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Grand unified string models and low energy couplings

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Recent calculations have shown that the string unification scale is naturally about two orders of magnitude larger than a typical supersymmetric grand unification scale compatible with the latest LEP data on $\sin^2\theta_w$ and α_3. Studying the evolution of the gauge couplings of the string derived models based on the $SU(4)\times SU(2)_L\times SU(2)_R$ and $SU(3)^3$ gauge groups, we examine the conditions that make consistent the high string unification scale with the low energy phenomenology. Assuming only the content of the minimal supersymmetric standard model below a reasonable grand unification scale, we derive the constraints on the string spectrum of the above models.

Recently, the unification idea has been revived in the light of LEP results which provided very accurate values for the parameters of the standard model and in particular for the weak angle [1]. Extrapolating the three gauge couplings at very high energies by the renormalization group equations (RGEs) and assuming the minimal particle content of the standard model, one finds that the strong coupling α_3 misses the crossing of the other two, by almost four orders of magnitude in the energy scale. However, in the presence of supersymmetry, the three couplings meet in a single point around 10^{16} GeV, assuming a supersymmetry breaking scale in the range between 100 GeV and a few TeV.

In the context of superstring theories, all interactions are unified at the string scale M_{su} which is close to the Planck scale. In fact the coupling constant g_i associated to a (non-abelian) gauge group factor G_i is given by $g_i^2 = g^2/k_i$ where g is the four-dimensional string coupling and k_i the (integer) Kac–Moody level. An automatic unification of all couplings is then obtained by setting $k_i = 1$ which also guarantees the absence of "exotic" massless representations, leading only to SU(3) triplets and/or SU(2) doublets.

The string unification scale is calculable in terms of the Planck mass M_{Pl} and the string threshold corrections [2–5]. Therefore in the minimal string unification, where only the supersymmetric standard model is derived below M_{Pl}, one finds two predictions; both $\sin^2\theta_w$ and the strong coupling α_3 are calculable in terms of the electromagnetic coupling α and the Planck mass [6,7]. Indeed assuming the standard model minimal content (supersymmetrized), the RGEs read

$$\ln\frac{M_{su}}{M_Z} = \frac{\pi}{10\alpha}\left(1 - \frac{8}{3}\frac{\alpha}{\alpha_3}\right), \tag{1a}$$

$$\sin^2\theta_w = \frac{1}{5} + \frac{7}{15}\frac{\alpha}{\alpha_3}, \tag{1b}$$

$$\frac{1}{\alpha_u} = \frac{1}{\alpha_3} + \frac{3}{2\pi}\ln\frac{M_{su}}{M_Z}. \tag{1c}$$

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At lowest order, the string unification scale can be taken to be equal to the square root of the inverse Regge slope

$$M_{su} \sim (\alpha')^{-1/2} = \tfrac{1}{2}\sqrt{\alpha_{su}}\, M_{Pl} \sim 10^{18}\,\text{GeV}\,. \qquad (2)$$

This scale is obviously two orders of magnitude larger than the minimal unification scale and leads to $\sin^2\theta_w = 0.215$ and $\alpha_3 = 0.24$, which are clearly unacceptable. String threshold corrections are in principle large because of the contribution of an infinite number of massive states. However, they turn to be unimportant in most cases, where an explicit calculation was performed or a lower bound on M_{su} could be obtained [5]. As a result, the minimal string unification seems unlikely to be realized in a phenomenologically viable way [6,7].

Modifying the "minimal" scenario the predictive power of the theory is reduced and there are two possible alternatives. The first is to consider additional matter representations at relatively low energies, besides those of the minimal content of the standard model [6]. The second is to introduce an ordinary grand unified (GUT) group which is broken to the standard model in a GUT scale M_X different from M_{su}. In this letter the latter possibility is investigated in the context of two viable string models based on the left–right symmetric GUT groups, namely the $SU(4)\times SU(2)_L\times SU(2)_R$ [8] derived in the fermionic construction of four dimensional strings, and the $SU(3)\times SU(3)_L\times SU(3)_R$ [9,10] derived via a Calabi–Yau compactification. Another possibility is the scale of flipped SU(5) [11], where it was found [5,6] that the existence of an intermediate $SU(5)\times U(1)$ phase decreases the value of $\sin^2\theta_w$ and it is therefore preferable, in this case, to have M_X close to M_{su}. The agreement with low energy data can then be achieved at the expense of introducing additional light matter representations.

We start by considering the model based on the $SU(4)\times SU(2)_L\times SU(2)_R$ gauge symmetry. For completeness we will briefly present here its massless spectrum. The n_G fermion generations are sitting in the $(\bar{4}, 1, 2)+(4, 2, 1)$ representations of the $SU(4)\times SU(2)_L\times SU(2)_R$ symmetry. Let n_H be the number of Higgs fields, which break $SU(4)\times SU(2)_R \rightarrow SU(3)\times U(1)_Y$, at some scale M_X, sitting in the $(4, 1, 2)$ and $(\bar{4}, 1, 2)$ representations, n_{22} the number of $(1, 2, 2)$ Higgs which break the standard model symmetry to $SU(3)\times U(1)_{EM}$ and n_6 the number of sextets $(6, 1, 1)$. In $k=1$ constructions (as in the case of this specific model), fractionally charged states also appear sitting in $(4, 1, 1)$, $(1, 2, 1)$ and $(1, 1, 2)$ representations. Let n_4, n_L and n_R be their number correspondingly. After the first symmetry breaking at M_X, in the most general case, one is left with exotic states (in addition to the standard content) which arise from the decompositions of the previous fields:

$$\begin{aligned}
&\bar{H}(\bar{4}, 1, 2) \rightarrow u^c_H(\bar{3}, 1, -\tfrac{4}{3}) + d^c_H(\bar{3}, 1, \tfrac{2}{3}) \\
&\quad + \nu^c_H(1, 1, 0) + e^c_H(1, 1, 2)\,, \\
&H(4, 1, 2) \rightarrow \bar{u}^c_H(\bar{3}, 1, \tfrac{4}{3}) + \bar{d}^c_H(3, 1, -\tfrac{2}{3}) \\
&\quad + \bar{\nu}^c_H(1, 1, 0) + \bar{e}^c_H(1, 1, -2)\,, \\
&D(6, 1, 1) \rightarrow D_3(3, 1, -\tfrac{2}{3}) + \bar{D}_3(\bar{3}, 1, \tfrac{2}{3})\,, \\
&H_4(4, 1, 1) \rightarrow d'_3(3, 1, -\tfrac{1}{3}) + e'_3(1, 1, -1)\,, \\
&\bar{H}_4(\bar{4}, 1, 1) \rightarrow d^{c\prime}_3(\bar{3}, 1, \tfrac{1}{3}) + e^{c\prime}_3(1, 1, 1)\,, \\
&h_L(1, 2, 1) \rightarrow h_L(1, 2, 0)\,, \\
&h_R(1, 1, 2) \rightarrow h^+_R(1, 1, 1) + h^-_R(1, 1, -1)\,. \qquad (3)
\end{aligned}$$

In eq. (3), the fields in the left-hand side appear with their quantum numbers under the $SU(4)\times SU(2)_L \times SU(2)_R$, while in the right-hand side under $SU(3) \times SU(2)_L \times U(1)_Y$.

Once we have derived the spectrum, we can write down the one-loop β function coefficients. In the range $M_X < M < M_{su}$ we get

$$\begin{aligned}
&b_4 = -12 + 2n_G + n_H + n_6 + \tfrac{1}{2}n_4\,, \\
&b_{2L} = -6 + 2n_G + n_{22} + \tfrac{1}{2}n_L\,, \\
&b_{2R} = -6 + 2n_G + n_{22} + \tfrac{1}{2}n_R + 2n_H\,, \qquad (4)
\end{aligned}$$

where n_G is the number of generations ($n_G = 3$). In the range $M_Z < M < M_X$ we have

$$\begin{aligned}
&b_3 = -9 + 2n_G + \tfrac{1}{2}n_3 + \tfrac{1}{2}n'_3 + \tfrac{1}{2}n'_{31}\,, \\
&b_2 = -6 + 2n_G + \tfrac{1}{2}n_2 + \tfrac{1}{2}n'_L\,, \\
&b_Y = 2n_G + \tfrac{3}{10}n_2 + \tfrac{3}{20}n' + \tfrac{1}{5}n_3 + \tfrac{1}{20}n'_3 + \tfrac{4}{5}n'_{31}\,, \qquad (5)
\end{aligned}$$

where n_3 is the number of the usual color triplets $(3, 1, \tfrac{1}{3})$, n'_3 is the number of triplets arising from H'_4, $\bar{H}'_4$ fields while n'_{31} is the number of u^c_H and $\bar{u}^c_H$ upquark-type Higgs. Also n' is the number of singlets with fractional charges $\pm\frac{1}{2}$, arising from h_R and H'_4, $\bar{H}'_4$. In particular, in the model under consideration, $n'_{31}=n'_3=2$, if none of these triplets get mass at M_X, and $n'=2n_R+2$.

Let us consider first the simplest case in the above model, where we have only one intermediate scale M_X between the superunification scale M_{su} and the weak scale M_w. Assume further that M_X is the SU(4) breaking scale and that below this energy we have the standard model gauge group with the minimal fermion and Higgs content. Above M_X, the spectrum contains the representations of the particular string model. Bearing in mind that at M_X we have the relation

$$\frac{1}{\alpha_Y}=\frac{2}{5}\frac{1}{\alpha_4}+\frac{3}{5}\frac{1}{\alpha_R}, \tag{6}$$

the RGEs yield

$$\frac{1}{\alpha_3}=\frac{1}{\alpha_{su}}+\frac{b_4}{2\pi}\ln\frac{M_{su}}{M_X}-\frac{3}{2\pi}\ln\frac{M_X}{M_Z}, \tag{7a}$$

$$\sin^2\theta_w=\frac{\alpha}{\alpha_3}+\frac{\alpha}{2\pi}(b_L-b_4)\ln\frac{M_{su}}{M_X}+\frac{2\alpha}{\pi}\ln\frac{M_X}{M_Z}, \tag{7b}$$

where

$$\ln M_X=\frac{k\ln M_{su}-36\ln M_Z-6\pi/\alpha-16\pi/\alpha_{su}}{k-36}, \tag{7c}$$

and $k=2b_4+3b_R+3b_L$. It is easy to check, that even in this simple picture of the model, one can find acceptable values of the relevant parameters satisfying eqs. (7). For example, if $n_H=n_6=n_{22}=4$ and $n_L=n_R=0$, we find that $M_X\sim2\times10^{14}$ GeV, $\sin^2\theta_w\sim0.233$, $\alpha_3\sim0.108$ and $\alpha_{su}\sim0.054$. In the case where $n_H=n_6=n_{22}=4$ and $n_L=2$ and $n_R=0$, we get $M_X\sim0.8\times10^{14}$ GeV, $\sin^2\theta_w\sim0.232$, $\alpha_3\sim0.108$ and $\alpha_{su}\sim0.057$. The resulting M_X scale, although somewhat small, does not create any problem, since in the case of SU(4) unification there are no dangerous gauge bosons leading to fast proton decay.

The above results show that in the case of the $SU(4)\times SU(2)_L\times SU(2)_R$ string model, there is a positive role of the intermediate GUT scale M_X. The different role of the M_X scale in this case compared to the one flipped SU(5) [5,6], can be understood by studying the value of $\sin^2\theta_w$ at M_X:

$$\sin^2\theta_w=\frac{\alpha}{\alpha_2}=\frac{3}{8+5\alpha_2(1/\alpha_Y-1/\alpha_2)}. \tag{8}$$

At M_X, in the case of flipped SU(5), one has the relation

$$\frac{1}{\alpha_Y}=\frac{1}{25}\frac{1}{\alpha_5}+\frac{24}{25}\frac{1}{\alpha_1}, \tag{9a}$$

which implies that

$$\frac{1}{\alpha_Y}-\frac{1}{\alpha_2}=\frac{24}{25}\left(\frac{1}{\alpha_1}-\frac{1}{\alpha_5}\right)$$
$$=\frac{1}{4\pi}\frac{24}{25}(b_1-b_5)\ln\left(\frac{M_{su}^2}{M_X^2}\right). \tag{9b}$$

Now notice that the contribution of gauge bosons (and gauginos) alone is large and positive: $b_1-b_5=15$. Thus, the value of $\sin^2\theta_w$ at M_X is in general reduced and it is preferable to raise the SU(5) breaking scale M_X closer to M_{su}. In the case of $SU(4)\times SU(2)_L\times SU(2)_R$ the situation is somewhat different. The relation (6) that holds between α_Y, α_4 and α_R at M_X, implies that

$$\frac{1}{\alpha_Y}-\frac{1}{\alpha_2}=\frac{2}{5}\frac{1}{\alpha_4}+\frac{3}{5}\frac{1}{\alpha_5}-\frac{1}{\alpha_L}$$
$$=\frac{1}{4\pi}\left(\tfrac{2}{5}b_4+\tfrac{3}{5}b_R-b_L\right)\ln\left(\frac{M_{su}^2}{M_X^2}\right). \tag{10}$$

In this case, the gauge boson and gaugino contributions turn out to be negative: $\frac{2}{5}b_4+\frac{3}{5}b_R-b_L=-\frac{12}{5}$. The value of $\sin^2\theta_w$ at M_X is thus increased and an appropriate value for α_Y at M_X leads to reasonable values of $\sin^2\theta_w$ at M_Z.

We have already mentioned that the most general picture of the model is more complicated. Firstly, there exists a scale M_A where the anomalous U(1) factor breaks down while some singlet fields acquire non-vanishing VEVs [8]. It is possible that some of the fields receive masses at that scale. Furthermore, after the $SU(4)\times SU(2)_R$ breaking, some of the exotic states may survive down to an intermediate scale M_I, at which some non-renormalizable interactions make them massive. In this case, eqs. (7) are generalized as follows:

$$\frac{1}{\alpha_3}=\frac{1}{\alpha_{su}}+\frac{3}{2\pi}\ln\frac{M_{su}}{M_A}+\frac{b_4}{2\pi}\ln\frac{M_A}{M_X}$$

$$+\frac{b_3}{2\pi}\ln\frac{M_X}{M_I}-\frac{3}{2\pi}\ln\frac{M_I}{M_Z}, \tag{11a}$$

$$\sin^2\theta_w=\frac{\alpha}{\alpha_3}+\frac{\alpha}{2\pi}6\ln\frac{M_{su}}{M_A}+\frac{\alpha}{2\pi}(b_L-b_4)\ln\frac{M_A}{M_X}$$

$$+\frac{\alpha}{2\pi}(b_2-b_3)\ln\frac{M_X}{M_I}+\frac{\alpha}{2\pi}4\ln\frac{M_I}{M_Z}, \tag{11b}$$

where M_I is given by

$$\ln M_I=\left(\frac{16\pi}{\alpha_{su}}-\frac{6\pi}{\alpha}+84\ln\frac{M_{su}}{M_A}+k\ln\frac{M_A}{M_X}\right.$$

$$\left.+k'\ln M_X-36\ln M_Z\right)(k'-36)^{-1}, \tag{11c}$$

with $k'=5b_Y+3b_2$. In the above formulae we have assumed the string content between M_{su} and M_A and the standard model one below M_I. Using eqs. (11) one can plot, in the parameter space $(\sin^2\theta_w, \alpha_3)$, contours of constant M_X. Alternatively, instead of eq. (11c) one can solve for M_X getting

$$\ln M_X=\left(\frac{16\pi}{\alpha_{su}}-\frac{6\pi}{\alpha}+84\ln\frac{M_{su}}{M_A}+k\ln M_A\right.$$

$$\left.-k'\ln M_I+36\ln\frac{M_I}{M_Z}\right)(k-k')^{-1}. \tag{11d}$$

Using now eqs. (11a), (11b), (11d) one can plot contours for constant M_I. In fig. 1 we show such contours using the following particle content:

$$n_L=4,\quad n_R=0,\quad n_{22}=n_4=n_2=n_H=2,$$

$$n_6=4,\quad n'_{31}=n'_3=n'_L=n'=2,\quad n_3=0.$$

From this figure we get that M_X varies approximately between 10^{15} GeV and $10^{15.8}$ GeV. These values are significantly higher than the ones obtained before in the simple picture of the model. The intermediate scale M_I lies in the range $10^8<M_I<10^{14}$ GeV. Of course a more detailed analysis should have taken into account the possibility that different exotic states acquire masses at different intermediate scales. Our analysis here estimates a rather average value of these possible intermediate scales.

Another gauge string model which has been derived from the compactification procedure, is based on the gauge symmetry $SU(3)\times SU(3)_L\times SU(3)_R$, which is a maximal subgroup of E_6. The standard quark and lepton fields are found in the 27 representation of the group, which under the maximal subgroup decomposes as follows:

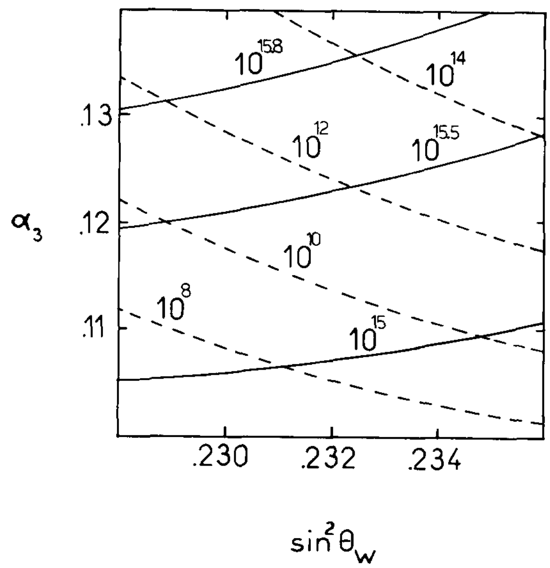

Fig. 1. Contours of constant M_X (solid lines) and M_I (dashed lines), labelled in GeV, in the parameter space $(\sin^2\theta_w, \alpha_3)$. The particle content is $n_L=4$, $n_R=0$, $n_{22}=n_4=n_2=n_H=2$, $n_6=4$ and $n'_{31}=n'_3=n'_L=n'=2$, $n_3=0$.

$$27\to(3,\bar{3},1)+(\bar{3},1,3)+(1,3,\bar{3}).$$

The accommodation of the various particles is

$$(3,\bar{3},1)=\begin{pmatrix}u\\d\\D\end{pmatrix},\quad(\bar{3},1,3)=\begin{pmatrix}u^c\\d^c\\D^c\end{pmatrix},$$

$$(1,3,\bar{3})=\begin{pmatrix}h^0&h&e^c\\h^-&\bar{h}^0&\nu^c\\e&\nu&N\end{pmatrix}, \tag{12}$$

where the charge operator is $Q=I_{3L}+I_{3R}+\frac{1}{2}(Y_L+Y_R)$. As in the previous model, in the case of a $k=1$ construction, the appearance of massless states with fractional charges is possible. We will see, however, that it is not necessary to complicate our analysis with such exotic states.

There are several symmetry breaking chains of the above gauge group down to the standard model. In the following we will assume the simplest one which includes only one intermediate scale:

$$\mathrm{E} \xrightarrow{M_{\mathrm{su}}} \mathrm{SU}(3)\times\mathrm{SU}(3)_{\mathrm{L}}\times\mathrm{SU}(3)_{\mathrm{R}}$$

$$\xrightarrow{M_{\mathrm{X}}} \mathrm{SU}(3)\times\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{Y}.$$

At M_{X} we have the relation

$$\frac{1}{\alpha_Y}=\frac{4}{5}\frac{1}{\alpha_{3\mathrm{R}}}+\frac{1}{5}\frac{1}{\alpha_{3\mathrm{L}}}, \tag{13a}$$

which implies that

$$\frac{1}{\alpha_Y}-\frac{1}{\alpha_2}=\frac{4}{5}\left(\frac{1}{\alpha_{3\mathrm{R}}}-\frac{1}{\alpha_{3\mathrm{L}}}\right)$$

$$=\frac{1}{5\pi}(b_{3\mathrm{R}}-b_{3\mathrm{L}})\ln\left(\frac{M_{\mathrm{su}}^2}{M_{\mathrm{X}}^2}\right). \tag{13b}$$

Note that in this case the gauge boson and gaugino contribution alone vanishes and the value of $\sin^2\theta_{\mathrm{w}}$ remains $\frac{3}{8}$ at M_{X}. The β-function coefficients, between M_{su} and M_{X} are

$$b_3=-9+\tfrac{3}{2}(n_{\mathrm{q}}+n_{\mathrm{Q^c}}),$$

$$b_{3\mathrm{L}}=-9+\tfrac{3}{2}(n_{\ell}+n_{\mathrm{q}}),$$

$$b_{3\mathrm{R}}=-9+\tfrac{3}{2}(n_{\ell}+n_{\mathrm{Q^c}}), \tag{14}$$

where n_{q} is the number of (3, 3, 1) representations, $n_{\mathrm{Q^c}}$ of the $(\bar{3}, 1, 3)$ and n_{ℓ} of the $(1, \bar{3}, 3)$ representations. Below M_{su} we have the standard model β-functions. Now, using the RGEs, the values of $\sin^2\theta_{\mathrm{w}}$ and α_3 at M_Z are given by

$$\sin^2\theta_{\mathrm{w}}=\tfrac{3}{8}+\tfrac{1}{2}(b_{3\mathrm{L}}-b_{3\mathrm{R}})\frac{\alpha}{2\pi}\ln\frac{M_{\mathrm{su}}}{M_{\mathrm{X}}}$$

$$-\tfrac{5}{8}(b_Y-b_2)\frac{\alpha}{2\pi}\ln\frac{M_{\mathrm{X}}}{M_Z},$$

$$\frac{\alpha}{\alpha_3}=\tfrac{3}{8}+[b_3-\tfrac{1}{2}(b_{3\mathrm{L}}+b_{3\mathrm{R}})]\frac{\alpha}{2\pi}\ln\frac{M_{\mathrm{su}}}{M_{\mathrm{X}}}$$

$$+(b_3-\tfrac{3}{8}b_2-\tfrac{5}{8}b_Y)\frac{\alpha}{2\pi}\ln\frac{M_{\mathrm{X}}}{M_Z}. \tag{15}$$

However, some extra string representations may remain massless below M_{X}, as in the previous model. In that case, one should also include the contribution of these states above some intermediate scale M_{I}. Since our purpose here is to show that there are some minimal requirements, which should be met in order to be consistent with a high ($\sim 10^{18}$ GeV) string unification scale, we are not complicating our analysis introducing such new scales M_{I}.

Now a careful examination reveals the following property. If $n_{\mathrm{q}}=n_{\mathrm{Q^c}}=n_{\ell}$, $b_{3\mathrm{L}}=b_{3\mathrm{R}}=b_3$ and the couplings $g_{3\mathrm{L}}$, $g_{3\mathrm{R}}$ and g_3 evolve identically from M_{su} down to M_{X}, then, eqs. (15) take exactly the form of eqs. (1) where now M_{X} is the $\mathrm{SU}(3)_{\mathrm{L}}\times\mathrm{SU}(3)_{\mathrm{R}}$ breaking scale. It is possible now, by a judicious choice of α_{su} at M_{su}, to obtain $\alpha_{3\mathrm{R}}=\alpha_{3\mathrm{L}}=\alpha_3\sim\frac{1}{25}$ at $M_{\mathrm{X}}\sim 10^{16}$ GeV, since we know that these values lead to the successful predictions for $\sin^2\theta_{\mathrm{w}}$ and α_3. However, in realistic string models one usually finds that under the group of honest symmetries and the flux breaking mechanism, different numbers of lepton, quark and antiquark superfields survive. Thus in general $b_{3\mathrm{L}}$, $b_{3\mathrm{R}}$ and b_3 are present in eqs. (15), affecting the low energy values of $\sin^2\theta_{\mathrm{w}}$ and α_3. In the following we examine this general case and put constraints on the possible numbers of quark, antiquark and lepton superfields which a string model with $\mathrm{SU}(3)\times\mathrm{SU}(3)_{\mathrm{L}}\times\mathrm{SU}(3)_{\mathrm{R}}$ symmetry should have, in order to be consistent with the low energy phenomenology.

Let us denote for simplicity $n=n_{\mathrm{q}}-n_{\mathrm{Q^c}}$ and $\bar{n}=n_{\mathrm{q}}-n_{\ell}$. Then, the experimentally acceptable range of $\sin^2\theta_{\mathrm{w}}$ and α_3 leads to the following conditions:

$$320<(10+n+2\bar{n})\ln\frac{M_{\mathrm{X}}}{M_Z}-(n+2\bar{n})\ln\frac{M_{\mathrm{su}}}{M_Z}<330,$$

$$151<(\tfrac{14}{3}+n)\ln\frac{M_{\mathrm{X}}}{M_Z}-n\ln\frac{M_{\mathrm{su}}}{M_Z}<157. \tag{16}$$

The above conditions can be satisfied for various values of n and $\bar{n}$, provided that we use in eqs. (16) the proper scale M_{X}. Thus, if $n=0$, which means that $n_{\mathrm{q}}=n_{\mathrm{Q^c}}$, we get $10^{16}<M_{\mathrm{X}}/\mathrm{GeV}<5.8\times10^{16}$. Inserting the above range of M_{X} in eqs. (16) we find $\bar{n}=0$ or 1, corresponding to $n_{\mathrm{Q^c}}=n_{\ell}$ or $n_{\mathrm{Q^c}}=n_{\ell}+1$. A list of various possible cases is presented in table 1, which shows that the $\mathrm{SU}(3)\times\mathrm{SU}(3)_{\mathrm{L}}\times\mathrm{SU}(3)_{\mathrm{R}}$ model is probably less constrained compared to the other two cases analyzed above and to the minimal superstring version of the standard model.

In conclusion, we have analyzed the possibility of obtaining the correct low energy values for the exper-

Table 1
The range of the scale M_X for various possible choices of n and $\bar{n}$, in the $SU(3)^3$ model.

n	$\bar{n}$	$M_{X_{min}}$ (GeV)	$M_{X_{max}}$ (GeV)
0	1	1.9×10^{16}	5.8×10^{16}
0	0	8.4×10^{15}	5.8×10^{16}
1	1	2.1×10^{16}	1.0×10^{17}
2	0	3.9×10^{16}	3.9×10^{16}
2	1	3.9×10^{16}	8.0×10^{16}
2	2	3.9×10^{16}	1.3×10^{17}
1	2	2.6×10^{16}	1.0×10^{17}
0	2	3.5×10^{16}	5.8×10^{16}

imentally determined parameters α, α_3 and $\sin^2\theta_w$ in two GUT-like string models based on the $SU(4)\times SU(2)_L\times SU(2)_R$ and $SU(3)^3$ gauge symmetries, after taking into account the modifications to the unification scale due to string threshold effects. We have found that the unification scenario is possible in these two cases under certain assumptions, related with the particle spectrum and the intermediate "grand unification" scale M_X. In the $SU(4)\times SU(2)_L\times SU(2)_R$ case, string unification at a high scale is possible even with the minimal particle content if $M_X\sim10^{14}$ GeV. Introduction of extra exotic states (as in realistic string versions) raises M_X by one to two orders of magnitude. The unification scenario is even less restrictive in the case of $SU(3)^3$ symmetry. There are numerous choices of the field content which may give phenomenologically acceptable models. However, a lot of work is needed until a model with the required spectrum appears.

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