ANSATZ FOR QUARK, CHARGED LEPTON, AND NEUTRINO MASSES IN SUSY GUTs

H. DREINER*, G. K. LEONTARIS† and N. D. TRACAS‡

*Department of Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK

†Physics Department, Theory Division, University of Ioannina, GR-45 110, Ioannina, Greece

‡Physics Department, National Technical University, GR-15 780, Zografou, Athens, Greece

We extend a fermion mass matrix ansatz by Giudice to include neutrino masses. The previous predictions are maintained. With two additional parameters, a large Majorana neutrino mass and a hierarchy factor, we have seven *further* low energy predictions: the masses of the neutrinos, the mixing angles and the phase in the leptonic sector. We choose a reasonable hierarchy of Majorana masses and fit the overall mass scale according to a solution of the solar neutrino problem via the MSW mechanism, which is in agreement with the ^{37}Cl, Kamiokande, SAGE and GALLEX data. We then also obtain a cosmologically interesting tau-neutrino mass.

One of the fundamental problems of particle physics is to understand the observed fermion mass spectrum. In the Standard Model it is described by thirteen parameters: nine masses, three quark mixing angles and a CP violating phase. Ultimately one hopes for a solution in terms of a more fundamental model, e.g. string theory, where the structure of the mass matrices is determined by a set of symmetries. Meanwhile any reduction in the set of required parameters could be a guide in finding the appropriate symmetries and also lead to a prediction of yet undetermined fermion masses in terms of experimentally known masses and angles.

There has been much fruitful work on this problem.[1-4] Recently, motivated by the observed merging of the Standard Model gauge coupling constants in supersymmetric grand unified theories (SUSY GUTs),[5] Dimopoulos, Hall and Raby (DHR),[2] Giudice[3] and other groups[4] have considered simple fermion mass matrices at the GUT scale and derived the resulting spectrum below the electroweak scale using the renormalization group equations (RGEs) for the case of minimal supersymmetry. In the DHR ansatz the fourteen parameters (including $\tan\beta$, the ratio of Higgs vevs in supersymmetry) are given in terms of eight input parameters; hence six predictions. In the modified ansatz studied by Giudice more structure is imposed on the mass matrices (by hand) leading to one input parameter less and thus one more prediction. Both are in good agreement with the known experimental values.[2-4,6]

In a large class of mass matrices these ansatz are the only consistent solutions.[7]

In the work of DHR and of Giudice neutrino mass matrices were not considered. Most GUT models. e.g. SO(10), SU(4)$\otimes$SU(2)^2, SU(5)$\otimes$U(1), and E_6, predict the existence of right-handed neutrinos leading to Dirac-neutrino matrix at the GUT scale. In this letter we extend the ansatz by Giudice to include neutrino masses. This modifies the analysis in the following points:

(i) New Higgs representations should provide heavy Majorana masses to the right-handed neutrinos, in order to realize the seesaw mechanism.[8]

(ii) The leptonic mixing angles are completely determined by the charged-lepton and up-quark masses. With one additional parameter we obtain the neutrino mass spectrum and the mixing angles in the lepton sector. We thus have seven further low-energy predictions. We obtain a possible solution to the solar neutrino problem via the MSW mechanism[9] in agreement with the combined data of the ^{37}Cl, Kamiokande, SAGE,[10] and GALLEX experiments[11] and the tau-neutrino mass is cosmologically relevant.

A realistic fit to the desired structure of the mass matrices in SUSY GUT models requires a rather complicated Higgs sector. In a more fundamental theory the structure of the matrices might arise more naturally. However, as an example we consider here the relevant couplings in an SO(10) SUSY GUT, assuming that additional discrete symmetries, as in Harvey *et al.* of Ref. 1, prevent the appearance of specific entries in the mass matrices.

Within an SO(10) SUSY GUT the down-like quark and charged lepton masses arise from the following terms:

$$\hat{f}(\underline{16}_1\cdot\underline{16}_2\cdot\underline{10}_{(5)})+(2\hat{d})(\underline{16}_2\cdot\underline{16}_3\cdot\underline{10}_{(5)})+\hat{d}(\underline{16}_2\cdot\underline{16}_2\cdot\underline{126}_{(\overline{45})})+\hat{c}(\underline{16}_3\cdot\underline{16}_3\cdot\underline{10}_{(5)}),\quad(1)$$

where $\underline{16}_i$ are the fermion families. The simple relation between the second and third coefficients is imposed by hand. The $\underline{126}$ Higgs representation has a nonzero vev along the $\langle\overline{45}\rangle$ direction of the SU(5). The structure of this vev results in a relative factor (-3) in the $(2,2)$-entry of the mass matrices M_d and M_e below[12,13] and leads to the successful Georgi-Jarlskog relations $m_b \simeq m_\tau$. $m_s \simeq m_\mu/3$ and $m_d \simeq 3m_e$ at the GUT-scale.[13] The up-quark and the Dirac-neutrino masses arise from

$$\hat{b}(\underline{16}_1\cdot\underline{16}_3\cdot\underline{10}_{(5)}+\underline{16}_2\cdot\underline{16}_2\cdot\underline{10}_{(5)})+\hat{a}(\underline{16}_3\cdot\underline{16}_3\cdot\underline{10}_{(5)}).\quad(2)$$

and the heavy Majorana-neutrino mass matrix from

$$\lambda_{\nu_i}\underline{16}_i\cdot\underline{16}_i\langle\underline{126}\rangle_{(1)}\rightarrow M_i\nu_i^c\nu_i^c,\quad(3)$$

where we have assumed a diagonal Majorana mass matrix, diag(M_1, M_2, M_3). In the following we shall make the simple hierarchical ansatz $M = M_3 = kM_2 = k^2M_1$, $k = 10$, motivated by the known fermion mass hierarchies. λ is a Yukawa coupling and $i = 1, 2, 3$ is a generation index. As we have already pointed out however, these are not the most general couplings allowed, but we assume that other relevant

couplings are not present due to some kind of discrete symmetry of the underlying theory.

Thus, in the present case, we obtain the Giudice mass matrices[3] augmented by a simple ansatz for the neutrino masses

$$M_u = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \qquad M_{\nu\nu^c} = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \tag{4}$$

$$M_d = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & d & 2d \\ 0 & 2d & c \end{pmatrix}, \qquad M_e = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & -3d & 2d \\ 0 & 2d & c \end{pmatrix}, \tag{5}$$

$$M_{\nu^c\nu^c} = M \operatorname{diag}(k^{-2}, k^{-1}, 1). \tag{6}$$

The entries in the mass matrices are given by the corresponding Yukawa coupling of Eqs. (1)–(3) multiplied with the appropriate vev. In general these entries are complex. However, the fields can be redefined such that all parameters are real and we only have the shown phases.[3] We have included a single phase in the charged lepton mass matrix, since the neutrinos are no longer degenerate. For simplicity we have chosen this phase to be identical with the phase in M_d.[a] We thus have eight parameters at the GUT scale (including k) in order to provide a total of 20 parameters at low energies: six quark masses; three charged lepton masses; three light neutrino masses; six mixing angles, three for the Cabbibo-Kobayashi-Maskawa mixing matrix and three for the corresponding leptonic mixing matrix; and two phases, one for each mixing matrix. Hence, we have seven predictions for the Standard Model parameters and seven predictions for the extension of the Standard Model due to the massive neutrinos.

Using the results of Ref. 14 we obtain the renormalization group equations for the Yukawa couplings at the one-loop level

$$16\pi^2 \frac{d}{dt}\lambda_U = \left(I \cdot \operatorname{Tr}\left[3\lambda_U \lambda_U^\dagger\right] + 3\lambda_U \lambda_U^\dagger + \lambda_D \lambda_D^\dagger - I \cdot G_U\right)\lambda_U, \tag{7}$$

$$16\pi^2 \frac{d}{dt}\lambda_N = \left(I \cdot \operatorname{Tr}\left[\lambda_U \lambda_U^\dagger\right] + \lambda_E \lambda_E^\dagger - I \cdot G_N\right)\lambda_N, \tag{8}$$

$$16\pi^2 \frac{d}{dt}\lambda_D = \left(I \cdot \operatorname{Tr}\left[3\lambda_D \lambda_D^\dagger + \lambda_E \lambda_E^\dagger\right] + 3\lambda_D \lambda_D^\dagger + \lambda_U \lambda_U^\dagger - I \cdot G_D\right)\lambda_D, \tag{9}$$

$$16\pi^2 \frac{d}{dt}\lambda_E = \left(I \cdot \operatorname{Tr}\left[\lambda_E \lambda_E^\dagger + 3\lambda_D \lambda_D^\dagger\right] + 3\lambda_E \lambda_E^\dagger - I \cdot G_E\right)\lambda_E, \tag{10}$$

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[a] In general the leptonic mixing matrix will include three phases.

where λ_a, $\alpha = U, N, \overline{D, E}$, represent the 3×3 Yukawa matrices which are defined in terms of the mass matrices given in Eqs. (4)–(6), and I is the 3×3 identity matrix. We have neglected one-loop corrections proportional to λ_N^2. $t \equiv \ln(\mu/\mu_0)$, μ is the scale at which the couplings are to be determined and μ_0 is the reference scale, in our case the GUT scale. The gauge contributions are given by

$$G_\alpha = \sum_{i=1}^{3} c_\alpha^i g_i^2(t) \,, \tag{11}$$

$$g_i^2(t) = \frac{g_i^2(t_0)}{1 - \dfrac{b_i}{8\pi^2} g_i^2(t_0)(t - t_0)} \,. \tag{12}$$

The g_i are the three gauge coupling constants of the Standard Model and b_i are the corresponding beta functions in minimal supersymmetry. The coefficients c_α^i are given by

$$\{c_U^i\}_{i=1,2,3} = \left\{ \frac{13}{15}, 3, \frac{16}{3} \right\}, \qquad \{c_D^i\}_{i=1,2,3} = \left\{ \frac{7}{15}, 3, \frac{16}{3} \right\}, \tag{13}$$

$$\{c_E^i\}_{i=1,2,3} = \left\{ \frac{9}{5}, 3, 0 \right\}, \qquad \{c_N^i\}_{i=1,2,3} = \left\{ \frac{3}{5}, 3, 0 \right\}. \tag{14}$$

We find it convenient to redefine the quark and lepton fields such that λ_U and λ_N are diagonal

$$\lambda_U \to \tilde{\lambda}_U = K^\dagger \lambda_U K \,, \qquad \lambda_N \to \tilde{\lambda}_N = K^\dagger \lambda_N K \,,$$
$$\lambda_D \to \tilde{\lambda}_D = K^\dagger \lambda_D K \,, \qquad \lambda_E \to \tilde{\lambda}_E = K^\dagger \lambda_E K \,. \tag{15}$$

The diagonalizing matrix is given by[3]

$$K = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \qquad \tan 2\theta = \frac{2b}{a}. \tag{16}$$

For the up-quarks and the corresponding Dirac-neutrinos we obtain at the GUT scale

$$m_u = m_{\nu_{D_1}} = m_0 \tan^2\theta \,, \qquad m_c = m_{\nu_{D_2}} = m_0 \tan\theta \,,$$
$$m_t = m_{\nu_{D_3}} = m_0 \,, \qquad m_0 = \frac{a}{1 - \tan^2\theta} \,. \tag{17}$$

We apply the field redefinitions (15) to the differential equations (7)–(10) and within the parenthesis on the right-hand side only retain the dominant Yukawa coupling $\tilde{\lambda}_{U_{33}}^2$

$$16\pi^2 \frac{d}{dt} \tilde{\lambda}_U = \left(\tilde{\lambda}_{U_{33}}^2 \begin{pmatrix} 3 & & \\ & 3 & \\ & & 6 \end{pmatrix} - G_U(t) I \right) \tilde{\lambda}_U \,, \tag{18}$$

$$16\pi^2 \frac{d}{dt}\tilde{\lambda}_N = \left(\tilde{\lambda}_{U_{33}}^2 - G_N(t)I\right)\tilde{\lambda}_N \, , \tag{19}$$

$$16\pi^2 \frac{d}{dt}\tilde{\lambda}_D = \left(\tilde{\lambda}_{U_{33}}^2 \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} - G_D(t)I\right)\tilde{\lambda}_D \, , \tag{20}$$

$$16\pi^2 \frac{d}{dt}\tilde{\lambda}_E = -G_E(t)I\tilde{\lambda}_E \, . \tag{21}$$

The equations for $\tilde{\lambda}_U$, $\tilde{\lambda}_D$, and $\tilde{\lambda}_E$ are thus unaffected by the neutrinos. We have solved this (approximate) system by first obtaining the exact solution for[15,3] $\tilde{\lambda}_{U_{33}}$

$$\tilde{\lambda}_{U_{33}}(t) = \tilde{\lambda}_{U_{33}}(t_0)\zeta^6 \gamma_U(t) \, , \tag{22}$$

where

$$\gamma_\alpha(t) = \exp\left(-\int G_\alpha(t)dt/(16\pi^2)\right) \tag{23}$$

$$= \prod_{j=1}^{3} \left(\frac{\alpha_{j,0}}{\alpha_j}\right)^{c_\alpha^j/2b_j} \tag{24}$$

$$= \prod_{j=1}^{3} \left(1 - \frac{b_{j,0}\alpha_{j,0}(t-t_0)}{2\pi}\right)^{c_\alpha^j/2b_j} \, , \tag{25}$$

$$\zeta = \exp\left(\frac{1}{16\pi^2}\int_{t_0}^{t}\lambda_{U_{33}}^2\,dt\right) \tag{26}$$

$$= \left(1 - \frac{\kappa}{8\pi^2}\lambda_{U_{33}}^2(t_0)\int_{t_0}^{t}\gamma_\alpha^2(t)dt\right)^{-1/(2\kappa)} \, , \tag{27}$$

where κ is the constant in front of the $\lambda_{U_{33}}$ in the differential equation for the λ_α Yukawa coupling. We then use this solution for $\tilde{\lambda}_{U_{33}}$ to solve the equations for the other couplings. We reproduce the results of Ref. 3, including

$$m_b = \eta_b \frac{\gamma_D}{\gamma_E}\zeta m_\tau \, , \tag{28}$$

$$m_t = \zeta_3 \frac{\eta_u}{\eta_c^2}\frac{m_c^2}{m_u} \, . \tag{29}$$

$\eta_q = m_q(1\text{ GeV})/m_q(m_t)$ for the light quarks and $\eta_q = m_q(m_q)/m_q(m_t)$ for the c and b quark. In our analysis we have taken $\eta_u = \eta_d = \eta_s = 2$, $\eta_c = 1.8$ and $\eta_b = 1.4$. $\gamma_D/\gamma_E = 2.1$ and $\gamma_U/\gamma_N = 2.4$. In order to produce the proper running bottom quark mass[16,17]

$$m_b(m_b) = 4.25 \pm 0.1 \text{ GeV} \tag{30}$$

we require $\zeta = 0.81 \pm 0.2$. Following[3] we obtain the range

$$125 \text{ GeV} \leq m_t \leq 170 \text{ GeV} \tag{31}$$

from bounds on $\tan \beta$ from electroweak breaking as well as on m_u.[17]

In order to determine the lepton mixing matrices we must find the matrix V_l such that $(V_l^\dagger \tilde{\lambda}_E^R (\tilde{\lambda}_E^R)^\dagger V_l)$ is diagonal. We parametrize the mixing matrix as

$$V_l = \begin{pmatrix} c_1 c_3 e^{i\phi} - s_1 s_2 s_3 & s_1 c_3 e^{i\phi} + c_1 s_2 s_3 & -c_2 s_3 \\ -s_1 c_2 & c_1 c_2 & s_2 \\ c_1 s_3 e^{i\phi} + s_1 s_2 c_3 & s_1 s_3 e^{i\phi} - c_1 s_2 c_3 & c_2 c_3 \end{pmatrix}, \tag{32}$$

where $c_1 = \cos \theta_1$, $s_1 = \sin \theta_1$, etc. Then we find

$$V_{\nu_\mu e} \simeq |s_1| = \sqrt{y}\left(1 - \frac{1}{2}y\right) \simeq 6.9 \times 10^{-2}, \tag{33}$$

$$V_{\nu_\mu \tau} = |s_2| = \frac{2}{3}x\left(1 - y - \frac{13}{9}x\right) \simeq (3.62 \pm 0.01)10^{-2}, \tag{34}$$

$$V_{\nu_e \tau} = |s_3| = \frac{m_c}{\eta_c m_t} \simeq (4.9 \pm 0.2)10^{-3}\left(\frac{145 \text{ GeV}}{m_t}\right), \tag{35}$$

where $x = m_\mu/m_\tau$, $y = m_e/m_\mu$ and we have used the values for the charged lepton masses given by the particle data group.[18] In the last equation we have used the constraint on the top quark mass (31) and $m_c = (1.27 \pm 0.05)$ GeV. Thus we predict the "leptonic Cabbibo angle" (θ_1) to be substantially smaller than the corresponding quark angle. This is different from phenomenological ansatz for the angle in Ref. 19, where the leptonic mixing angles are approximated by the corresponding quark angles. It is directly due to the factor of (-3) in the down-quark mass matrix, which is *necessary* to obtain the proper mass predictions. In Ref. 20 a similar value is obtained from a different phenomenological ansatz.

The angle ϕ determines a CP violating quantity J discussed in Ref. 3, which is experimentally only poorly determined. The corresponding J_l for the leptonic mixing matrix is smaller by about a factor of 3, corresponding to $s_1^l \simeq \frac{1}{3}s_1^q$.

From Eqs. (33)–(35) we obtain the following neutrino mixing angles which we choose to write in the form $\sin^2 2\theta_{ex}$, motivated by the solutions to the solar neutrino problem

$$\sin^2 2\theta_{e\mu} = 1.9 \times 10^{-2} \tag{36}$$

$$\sin^2 2\theta_{\mu\tau} = (5.2 \pm 0.1)10^{-3} \tag{37}$$

$$\sin^2 2\theta_{e\tau} = (9.6 \pm 0.4)10^{-5} \tag{38}$$

and we have estimated the last angle by θ_3 since this ansatz does not determine the phase ϕ. We thus obtain a mixing angle $\theta_{\mu\tau}$ which is smaller than the phenomenological ansatz in Ref. 20.

Renormalizing λ_N down to m_t and expressing the eigenvalues in terms of the up-quark masses, the Dirac-neutrino masses are

$$m_{\nu_{D1}} = \frac{\gamma_N}{\gamma_U} \frac{1}{\eta_u \zeta^2} m_u,$$

$$m_{\nu_{D2}} = \frac{\gamma_N}{\gamma_U} \frac{1}{\eta_c \zeta^2} m_c, \tag{39}$$

$$m_{\nu_{D3}} = \frac{\gamma_N}{\gamma_U} \frac{1}{\zeta^5} m_t.$$

In order to obtain realistic neutrino masses at the weak-scale we implement the seesaw mechanism[8] via the universal heavy Majorana mass matrix (6). The matrix K, which diagonalizes the Dirac-neutrino mass matrix produces off-diagonal elements in the heavy Majorana mass matrix $M_{\nu^c \nu^c}$. However, to a good approximation the eigenvalues of the complete 6×6 neutrino mass matrix are unaffected by this rotation. For each generation we thus have the neutrino mass matrix

$$
\begin{array}{c|cc}
 & \nu_i & \nu_i^c \\
\hline
\nu_i & 0 & \frac{1}{2} m_{\nu_{Di}} \\
\nu_i^c & \frac{1}{2} m_{\nu_{Di}} & M_i
\end{array}
\tag{40}
$$

Diagonalizing this matrix we obtain the light neutrino masses $m_{\nu_i} = m_{\nu_{D_i}}^2/(4M_i)$, which we can express in terms of the up-quark masses using Eq. (39)

$$m_{\nu_e} = \frac{1}{4}\left(\frac{\gamma_N}{\gamma_U} \frac{1}{\eta_u \zeta^2}\right)^2 \frac{k^2 m_u^2}{M} = (2.4 \pm 0.2)\frac{m_u^2}{M}, \tag{41}$$

$$m_{\nu_\mu} = \frac{1}{4}\left(\frac{\gamma_N}{\gamma_U} \frac{1}{\eta_c \zeta^2}\right)^2 \frac{k m_c^2}{M} = (3.0 \pm 0.3)10^{-1}\frac{m_c^2}{M}, \tag{42}$$

$$m_{\nu_\tau} = \frac{1}{4}\left(\frac{\gamma_N}{\gamma_U} \frac{1}{\zeta^5}\right)^2 \frac{m_t^2}{M} = (0.35 \pm 0.09)\frac{m_t^2}{M}. \tag{43}$$

Recall that we have made the simple hierarchy ansatz $M = M_3 = kM_2 = k^2 M_1$, $k = 10$, which also gives three heavy neutrinos which are inaccessible to experiment. The ratio of the light neutrino masses is predicted to be

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = (10 m_u/\eta_u)^2 : (3.1 m_c/\eta_c)^2 : (m_t^2/\zeta^3). \tag{44}$$

This is what one would expect in a general GUT seesaw mechanism.[8] It is similar to the phenomenological ansatz discussed in Ref. 20. Due to our restrictive ansatz (4)–(6) we have the additional constraint (29) and m_t is *not* a free parameter. As seen above, the mixing angles in the leptonic sector are determined through known quantities. So with just one additional parameter, M, and a hierarchy, k, we fix the scale of the neutrino masses and have a completely determined neutrino sector.

We chose to fix M so as to obtain a solution to the solar neutrino problem via the MSW mechanism.[9] We assume a two generation mixing solution to this problem. From Fig. 1 in the second reference of Ref. 11, which includes ^{37}Cl, the Kamiokande and the GALLEX data, we have two possible solutions[b]

$$\sin 2\theta_{ex} = (0.39 - 2.2)10^{-2}, \qquad \Delta m^2 = (0.29 - 2.0)10^{-5} \text{ eV}^2, \qquad (45)$$

and

$$\sin 2\theta_{ex} = 0.2 - 0.8, \qquad \Delta m^2 = (0.31 - 9.1)10^{-5} \text{ eV}^2. \qquad (46)$$

Comparing with Eqs. (36)–(38), we have one possible solution in the range of (45) for $\nu_e - \nu_\mu$ mixing. Using Fig. 1 of Ref. 11 this requires

$$m_{\nu_\mu} = (1.7 - 1.9)10^{-3} \text{ eV}. \qquad (47)$$

This fixes M to be

$$M = (2.7 \pm 0.5)10^{11} \text{ GeV}. \qquad (48)$$

Using Eqs. (41) and (43) we then predict the other neutrino masses

$$m_{\nu_e} = (2.3 \pm 0.6)10^{-7} \text{ eV} \qquad (49)$$

$$m_{\nu_\tau} = (27.3 \pm 8.7) \text{ eV} \left(\frac{m_t}{145 \text{ GeV}}\right)^2. \qquad (50)$$

The resulting tau-neutrino mass is in a cosmologically interesting range.[22] Using the formula $\Omega_\nu h^2 = m_\nu/91.5$ eV, where $h = 0.5$–1 is the Hubble parameter,[22] for the relic density of the neutrinos we obtain

$$\Omega_\nu h^2 = 0.27 \pm 0.07. \qquad (51)$$

This is in the range one would expect from the recent COBE data.

The above predicted ranges of neutrino masses and leptonic mixing angles should also respect the stringent bounds which arise from neutrino double beta decay searches. Indeed. it is well known that experimental limits put severe bounds on the effective neutrino mass parameter, related to this process,[23] and given by

$$\langle m_\nu \rangle = \sum_{i=1}^{3} V_{ei} V_{ei}^* \xi_i |m_{\nu_i}|, \qquad (52)$$

[b]A complete analysis for the allowed regions using all available data, may be found in Ref. 21.

where ξ_i are the CP-phases of the neutrino mass eigenstates. Substituting (33)–(35) and (41)–(43) in the above formula, we find that

$$\langle m_\nu \rangle \approx \left(\frac{m_c}{\eta_c m_t} \right)^2 \frac{1}{4} \left(\frac{\gamma_N}{\gamma_U} \frac{1}{\zeta^5} \right)^2 \frac{m_t^2}{M} \tag{53}$$

$$= \left(\frac{\gamma_N}{2\gamma_U} \frac{1}{\zeta^5} \right) \frac{m_c^2}{\eta_c^2 M}. \tag{54}$$

The importance of this result lies in the fact that, although both the mixing angle as well as the ν_τ mass depend explicitly on m_t which lies in a wide range from 125 to 175 GeV, the experimentally measured parameter $\langle m_\nu \rangle$ in $\beta\beta_{0\nu}$-decay, turns out to be independent of the top-mass, leading therefore to a definite prediction for this process. Substituting the numerical values for the masses and the various parameters we find

$$\langle m_\nu \rangle \approx (0.6 \pm 0.2) \times 10^{-3} \text{ eV} \tag{55}$$

which respects the experimental limit. We should note, however, that although there is a large ν_τ mass which might enhance the above lepton number violating process, the corresponding mixing angle is tiny and therefore it is not possible to have a sizable effect on $\beta\beta_{0\nu}$-decay to be tested in proposed experiments which are expected to be sensitive to $\langle m_\nu \rangle \approx 0.2$–$0.3$ eV.[24]

Our predictions for the quantity $\sin^2 2\theta_{\mu\tau}$, relevant for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, together with the predicted range for the ν_τ-mass, are in the expected range of sensitivity of the NOMAD and CHORUS experiments.[25] Our prediction, however, for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations are marginally allowed by the results of Fermilab E531 collab.[26] where at maximum mixing they give the mass difference bound $\Delta m^2 \leq 0.9$ eV^2, and at maximum sensitivity, the mixing angle limit $\sin^2 2\theta_{\mu\tau} \leq 4.0 \times 10^{-3}$. On the other hand, $\nu_e \leftrightarrow \nu_\tau$ oscillations are found to be far below the limit reported by the same experiment.

In conclusion we have expanded an ansatz by Giudice for the fermion mass matrices at the GUT scale to include massive neutrinos, since these are predicted by most GUTs. Giudice's predictions for the Standard Model parameters are retained. Without any additional parameters we predict the neutrino mixing angles completely in terms of known fermion masses. In addition with one parameter and a simple hierarchy we predict a neutrino mass spectrum, which gives a possible solution to the solar neutrino problem via the MSW mechanism and gives a significant contribution to the hot dark matter in agreement with recent interpretations of the COBE data.

Note Added

While this letter was being written we received two preprints[27] dealing with the same problems, but in different ansatz.

We also just received a copy of Ref. 28 where among other topics the constraint on the Giudice ansatz due to CP-asymmetries in B-meson decays is considered. This

leads to stronger bounds on the quark mixing angles and indirectly to a stronger bound on m_t than our Eq. (31): $125 \leq m_t \leq 155$ GeV.[28]

Acknowledgments

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