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One loop corrections to the neutralino sector and radiative electroweak breaking in the MSSM

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Abstract

We compute one loop radiative corrections to the physical neutralino masses in the MSSM considering the dominant top–stop contributions. We present a numerical renormalization group analysis of the feasibility of the radiative gauge symmetry breaking parametrized by the standard soft supersymmetry breaking terms. Although the above computed effects can be, in principle, large for extreme values of the Yukawa couplings, they do not, in general, exceed a few per cent for most of the parameter space. Therefore tree level constraints imposed on the gluino mass $m_{\tilde{g}}$, and on the superpotential parameter μ by LEP1 and CDF experiments, are not upset by the heavy top/stop sector.

Supersymmetry seems to be the only framework which allows unification of the three gauge coupling constants at a common energy scale, while at the same time respects their low energy values as well as the lower bounds on proton decay [1–3]. Moreover, softly broken supersymmetry (resulting possibly from an underlying superstring framework) could lead to $SU(2)_L \times U(1)_Y$ gauge symmetry breaking through radiative corrections for a certain range of values of the parameters [4–7]. Thus, the elegant ideas of supersymmetry, gauge coupling unification and radiative symmetry breaking could be realized within the same framework. The Minimal Supersymmetric Standard Model (MSSM) incorporates all of the above. It has recently attracted a lot of attention and it has been the subject of numerous analyses based on the renormalization group [8–12]. It has also recently become evident that, due to the largeness of the top quark mass, the one loop contributions to the Higgs potential and to the Higgs physical masses could be important [13–17]. Then it is possible that the sparticle masses could also acquire non-negligible radiative corrections from the top–stop contributions. Since the neutralino sector is, in general, the lightest sector of the theory (accommodating the LSP), with a possible exception of a Higgs, it seems to be a good place to search for substantial radiative corrections.

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In the present paper, we perform a one loop calculation of the neutralino physical masses in the context of the MSSM. Only the dominant top–stop contributions are taken into account. We also perform a renormalization group analysis of the model to determine the range of parameters that leads to radiative electroweak breaking with the correct value of M_Z. Our numerical analysis follows that of Refs. [8,12]. As inputs we consider the parameters A_0, m_0 and $m_{1/2}$, which parametrize the soft supersymmetry breaking terms, $\tan\beta(M_Z) = v_2/v_1$ (the ratio of the two Higgs field v.e.v.'s) and the running top quark mass $m_t(M_Z)$ at the scale M_Z. The parameters $B(M_Z)$ and $\mu(M_Z)$, which set the size of the mixing of the Higgs scalars and Higgsinos, can be derived by minimizing the Higgs potential. The value of the parameter μ, which is essential for the neutralino masses, is sensitive to radiative effects and thus the one loop effective potential should be used in the minimization procedure.

LEP1 and Tevatron CDF experiments put constraints on the parameter space $(m_{1/2}, \mu)$, or equivalently $(m_{\tilde{g}}, \mu)$ where $m_{\tilde{g}}$ is the gluino mass. In these analyses the effects of the radiative corrections to the neutralino masses and especially to the LSP $\tilde{Z}_1$, as produced in $Z \to \tilde{Z}_1\tilde{Z}_1$, have been ignored. The possible existence of a region in the parameter space where the top–stop radiative corrections become important, means that the experimental bounds should be reconsidered taking into account those effects. On the other hand, if these radiative effects are small the tree level bounds on $m_{1/2}$ and μ can be trusted [18,19].

The superpotential of the MSSM is [20,21]

$$\mathscr{W} = (h_U \hat{Q}^i \hat{H}_2^j \hat{U}^c + h_D \hat{Q}^i \hat{H}_1^j \hat{D}^c + f_L \hat{L}^i \hat{H}_1^j \hat{E}^c + \mu \hat{H}_1^i \hat{H}_2^j)\epsilon_{ij}, \quad \epsilon_{12} = +1\,, \tag{1}$$

where $\hat{Q}$, $\hat{D}^c$, $\hat{U}^c$, $\hat{L}$, $\hat{E}^c$, $\hat{H}_1$ and $\hat{H}_2$ stand for the $(3, 2, 1/6)$, $(\bar{3}, 1, 1/3)$, $(\bar{3}, 1, -2/3)$, $(1, 2, -1/2)$, $(1, 1, 1)$, $(1, 2, -1/2)$ and $(1, 2, 1/2)$ matter chiral superfields. Colour and family indices are suppressed. The only dimensionful parameter is μ. Our analysis will be independent of the exact dynamical origin of this parameter [22,23] as long as it has the right order of magnitude ($\mathscr{O}(M_W)$).

The scalar potential involves soft SUSY breaking terms given by

$$\begin{aligned} V_{\rm sb} &= m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_Q^2 |\tilde{Q}|^2 + m_{U^c}^2 |\tilde{U}^c|^2 + m_{D^c}^2 |\tilde{D}^c|^2 + m_L^2 |\tilde{L}|^2 + m_{E^c}^2 |\tilde{E}^c|^2 \\ &\quad + (h_U A_U \tilde{Q} H_2 \tilde{U}^c + h_D A_D \tilde{Q} H_1 \tilde{D}^c + h_L A_L \tilde{L} H_1 \tilde{E}^c + \text{h.c.}) + (B\mu H_1 H_2 + \text{h.c.})\,. \end{aligned} \tag{2}$$

We also have the soft breaking Majorana masses for the gauginos,

$$\mathscr{L}_{\rm sb} = -\tfrac{1}{2} \sum_A M_A \bar{\lambda}_A \lambda_A\,. \tag{3}$$

Since radiative corrections are expected to be small, whenever the large top quark Yukawa coupling is not involved, a reasonable approximation is to keep only loops which involve the top-stop system. In that case the one loop corrections to the scalar potential are

$$\Delta V_1 = \frac{3}{64\pi^2} \sum_{+,-} m_{\pm}^4 \left(\ln \frac{m_{\pm}^2}{Q^2} - \frac{3}{2}\right) - \frac{3}{32\pi^2} m_t^4 \left(\ln \frac{m_t^2}{Q^2} - \frac{3}{2}\right), \tag{4}$$

where the $\overline{\rm DR}$ regulation scheme has been used. Taking all fields to vanish, except the neutral Higgses, we have

$$m_t \equiv h_t H_2^0, \quad m_{\pm}^2 \equiv \tfrac{1}{2}\{m_{\rm LL}^2 + m_{\rm RR}^2 \pm [(m_{\rm LL}^2 - m_{\rm RR}^2)^2 + 4m_{\rm LR}^4]^{1/2}\}\,, \tag{5}$$

where

$$\begin{aligned} &m_{\rm LL}^2 \equiv m_{\tilde{t}}^2 + m_t^2 + (\tfrac{1}{12}g'^2 - \tfrac{1}{4}g^2)(|H_2^0|^2 - |H_1^0|^2), \quad m_{\rm RR}^2 \equiv m_{\tilde{t}^c}^2 + m_t^2 - \tfrac{1}{3}g'^2(|H_2^0|^2 - |H_1^0|^2)\,, \\ &m_{\rm LR}^2 \equiv h_t(H_2^0 A + \mu H_1^{0*})\,. \end{aligned} \tag{6}$$

All parameters are considered to be running ones, depending on the scale Q appearing in Eq. (4).

Minimization of the scalar potential yields two conditions on the Higgs v.e.v.'s $v_2 \equiv \langle H_2 \rangle$ and $v_1 \equiv \langle H_1 \rangle$ [#1]

[#1] We follow the conventions of J. Ellis and F. Zwirner, Nucl. Phys. B 338 (1990) 317, where B and μ have opposite signs.

$$\frac{M_Z^2}{2} = \frac{\bar{m}_1^2 - \bar{m}_2^2 \tan^2\beta}{(\tan^2\beta - 1)} \quad (M_Z = 91.2 \text{ GeV}) , \tag{7}$$

$$\sin 2\beta = -\frac{2B\mu}{\bar{m}_1^2 + \bar{m}_2^2} . \tag{8}$$

In Eqs. (7,8) we have defined [12,24]

$$\bar{m}_{1,2}^2 \equiv m_{1,2}^2 + \frac{\partial \Delta V}{\partial v_{1,2}^2} , \quad \text{with } m_{1,2}^2 \equiv m_{H_{1,2}}^2 + \mu^2 . \tag{9}$$

The key features of the radiative symmetry breaking could be seen even with the tree-level potential. This breaking occurs at the scale Q_b where the (Q-dependent) expression $m_1^2 m_2^2 - (\mu B)^2$ becomes negative. The free parameters of the model should be chosen in a manner consistent with the observed value of M_Z. At another scale $Q_c < Q_b$ the expression $m_1^2 + m_2^2 - 2\mu B$ becomes negative. This makes the tree-level potential unbounded from below and therefore untrustworthy. The above consideration makes the choice of the scale Q critical. In contrast, the inclusion of the one loop correction to the potential makes this choice irrelevant, as long as it is near M_Z. The reason is that the one loop corrected potential is to this order to computation, renormalization (i.e. Q) independent, up to constant (but Q-dependent) terms which do not appear in the minimization equations. Therefore, using the one loop corrected potential, we consider the minimization conditions, Eqs. (7), (8) at $Q = M_Z$. Given a set of values for the input parameters at some scale M_{GUT}, the validity of Eqs. (7), (8) ensures that the electroweak breaking occurs with the correct value for M_Z. As we have already mentioned above, we take $\tan\beta(M_Z)$ and $m_t(M_Z)$ as inputs. In that case the parameters $B(M_Z)$ and $\mu(M_Z)$ are derived from the Eqs. (7), (8). The connection between the values of the parameters at the scale M_{GUT} and those at M_Z is carried out by the renormalization group equations. More details on the numerical procedure followed will be given below.

The neutral gauge-fermion and neutral Higgsino (collectively "neutralino") mass matrix is easily seen to be

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & g'v_1/\sqrt{2} & -g'v_2/\sqrt{2} \\ 0 & M_2 & -gv_1/\sqrt{2} & gv_2/\sqrt{2} \\ g'v_1/\sqrt{2} & -gv_1/\sqrt{2} & 0 & -\mu \\ -g'v_2/\sqrt{2} & gv_2/\sqrt{2} & -\mu & 0 \end{pmatrix} , \tag{10}$$

where we have used a $\psi^i \equiv (\tilde{B}, \tilde{W}_3, \tilde{\chi}_1, \tilde{\chi}_2)$ Weyl basis, with $\tilde{\chi}_{1,2} \equiv \mathrm{i}\tilde{H}_{1,2}$ to make $\mathcal{M}$ real and symmetric. Radiative corrections to the neutralino two point function take the form $-\mathrm{i}(\not{p}^{\alpha\dot{\beta}})a_{ij}$ and $-\mathrm{i}\epsilon^{\beta\alpha}b_{ij}$ ($\not{p}^{\alpha\dot{\beta}} \equiv p^\mu \sigma_\mu^{\alpha\dot{\beta}}$). Taking into account only the dominant top–stop contributions (Fig. 1) we obtain the following expressions:

$$a_{\tilde{B}\tilde{W}} = \frac{1}{2}\sqrt{\frac{3}{5}} \frac{(\alpha_1\alpha_2)^{1/2}}{4\pi} (\Lambda_+ \cos^2\phi + \Lambda_- \sin^2\phi) = \left(\frac{\alpha_1}{15\alpha_2}\right)^{1/2} a_{\tilde{W}\tilde{W}} , \tag{11}$$

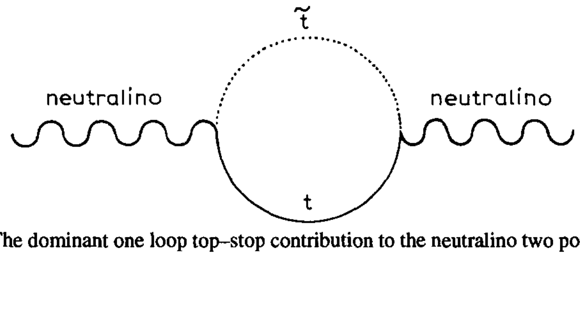

Fig. 1. The dominant one loop top–stop contribution to the neutralino two point function.

$$a_{\tilde B\tilde\chi_2} = -3\sqrt{\frac{3}{10}}\frac{(\alpha_1\alpha_t)^{1/2}}{4\pi}\cos\phi\sin\phi(\Lambda_+ - \Lambda_-) = -\sqrt{\frac{3\alpha_1}{5\alpha_2}}\,a_{\tilde W\tilde\chi_2}\,,$$

$$a_{\tilde\chi_2\tilde\chi_2} = \frac{3}{2}\frac{\alpha_t}{4\pi}(\Lambda_+ + \Lambda_-)\,,$$

$$a_{\tilde B\tilde B} = \frac{8}{15}\left(\frac{\alpha_1}{\alpha_t}\right)a_{\tilde\chi_2\tilde\chi_2} - \left(\frac{15\alpha_1}{\alpha_2}\right)^{1/2} a_{\tilde B\tilde W} \qquad \text{(11 cont'd)}$$

and

$$b_{\tilde B\tilde B} = \frac{4}{5}\frac{\alpha_1}{4\pi}m_t\cos\phi\sin\phi(L_+ - L_-) = -\frac{2}{15}\frac{\alpha_1}{\alpha_t}b_{\tilde\chi_2\tilde\chi_2} = \frac{2}{\sqrt{15}}\left(\frac{\alpha_1}{\alpha_2}\right)^{1/2} b_{\tilde B\tilde W}\,,$$

$$b_{\tilde B\tilde\chi_2} = \sqrt{\frac{3}{10}}\frac{(\alpha_1\alpha_t)^{1/2}}{4\pi}m_t[4(L_+ + L_-) - 5(L_+\cos^2\phi + L_-\sin^2\phi)]\,,$$

$$b_{\tilde W\tilde\chi_2} = -\frac{3}{\sqrt{2}}\frac{(\alpha_2\alpha_t)^{1/2}}{4\pi}m_t(L_+\cos^2\phi + L_-\sin^2\phi)\,. \qquad (12)$$

Note that ji and $b_{ij}=b_{ji}$. Entries which are not shown vanish. In the above formulae $\alpha_{1,2}$ are the gauge couplings, $\alpha_t \equiv h_t^2/(4\pi)$, ϕ is the angle diagonalizing the stop matrix given by

$$\cos^2\phi = \frac{m_+^2 - m_{\tilde t^c}^2 - m_t^2 - \frac{2}{3}\sin^2\theta_W M_Z^2\cos 2\beta}{m_+^2 - m_-^2}\,,$$

and

$$\Lambda_\pm = \mathrm{Re}\int_0^1 \mathrm{d}x\, x\ln\left(\frac{p^2}{Q^2}x^2 + \frac{m_\pm^2 - m_t^2 - p^2}{Q^2}x + \frac{m_t^2}{Q^2}\right),$$

$$L_\pm = \mathrm{Re}\int_0^1 \mathrm{d}x\ln\left(\frac{p^2}{Q^2}x^2 + \frac{m_\pm^2 - m_t^2 - p^2}{Q^2}x + \frac{m_t^2}{Q^2}\right).$$

Finally $m_\pm$ are the physical stop masses and m_t is the running top mass. They can be read off from Eqs. (5), (6) by putting H_1 and H_2 on their v.e.v.'s. We have evaluated them at the scale Q appearing in the integrals Λ and L. The choice of Q is not critical, since physical masses, that is poles of the propagators, should be scale-independent, up to this order of approximation. In our numerical analysis we have chosen this scale to be the heaviest of the thresholds involved which is either the gluino mass $m_{\tilde g}$ or the mass of the $\tilde d_L^c$ squark, depending on the values of the inputs $m_{1/2}$ and m_0. We have numerically checked that our results remain independent of the scale, as far as it lies between M_Z and the heaviest threshold.

The chirality flipping part of the one loop propagator is found to be

$$\mathrm{i}\epsilon_{\alpha\beta}[(\mathcal{M} + b + a\mathcal{M} + \mathcal{M}a)(p^2 - \mathcal{M}^2 - D(p^2))^{-1}]_{ij}\,,$$

with

$$D(p^2) = \mathcal{M}b + b\mathcal{M} + \mathcal{M}a\mathcal{M} + \mathcal{M}^2 a\,.$$

Then, physical masses are determined by the poles of the propagator or equivalently

$$\det(p^2 - \mathcal{M}^2 - D(p^2)) = 0\,. \qquad (13)$$

This is also the position of the poles of the chirality conserving part. Eq. (13) can be solved perturbatively giving corrections δm_n^2 to the n^{th} neutralino mass squared m_n^2,

$$\delta m_n^2 = 2 \sum_{i,j} R_{ni} R_{nj} (m_n^2 a_{ij} + m_n b_{ij}) \,. \tag{14}$$

In Eq. (14) the matrix $\boldsymbol{R}$ diagonalizes the tree-level neutralino mass matrix, $\boldsymbol{R}\mathcal{M}\boldsymbol{R}^{\mathrm{T}}$ = diagonal while a_{ij} and b_{ij} are evaluated for $p^2 = m_n^2$. Although it suffices to know the parameters entering $\mathcal{M}$ and the stop mass matrix at the scale Q for the determination of the corrections given by Eq. (14), a systematic renormalization group analysis that is consistent with the radiative breaking scenario and takes into account all existing experimental and theoretical constraints has to be performed.

The corrections under discussion, if one neglects the aforementioned constraints, can be as large as 10 per cent or even more in some special cases. In order to understand qualitatively how this may arise, consider a simplified picture in which the tree-level stop mass matrix is given by

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\mathrm{L}}^2 & \Delta \\ \Delta & m_{\mathrm{R}}^2 \end{pmatrix} \tag{15}$$

with $m_{\mathrm{L}}^2 \simeq m_{\mathrm{R}}^2 \simeq m_t^2 + m_{\mathrm{SUSY}}^2$ and $\Delta = m_t(A + \mu \cot\beta)$. The dominant mass renormalization, as can be seen from Eq. (12), is provided by the $\tilde{\chi}_2\tilde{\chi}_2$ term. Obviously sizeable corrections can be obtained if $b_{\tilde{\chi}_2\tilde{\chi}_2}$ becomes large. The latter is true if we have a large top mass, large mixing ($\cos\phi \sim \sin\phi = 1/\sqrt{2}$) and large stop mass splitting (giving $(L_+ - L_-) \sim \ln(m_+^2/m_-^2) \sim \mathcal{O}(1)$). The second condition is always satisfied since $m_{\mathrm{L}}^2 \sim m_{\mathrm{R}}^2$. The effect of the radiative corrections is expected to be enhanced in the case of light neutralinos ($< M_W$). If we assume $\tan\beta = 1$, one of the eigenstates has mass $-\mu$. Thus with $|\mu| < M_W$, we always have a light ($< M_W$) neutralino state. If in addition $M_{1,2} \gg M_W$, the condition $\mathrm{Tr}\,\mathcal{M} = M_1 + M_2$ guarantees that a second light state exists as well. Thus, in this case we obtain two light neutralinos. In Table 1 we present a typical example of this situation with the resulting radiative corrections to the neutralino tree-level masses, as they are obtained using Eq. (14) [#2], where all wave function (Eq. (11)) and mass (Eq. (12)) renormalization effects have been taken into account.

One observes a substantial correction (10 per cent) to the lightest neutralino state. Therefore, large radiative effects to the light neutralinos cannot be excluded a priori. Admittedly this is an oversimplified picture. We know that $m_{\mathrm{L}}^2 \neq m_{\mathrm{R}}^2$ since soft masses $m_{\tilde{t}}$ and $m_{\tilde{t}^c}$ are different due to their different dependence on

$$\Delta_t = \int_0^{t_0} \mathrm{d}t \, \frac{h_t^2}{(4\pi)^2} (A^2 + m_{Q_3}^2 + m_{U_3}^2 + m_{H_2}^2), \quad t_0 \equiv \ln \frac{m_{\mathrm{GUT}}^2}{Q^2}$$

$m_{\mathrm{L}}^2 \sim m_{\mathrm{R}}^2$ can only occur provided that $m_0^2 + 7m_{1/2}^2 \gg 2\Delta_t$, that is when the m_0 and $m_{1/2}$ dependence of $m_{\tilde{t},\tilde{t}^c}^2$

Table 1
Tree-level (m_i) and radiatively corrected (m_i') neutralino masses (absolute values are shown) for the input values m_t, A_0, m_0, $m_{1/2}$, $\tan\beta$ and μ shown below the table

m_i	m_i'	$m_i - m_i'$	Mass correction (%)
670.64	669.01	1.63	0.24
346.59	346.39	0.20	0.06
70.00	71.98	−1.98	−2.84
52.77	47.65	5.12	9.71

$m_t = 170$ GeV, $A_0 = 300$ GeV, $m_0 = 250$ GeV, $m_{1/2} = 800$ GeV, $\tan\beta = 1$, $\mu = 700$ GeV; stop masses: 393 GeV, 169 GeV.

[#2] We are aware of the fact that such large values of m_t are not compatible with $\tan\beta = 1$ within the perturbatively regime.

overwhelms that of Δ_t. This constraints restrict considerably the parameter space. Besides, the large mixing condition we have assumed imposes a further constraint, namely $m_{\tilde{L}}^2 - m_{\tilde{R}}^2 \ll 2m_t(A + \mu \cot \beta)$, reducing even more the allowed region. Even if there is a window where these conditions are satisfied, consistency with radiative breaking and small value of μ (where the radiative effects are important) is not certain. There is a strong correlation of the parameters entering in the radiative corrections under discussion and therefore they cannot be chosen at will. In order to see whether such a picture can really emerge, we have therefore to perform a systematic renormalization group analysis that takes into account all theoretical constraints and experimental bounds put on physical parameters.

The renormalization group equations (RGE) of the running parameters involved can be found in the literature and will not be repeated here [5,20]. As free parameters of the model we take the soft breaking parameters A_0, m_0, $m_{1/2}$ at the unification scale $M_{\mathrm{GUT}}(\simeq 10^{16}\,\mathrm{GeV})$ [#3] and the values of $\tan\beta(M_Z)$ and $m_t(M_Z)$ as we already mentioned. This is the same procedure adopted by other authors as well. The value of the Yukawa coupling $h_t(M_Z)$ can easily be evaluated and the numerical routines for the gauge and Yukawa couplings provide the value $h_0 = h_t(M_{\mathrm{GUT}})$ at the unification scale M_{GUT}. We shall limit ourselves to the case when the bottom and the tau Yukawa couplings are small compared to h_t. This, of course, excludes large values for $\tan\beta$ (some models seem to prefer values in the range $3 < \tan\beta < 15$ though higher values $\leqslant 85$ cannot be excluded) [25]. In any case, our numerical results show that accepting nonvanishing bottom and tau Yukawa couplings produces minor changes in the radiatively corrected neutralino masses, and therefore we ignore them in the following.

Starting with A_0, m_0, $m_{1/2}$ and h_0 at M_{GUT}, we run the RGE for all soft masses and A down to the scale M_Z. The values $B_0 = B(M_{\mathrm{GUT}})$ and $\mu_0 = \mu(M_{\mathrm{GUT}})$ are not considered as free parameters, as we have repeatedly remarked, but their values at M_Z can be extracted from the minimization conditions, Eqs. (7), (8). Their values at any other scale can be found by running the RGE for these parameters. Solving Eqs. (7), (8) is a straightforward task, if the radiative corrections to the potential are ignored. However it is important that these corrections should be included. In that case, the determinations of $B(M_Z)$ and $\mu(M_Z)$ is not that easy since the dominant top–stop contributions depend on μ through the field dependent masses in Eqs. (5), (6). Several runs are required to achieve convergence and get the desired corrected values. It is well known that the values of μ obtained by that way could differ substantially from their tree-level ones in some regions of the parameter space. As far as the boundary conditions at the unification scale M_{GUT} are concerned, various types are possible depending on the underlying theory. The simplest choice at M_{GUT}, corresponding to unification within the minimal supergravity context, is

$$m_Q^2 = m_{U^c}^2 = m_{D^c}^2 = m_{E^c}^2 = m_L^2 = m_{H_1}^2 = m_{H_2}^2 = m_0^2 .$$

and

$$M_1 = M_2 = M_3 = m_{1/2} ,$$

which we shall assume.

Due to an infrared fixed point [26,27] of the Yukawa coupling, $\tan\beta(M_Z)$ is forced to a minimum value, given the input value $m_t(M_Z)$. If this value is exceeded, h_t becomes nonperturbatively large as we increase the scale. We shall therefore impose the perturbative requirement $h_t^2/(4\pi) \leqslant \mathcal{O}(1)$ at all scales. The running mass m_t and the physical (pole of the propagator) top quark mass M_t are related by [28]

$$M_t = \frac{m_t(M_t)}{1 + 4\alpha_3(M_t)/3\pi + \dots} .$$

This takes into account the QCD corrections to the top quark propagator. In our numerical studies we have taken $M_t > 110$ GeV which is the lower experimental bound put on M_t.

The choice of the scale Q involved in Eqs. (11), (12) through the integrals A and L, is not important as physical masses should not depend on it. We have numerically checked that this is the case. In our calculation we have

[#3] Our numerical analysis reveals that the corrections are not sensitive to the precise value of M_{GUT} as long as it is in the range of 10^{16} GeV. In the rest of our discussion, we take $M_{\mathrm{GUT}} = 10^{16}$ GeV.

chosen Q to be the largest of the thresholds encountered as we run from M_{GUT} down to M_Z. Following Refs. [8,12], we have ignored the small electroweak breaking effects and define the threshold μ_i, for the particle i, to be the scale where

$$m_i^2(\mu_i^2) = \mu_i^2 .$$

The scale Q turns out to be the heaviest of the gluino and the $\tilde{d}_L^c$ squark mass, depending on the initial values of m_0 and $m_{1/2}$. The electroweak breaking effects do not alter substantially the value of Q, provided it stays in the TeV range.

In Tables 2–5, we present sample results for $m_t(M_Z) = 150$ and 180 GeV and for various values of m_0 and $m_{1/2}$ such that $m_0 \simeq m_{1/2}$, $m_0 > m_{1/2}$ and $m_0 < m_{1/2}$. Cases where $A_0 = 0$ or $A_0 \neq 0$ are shown. As far as the values of

Table 2
Tree-level neutralino masses (absolute values are shown) and the resulting radiative corrections percentage for $A_0 = 0$ GeV and $A_0 = 200$ GeV respectively and for values of m_0, $m_{1/2}$, $m_t(M_Z)$ and tan β as shown below the Table

$\mu > 0$		$\mu < 0$	
Tree level mass	Correction (%)	Tree level mass	Correction (%)
162.67 (194.88)	+0.06 (−0.45)	153.33 (184.77)	−0.04 (−0.44)
115.73 (164.78)	+0.45 (+0.22)	123.10 (171.07)	+0.37 (+0.17)
56.30 (62.71)	+0.04 (−0.16)	59.56 (70.97)	−0.20 (−0.40)
23.00 (33.97)	−1.60 (−1.00)	37.02 (42.10)	−0.68 (−0.19)

$m_0 = 200$ GeV, $m_{1/2} = 100$ GeV, $A_0 = 0$ (200)GeV, $m_t(M_Z) = 150$ GeV, tan $\beta(M_Z) = 10$. The absolute value of $\mu(Q) = 96$ (150)GeV. See main text for details.

Table 3
Same as in Table 2 for the no-scale case: $A_0 = 0$ and $m_0 = 0$ and for $m_{1/2}$, $m_t(M_Z)$ and tan β as shown below the Table

$\mu > 0$		$\mu < 0$	
Tree level mass	Correction (%)	Tree level mass	Correction (%)
285.29	+0.39 (−0.11)	275.30	−0.41 (−0.11)
254.40	−0.29 (−0.40)	258.38	−0.30 (−0.38)
142.18	−0.52 (−0.27)	151.81	−0.49 (−0.26)
81.67	−0.33 (−0.20)	86.01	−0.17 (−0.05)

$m_0 = 0$ GeV, $m_{1/2} = 200$ GeV, $A_0 = 0$ GeV, $m_t(M_Z) = 150$ GeV, tan $\beta(M_Z) = 10$. The absolute value of $\mu(Q) = 246$ GeV. The numbers in brackets are the mass corrections in the absence of wave function renormalization effects.

Table 4
Same as in Table 2, for different input values for the parameters A_0, m_0, $m_{1/2}$, m_t and tan β

$\mu > 0$		$\mu < 0$	
Tree level mass	Correction (%)	Tree level mass	Correction (%)
347.20 (377.03)	−0.63 (−0.92)	326.00 (359.74)	−0.47 (−0.56)
313.99 (348.52)	−0.42 (−0.52)	319.11 (352.40)	−0.68 (−0.85)
142.38 (145.77)	−0.44 (−0.45)	171.71 (172.28)	−0.18 (−0.16)
79.14 (80.46)	−0.44 (−0.40)	89.92 (89.80)	−0.04 (−0.04)

$m_0 = 200$ GeV, $m_{1/2} = 200$ GeV, $A_0 = 0$ (200)GeV, $m_t(M_Z) = 150$ GeV, tan $\beta(M_Z) = 2.5$. The absolute value of $\mu(Q) = 311$ (346)GeV.

Table 5. Same as in Table 2, for different input values for the parameters A_0, m_0, $m_{1/2}$, m_t and tan β

$\mu>0$		$\mu<0$	
Tree level mass	Correction (%)	Tree level mass	Correction (%)
429.69 (435.28)	−2.50 (−2.47)	427.14 (432.80)	−2.16 (−2.18)
416.66 (422.44)	−2.00 (−2.00)	417.33 (423.09)	−2.62 (−2.60)
33.75 (33.84)	−0.23 (−0.22)	49.88 (49.80)	+0.17 (+0.17)
16.94 (17.04)	−2.39 (−2.42)	23.65 (23.63)	+0.19 (+0.18)

$m_0=500$ GeV, $m_{1/2}=50$ GeV, $A_0=300$ (700)GeV, $m_t(M_Z)=180$ GeV, tan $\beta(M_Z)=3$. The absolute value of $\mu(Q)=413$ (419)GeV.

tan $\beta(M_Z)$ are concerned, we present results for the smallest allowed value and for tan $\beta(M_z)=10$. These cases give a clear general picture. We have actually scanned a large region of the parameter space confirming the above statement. Therefore these results should be considered as representatives of the general situation. Deliberately, and in order to explore a large region of the parameter space, we have not imposed any constraints on the trilinear coupling A resulting from the tree-level study of the scalar potential. These constraints, which are imposed to avoid charge and color breaking minima, are known to be too restrictive and the picture could very well change when loop effects are taken into account [29]. We also do not exclude cases where the light Higgs gets lower than the ALEPH bound of $\simeq 58$ GeV [30] or chargino masses lower than 45 GeV.

As one can observe, the correction to the neutralino masses are quite small. They are less than 1 per cent even in cases where light states $\simeq 40$ GeV appear in the spectrum. In the case of a very light $\simeq 20$ GeV neutralino, the corrections are slightly enhanced to 2.5 per cent. The smallness of these corrections is not a result of a mere destructive interference of wave function and mass renormalization effects. We can switch off the wave function renormalization contribution to the neutralino propagator and no enhancement is observed [#4]. Such a case is shown in Table 3. We have also allowed for a scale Q different from the heaviest threshold. No significant change, up to this order of approximation, was observed. The smallness of the radiative corrections under discussion leaves no hope that the inclusion of the remaining particle contributions of the effective potential with alter the situation. Two simple arguments are in favour of this statement: (a) it is known that such contributions could not be larger than those of the top-stop system, and (b) the scale Q could not play any critical rôle since our results do not depend on that scale.

Let us summarize by repeating our main conclusions. We have systematically computed the top-stop one loop corrections to the physical neutralino masses in the MSSM. The heaviness of the top quark ensures that these corrections are the dominant ones. Although large corrections, of the order of 10 per cent cannot be excluded, these corrections turn to be very small if the radiative electroweak breaking scenario is adopted. This scenario, along with the universal boundary conditions imposed at M_{GUT} (making the number of the free parameters small), strongly correlates the running parameters and diminishes the effects of the radiative corrections. In most of the parameter space these corrections never exceed a few per cent $<\mathcal{O}(2\%–3\%)$, even in cases where some of the neutralino masses are low $\mathcal{O}(20\text{ GeV})$ and the effect would have been expected to be enhanced. This implies that the relevant parameter space for the phenomenological study of the neutralino sector is consistently described by the $(m_{\tilde{g}}, \mu)$ pair, since these two quantities are very weakly correlated to the other parameters of the model and especially to the heavy top quark mass. This natural suppression of the radiative effects in the neutralino sector from the top-stop system, may be welcome since it shows that LEP and CDF phenomenological studies and constraints imposed on $m_{\tilde{g}}$ and μ are stable against radiative corrections.

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[#4] The wave function and mass renormalization effects are actually found to be of the same order of magnitude.

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