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Analytic results for the Gross-Neveu model β function in three loops

N. D. Tracas
Physics Department, National Technical University, 157 73 Zografou, Athens, Greece

N. D. Vlachos
Department of Theoretical Physics, University of Thessaloniki, 540 06 Thessaloniki, Greece
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We calculate the three-loop β function for the Gross-Neveu model, showing explicitly the techniques used to minimize the number of diagrams needed to be evaluated. The lack of a term quadratic in N suggests that an N-independent coupling rescaling could possibly eliminate all but the first two terms of the β function.

I. INTRODUCTION

The Gross-Neveu (GN) model[1] has been, for a long time, a very useful testing ground for investigating various interesting phenomena in field theory. It is an asymptotically free two-dimensional fermion field theory with a quartic interaction. The model has been analyzed in the $1/N$ approximation, where N is the number of fermions, and found to exhibit dynamical breaking of the discrete chiral symmetry of the model and a rich bound-state spectrum. The symmetry-breaking sign is a nonvanishing vacuum expectation value for the composite field $\bar{\psi}\psi$. The dynamical mass depends, in a nontrivial way, on the coupling constant, vanishing exponentially with it. The last statement is a more general one for asymptotically free gauge theories where all masses are generated dynamically. Lately the GN model was analyzed using the so-called δ expansion,[2] and it is argued that the dynamical symmetry breaking is being achieved for finite N as well as for $N\to\infty$. The model is also classically integrable[3] and the quantum S matrix has been known for some time.[4]

The Lagrangian of the model is

$$L=\bar{\psi}(i\partial\!\!\!/)\psi+\lambda(\bar{\psi}\psi)^2/2\ , \tag{1.1}$$

where a summation over the flavor index of the fermion field ψ^i, $i=1,\dots,N$, is understood. When $N=1$ the two-dimensional Fierz identity

$$2(\bar{\psi}\psi)(\bar{\psi}\psi)=-(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$$

ensures that the Lagrangian above is equivalent to

$$L_{\text{Th}}=\bar{\psi}(i\partial\!\!\!/)\psi-\lambda(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi)/2\ , \tag{1.2}$$

which corresponds to the (massless) Thirring model, known to be a finite field theory (i.e., the β function vanishes). The β function of the GN model, up to order λ^3, was evaluated some time ago.[5] In this paper,[6] exploring properties of the specific model and of the renormalization procedure, we evaluate the β function to order λ^4 in an easy way.

The structure of the contributing Feynman graphs is best understood in the σ-field formulation where the Lagrangian is written as[1]

$$L_\sigma=\bar{\psi}(i\partial\!\!\!/)\psi+\sigma^2/2-g(\bar{\psi}\psi)\sigma,\quad g^2=\lambda\ , \tag{1.3}$$

where σ is a nonpropagating scalar field. This Lagrangian gives fermion Green's functions identical to the original one. The Feynman rules, for both Lagrangians in Eqs. (1.1) and (1.3), are shown in Fig. 1.

The quantity to be evaluated is the 3-loop coupling renormalization constant Z_λ, defined through $\lambda_B=Z_\lambda\lambda_R$, where $B(R)$ stands for bare (renormalized). Now Z_λ can be given in terms of the 2- and 4-point function renormalization constants Z_4 and Z_2, as

$$Z_\lambda=Z_4Z_2^{-2}\ , \tag{1.4}$$

where Z_4 and Z_2 are defined through $G^R{}_{2(4)}=Z_{2(4)}G^B{}_{2(4)}$. Working in the minimal subtraction (MS) scheme, the β function is only associated with the residues (i.e., with the coefficient of $1/\varepsilon$ in dimensional regularization) of the renormalization constants[7]

$$\begin{aligned}\beta(\lambda)&=2\lambda^2\frac{d}{d\lambda}(\text{Res}Z_\lambda)\\&=2\lambda^2\frac{d}{d\lambda}[\text{Res}(Z_4-2Z_2)]\ .\end{aligned} \tag{1.5}$$

The calculation of Z_4 requires an off-shell evaluation of the corresponding 4-point function that diverges logarithmically. The infinite part therefore cannot depend on any

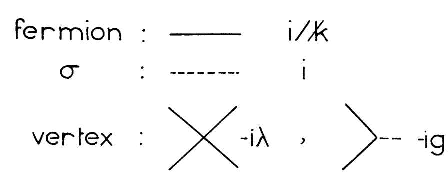


FIG. 1. The Feynman rules for the Gross-Neveu model.

dimensional parameter, such as external momenta or mass parameters. In order to keep computational complications down to a minimum, we choose to calculate the 4-point function at zero external momenta and add a fermion mass parameter to be used as an infrared regulator. Mass renormalization effects must now be taken into account; however, the final result does not depend on this particular choice, as should be the case. Moreover the very existence of this mass parameter allows the summation of classes of diagrams by means of Ward-like identities.

The structure of the paper is as follows. In Sec. II we explain in detail the ideas and the shortcuts used in order to reduce the tedium of the calculations. In Secs. III–V we evaluate the relevant graphs for the wave function and the vertex, the σ propagator and the contribution of the two-loop skeleton graphs correspondingly. In Sec. VI we evaluate the desired renormalization constants and finally the β function. The conclusion and several remarks are stated in Sec. VII.

II. GENERAL CONSIDERATIONS

In this section we present the ideas that allow us to reduce considerably the amount of work to be done. There are four main considerations.

If we write the β function in the usual way

$$\beta(\lambda)=\beta_0\lambda^2+\beta_1\lambda^3+\beta_2\lambda^4 \qquad (2.1)$$

we expect the coefficients β_0, β_1, and β_2 to be polynomials in N. Note that factors of N are associated with the presence of fermionic loops. The connection with the Thirring model (vanishing β function), on the other hand, allows us to factor out an $N-1$ term:

$$\begin{aligned}\beta_2&=(N-1)(\beta_{22}N^2+\beta_{21}N+\beta_{20})\\&=\beta_{22}N^3+(\beta_{21}-\beta_{22})N^2+(\beta_{20}-\beta_{21})N-\beta_{20}\ ,\end{aligned} \qquad (2.2)$$

where β_{2i}, $i=0,1,2$, are constants. It is obvious from Eq. (2.2) that we only need to evaluate three out of the four coefficients of the third-order polynomial appearing. The most convenient choice is to avoid calculating β_{20}, corresponding to diagrams without a fermionic loop, which are the most cumbersome.

Next we note that diagrams relevant to Z_4 and corresponding to "non-one-particle-irreducible- (1PI-) like graphs," in the σ formulation, do not contribute, since the loops are disconnected. Some examples are shown in Fig. 2. In particular the first of these diagrams is the only one proportional to N^3. This diagram can be renormalized using the 1-loop counterterm, leading therefore to the statement that β_{22} in Eq. (2.2) is zero.

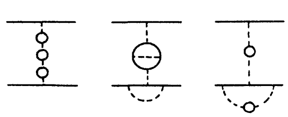


FIG. 2. Non-1PI-like diagrams, in the σ formulation, which do not contribute. The first is the only one proportional to N^3.

The third point is the following. All diagrams contributing to the 4-point function are, in general, overall logarithmically divergent. If this overall divergence is missing, either in a diagram or in a sum of appropriate diagrams, we do not have a contribution, since the subdivergences are canceled by the lower-order counterterms, rendering thus the diagram finite at this level.

There are two cases where the statement above can be applied. The first case refers to diagrams having a fermion loop with an odd number of fermion propagators (dressed or not). This loop is obviously proportional to m and the overall divergence has disappeared. For the second case consider a diagram where a subdiagram consists of a fermion-line correction. We expect this correction to have a term proportional to the momentum (slashed) of this line and a term proportional to the mass m of the fermion. This last term cannot contribute since, having lost a power of momentum, the overall divergence is lost. This point gets extremely useful in the case of a 1-loop fermion-line correction where there is only an m-proportional term, rendering the whole diagram uninteresting.

The last point we would like to consider is the following. If we differentiate a fermion propagator with respect to m we create a vertex carrying a zero-momentum σ line. By analogy, differentiating a diagram we take the sum of diagrams where one σ line is coming out of each fermion propagator. Applying this idea to the diagrams contributing to the mass renormalization we get graphs contributing to the vertex renormalization.

The ensemble of 3-loop diagrams contributing to the 4-point function, shown in Fig. 3, can be generated by the zero-, one-, and two-loop skeleton diagrams dressed in all possible ways (propagators and vertices) plus the new structures appearing at this loop level. Let us go through each group shown in the figure. Groups II and VI do not contribute since, adding them in pairs, the diagrams lose their overall divergence. Group IV also does not contribute since an odd-number fermion propagator loop appears. As a general rule we can state that graphs vanishing for $m \to 0$ do not contribute. In group I we have either a vertex or a σ-propagator correction (if we have both, the diagram is a "non-1PI-like" one). The vertex correction diagrams can be taken, as a bonus, by differentiating with respect to the mass m, the fermion 2-point function corrections which we evaluate in Sec. III. The σ-propagator corrections can be found, as is shown in Sec. IV, by differentiating vacuum-to-vacuum diagrams twice with respect to m. Finally group III is evaluated in Sec. V.

All calculations have been performed using dimensional regularization and the MS scheme. The general structure of the infinite parts of the relevant diagrams and the corresponding one- and two-loop counterterm graphs is shown below:

3-loop graph:

$$(a+b\varepsilon+c\varepsilon^2)I^3\ ; \qquad (2.3a)$$

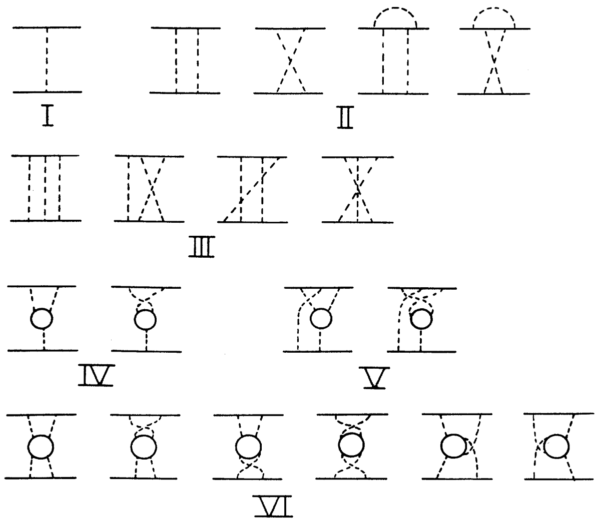


FIG. 3. Diagrams contributing to the 4-point function in the 3-loop order. Groups I, II, and III are supposed to be dressed in all possible ways (vertices and propagators).

2-loop counterterm graph:

$$(A_2/\varepsilon^2+B_2/\varepsilon)(a_2+b_2\varepsilon+c_2\varepsilon^2)I\ ; \tag{2.3b}$$

1-loop counterterm graph:

$$(A_1/\varepsilon)(a_1+b_1\varepsilon+c_1\varepsilon^2)I^2\ , \tag{2.3c}$$

where $\varepsilon=1-n/2$ and $I=\int d^nk/(k^2-m^2)$. Renormalizability of the theory requires the following relations to hold true:

$$3a+a_1A_1=0,\quad a_2A_2+a_1A_1=0\ ,$$

$$3b+a_2B_2+b_2A_2+2b_1A_1=0\ . \tag{2.4}$$

These relations ensure that terms of the form $\ln^a m/\varepsilon^b$, as well as some unwanted terms, such as $\ln 4\pi$, γ, and $\psi(0)$, cancel out between graphs and countergraphs. The interesting $1/\varepsilon$, $1/\varepsilon^2$, and $1/\varepsilon^3$ terms are given by

$$1/\varepsilon \text{ term: } c+A_1c_1+A_2c_2+b_2B_2\ , \tag{2.5a}$$

$$1/\varepsilon^2 \text{ term: } b+A_1b_1+b_2A_2+a_2B_2\ , \tag{2.5b}$$

$$1/\varepsilon^3 \text{ term: } a+a_1A_1+a_2A_2\ . \tag{2.5c}$$

Although all infinite parts could be written in the form of Eqs. (2.3), in a number of diagrams we have been faced with terms of the form

$$m^{2a}I\int\frac{d^nk\,d^np}{(k^2-m^2)[(k+p)^2-m^2](p^2-m^2)^a},\quad a=1,2\ ,$$

which cancel out completely when counter diagrams are included. We have neither put these terms in the form of Eqs. (2.3), nor included their contribution in the following tables, in order to keep the picture as clear as possible. Tables I and II give all the infinite parts needed for the evaluation of the counterterms in the three-loop approximation.

TABLE I. Infinite part of the 1-loop diagrams. A factor of $1/(4\pi)$ is understood.

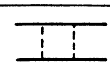
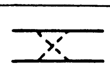
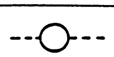
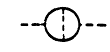
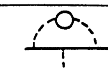
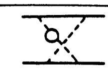
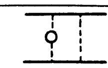
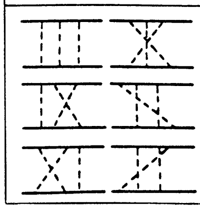
	ig^2m/ε	(1.1)
	ig^3/ε	(1.2)
	$[ig^4/(2\varepsilon)]\ (\gamma^\mu)(\gamma_\mu)$	(1.3)
	$-[ig^4/(2\varepsilon)]\,(\gamma^\mu)(\gamma_\mu)$	(1.4)
	$-ig^2 2N/\varepsilon$	(1.5)

TABLE II. Infinite part of the 2-loop diagrams. A factor of $1/(4\pi)^2$ is understood.

	$-ig^4m(-1/\varepsilon^2+1/\varepsilon)$	(2.1)
	$-ig^4\{\not k/(2\varepsilon)+m(-1/(2\varepsilon^2)-1/\varepsilon)\}$	(2.2)
	$ig^4N\{\not k/\varepsilon+m(-1/(\varepsilon^2)-1/\varepsilon)\}$	(2.3)
	$ig^4N(-2/(\varepsilon^2)+2/\varepsilon)$	(2.4)
	$ig^5N(-1/(\varepsilon^2)-1/\varepsilon)$	(2.5)
	$ig^6N(1/(2\varepsilon^2)+1/\varepsilon)\ (\gamma^\mu)(\gamma_\mu)$	(2.6)
	$-ig^6N(1/(2\varepsilon^2)+1/\varepsilon)\ (\gamma^\mu)(\gamma_\mu)$	(2.7)
	$-(ig^6/\varepsilon)(I)(I)$	(2.8)

III. FERMION WAVE FUNCTION AND VERTEX RENORMALIZATION

We have evaluated all N^2- and N-proportional diagrams contributing to the wave-function renormalization of the fermion with s (external) momentum. The results, following the notation of Eq. (2.3), are shown in Table III. The $1/\varepsilon$ term is then easily evaluated, using Eq. (2.5a):

$$i[g^6/(4\pi)^3][N^2(-8\not s/3+4m/3)+N(-6\not s-4m)]/\varepsilon. \tag{3.1}$$

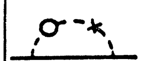
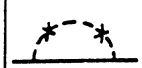
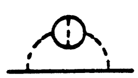
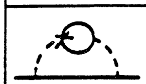
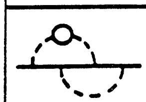
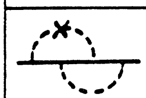
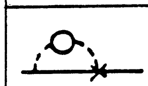
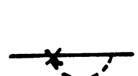
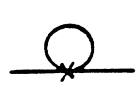
By differentiating with respect to m we get the corresponding contribution of the $\bar{\psi}\psi\sigma$ vertex to the 4-point function, appropriately multiplied by $2g^2$,

$$i[g^8/(4\pi)^3](8N^2/3-8N)/\varepsilon \ . \tag{3.2}$$

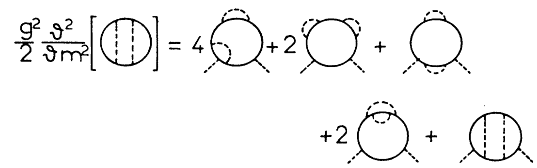
IV. σ PROPAGATOR

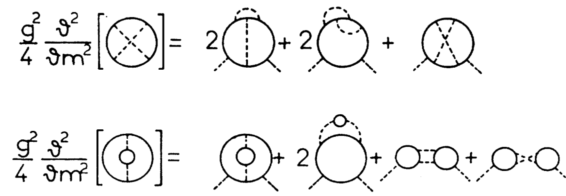
The diagrams contributing to the σ propagator can be easily evaluated by applying twice the differentiation technique to three vacuum diagrams. Diagrammatically

TABLE III. 3-loop diagrams and counter diagrams contributing to the fermion wave-function renormalization. The external momentum of the fermion line is s. A factor of ig^6 is understood.

			$\not{s}$			m			
DIAGRAM	$1/\varepsilon^2$	$1/\varepsilon$	ε^0	ε	ε^2	ε^0	ε	ε^2	FACTOR
	-	-	0	8/3	-8/3	4/3	-8/3	4/3	N^2
	-	2	0	1	1	1	-1	0	$-2N^2/(4\pi)$
	4	0	0	0	1	1	0	0	$N^2/(4\pi)^2$
	-	-	0	4/3	0	2/3	-2/3	0	$-N$
	-	1	0	1	0	1	-1	0	$2N/(4\pi)$
	2	-2	0	0	1	1	0	0	$-2N/(4\pi)^2$
	-	-	0	4/3	0	2/3	-2/3	0	$-2N$
	-	2	0	1/2	1	1/2	0	0	$2N/(4\pi)$
	-	-1	0	1	0	1	-1	0	$-2N/(4\pi)$
	2	0	0	0	2	1	0	0	$-2N/(4\pi)^2$
	1	1	0	0	0	1	0	0	$-2N/(4\pi)^2$
	-1/2	-1	0	0	0	2	-2	0	$-2N/(4\pi)^2$

this can be written as





The results for the three vacuum diagrams are given in

TABLE III. (*Continued*).

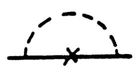
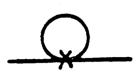
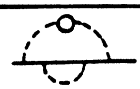
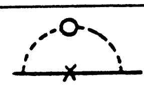
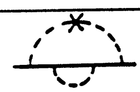
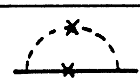
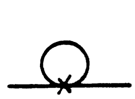
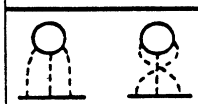
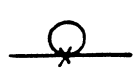
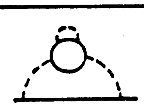
	-	-	0	0	0	2/3	-2	4/3	-N
	-	2	0	0	0	1	-2	0	N/(4π)
	0	-1	0	0	0	2	-2	0	$-N/(4\pi)^2$
	1	1	0	0	0	1	-2	0	$-N/(4\pi)^2$
	1/2	1	0	0	0	2	-2	0	$-N/(4\pi)^2$
	-	-	0	0	0	1	-5	4	-N
	-	-1	0	0	0	1	-5	4	-N/(4π)
	-	2	0	0	0	1	-2	0	N/(4π)
	-2	0	0	0	0	1	-2	0	$N/(4\pi)^2$
	1/2	1	0	0	0	2	-2	0	$-N/(4\pi)^2$
	-	-	0	0	0	0	2/3	2/3	-N
	0	-1	0	0	0	2	-2	0	$-N/(4\pi)^2$
	IT DOES NOT CONTRIBUTE								

Table IV. We note that these diagrams have (mass) dimension 2, expecting them to be proportional to m^2.

Finally the contribution of the σ propagator to the desired 4-point function is

$$i[g^8/(4\pi)^3](8N^2/3-10N/3)/\varepsilon\ . \tag{4.1}$$

V. 2-LOOP SKELETON DRESSED

Since we do not evaluate graphs independent of N, the only dressing we can have in these diagrams is a fermion loop in one of the three σ lines appearing in all possible ways. In Fig. 4 we show all these graphs which give

$$\text{Group I:}\quad -i[g^8/(4\pi)^3](8N)/\varepsilon\ , \tag{5.1a}$$

$$\text{Group II:}\quad -i[g^8/(4\pi)^3](-4N/3)/\varepsilon\ , \tag{5.1b}$$

$$\text{Group III:}\quad -i[g^8/(4\pi)^3](40N)/\varepsilon\ . \tag{5.1c}$$

TABLE IV. As in Table III for the 3-loop vacuum-vacuum diagrams used for the σ propagator corrections. A factor of ig^4m^2 is understood.

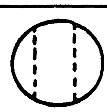
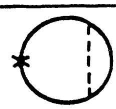
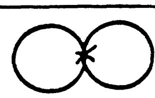
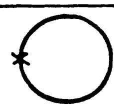
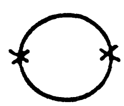
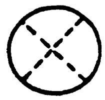
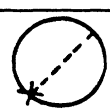
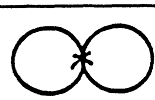
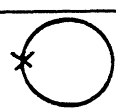
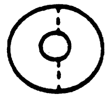
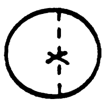
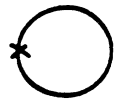
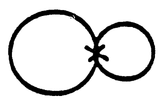
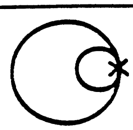
DIAGRAM	$1/\varepsilon^2$	$1/\varepsilon$	ε^0	ε	ε^2	FACTOR
	-	-	2	-6	4	$-N$
	-	-1	2	-6	4	$-2N/(4\pi)$
	-	-1/2	4	-8	4	$-N/(4\pi)$
	1	-1	2	-2	0	$-2N/(4\pi)^2$
	1	0	2	-6	4	$-N/(4\pi)^2$
	-	-	4/3	0	8/3	$-N$
	-	-1	2	-2	0	$-4N/(4\pi)$
	-	1/2	4	-8	4	$-2N/(4\pi)$
	0	-1/2	2	-2	0	$-4N/(4\pi)^2$
	1/2	1	2	-2	0	$-4N/(4\pi)^2$

TABLE IV. (*Continued*).

	-	-	8/3	-8/3	16/3	N^2
	-	2	2	-2	0	$-2N^2/(4\pi)$
	0	-1	2	-2	0	$4N^2/(4\pi)^2$
	1	1	2	-2	0	$4N^2/(4\pi)^2$
	-	-1/2	0	0	0	$N^2/(4\pi)$
	-	1/2	0	0	0	$N^2/(4\pi)$

VI. THE Z_4, Z_2, AND Z_λ RENORMALIZATION CONSTANTS, THE β FUNCTION

Using Eqs. (3.2), (4.1), and (5.1) we get the complete $1/\varepsilon$ term of the diagrams contributing to Z_4:

$$i[g^8/(4\pi)^3](16N^2/3-58N)/\varepsilon \ . \tag{6.1}$$

Therefore the 3-loop order of Z_4 is

$$[g^6/(4\pi)^3](16N^2/3-58N)/\varepsilon \ . \tag{6.2}$$

From Eq. (3.1) we can find the corresponding term for Z_2:

$$[g^6/(4\pi)^3](8N^2/3+6N)/\varepsilon \ . \tag{6.3}$$

Finally from Eq. (1.4) we get Z_λ:

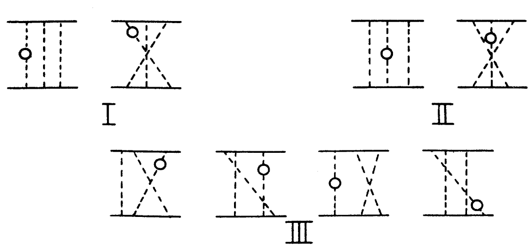


FIG. 4. The 2-loop skeleton diagrams dressed with a fermion loop.

$$[\lambda^3/(4\pi)^3](-70N)/\varepsilon \tag{6.4}$$

where we have switched back to the λ coupling. Amazingly enough, the N^2 dependence is canceled and the only nonzero coefficient of Eq. (2.2) is β_{20}. This way, Z_λ is fully written as

$$-[\lambda^3/(4\pi)^3]70(N-1)/\varepsilon \ . \tag{6.5}$$

Using Eq. (1.5) we evaluate β_2, which turns out to be

$$\beta_2=-[1/(4\pi)^3]420(N-1) \ , \tag{6.6}$$

and the β function up to this order now reads

$$\beta(\lambda)=(N-1)\left[-\frac{4\lambda^2}{4\pi}+\frac{8\lambda^3}{(4\pi)^2}-\frac{420\lambda^4}{(4\pi)^3}\right] . \tag{6.7}$$

VII. CONCLUSIONS AND REMARKS

The interesting and unexpected point of Eq. (6.7) is the lack of the N^2-proportional term in the third-loop contribution. One may argue that this contribution, as well as the higher ones, is renormalization-scheme dependent. Careful examination of this point reveals that the scheme dependence of the β_2 term is proportional to $(N-1)$, which shows that the N^2-proportional term is scheme independent, provided that the coupling rescaling is N independent. Therefore, the whole 3-loop contribution to the β function can be eliminated by a suitable rescaling of the coupling, which is N *independent,* and therefore *unique for all N.*

The next question is now obvious: Does the lack of the scheme-independent terms persist in higher orders too? In this case all coefficients β_n, $n>1$ could be eliminated in an N-independent way, leaving only the first two terms. We believe that this fact, if present, could shed light on the understanding of the mass-generation mechanism of the model.

We note also that even in the next term of the β function (4 loop), the scheme dependence is again proportional to $N-1$. The speculation above requires the vanishing of both terms proportional to N^3 and N^2. This subject is under investigation presently.

ACKNOWLEDGMENTS

We would like to thank J. Iliopoulos for stimulating discussions.

APPENDIX

We give the infinite part for a number of integrals needed in the calculations.

Integral 1:

$$\int \frac{d^np\, d^nk\, d^nl\, A(p,k,l,m)}{(p^2-m^2)^a(k^2-m^2)(l^2-m^2)[(p+k)^2-m^2][(p+l)^2-m^2]} = A(p,k,l,m)/(a)$$

$m^2(pk)(pl)/(2)\ldots 0$

$m^2(pk)(kl)/(2)\ldots -I(m^2\zeta_1+m^4\zeta_2)/4+$finite

$m^2(kp)^2/(2)\ldots I(m^2\zeta_1+m^4\zeta_2)/2+$finite

$(lk)^2/(1)\ldots [Im^2\zeta_1+(7n-6)I^3/12]/(n-1)+$finite

$(pk)(pl)/(1)\ldots I^3/3$

$(pk)/(1)\ldots$ finite

$(kl)/(1)\ldots$ finite

$(kl)(pk)/(1)\ldots -5I^3/12-Im^2\zeta_1/4+$finite

where ζ_a, $a=1,2$, are given by

$$\int \frac{d^nk\, d^np}{(k^2-m^2)[(k+p)^2-m^2](p^2-m^2)^a}, \quad a=1,2\ .$$

Integral 2:

$$\int \frac{d^np\, d^nk\, d^nl\, A(p,k,l,m)}{(p^2-m^2)(k^2-m^2)(l^2-m^2)[(p+k+l)^2-m^2]^a} = A(p,k,l,m)/(a)$$

$1/(1)\ldots$ finite

$k^2/(1)\ldots I^3+$finite

$(kl)/(1)\ldots -I^3/3+$finite

$p^2(kp)/(2)\ldots 3(1-n/2)I^3/2+$finite

$l^2(kp)/(2)\ldots (1-n/2)I^3/2+$finite

$k^4/(2)\ldots (2n-3)I^3+$finite

$p^2l^2/(2)\ldots -2(1-n/2)I^3+$finite

$(kl)^2/(2)\ldots -(1-n/2)I^3/3+$finite

$(kp)(kl)/(2)\ldots -(1-n/2)I^3/3+$finite

$m^2(kl)^2/(3)\ldots (1-n/2)(1-n/8)I^3+$finite

$m^2(kp)(kl)/(3)\ldots -(1-n/2)(n-3)I^3/8+$finite

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