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# Fractional charges and the renormalization group in the $SU(4) \times O(4)$ string model

G.K. Leontaris[1] and N.D. Tracas[2]

[1] Theoretical Physics Division, University of Ioannina, GR-451 10 Ioannina, Greece
[2] Physics Department, National Technical University, GR-157 73 Zografou, Greece

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**Abstract.** We study the renormalization group equations of the gauge couplings in the $SU(4) \times O(4) \sim SU(4) \times SU(2)_L \times SU(2)_R$ string model, derived in the context of the free fermionic formulation of the four dimensional superstring. We calculate the effective string unification scale taking into account string threshold corrections and we consider the consequences of the $n_L$ and $n_R$ fractionally charged states, sitting in the (1, 2, 1) and (1, 1, 2) representations correspondingly, of the gauge symmetry of the model. Some of these states become massive at a very high scale, when a number of singlet fields acquire vev's. However, many of them (the precise number depends on the specific choice of the flat direction) remain in the massless spectrum. We consider various cases and find that, for specific choices of flat directions, the physical parameters of the model, like the grand unification scale and the low energy parameters $\sin^2 \theta_W$ and $\alpha_3$, depend only on the difference $n_- = n_L - n_R$. We study more general cases where remnants of the exotic doublets remain below the $SU(4)$ breaking scale. We also solve the coupled differential system of the renormalization group equations for the gauge and the Yukawa couplings and estimate the range of the top quark mass which is found to lie in the range $140\,\mathrm{GeV} < m_t < 190\,\mathrm{GeV}$.

## 1 Introduction

Nowadays string theories [1] appear to be the only promising candidates for a fundamental theory of elementary particles. They face, however, a number of unsolved problems, while many questions are still open. Among them, the problem of finding a phenomenologically viable string model has fascinated many particle physicists today. Indeed, the last few years, there has been a lot of excitement about the possibility of constructing a model with string origin leading at low energies to the standard model, which is in agreement with all experimental observations. Nevertheless, the standard model cannot predict the Yukawa couplings and consequently the masses and mixing angles of the low energy particles. On the contrary, in string theories one can predict the Yukawa couplings and calculate all trilinear and non-renormalizable superpotential terms in any superstring model. Thus, in principle, one can obtain all the low energy parameters (like masses and mixing angles) from the superstring theory. In early approaches, effective low energy models were based on ten dimensional constructions [2], where six of the spatial dimensions were compactified on a Calabi-Yau manifold. In recent approaches, model building is based on string theories formulated directly in four space-time dimensions [3]. The last few years, a large number of models has been derived from Calabi-Yau compactifications [4], orbifolds [5] and four dimensional superstrings [6, 7]. These models can be divided in two classes. The ones which are based on the standard gauge group with some additional $U(1)$ factors, i.e. on $SU(3) \times SU(2) \times U(1)^n$ gauge groups, and the others which incorporate modifications of the old Grand Unified Theories (GUT's). However, there are certain theoretical difficulties in both classes of models. In the case where the Standard Model is derived without invoking any intermediate breaking scale, one can hardly obtain the right Yukawa couplings which are going to give the observed mass hierarchy on the one hand, and forbid fast proton decay and flavour changing neutral currents on the other. Many of these problems can be solved elegantly in the case of superstring modifications of GUT's. Nevertheless, the situation is much more restrictive than in the case of old GUT's. For instance one cannot have Higgs in the adjoint or any higher represenation, at least as long as only $k=1$ level of Kac-Moody constructions are considered. On the other hand, we know that traditional GUT's use the adjoint represenation to break down to the standard model. It was shown that in the four dimensional fermionic construction of the heterotic string, a model [7] based on the $SU(4) \times O(4)$ gauge symmetry (which is isomorphic to $SU(4) \times SU(2)_L \times SU(2)_R$ [8]), does not need Higgses in the adjoint representation to break down. The model has been found to possess other advantages too. Among them, one should notice the absence of dimension five baryon violating operators in the trilinear superpotential [9]. In addition the colour triplets become superheavy, avoiding the danger of fast proton decay through dimension six operators. Finally there is a natural see-saw mechanism for all neutrino species, which gives superheavy masses to the right handed components and tiny (but possibly observable in the near future) masses to the left handed ones [10].

A discouraging fact however, in most of the models constructed at level $k=1$ of the Kac-Moody algebras, is the appearance of fractionally charged particles (FCP) in the massless spectrum. Since no violation of charge quantization has been observed, at first sight, such particles might seem to rule out these models. In fact, it was shown [11] that in order to avoid FCP's, one has either to deal with small values of the Weinberg angle at the unification scale, or to consider constructions realized at higher level of Kac-Moody algebras [12]. Since in the second case no realistic model has appeared as yet, it would be desirable to investigate the possibility that these unwanted states decouple from the light spectrum. Indeed, one can find such mechanisms that render most of these particles superheavy. As an example, in this particular model [7] (as well as in other models derived in the fermionic construction [6]), the classical vacuum is unstable due to the existence of an anomalous $U(1)$ which generates a $D$-term in the scalar potential. In order to restore supersymmetry and stabilize the vacuum, one should introduce non vanishing vev's for some of the singlet fields which appear in the massless spectrum. These vev's, however, may give masses to a large number of the unwanted FCP's, at a high scale $M_A$ of the order $M_A \sim \mathcal{O}(10^{17})$ GeV. Other mechanisms may cooperate [7, 13], and make all of them dissapear from the light spectrum.

FCP's may affect decisively the renormalization group scaling of the various physical quantites. Even if they become massive by some mechanism like the one described above, their contribution to the $\beta$-function coefficients should be taken into account for energies higher than the scale they acquire mass and decouple from the massless spectrum. In the present work, extending our previous analysis [14], we are going to investigate the consequences of fractionally charged states on the various mass scales of our superstring model. In the presence of FCP's, we consider the renormalization group equations for the gauge couplings in order to find the constraints imposed on the Grand Unification and other possible intermediate scales, by allowing the low energy parameters $\sin^2\theta_W$ and $\alpha_3$, to vary in their experimantally acceptable ranges. It is understood however, that the number of the massless FCP's will depend on the specific choice of the non zero singlet vev's (usually there are more than one flat directions in any particular model). Thus, in order to make our analysis as general as possible, we treat the number of FCP's as a free parameter.

Our paper is organized as follows. In Sect. 2 we give a brief description of the string $SU(4)\times O(4)$ model and present its massless spectrum. In Sect. 3 we make a general analysis of the renormalization group flow of the gauge couplings in the one and two loop level, in the presence of FCP's, and finally in Sect. 4, we discuss the results and present our conclusions.

## 2 Description of the model

Our string model [7] has been derived using the free fermionic formulation of the heterotic string [15]. We have used nine vectors of boundary conditions for the world sheet fermions to construct it. A remarkable feature of this model is that one imposes only periodic and antiperiodic boundary conditions for all the world sheet fermions to derive it. The resulting gauge symmetry of the model is

$$SU(4)\times O(4)\times U(1)^4\times[SU(8)\times U(1)']_{\text{Hidden}}.$$

It consists of the anticipated $SU(4)\times O(4)\sim SU(4)\times SU(2)_L\times SU(2)_R$ observable part accompanied by four surplus $U(1)$ factors and the Hidden $SU(8)\times U(1)'$ gauge group. The massless spectrum generated by the above nine element basis, is listed in Tables 1, 2 and 3.

In Table 1, we have included all the standard fermion and Higgs content of the model together with a number of singlet fields. There are three states sitting in $(4, 2, 1)$ representation of the observable gauge group, namely $F_{1L}$, $F_{3L}$ and $F_{4L}$, which are going to accommodate the left handed fields of the three fermion generations.

There are five fields sitting in $(\bar{4}, 1, 2)$ representation, namely $\bar{F}_{1R}$, $\bar{F}_{2R}$, $\bar{F}'_{2R}$, $\bar{F}_{3R}$, and $\bar{F}_{5R}$. Three linear combinations of them have been interpreted [7] as the right handed partners of the fermion generations. The other two linear combinations, together with $F_{4R}$ and $F_{5R}$ sitting in $(4, 1, 2)$, are going to acquire vev's in order to break the $SU(4)\times SU(2)_R$ part of the observable gauge group down to $SU(3)\times U(1)_Y$. Notice that $F_{iL}+\bar{F}_{iR}$ makes up

**Table 1.** Analysis under $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)^4$ of the fields belonging to the observable sector (i.e trivial transformations under the hidden $SU(8)$ group and zero charges under $U(1)'$)

$$\bar{F}_{1R}=(\bar{4},1,2)_{(1/2,0,0,0)},\qquad F_{1L}=(4,2,1)_{(1/2,0,0,0)}$$
$$\bar{F}_{2R}=(\bar{4},1,2)_{(0,1/2,0,0)},\qquad \bar{F}'_{2R}=(\bar{4},1,2)_{(0,-1/2,0,0)},$$
$$\bar{F}_{3R}=(\bar{4},1,2)_{(0,0,-1/2,1/2)},\qquad F_{3L}=(4,2,1)_{0,0,-1/2,-1/2)},$$
$$F_{4R}=(4,1,2)_{(1/2,0,0,0)},\qquad F_{4L}=(4,2,1)_{(-1/2,0,0,0),},$$
$$\bar{F}_{5R}=(\bar{4},1,2)_{(0,-1/2,0,0)},\qquad F_{5R}=(4,1,2)_{(0,-1/2,0,0)}.$$
$$h_3=(1,2,2)_{(0,0,1,0)},\qquad \bar{h}_3=(1,2,2)_{(0,0,-1,0)}.$$
$$\Phi_1=(1,1,1)_{(0,0,0,0)},\qquad \Phi_2=(1,1,1)_{(0,0,0,0)},$$
$$\Phi_3=(1,1,1)_{(0,0,0,0)},\qquad \Phi_4=(1,1,1)_{(0,0,0,0)},$$
$$\Phi_5=(1,1,1)_{(0,0,0,0)}.$$
$$\Phi_{12}=(1,1,1)_{(1,1,0,0)},\qquad \bar{\Phi}_{12}=(1,1,1)_{(-1,-1,0,0)},$$
$$\Phi_{\bar{1}2}=(1,1,1)_{(1,-1,0,0)},\qquad \bar{\Phi}_{\bar{1}2}=(1,1,1)_{(-1,1,0,0)}.$$
$$D_1=(6,1,1)_{(1,0,0,0)},\qquad \bar{D}_1=(6,1,1)_{(-1,0,0,0)},$$
$$D_2=(6,1,1)_{(0,1,0,0)},\qquad D_2=(6,1,1)_{(0,-1,0,0)}.$$
$$h_{12}=(1,2,2)_{(1/2,1/2,0,0)},\qquad \bar{h}_{12}=(1,2,2)_{(-1/2,-1/2,0,0)},$$
$$\zeta_1=(1,1,1)_{(1/2,-1/2,0,0)},\qquad \bar{\zeta}_1=(1,1,1)_{(-1/2,1/2,0,0)},$$
$$\zeta_2=(1,1,1)_{(1/2,-1/2,0,0)},\qquad \bar{\zeta}_2=(1,1,1)_{(-1/2,1/2,0,0)},$$
$$\xi_1=(1,1,1)_{(1/2,1/2,1,0)},\qquad \bar{\xi}_1=(1,1,1)_{(-1/2,-1/2,-1,0)},$$
$$\xi_2=(1,1,1)_{(1/2,-1/2,0,1)},\qquad \bar{\xi}_2=(1,1,1)_{(-1/2,1/2,0,-1)},$$
$$\xi_3=(1,1,1)_{(-1/2,1/2,0,1)},\qquad \bar{\xi}_3=(1,1,1)_{(1/2,-1/2,0,-1)},$$
$$\xi_4=(1,1,1)_{(1/2,1/2,-1,0)},\qquad \bar{\xi}_4=(1,1,1)_{(-1/2,-1/2,1,0)}.$$

**Table 2.** Analysis under $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)^4\times U(1)'$ of the fields being $SU(8)$ singlets but having non-zero charges under $U(1)'$

| | |
|---|---|
| $h_{1L}^{+}=(1,2,1)_{(0,-1/2,0,1/2)(-1)}$, | $h_{1L}'^{+}=(1,2,1)_{(0,-1/2,0,-1/2)(-1)}$ |
| $h_{1R}^{+}=(1,1,2)_{(0,1/2,0,1/2)(-1)}$, | $h_{1R}'^{+}=(1,1,2)_{(0,1/2,0,-1/2)(-1)}$ |
| $h_{2L}^{-}=(1,2,1)_{(1/2,0,0,1/2)(+1)}$, | $h_{2L}'^{-}=(1,2,1)_{(-1/2,0,0,1/2)(+1)}$ |
| $h_{2R}^{-}=(1,1,2)_{(1/2,0,0,-1/2)(+1)}$, | $h_{2R}'^{-}=(1,1,2)_{(-1/2,0,0,-1/2)(+1)}$ |
| $h_{3L}^{+}=(1,2,1)_{(1/2,-1/2,1/2,0)(-1)}$, | $h_{3L}^{-}=(1,2,1)_{(1/2,-1/2,-1/2,0)(+1)}$ |
| $h_{3R}^{-}=(1,1,2)_{(-1/2,1/2,1/2,0)(+1)}$, | $h_{3R}^{+}=(1,1,2)_{(-1/2,1/2,-1/2,0)(-1)}$ |
| $h_{4L}^{+}=(1,2,1)_{(0,1/2,0,1/2)(-1)}$, | $h_{4L}^{-}=(1,2,1)_{(0,-1/2,0,1/2)(+1)}$ |
| $h_{4R}^{-}=(1,1,2)_{(0,1/2,0,-1/2)(+1)}$, | $h_{4R}^{+}=(1,1,2)_{(0,-1/2,0,-1/2)(-1)}$ |
| $h_{5L}^{-}=(1,2,1)_{(1/2,0,0,1/2)(+1)}$, | $h_{5L}^{+}=(1,2,1)_{(1/2,0,0,1/2)(-1)}$ |
| $h_{5R}^{+}=(1,1,2)_{(-1/2,0,0,1/2)(-1)}$, | $h_{5R}^{-}=(1,1,2)_{(-1/2,0,0,-1/2)(+1)}$ |
| $H_4=(4,1,1)_{(0,0,0,1/2)(-1)}$, | $\bar{H}_4=(\bar{4},1,1)_{(0,0,0,-1/2)(+1)}$ |

**Table 3.** Analysis under $SU(8)\times U(1)'\times U(1)^4$ of the fields being singlets under the observable sector

| | |
|---|---|
| $\bar{Z}_1=(\bar{8},1/2)_{(0,-1/2,-1/2,0)}$, | $Z_1=(8,-1/2)_{(0,-1/2,-1/2,0)}$ |
| $Z_2=(8,-1/2)_{(1/2,0,-1/2,0)}$, | $Z_2'=(8,-1/2)_{(-1/2,0,-1/2,0)}$ |
| $\bar{Z}_3=(\bar{8},1/2)_{(0,1/2,-1/2,0)}$, | $\bar{Z}_3'=(\bar{8},1/2)_{(0,-1/2,1/2,0)}$ |
| $\bar{Z}_4=(\bar{8},1/2)_{(1/2,0,1/2,0)}$, | $Z_4=(8,-1/2)_{(1/2,0,-1/2,0)}$ |
| $Z_5=(8,-1/2)_{(1/2,-1/2,0,1/2)}$, | $\bar{Z}_5=(\bar{8},1/2)_{(1/2,-1/2,0,-1/2)}$ |

the complete $\underline{16}$ spinorial represenation of the $SO(10)$ group, while $F_{iR}=(4,1,2)$ is just half a piece of the $\overline{16}$ of the $SO(10)$. The spectrum in Table I is completed with four sextet fields $\overset{(-)}{D}_i=(6,1,1)$ and left-right doublet fields $h_i=(1,2,2)$. Both kinds of representations arise from the decomposition of the 10 dimensional representation of the $SO(10)$ symmetry $((10)\to(6,1,1)+(1,2,2))$.

In Table 2 there are 10 $(h_L+h_R)$ doublet fields sitting in $(1,2,1)+(1,1,2)$ representations and two fourplets, namely $H=(4,1,1)$, $\bar{H}=(\bar{4},1,1)$. All these states carry fractional electric charges. Indeed, the charge operator may be written

$$Q=\tfrac{1}{6}T_{15}+\tfrac{1}{2}T_{3L}+\tfrac{1}{2}T_{3R}, \tag{2.1}$$

where $T_{15}=\mathrm{diag}(1,1,1,-3)$ and $T_{3L}=T_{3R}=\mathrm{diag}(1,-1)$. Thus the charge of the doublets is $\pm\frac{1}{2}$. Notice however that fractionally charged states are generic in all $k=1$ constructions. We could avoid them, either by going to higher level Kac-Moody algebras, or by allowing small values for $\sin^2\theta_W$ at the unification scale.

Finally in Table 3 we collected all the states having non trivial properties only under the hidden gauge group $SU(8)\times U(1)'$, giving also their $U(1)^4$ charges.

In this particular free fermionic formulation of the heterotic string one can compute unambiguously the Yukawa couplings of any model. In our case, the trilinear superpotential consists of the following terms

$$\begin{aligned}
W=g\sqrt{2}\,[&F_{4L}\bar{F}_{5R}h_{12}+\tfrac{1}{\sqrt{2}}F_{4R}\bar{F}_{5R}\zeta_2+\bar{F}_{3R}F_{3L}h_3\\
&+D_1D_2\bar{\Phi}_{12}+D_1\bar{D}_2\bar{\Phi}_{\overline{12}}+\bar{D}_1D_2\Phi_{\overline{12}}+\bar{D}_1\bar{D}_2\Phi_{12}\\
&+\tfrac{1}{2}(F_{1L}F_{1L}+\bar{F}_{1R}\bar{F}_{1R}+F_{4R}F_{4R})\bar{D}_1+\tfrac{1}{2}F_{4L}F_{4L}D_1\\
&+\tfrac{1}{2}(\bar{F}_{2R}'\bar{F}_{2R}'+F_{5R}F_{5R}+\bar{F}_{5R}\bar{F}_{5R})D_2+\tfrac{1}{2}\bar{F}_{2R}\bar{F}_{2R}\bar{D}_2\\
&+\tfrac{1}{2}(h_{12}\bar{h}_{12}+\zeta_1\bar{\zeta}_1+\zeta_2\bar{\zeta}_2+\xi_1\bar{\xi}_1+\xi_2\bar{\xi}_2+\xi_3\bar{\xi}_3\\
&+\xi_4\bar{\xi}_4)\Phi_3+(\bar{\zeta}_1\zeta_2+\bar{\zeta}_2\zeta_1)\Phi_4+(\tfrac{1}{2}h_{12}h_{12}+\xi_1\xi_4)\bar{\Phi}_{12}\\
&+(\tfrac{1}{2}\bar{h}_{12}\bar{h}_{12}+\bar{\xi}_1\bar{\xi}_4)\Phi_{12}+(\tfrac{1}{2}\zeta_1\zeta_1+\tfrac{1}{2}\zeta_2\zeta_2+\xi_2\bar{\xi}_3)\bar{\Phi}_{\overline{12}}\\
&+(\tfrac{1}{2}\bar{\zeta}_1\bar{\zeta}_1+\tfrac{1}{2}\bar{\zeta}_2\bar{\zeta}_2+\bar{\xi}_2\xi_3)\Phi_{\overline{12}}+(\xi_4\bar{h}_{12}h_3+\bar{\xi}_1h_{12}\bar{h}_3\\
&+\bar{\xi}_4h_{12}\bar{h}_3+\xi_1\bar{h}_{12}\bar{h}_3)+H_4\bar{H}_4\Phi_3+\bar{F}_{1R}h_{2R}'^{-}H_4\\
&+\bar{F}_{2R}'h_{4R}^{-}H_4+h_{3L}^{+}h_{3R}^{-}\bar{h}_3+h_{3L}^{-}h_{3R}^{+}h_3+h_{3L}^{-}h_{3L}^{+}\bar{\Phi}_{\overline{12}}\\
&+\Phi_{\overline{12}}h_{3R}^{-}h_{3R}^{+}+h_{1L}^{+}h_{5R}^{-}h_{12}+h_{1R}'^{+}h_{5L}^{-}\bar{h}_{12}+\bar{\xi}_2h_{1L}^{+}h_{5L}^{-}\\
&+\xi_2h_{1R}'^{+}h_{5R}^{-}+\tfrac{1}{\sqrt{2}}(h_{1L}'^{+}h_{5L}^{-}\bar{\zeta}_1+h_{1R}^{+}h_{5R}^{-}\zeta_1)+Z_5\bar{Z}_5\Phi_{\overline{12}}\\
&+Z_3\bar{Z}_4\bar{\xi}_4+\tfrac{1}{\sqrt{2}}\bar{Z}_3'Z_4\bar{\zeta}_2].
\end{aligned} \tag{2.2}$$

Let us discuss now in brief the gauge symmetry breaking. The observable gauge group $SU(4)\times O(4)\times U(1)^4$ at the first stage breaks down to $SU(4)\times O(4)$, where some of the singlet fields, possessing non-zero charges under these four surplus $U(1)$ factors, develop vacuum expectation values. Note however that three of them are not traceless. We can, nevertheless, define new linear orthogonal combinations, namely

$$U(1)'=U(1)_1+U(1)_2,\quad U(1)_4'=U(1)_4,$$

$$U(1)_2'=U(1)_1-U(1)_2+2U(1)_3,$$

$$U(1)_A=U(1)_1-U(1)_2-U(1)_3,$$

where only one of them, $U(1)_A$ is found to be anomalous, with $\mathrm{Tr}[U(1)_A]=72$. This particular $U(1)_A$ symmetry is broken by the Dine-Seiberg-Witten mechanism [16]. According to that mechanism, the anomalous $D$-term generated by a vev of the dilaton field is cancelled by singlet vev's that break the other non-anomalous $U(1)$ symmetries, so that supersymmetry is preserved. Any choice of these vev's should be a consistent solution of the $F$- and $D$-flatness conditions of the model.

A possible choice of non-zero vev's, consistent with the $F$- and $D$- flatness conditions, is [7]

$$\langle\Phi_{12}\rangle,\quad\langle\bar{\Phi}_{\overline{12}}\rangle,\quad\langle\xi_1\rangle,\quad\langle\bar{\xi}_2\rangle\neq 0,$$

for the singlets, and

$$\langle Z_5\rangle,\quad\langle\bar{Z}_3'\rangle\neq 0,$$

for the representations of the hidden gauge group. The $F$-flatness conditions are trivially satisfied, while the $D$-flatness conditions read

$$2|\Phi_{12}|^2+|\xi_1|^2-\tfrac{1}{2}|\bar{Z}_3'|^2=0, \tag{2.3}$$

$$-2|\bar{\Phi}_{\overline{12}}|^2+2|\xi_1|^2-|\bar{\xi}_2|^2+|Z_5|^2+\tfrac{3}{2}|\bar{Z}_3'|^2=0, \tag{2.4}$$

$$-|\bar{\xi}_2|^2+\tfrac{1}{2}|Z_5|^2=0, \tag{2.5}$$

$$-2|\bar{\Phi}_{\overline{12}}|^2-|\xi_1|^2-|\bar{\xi}_2|^2+|Z_5|^2+\frac{3\alpha_u}{2\pi}=0, \tag{2.6}$$

The above equations give

$$|2\Phi_{12}|^2=\tfrac{1}{2}|\bar{Z}_3'|^2-\frac{1}{4}\frac{3\alpha_u}{2\pi}, \tag{2.7}$$

$$|\bar{\Phi}_{\bar{1}2}|^2=\frac{1}{2}\frac{3\alpha_u}{2\pi}+\tfrac{1}{4}|Z_5|^2+\tfrac{1}{4}|\bar{Z}_3'|^2, \tag{2.8}$$

$$|\xi_1|^2=\frac{1}{2}\left(\frac{3\alpha_u}{2\pi}-|\bar{Z}_3'|^2\right), \tag{2.9}$$

$$|\bar{\xi}_2|^2=\tfrac{1}{2}|Z_5|^2. \tag{2.10}$$

Thus, $|\bar{Z}_3'|$ should satisfy

$$\frac{\alpha_u}{2\pi}<|\bar{Z}_3'|^2<\frac{\alpha_u}{\pi}. \tag{2.11}$$

The $SU(4)\times SU(2)_R$ breaking down to $SU(3)\times U(1)_Y$ occurs when the $F_{4R}$, $\bar{F}_{5R}$ tetraplets together with two linear combinations of the fields $\bar{F}_{1R}$, $\bar{F}_{2R}$, $\bar{F}_{\bar{2}R}$ and $\bar{F}_{3R}$ develop vev's. The above choice has the following phenomenological advantages:

i) All the "conventional" colour triplets in (3, 1, 1) and ($\bar{3}$, 1, 1) representations, which could in principle mediate proton decay processes, become superheavy as one can see from the superpotential.
ii) The fermions of the heaviest generation, $F_{4L}$ and $\bar{F}_{5R}$, receive masses from the first term of the superpotential when the Higgs field $h_{12}$ acquires vev. The second term $F_{4R}\bar{F}_{5R}\bar{\zeta}_2$, provides a see-saw mechanism for the neutrinos of this generation. The two lightest generations do not receive any mass at the tree level, but higher order non-renormalizable terms are expected to provide the right order of magnitude masses.
iii) All components of the extra electroweak type doublets $\bar{h}_{12}$ and $\bar{h}_3$ receive masses at a high scale. In addition four of the "exotic" fractionally charged doublets of Table 2 also receive masses from the trilinear superpotential. At this stage, the rest of the "exotic" doublets remain massless. However, some of them are expected to acquire large masses through dimension-five operators. Of course, one could also take into account the effects of the condensation of the "hidden" group $SU(8)$, which most likely takes place. In that case it is easy to check that many of the right handed "exotic" doublets receive superheavy masses, assuming non-zero vev's for $\langle Z_i\bar{Z}_j\rangle$ bilinears of the states appearing in Table 3.

It is important to remember at this point that our choice of non-zero vev's considered above is not the only one acceptable. In fact, the $F$- and $D$-flatness constraints leave considerable freedom to choose some other singlet fields to develop non-zero vev's. Any other choice will have different phenomenological consequences for the model. One, of course, cannot decide which of these choices is the most promising one, until a detailed study of the model is done. Note, however, that a decisive role on the choice of the particular flat direction is played also by the non-renormalizable contributions to the superpotential [17].

Indeed, the number of the "exotic" doublets that become massive from the trilinear superpotential depends on the specific choice of the flat direction. As we already mentioned in the introduction, the "exotic" doublets play also important role in the evolution of the gauge couplings through the renormalization group equations. In the next section we will examine their consequences in the low energy parameters $\sin^2\theta_W$ and $\alpha_3$.

## 3 One and two loop analysis

Let us start by specifying the energy scales. In the most general case we have: i) $M_{SU}$ where $\alpha_4=\alpha_{2L}=\alpha_{2R}\equiv\alpha_u$; ii) $M_A$, where the anomalous $U(1)$ breaks and a number of fields acquires a mass through some singlet fields developing non zero vev's. Between $M_{SU}$ and $M_A$ we assume the full string content of the model; iii) $M_G$, where the group $SU(4)\times SU(2)_L\times SU(2)_R$ breaks down to $SU(3)\times SU(2)_L\times U(1)_Y$ in the way already mentioned and the relation between the couplings, at that scale, is given by

$$\frac{1}{\alpha_Y}=\frac{1}{\frac{5}{3}\alpha_{2R}}+\frac{1}{\frac{5}{2}\alpha_4}, \tag{3.1}$$

and finally iv) $M_I$, where we assume that below this scale we only have the standard model content. Between $M_G$ and $M_I$ some exotic remnants could survive.

The first two scales, $M_{SU}$ and $M_A$, could be expressed, once the string content is known, in terms of the string scale $M_{\text{string}}$ and the string threshold corrections $\Delta_\alpha$ [18, 19], which should be calculated in the $\overline{DR}$-scheme, closely related to the $\overline{MS}$-scheme ($\alpha^{-1}_{i\overline{DR}}=\alpha^{-1}_{i\overline{MS}}-c_{Ai}/2\pi$).

The string scale, in the Pauli-Villars (PV) scheme, is found to be [19]

$$M_{\text{PV}}=M_{\text{string}}=\left(\frac{2e^{(1-\gamma)}}{3\sqrt{3}\pi\alpha'}\right)^{1/2}=0.7\,g_{\text{string}}\,10^{18}\text{ GeV},$$

where $\alpha'=16\pi^2/g^2_{\text{string}}M^2_{\text{Pl}}$ and $\gamma$ is the Euler-Macheroni constant. However, according to arguments presented in [19], $M_{\text{PV}}=M_{\overline{DR}}$.

Using modular invariance properties of the low energy action, the threshold corrections to the inverse gauge coupling, in the free fermionic models, are [18, 19]

$$\Delta^\alpha=-b_\alpha\ln[|n(iT)|^4\,\text{Re}(T)]+c_\alpha+Y.$$

Here, $b_\alpha$ are the $\beta$-function coefficients, $T$ are the untwisted moduli fields while $c_\alpha$ and $Y$ are gauge dependent and gauge independent pieces respectively [18, 19]. Using the methods of [19] one can find that the $SO(6)\times O(4)$ gauge group gives

$$\delta\Delta=23\left(\frac{\pi}{3}+\delta c\right),$$

where $\delta c=c_{O(4)}-c_{SO(6)}$. This quantity is not determined by this method but explicit calculations on a similar model [19], the flipped $SU(5)$, based on the free fermionic construction as well as in a $Z_3$ model [18], have shown that this quantity is very small ($\delta c\sim 0.025$). Since our model is also a free fermionic construction, while any free fermionic model is related to a $Z_2\times Z_2$ orbifold, we conclude that the possible non-moduli corrections $c_i$ are small in our model too. Thus the effective scale $M_{SU}$ is found to be

$$M_{SU}=M_{\text{string}}\,e^{(\frac{\pi}{3}+\delta c)/2}=1.2\,g_{\text{string}}\,10^{18}\,\text{GeV}.$$

Thresholds effects from other superheavy particles [20], when included, may also change the intermediate scales $M_I$ and $M_X$. However in most of the cases considered in our analysis, they are not important and we ignore them.

Finally, taking into account the solution of the $D$-flatness constraints, we adopt the following value for the $U(1)_A$ breaking scale

$$M_A = 7.8 g_{\text{string}}\, 10^{17}\ \text{GeV},$$

which corresponds to the choice

$$|\bar{Z}_5|^2 \sim |\bar{Z}_3|^2_{\max} \sim \frac{\alpha_U}{\pi}.$$

A lower value of $M_A = 6 g_{\text{string}}\ 10^{17}$ GeV is suggested through (2.8) by the choice

$$|\bar{Z}_5|^2 \sim |\bar{Z}_3|^2_{\max} \sim \frac{\alpha_U}{2\pi}.$$

Thus the $M_A$ range is rather enough constrained and our specific choice will not introduce any significant uncertainty in our calculations.

The evolution for the gauge couplings is given by the general formula

$$\frac{d\alpha_i}{dt} = \frac{\alpha_i^2}{2\pi}\left(b_i + \frac{1}{4\pi}\sum b_{ij}\alpha_j\right), \tag{3.2}$$

where $b_i$ are the one-loop and $b_{ij}$ are the two-loop $\beta$-functions. Above $M_G$ they are given by [21]

$$b_i = -\begin{pmatrix}6\\6\\12\end{pmatrix} + \begin{pmatrix}2\\2\\2\end{pmatrix} n_G + \begin{pmatrix}2\\0\\1\end{pmatrix} n_H + \begin{pmatrix}1\\1\\0\end{pmatrix} n_{22} + \begin{pmatrix}0\\0\\1\end{pmatrix} n_6 + \begin{pmatrix}0\\0\\\frac{1}{2}\end{pmatrix} n_4$$
$$+ \begin{pmatrix}0\\\frac{1}{2}\\0\end{pmatrix} n_L + \begin{pmatrix}\frac{1}{2}\\0\\0\end{pmatrix} n_R, \tag{3.3a}$$

$$b_{ij} = \begin{pmatrix}-24 & 0 & 0\\ 0 & -24 & 0\\ 0 & 0 & -96\end{pmatrix} + \begin{pmatrix}14 & 0 & 15\\ 0 & 14 & 15\\ 3 & 3 & 31\end{pmatrix} n_G + \begin{pmatrix}14 & 0 & 15\\ 0 & 0 & 0\\ 3 & 0 & \frac{31}{2}\end{pmatrix} n_H$$
$$+ \begin{pmatrix}7 & 3 & 0\\ 3 & 7 & 0\\ 0 & 0 & 0\end{pmatrix} n_{22} + \begin{pmatrix}\frac{7}{2} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} n_R + \begin{pmatrix}0 & 0 & 0\\ 0 & \frac{7}{2} & 0\\ 0 & 0 & 0\end{pmatrix} n_L$$
$$+ \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \frac{31}{4}\end{pmatrix} n_4 + \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 18\end{pmatrix} n_6, \tag{3.3b}$$

where $i = (R, L, 4)$, while below $M_G$ we have

$$b_i = -\begin{pmatrix}0\\6\\9\end{pmatrix} + \begin{pmatrix}2\\2\\2\end{pmatrix} n_G + \begin{pmatrix}\frac{3}{10}\\\frac{1}{2}\\0\end{pmatrix} n_2 + \begin{pmatrix}\frac{1}{5}\\0\\\frac{1}{2}\end{pmatrix} n_3 + \begin{pmatrix}\frac{1}{20}\\0\\\frac{1}{2}\end{pmatrix} n'_3 + \begin{pmatrix}\frac{4}{5}\\0\\\frac{1}{2}\end{pmatrix} n'_{31}$$
$$+ \begin{pmatrix}\frac{3}{20}\\0\\0\end{pmatrix} n' + \begin{pmatrix}0\\\frac{1}{2}\\0\end{pmatrix} n'_L, \tag{3.4a}$$

$$b_{ij} = \begin{pmatrix}0 & 0 & 0\\ 0 & -24 & 0\\ 0 & 0 & -54\end{pmatrix} + \begin{pmatrix}\frac{38}{15} & \frac{6}{5} & \frac{88}{15}\\ \frac{2}{5} & 14 & 8\\ \frac{11}{15} & 3 & \frac{68}{3}\end{pmatrix} n_G + \begin{pmatrix}\frac{9}{50} & \frac{9}{10} & 0\\ \frac{3}{10} & \frac{7}{2} & 0\\ 0 & 0 & 0\end{pmatrix} n_2$$
$$+ \begin{pmatrix}\frac{4}{75} & 0 & \frac{16}{15}\\ 0 & 0 & 0\\ \frac{2}{15} & 0 & \frac{17}{3}\end{pmatrix} n_3 + \begin{pmatrix}0 & 0 & 0\\ 0 & \frac{7}{2} & 0\\ 0 & 0 & 0\end{pmatrix} n'_L + \begin{pmatrix}\frac{1}{100} & 0 & \frac{4}{15}\\ 0 & 0 & 0\\ \frac{1}{30} & 0 & \frac{17}{3}\end{pmatrix} n'_3$$
$$+ \begin{pmatrix}\frac{9}{100} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} n' + \begin{pmatrix}\frac{64}{75} & 0 & \frac{64}{15}\\ 0 & 0 & 0\\ \frac{8}{15} & 0 & \frac{17}{3}\end{pmatrix} n'_{31}, \tag{3.4b}$$

where now $i = (Y, 2, 3)$. We have assumed that supersymmetry is effective through all the range between $M_{SU}$ and $M_Z$. Although the mechanism which breaks sypersymmetry in string theory is still unknown, the latter is expected to occur at the electroweak scale in order to protect gauge hierarchy. In the above equations $n_6$ is the number of sextet fields (6, 1, 1), $n_4$ stands for the number of (4, 1, 1) representations, $n_H$ counts the (4, 1, 2) Higgses which break $SU(4)$, $n_{22}$ is the number of the (1, 2, 2) Higgses, $n_{L(R)}$ are the (1, 2, 1) and (1, 1, 2) representations respectively while $n_G$ is the number of generations. $n_3$ is the number of colour triplets, while $n_2$ is the number of $W$–$S$ doublets (the particular choice of non-zero vev's in (2.3), provides all $n_3$ triplets with heavy masses and they are not present below $M_G$; however, other vev choices which leave few of them massless down to some intermediate scale $M_I < M_G$, consistent with the $F$- and $D$- flatness constraints, are also possible). $n'_3$ is the number of triplets arising from $H'_4, \bar{H}'_4$ fields while $n'_{31}$ is the number of $u^c_H$ and $\bar{u}^c_H$ up-quark-type Higgs. Also $n'$ is the number of singlets with fractional charges $\pm\frac{1}{2}$, arising from $h_R$ and $H'_4, \bar{H}'_4$. In particular, in the model under consideration, $n'_{31} = n'_3 = 2$, if none of these triplets get mass at $M_G$, and $n' = 2n_R + 2$.

We are considering initially the case where $M_A \sim M_{SU}$ with a pair of triplets ($n_3 = 2$) surviving below $M_G$ down to the scale $M_I$. The one loop equations for the gauge couplings are given by

$$\frac{1}{\alpha_3} = \frac{1}{\alpha_u} + \frac{b_4}{2\pi}\ln\frac{M_{SU}}{M_G} + \frac{b'_3}{2\pi}\ln\frac{M_G}{M_I} + \frac{b_3}{2\pi}\ln\frac{M_1}{M_W}, \tag{3.5a}$$

$$\frac{1}{\alpha_2} = \frac{1}{\alpha_u} + \frac{b_L}{2\pi}\ln\frac{M_{SU}}{M_G} + \frac{b'_2}{2\pi}\ln\frac{M_G}{M_I} + \frac{b_2}{2\pi}\ln\frac{M_I}{M_W}, \tag{3.5b}$$

$$\frac{1}{\alpha_Y} = \frac{1}{\alpha_Y(M_G)} + \frac{b'_Y}{2\pi}\ln\frac{M_G}{M_I} + \frac{b_Y}{2\pi}\ln\frac{M_I}{M_W}. \tag{3.5c}$$

It is straightforward to use the above relations and derive the following formula

$$\sin^2\theta_W = \frac{1}{2} - \frac{1}{3}\frac{\alpha_{\text{em}}}{\alpha_3} + \frac{\alpha_{\text{em}}}{2\pi}\left[\left(\frac{n_-}{4} - n_H\right)\ln\frac{M_{SU}}{M_G} - 6\ln\frac{M_G}{M_W}\right], \tag{3.6}$$

where $n_- = n_L - n_R$. If we ignore the dependence of $M_{SU}$ on $\alpha_u$, the relation between the low energy parameters $\sin^2\theta_W$ and $\alpha_3$ depends only on the difference $n_- - 4n_H$ and $M_G$, once the correct value of $\alpha_{\text{em}}$ is ensured. But even when the $\alpha_u$ dependence of $M_{SU}$ is taken into account, the range of $\alpha_u$ and the weak dependence ($\ln M_{SU} \sim \ln\sqrt{\alpha_u}$) are such that the above statement holds true. As we shall see later this result persists even in the two loop level.

Let us now treat the gauge coupling evolution at the two loop level. We start considering all four scales $M_{SU}, M_A, M_G$ and $M_I$ but only admitting the pair of triplets we mentioned above in the range between $M_G$ and $M_I$. We run the R.G.E's and plot contours of constant $M_G$ in the parameter space of ($\sin^2\theta_W, \alpha_3$), adopting the following range

$$0.228<\sin^2\theta_W<0.236, \quad 0.10<\alpha_3<0.14.$$

In Fig. 1 we show three such plots. The content between $M_A$ and $M_G$ is $n_6=4$, $n_{22}=n_4=n_H=2$, while the pair $(n_L, n_R)$ takes the values (4, 0), (2, 0) and (4, 2) corresponding to the three figures a, b and c. We can observe that the low energy parameters $\sin^2\theta_W$ and $\alpha_3$ put limits on the number of the exotic doublets $n_L$ and $n_R$ that may remain massless below $M_A$. We remind the reader, that between $M_{SU}$ and $M_A$ we always have the full massless string content, i.e. $n_6=n_{22}=n_H=2n_4=4$, $n_G=3$ and $n_L=n_R=10$.

Indeed, comparing the figures with the same left-handed exotic doublets we notice that as $n_R$ increases the GUT scale decreases. If we still insist in a reasonable value for the GUT scale ($M_G>10^{15}$ GeV), then for $n_L>4$, $n_R$ should not be greater than 2. In fact, it is the difference $n_-$ which plays important role in these figures, as we have already seen from the one loop formula in (3.6). Comparing for example the (2, 0) and (4, 2) contours we notice that they are essentially the same. These figures differ only in the corresponding values of the intermediate scale $M_I$ (dotted lines). Therefore, the precise number of the exotic doublets $n_L$ and $n_R$ have a significant impact on the scale $M_I$. The higher the values of $n_L$ and $n_R$ the lower the scale $M_I$.

In Fig. 2 we have plotted the contours of constant $M_G$ (solid lines) and constant $M_I$ (dotted ones) in the same parameter space, using the field content $n_6=4$, $n_{22}=n_4=n_H=2$, $n'=n'_3=n'_{31}=2$, $n'_L=n_3=0$, for the three cases $(n_L, n_R)=(4,0)$, (2, 0) and (4, 2). From these figures we notice that the values of the GUT scale are lower compared to those where no exotic states are present below $M_G$. A resonable GUT scale is found in the case where $n_R\leq 2$.

From the above figures it is clear that the fractionally charged states play an important role in the various mass scales and the low energy parameters. In particular we observe a significant change in the Grand Unification scale $M_G$ as the number of the doublets FCP's changes. In particular, if we adopt a conservative lower value for the latter ($M_G>10^{15}$ GeV) in order to avoid proton decay problems from the colour triplet fields, various bounds on the allowed number of these states, as well as on the rest of the particle content of the model, can be extracted. Within the recently proposed region of the low energy parameters $\alpha_3=0.108\pm0.005$ and $\sin^2\theta_W=0.233\pm0.001$ [22], from the above figures it is clear that in the range ($M_A$, $M_G$), we

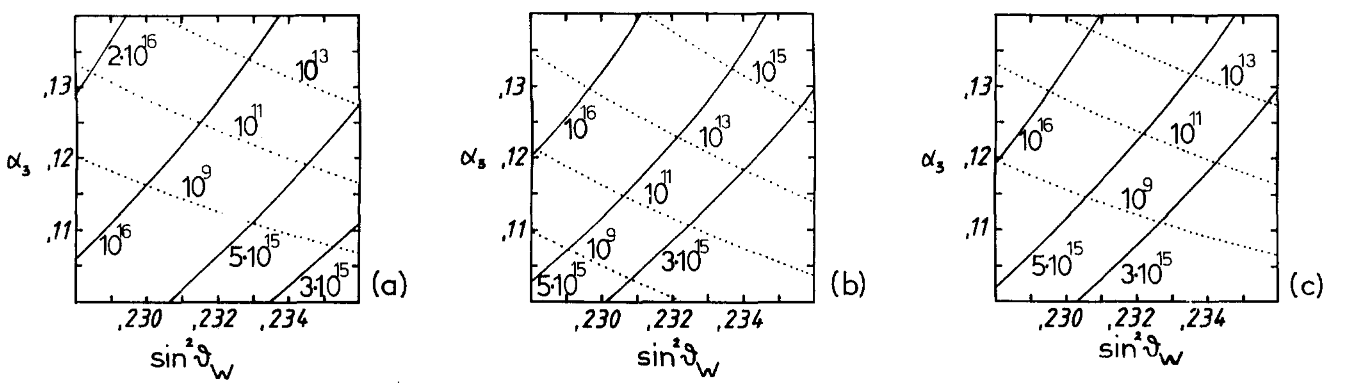


**Fig. 1a–c.** Contours of constant $M_G$ (solid lines) and constant $M_I$ (dotted lines), labelled in GeV, in the parameter space ($\sin^2\theta_W$, $\alpha_3$). The content between $M_A$ and $M_G$ is $n_6=4$, $n_{22}=n_4=n_H=2$ while the pair $(n_L, n_R)$ takes the values (4, 0), (2, 0) and (4, 2) corresponding to the figures **a**, **b** and **c**. Between $M_G$ and $M_Z$ we assumed the standard model content with a pair of triplets ($n_3=2$) down to $M_I$

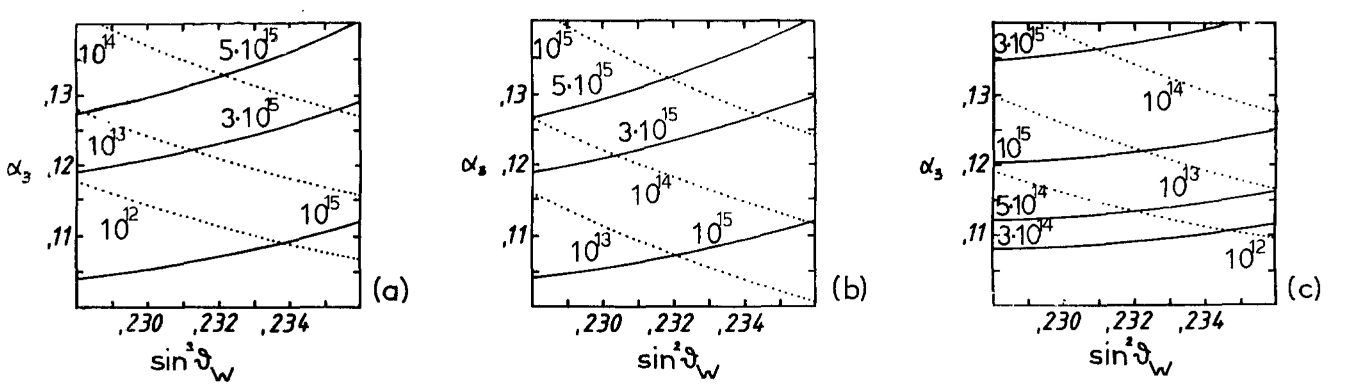


**Fig. 2a–c.** The same as in Fig. 1. The content between $M_G$ and $M_I$ is $n'=n'_3=n'_{31}=2$, $n'_L=n_3=0$

should have $n_L > n_R$, while $n_R$ should be small. On the contrary, the cases where $n_L < n_R$ are not favourable in the string case.

For completeness, we also run the coupled system of renormalization group equations for the gauge and the Yukawa couplings [21], and estimate the mass of the top quark, for a single case corresponding to Fig. 1a. We choose $M_G \sim 3 \times 10^{15}$ GeV and evaluate $m_t$ in terms of the ratio $\bar{v}/v$, with $v$ and $\bar{v}$ giving masses to the bottom and the top quarks correspondingly. Finally we choose the values of the Yukawa couplings, at $M_{SU}$, to be $\lambda_t = \lambda_b = 0.5 g_u$ as a central value. This choice of the Yukawa couplings and the experimentally known value of the bottom mass fix the vev ratio to be $\sim 34$. Our results show that $m_t$ lies in the range (140, 190) GeV. This range is essentially the same as in the case of the minimal version of the model [21]. We should note, however, that for a more complete calculation of the top quark mass one should include the string threshold corrections to the Yukawa couplings [23].

## 4 Conclusions

In this paper we have examined the evolution of the gauge couplings in the context of the superstring $SU(4) \times SU(2)_L \times SU(2)_R$ model. We have examined the possibility of obtaining the correct low energy values for the experimentally determined parameters $\alpha$, $\alpha_3$ and $\sin^2\theta_W$. We calculated the string unification scale $M_{SU}$ for this particular model, taking into account the modifications due to string threshold corrections, and we found that it is given by $M_{SU} = 1.2 g_{\rm string} 10^{18}$ GeV. Using this value for $M_{SU}$ and the aforementioned low energy constraints, we have considered possible cases of allowing additional matter representations below the $SU(4)$ breaking scale $M_G$, which permit the unification of the gauge couplings at such a high scale.

First we considered the case where only two colour triplets survive down to some intermediate scale $M_I$ and we have shown, at the one loop level, that the low energy value of $\sin^2\theta_W(M_Z)$ can be expressed as a function of $\alpha_3(M_Z)$, the scale $M_G$ and the difference $n_- = n_L - n_R$, where $n_L$ is the number of "left" fractionally charged doublets (1, 2, 1) and $n_R$ is the number of "right" fractionally charged doublets (1, 1, 2) which remain massless down to the scale $M_G$. The importance of this result is that the presence of FCP's in the massless spectrum is not going to destroy the renormalization group flow of the gauge couplings. On the contrary, a small number of them is necessary in order to realize naturally the unification scenario in this model, provided that their difference $n_- = n_L - n_R$ is positive. By a two loop calculation we have also shown that this dependence persists at this level too. Subsequently we considered cases where remnants of the exotic (1, 1, 2) and (1, 2, 1) fractionally charged states remain below the scale $M_G$. In these cases, we found that in order to have a reasonable $M_G$ scale, a certain number of them should remain down to some intermediate scale $M_I$. The $M_G$ value is intimately related to the number of these remnants as well as to the scale $M_I$.

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