h_t). If large bottom corrections arizing from finite contributions of sparticle loops were ignored the $\tan\beta$ and m_t predictions would be rather high. Particular cases, consistent with the $m_b(m_b)$ range, where this is achieved are shown in the Table 1.

Trying to achieve those large values of h_t, h_b and h_τ with a lower value of M_t, one needs to choose either a lower value of $\alpha_3(M_Z)$ or a smaller $\tan\beta$. The latter

Table 1
Input values for three cases where all three Yukawa couplings are near their perturbative limit at the gauge coupling unification energy E_G, ignoring large corrections to the bottom mass (Masses and energies are in GeV).

M_t	E_S	$\alpha_3(M_Z)$	$\tan\beta$	E_G
180	250	0.120	63.0	$10^{16.3}$
180	185	0.119	62.3	$10^{16.3}$
180	185	0.118	62.3	$10^{16.0}$

does not help since, at such large values, $\sin\beta$ does not change much while $\cos\beta$ does, preventing h_b and h_τ to reach the perturbative limit. On the other hand, the change of $\alpha_3(M_Z)$ does not affect h_τ in contrast with h_t and h_b.

The analysis of the large $\tan\beta$ regime becomes more complicated however, due to large finite corrections mentioned above [14,17,18]. In particular, for the case of t–b–τ unification, the uncertainties for the m_t and a_3 predictions are found to be substantially large. Thus, for $a_3(m_Z) \approx 0.12$ as in the case of Table 1, one may show [18] that there is an upper bound on the above predictions namely $M_t \leq 150$ GeV and $\tan\beta \leq 41$. A way out of this difficulty was proposed in Ref. [14] where a Peccei-Quinn as well as an approximate continuous R symmetry were imposed to keep these corrections small. However, as stated in [18] the approximate preservation of such a symmetry (whose breaking is characterized by the parameter $\epsilon_R = m_{\tilde{g}}/m_{\tilde{q}}$) in general is not consistent with the radiative symmetry breaking scenario. In particular when b–t unification and universality of the scalar mass spectrum at the GUT scale is assumed, it is necessary that $m_{1/2}^2 \geq m_0^2$ which in turn implies that gluinos have approximately a mass of the order of the squark masses and therefore ϵ_R cannot be small. In this case the corrections to the bottom mass are relatively large. On the other hand, the unified Yukawa coupling $h_t = h_b = h_\tau$ at M_{GUT} is forced to values substantially lower from its non-perturbative region. For the case of Table 1 for example, we find that bottom mass corrections are about 18–20% implying M_t and $\tan\beta$ predictions as those of Table 2.

Thus, there is a significant lowering of the top-mass prediction compared to the case of Table 1 where no corrections were included. The precise effect of the above corrections however would require a bet-

Table 2
Prediction for M_t and $\tan \beta$ in the b–t–τ Yukawa coupling unification scenario, including large corrections to the bottom mass.

M_t	E_S	$\alpha_3(M_Z)$	$\tan \beta$	E_G
149	250	0.120	40.5	$10^{16.20}$
149	250	0.117	40.5	$10^{16.15}$

ter knowledge of the initial parameters m_0, A_t etc at the GUT scale. The specific choice of the m_0, $m_{1/2}$, the sign of μ etc, play a very important role and we need a better understanding of the underlying theory which determines the above parameters. A systematic approach to this problem appears in Ref. [18]. A detailed analysis of these effects however, is beyond of the scope of this work.

In conclusion, in this paper we examined the implications of a heavy top quark, and bottom tau unification at the GUT scale, implied by popular unified models, in the minimal supersymmetric standard model. We have assumed a top Yukawa coupling close to its infrared fixed point and we have given analytic forms of the t-squark masses and the Higgs mass parameter responsible for the radiative electroweak symmetry breaking scenario. We have found that $m_{\tilde{t}_R}$ does not depend on the m_0 mass parameter, while all masses under consideration are very weakly dependent on the trilinear scalar parameter A. The bottom-tau unification turned out to be very restrictive. We have found only small ranges in the $(\tan \beta, \alpha_3)$-plane where this condition can be satisfied. Moreover, this condition demands a heavy top with a mass close to its infrared fixed point, as was previously assumed. In the case of the large $\tan \beta$ scenario, if large corrections to the bottom were ignored the above requirements could be satisfied only in a tiny region with $\tan \beta \approx 63$. Large bottom corrections however, invalidate these predictions. In the case of t–b–τ Yukawa unification and assuming universality of scalar masses at the GUT scale, it is found that top mass is less than 150 GeV while $\tan \beta \leq 41$.

Acknowledgements

References

[1] L. Ibáñez and G.G. Ross, Phys. Lett. B 110 (1982) 215.
[2] B. Pendleton and G.G. Ross, Phys. Lett. B 98 (1981) 291.
[3] K. Inoue et al., Prog. Theor. Phys. 68 (1982) 927;
    L. Alvarez-Gaumé, M. Claudson and M.B. Wise, Nucl. Phys. B 221(1983) 495;
    L. Ibáñez and C. Lopez, Phys. Lett. B 126 (1983) 54;
    J. Ellis, J.S. Hagelin and K. Tamvakis, Phys. Lett. B 125 (1983) 275;
    L. Ibáñez and C. Lopez, Nucl. Phys. B 233 (1984) 511;
    C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quirós, Phys. Lett. B 132 (1983) 95; Nucl. Phys. B 236(1984) 438;
    L. Ibáñez and C. Lopez and C. Muñoz, Nucl. Phys. B 256 (1985) 218;
    A. Bouquet, J. Kaplan and C. Savoy, Nucl. Phys. B 262 (1985) 299;
    R. Barbieri and J. Giudice, Nucl. Phys. B 306 (1988) 63.
[4] G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. B 331 (1990) 331;
    C. Kounnas, in: Properties of SUSY particles, p. 496, World Scientific, Erice proceedings, 1994.
[5] R. Arnowitt and P. Nath, Phys. Rev. D 46 (1992) 3981.
[6] D. Pierce and A. Papadopoulos, John Hopkins University preprints, JHU-TIPAC-930030, JHU-TIPAC-940001;
    A.B. Lahanas, K. Tamvakis and N.D. Tracas, CERN preprint, CERN-TH 7089/93 (Phys. Lett. B, to appear).
[7] G.K. Leontaris, Phys. Lett. B 317 (1993) 569.
[8] C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quirós, Phys. Lett. B 132 (1983) 95; Nucl. Phys. B 236 (1984) 438;
    J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B 241(1984) 406.
[9] M. Carena, T.E. Clark, C.E.M. Wagner, W.A. Bardeen and K. Sasaki, Nucl. Phys. B 369 (1992) 33;
    S. Kelley, J.L. Lopez and D.V. Nanopoulos, Phys. Lett. B 278 (1992) 140;
    M. Carena, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B 406 (1993) 59;
    W.A. Bardeen, M. Carena, S. Pokorski and C.E.M. Wagner, Phys. Lett. B 320 (1994) 110;
    M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, MPI preprints, MPI-Ph/93-66 and MPI-Ph/93-103.
[10] G.K. Leontaris, J. Rizos and K. Tamvakis, Phys. Lett. B 243 (1990) 220;
    B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Rev. D 44 (1991) 1613;
    H. Arason et al., Phys. Rev. Lett. 67 (1991) 2933;
    V. Barger, M.S. Berger and P. Ohmann and R.J.N. Phillips, Phys. Lett. B 314 (1993) 351;
    V. Barger, M.S. Berger and P. Ohmann, Phys. Rev. D 47 (1993) 1093;

G.L. Kane et al., UM-TH-93-24, October 1993;
G.K. Leontaris and N.D. Tracas, Phys. Lett. B 303 (1993) 50;
J. Lopez, D.V. Nanopoulos and A. Zichichi, CERN preprint CERN-TH. 7138/94;
B. Ananthanarayan, K.S. Babu and Q. Shafi, Bartol Research Institute preprint, BA-94-02;
P. Langacker and N. Polonsky, Phys. Rev 49 (1994) 1454; UPR-0594T (1994) preprint;
B.C. Allanach and S. King, preprint SHEP-93/94-15;
D. Kapetanakis, M. Mondragón and G. Zoupanos, Z. Phys. C 60 (1993) 181;
M. Mondragón and G. Zoupanos, CERN preprint CERN-TH 7098/93;
J. Kubo, M. Mondragón and G. Zoupanos, Nucl. Phys. B 424 (1994) 291.
[11] G. Andersom, S. Raby, S. Dimopoulos, L. Hall and G. Starkman, LBL preprint 33531.
[12] I. Antoniadis et al., Phys. Lett. B 216 (1989) 333; 245 (1990) 161;

G.K. Leontaris and N.D. Tracas, Phys. Lett. B 260 (1991) 339;
A Murayama and A. Toon, Phys. Lett. B 318 (1993) 298;
O. Korakianitis and N.D. Tracas, Phys. Lett. B 319 (1993) 145;
S. King, Phys. Lett. B 325 (1994) 129;
G. Lazarides and C. Panagiotakopoulos, University of Thessaloniki preprint, 3/1994.
[13] P. Langacker and N. Polonsky, Phys. Rev. D 47 (1993) 4028; M. Carena et al., Ref. [9].
[14] L.J. Hall, R. Rattazzi and U. Sarid, Lawrence Berkeley Lab preprint LBL-33997 (1993), hep-ph 9306309.
[15] G.K. Leontaris and N.D. Tracas, Low energy thresholds and the scalar mass spectrum in minimal supersymmetry, Ioannina preprint 307/94, August 1994.
[16] N. Polonsky and A. Pomarol, UPR-0616T/94.
[17] R. Hempfling, Phys. Rev. D 49 (1994) 6168.
[18] M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B 406 (1994) 269.