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GUT, supersymmetry and top threshold corrections in the $SU(4)\times SU(2)_L\times SU(2)_R$ model

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We investigate the interplay between GUT, SUSY and top threshold effects in the context of the string-derived model based on $SU(4)\times SU(2)_L\times SU(2)_R$. The SUSY and top thresholds demand a top mass somewhat high ($m_t > 140$ GeV) and a GUT scale $\sim 10^{15}$ GeV. This is lower than the GUT scale demanded in the model, where these thresholds are not taken into account.

1. Introduction

The success of the Standard Model (SM) is beyond question. It provides understanding of the strong and the electroweak interactions and it has succeeded in passing all experimental tests so far (within the experimental and theoretical uncertainties). Nevertheless, the plethora of free parameters, in conjunction with the question of unification at high energies, remains an unsolved problem. As far as the second issue is concerned, supersymmetric (SUSY) models show, in general, a better attitude, providing coupling constant unification at scales of the order of 10^{16-17} GeV, while at the same time reproducing the experimental values of the low-energy parameters.

The non-observation of SUSY particles forces us to admit the existence of an energy region, above M_Z, where the (non-SUSY) SM is effective. The present accuracy of measuring the low-energy parameters (α^{-1}, $\sin^2\theta_w$ and α_3) permits us to check the limits of successfulness of the SUSY models through the threshold effects [1-5] of the SUSY particles to the running of the coupling constants. Nevertheless, as pointed out in ref. [6], GUT-dependent threshold corrections at the unification scale could be important. Therefore, the presence of a specific GUT model, or an estimate of these threshold corrections, make our results, although model-dependent, safer.

In this letter, after a quick overview of the threshold corrections, we concentrate on a successful Grand Unified model, namely the one based on the $SU(4)\times SU(2)_L\times SU(2)_R$ symmetry and check its ability to reproduce the low-energy parameters, when GUT, SUSY and top threshold effects are taken into account.

2. The threshold corrections

The Renormalization Group Equations (RGEs) govern the running of the gauge couplings through the corresponding β-functions of the groups on which our theory is based. These β-functions are determined by the light-particle content of the model. If at some energy scale our symmetry breaks to a smaller one, some of the particles of the initial symmetry eventually become massive and do not contribute to the β-function below that scale. The threshold corrections take into account the contribution of these massive states to the running of the gauge couplings, since they could appear as virtual particles even below the symmetry-breaking scale. The effect is the same as if we had subtracted each particle contribution to the β-function(s) at the energy scale that is equal to its mass. Following the formalism of refs. [1,2], in the vicinity

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of the symmetry-breaking scale, the coupling constants of the unbroken α_G and of the broken α_{G_k} regions are related by

$$\frac{1}{\alpha_{G_k}(\mu)} \equiv \frac{1}{\alpha_G(\mu)} - \delta(\alpha_{G_k}^{-1}) = \frac{1}{\alpha_G(\mu)} - 4\pi\lambda_k(\mu) .$$

It is easy to see that these corrections are of the same order with the two-loop solution of the RGEs for the gauge couplings. The "matching" function λ is given by the general formula [3,5,7]

$$\lambda(\mu) = 2\sum_i b_i \ln\frac{M_i}{\mu} + C ,$$

where b_i is the contribution to the β-function of the particle i with mass M_i. The constant term depends on the renormalization scheme. In the $\overline{\rm DR}$ scheme $C=0$ [8], while in the $\overline{\rm MS}$ scheme it is given by

$$C = -\tfrac{2}{21} b_i' ,$$

where b_i' is the contribution to the β function of the gauge bosons (plus corresponding ghosts and would be Goldstone bosons) acquiring mass. This term appears in the $\overline{\rm MS}$ scheme from the product of the $1/\epsilon$ term of the momentum integral with the ϵ term of the γ-matrix algebra. The latter is missing in the $\overline{\rm DR}$ scheme. The connection between the two schemes comes through the conversion factor

$$\frac{1}{\alpha_{\overline{\rm DR}}} = \frac{1}{\alpha_{\rm MS}} - \frac{C_2(G)}{12\pi} ,$$

where $C_2(G)$ is the quadratic Casimir operator of the group G. By changing from one scheme to the other and back, the difference of the two conversion factors gives the constant term of the $\overline{\rm MS}$ scheme.

An equivalent formula for the matching function is [2,3]

$$\lambda(\mu) = \frac{1}{48\pi^2}\left(\mathrm{Tr}[t_{iV}^2] - 21\,\mathrm{Tr}\left[t_{iV}^2 \ln\frac{M_V}{\mu}\right] + C_F\mathrm{Tr}\left[t_{iF}^2 \ln\frac{M_F}{\mu}\right] + C_S\,\mathrm{Tr}\left[t_{iS}^2 \ln\frac{M_S}{\mu}\right]\right),$$

where V, F, and S stand for vector boson, fermion and scalar, while t is the group generator in the appropriate representation. The index i runs over the particles that become massive. The factor for the fermions takes the value $C_F=8$ for Dirac and $C_F=4$ for two-component spinors or Majorana fermions (the latter is applicable when the L and R components are treated separately). For the scalars the values are $C_S=1$ for complex representations and $C_S=2$ for real ones. For a SUSY theory the above formula takes the form

$$\lambda(\mu) = \frac{1}{48\pi^2}\left(\mathrm{Tr}[t_{iV}^2] - 13\,\mathrm{Tr}\left[t_{iV}^2 \ln\frac{M_V}{\mu}\right] + 6\,\mathrm{Tr}\left[t_{iC}^2 \ln\frac{M_C}{\mu}\right]\right),$$

where V and C stand for the vector and chiral multiplets. Note the unchanged constant term that comes from the spin-1 momentum integral. As mentioned before, in the $\overline{\rm DR}$ scheme this constant term is missing.

3. Threshold effects in the SU(4)×SU(2)$_L$×SU(2)$_R$ model

We are going to investigate the interplay between GUT, top and supersymmetry threshold effects in a string-derived model based on SU(4)×SU(2)$_L$×SU(2)$_R$. The symmetry group, derived from the free fermionic formulation [9], is SU(4)×O(4)×U(1)^4×{SU(8)×U(1)'}$_{\rm Hidden}$ [10]. For more information on the spectrum and the properties of the model see refs. [10,11]. We quickly review the different scales appearing in the running of the RGEs:

- M_U, where $\alpha_4 = \alpha_{2L} = \alpha_{2R} \equiv \alpha_U$,
- M_A, where one of the U(1)'s, which is anomalous, breaks and where a number of fields acquire masses through some singlet fields. Between M_U and M_A we assume the full string content of the model. Both M_U and M_A can be fairly well approximated by the simple expressions

$$M_U \sim 1.7\sqrt{4\pi\alpha_U}\times 10^{18}\ \mathrm{GeV} ,$$

$$M_A \sim 7.8\sqrt{4\pi\alpha_U}\times 10^{17}\ \mathrm{GeV} ,$$

- M_X, where the group SU(4)×SU(2)$_L$×SU(2)$_R$ breaks down to the MSSM and the relations among the couplings are

$$\alpha_3 = \alpha_4 ,\quad \alpha_2 = \alpha_{2L} ,\quad \frac{1}{\alpha_1} = \frac{1}{\frac{5}{3}\alpha_{2R}} + \frac{1}{\frac{5}{2}\alpha_4} , \qquad (1)$$

and finally

$-M_I$, where only the Standard Model content is present. Between M_X and M_I some exotic remnants could survive.

We give the breaking pattern at M_X for the different multiplets (the quantum numbers on the left correspond to $SU(4)\times SU(2)_L\times SU(2)_R$, while those on the right correspond to $SU(3)\times SU(2)_L\times U(1)$):

$$n_H(\overset{(-)}{4},1,2)\to n'_3(\overset{(-)}{3},1,\pm\tfrac{2}{3})+n_3(\overset{(-)}{3},1,\pm\tfrac{1}{3})$$
$$+n_s(1,1,\pm1)+n'_s(1,1,0)\,,$$
$$n_4(\overset{(-)}{4},1,1)\to n'_3(\overset{(-)}{3},1,\pm\tfrac{1}{6})+n'(1,1,\pm\tfrac{1}{2})\,,$$
$$n_{22}(1,2,2)\to n_2(1,2,\pm\tfrac{1}{2})\,,$$
$$n_L(1,2,1)\to n'_L(1,2,0)\,,$$
$$n_R(1,1,2)\to n'(1,1,\pm\tfrac{1}{2})\,,$$
$$n_6(6,1,1)\to n_3(\overset{(-)}{3},1,\pm\tfrac{1}{3})\,.$$

The number of generations is always 3. The full string content is

$$n_g=3\,,\quad n_l=n_R=10\,,$$
$$n_6=n_{22}=n_H=2n_4=4\,.$$

In a previous work [13] the threshold effects of the fields becoming massive at M_X were taken into account, assuming a degenerate mass of the order of M_X. The MSSM was effective down to M_Z. It was found that, for M_X in the region 3×10^{15}–10^{16} GeV, the low-energy parameters stay within the experimental limits for a wide range of particle content between M_A and M_X and with a remnant ($n'_3=n'=2$) between M_X and M_I.

We now turn to the SUSY thresholds. For completeness we write down the three matching functions corresponding to the groups SU(3), SU(2) and U(1):

$$\lambda_3(\mu)=\frac{1}{48\pi^2}\left(12\ln\frac{M_{\tilde g}}{\mu}+12\ln\frac{M_{\tilde q}}{\mu}-3\right),\qquad(2)$$

$$\lambda_2(\mu)=\frac{1}{48\pi^2}\left(9\ln\frac{M_{\tilde q}}{\mu}+8\ln\frac{M_{\tilde W}}{\mu}+3\ln\frac{M_{\tilde l}}{\mu}\right.$$
$$\left.+4\ln\frac{M_{\tilde H}}{\mu}+\ln\frac{M_H}{\mu}-2\right),\qquad(3)$$

$$\lambda_1(\mu)=\frac{1}{48\pi^2}\left(\tfrac{33}{5}\ln\frac{M_{\tilde q}}{\mu}+\tfrac{27}{5}\ln\frac{M_{\tilde l}}{\mu}+\tfrac{12}{5}\ln\frac{M_{\tilde H}}{\mu}\right.$$
$$\left.+\tfrac{3}{5}\ln\frac{M_H}{\mu}\right),\qquad(4)$$

where the subscripts $\tilde q$, $\tilde g$, $\tilde l$, $\tilde W$ and $\tilde H$ stand for the squark, gluino, slepton, wino and higgsino, while H stands for the heavy Higgs doublet. In the above formulae, we have incorporated the constant term coming from the conversion from $\overline{\rm DR}$ above the SUSY breaking scale, to $\overline{\rm MS}$, below that scale.

For the masses of the sparticles we shall assume a simplified version of the $m_{1/2}$ and m_0 scenario, where $m_{1/2}$ is a universal gaugino mass and m_0 a universal scalar mass. The masses of the gauginos, squarks and sleptons are given by the (one-loop) equations [12]

$$m_{\tilde g}=\frac{\alpha_3(m_{\tilde g})}{\alpha_X}m_{1/2}\,,\quad m_{\tilde W}=\frac{\alpha_2(m_{\tilde W})}{\alpha_X}m_{1/2}\,,$$

and

$$m_{\tilde q}^2=m_0^2+7m_{1/2}^2\,,\quad m_{\tilde l}^2=m_0^2+0.3m_{1/2}^2\,,$$

where we have assumed a common squark mass and a common slepton mass (and also that $\tan\beta=\langle\bar v\rangle/\langle v\rangle=1$). The coupling α_X is at the scale M_X.

Finally the top threshold corrections to α_3 and α^{-1} are given by

$$\delta^{\rm top}(\alpha_3^{-1})=\frac{1}{3\pi}\ln\frac{m_t}{M_Z}\,,\quad \delta^{\rm top}(\alpha^{-1})=\frac{8}{9\pi}\ln\frac{m_t}{M_Z}\,.$$

As far as the correction to $s^2\equiv\sin^2\theta_w$ is concerned, following ref. [5] we use the formula

$$\delta^{\rm top}(s^2)=1.05\times10^{-7}\,{\rm GeV}^{-2}\,(m_t^2-M_Z^2)\,.$$

The (SUSY and top) threshold-corrected low-energy parameters are given by

$$\alpha_3+\delta^{\rm thr}(\alpha_3)\,,\quad s^2-\delta^{\rm thr}(s^2)\,,\quad \alpha^{-1}-\delta^{\rm thr}(\alpha^{-1})\,.\qquad(5)$$

Our aim is to find the allowed regions in the ($m_{\tilde g}$, m_0) space – where we have traded $m_{1/2}$ for $m_{\tilde g}$ – which can lead to experimentally accepted values of the low-energy parameters. These regions will give us the corresponding allowed masses for the sparticles. The strategy is the following: we run the RGE, including M_X threshold effects (varying the ratio $r=M/M_X$, where M is the degenerate mass of the fields becoming massive at M_X), and top threshold effects (for m_t = 120–200 GeV). We keep SUSY β-functions down to M_Z. This will give us the ranges of the required corrections to α^{-1}, s^2 and α_3, at M_Z, in order to have experimentally allowed values at M_Z. Finally from the δ^{thr} of the sparticles (and the heavy Higgs) we search for a region in the $(m_{\tilde g}, m_0)$ space giving the required corrections to *all three* parameters.

In order to isolate and concentrate on the interplay among the three types of threshold corrections (GUT, SUSY and top), we choose all GUT parameters, except r, to be constant:

$$\alpha_U = 0.053\,,$$

$$M_X = 10^{15}\,\text{GeV}\,,\quad M_I = 3\times10^{12}\,\text{GeV}\,,$$

$$n_L = n_R = 4\,,\quad n_H = n_{22} = n_4 = \tfrac{1}{2}n_6 = 2\,,$$

$$n'_3 = n' = 2\,,\quad n'_{31} = n'_1 = 0\,.$$

At this point we should note that (i) we have taken into account the fact that the three gauge couplings are no longer unified at M_X. However this amounts to a sizeable change only in $m_{\tilde q}$: the coefficient of $m_{1/2}^2$ varies between 7.0 and 8.5. In fig. 1 we plot contours of constant mass for $m_{\tilde H}$, $m_{\tilde l}$ and $m_{\tilde q}$. For the latter, we have plotted the corresponding bands for the whole range of the $m_{1/2}^2$ coefficient mentioned above. (ii) The inclusion of the SUSY and the top thresholds forces us to reduce M_X to somehow lower values relative to the range mentioned in ref. [13]. (iii) In all our calculations we have taken $m_H = m_{\tilde H} = 100$ GeV.

In fig. 2 we plot the allowed regions in the $(m_{\tilde g}, m_0)$ space for different r and m_t values. The ranges of $m_{\tilde g}$ and m_0 were chosen so that sparticle masses stay below 1 TeV while they are above their experimentally allowed values. Before trying to explain the tendencies we see in the figures, let us state some facts. With our definitions, eq. (5), $\delta^{\text{thr}}(\alpha_3)$, $\delta^{\text{thr}}(s^2)$ and $\delta^{\text{thr}}(\alpha^{-1})$ are positive. The conversion factors C_3 and C_2 are too small to render the corresponding matching functions negative. This could only happen for very low $(m_{\tilde g}, m_0)$ values being outside our space. Also, when $m_{\tilde H}$ or $m_{\tilde l}$ get smaller than M_Z they decouple from the matching functions. This means that, before the inclusion of the SUSY and top thresholds, α_3 should stay below its maximum allowed experimental value, while s^2 and α^{-1} should stay above their lowest experimental values. Our GUT model has the tendency of giving high α_3 values (this is the main reason for keeping n'_3 different from zero below M_X). Any change in the GUT parameters producing an increase in the α_3 value, should lead to lower SUSY and top thresholds. Let us ow turn to the figures. For constant m_t, the sparticle masses get smaller when r is increased (GUT thresholds increase with r).

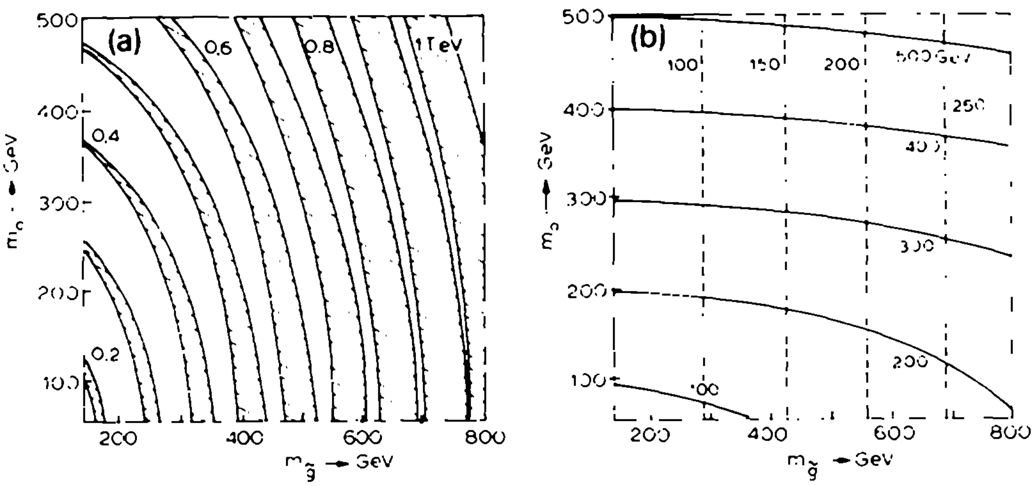

Fig. 1. Contours of constant $m_{\tilde q}$ (a) and $m_{\tilde l}$ and $m_{\tilde H}$ (b). In the latter, the band for each value of $m_{\tilde q}$ corresponds to the range of the $m_{1/2}^2$ coefficient: $m_{\tilde q}^2 = m_0^2 + (7-8.5)m_{1/2}^2$.

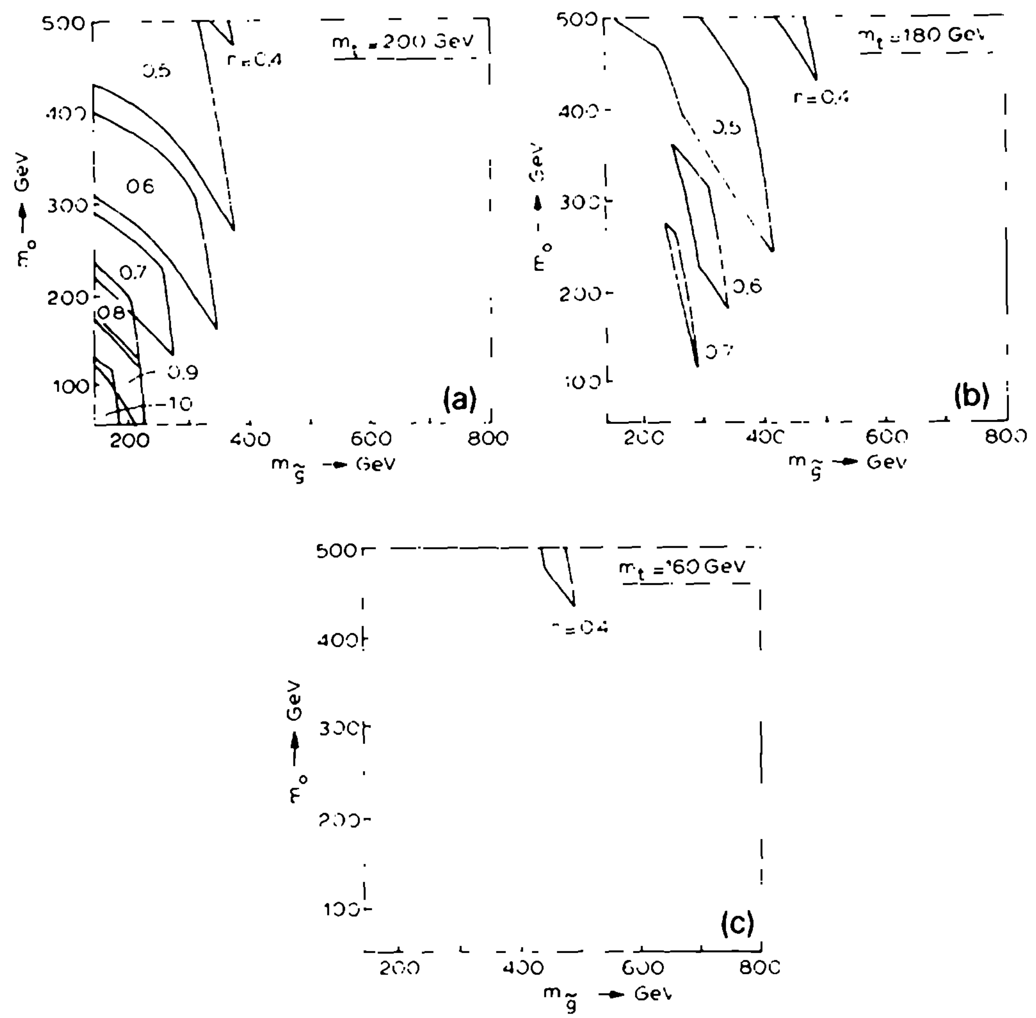

Fig. 2. Allowed regions in the $(m_{\tilde{g}}, m_0)$ space for different values of r and for $m_t = 200$ GeV (a), $m_t = 180$ GeV (b) and $m_t = 160$ GeV (c).

Therefore, for each m_t, we expect an allowed band in the r values. The limits on r are dictated by the allowed sparticle masses. This band becomes narrower as the top mass becomes smaller. For the specific choice of the GUT parameters we obtain the values shown in table 1.

For $m_t \leq 140$ GeV no allowed r region exists. Lowering m_t and increasing r could, in principle, compensate the two effects. Remember, however, that although there may exist regions rendering each low-energy parameter experimentally acceptable, these regions may not overlap. To show the complexity of the situation, we present two cases in fig. 3. In the first, overlapping of the allowed regions exists while in the second it does not.

Table 1

m_t (GeV)	r
200	0.4–1
180	0.4–0.7
160	0.4

Some comments on the GUT parameters are in order. When M_1 varies between 10^{12} GeV and 10^{14} GeV,

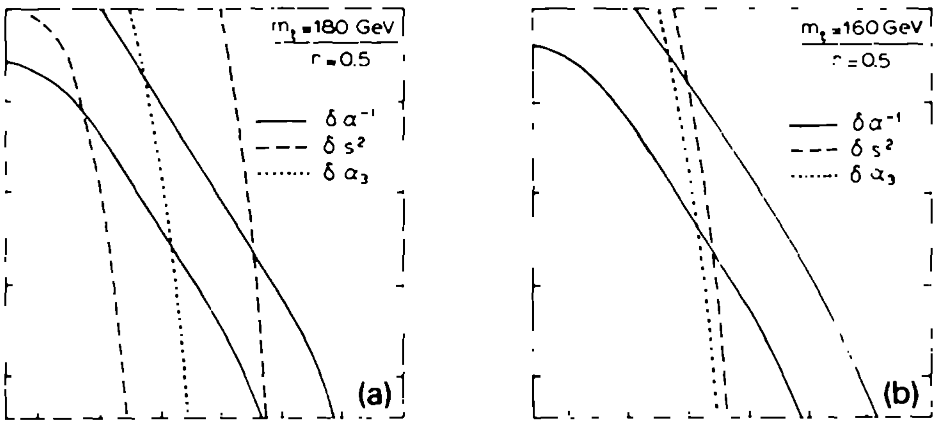

Fig. 3. The regions of the allowed SUSY thresholds for two cases. (a) $r=0.5$, $m_t=180$ GeV: $\delta(\alpha_3)\leqslant 0.014$ (dotted line), $1.4\times10^{-3}\leqslant\delta(s^2)\leqslant 2.6\times10^{-3}$ (dashed line) and $1.15\leqslant\delta(\alpha^{-1})\leqslant 1.35$ (continuous line). The overlapping region is shown in fig. 2b. (b) $r=0.5$, $m_t=160$ GeV $\delta(\alpha_3)\leqslant 0.014$ (dotted line), $\delta(s^2)\geqslant 2.1\times10^{-3}$ (dashed line) and $1.38\leqslant\delta(\alpha^{-1})\leqslant 1.58$ (continuous line). No overlapping region exists.

the allowed regions do not change significantly, and our results have the same qualitative features. For $M_1=M_X$, or equivalently with $n'_3=n'=0$, the model cannot give acceptable low-energy results [13]. Therefore $n'_3=n'=2$ is the simplest choice. Changing M_X or α_U results in more complicated situations. If M_X and/or α_U increase, the values of the gauge couplings at M_X also increase. This causes a significant increase of the squark masses (the coefficient of $m^2_{1/2}$ can be as large as 12 for $M_X=10^{16}$ GeV and $\alpha_U=0.053$). This fact considerably reduces the allowed $(m_{\tilde{g}}, m_0)$ space since we demand $m_{\tilde{q}}\leqslant 1$ TeV. Furthermore, as mentioned above, the corresponding increase in α_3 requires light sparticle masses. Hence, any increase in the values of M_X and/or α_U forces the allowed, if any, $(m_{\tilde{g}}, m_0)$ region to confine in the bottom-left corner of our graph.

Finally, some crude upper limits can be set on the sparticle masses. From our analysis it is obvious that these limits are achieved for the lowest possible r and m_t values

$$m_{\tilde{g}}\leqslant 500\ \mathrm{GeV},\quad m_{\tilde{q}}\leqslant 800\ \mathrm{GeV}\ ,\quad m_{H^{\pm}}\leqslant 175\ \mathrm{GeV}\ .$$

No actual limit can be set for the sleptons since for large values of m_0, $m_{\tilde{l}}\sim m_0$. The threshold effects of the strong coupling α_3 are responsible for these upper limits.

4. Conclusions

We have evaluated the GUT, SUSY and top thresholds, which should be taken into account when the two-loop RGEs are being run down to M_Z, in the context of the $SU(4)\times SU(2)_L\times SU(2)_R$ string-derived model. A general remark is that the GUT scale should be $\sim 10^{15}$ GeV, which is lower than the scale found in the case where no (SUSY and top) thresholds were taken into account. The strong coupling α_3, which tends to run high at M_Z, seems to dictate the range of the parameters in order to stay within experimentally allowed regions. The threshold effects enhance this situation. This α_3 dominance can set upper limits on the sparticle masses. The mass of the top stays above 140 GeV for a wide range of our parameters.

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