THREE LOOP CALCULATION OF THE β-FUNCTION FOR THE GROSS–NEVEU MODEL

N.D. TRACAS
Physics Department, National Technical University, GR-157 73 Zografou, Athens, Greece

and

N.D. VLACHOS
Department of Theoretical Physics, University of Thessaloniki, GR-540 06 Thessaloniki, Greece

Received 23 November 1989

We calculate the β-function for the Gross–Neveu model up to three loop order. The result found is $\beta(\lambda)=(N-1)[-4\lambda^2/4\pi+8\lambda^3/(4\pi)^2-276\lambda^4/(4\pi)^3]$ showing an unexpected lack of a term proportional to N^2 in the last term. This suggests the existence of an N-independent rescaling of the coupling that eliminates all but the first two terms of the β-function.

The Gross–Neveu model [1] belongs to the class of two-dimensional model field theories which exhibit many interesting properties such as asymptotic freedom, non-perturbative mass generation as a result of spontaneous breaking of the discrete chiral symmetry as well as a rich bound state spectrum. The model is classically integrable [2] while the quantum S-matrix for the different sectors of the theory has been known for some time [3].

The lagrangian of the model is

$$L=\bar{\psi}(\mathrm{i}\not{\partial})\psi+\tfrac{1}{2}\lambda(\bar{\psi}\psi)^2, \tag{1}$$

where a summation over the flavour index of the fermionic field ψ^i, $i=1, ..., N$, is understood. When $N=1$ the two-dimensional Fierz identities ensure that the lagrangian above is equivalent to the lagrangian corresponding to the massless Thirring model, which is known to be a finite field theory (i.e. the β-function vanishes).

In this letter we calculate the β-function up to order λ^4, one order higher than existing in the literature [4].

The quantity to be calculated is the three-loop coupling renormalization constant Z_λ where $\lambda_{\mathrm{B}}=Z_\lambda\lambda_{\mathrm{R}}$. Now, Z_λ can be given in terms of the 2- and 4-point function renormalization constants as

$$Z_\lambda=Z_4Z_2^{-2}, \tag{2}$$

where $G^{\mathrm{R}}_{2(4)}=Z_{2(4)}G^{\mathrm{B}}_{2(4)}$.

In the minimal subtraction (MS) scheme, the β-function is only associated with the residues (i.e. the coefficients of $1/\epsilon$) of the renormalization constants [5].

The structure of the contributing Feynman graphs is best understood in the σ-field formulation where the lagrangian is written as [1]

$$L_\sigma=\bar{\psi}(\mathrm{i}\not{\partial})\psi+\tfrac{1}{2}\sigma^2-g\bar{\psi}\psi\sigma,\quad g^2=\lambda, \tag{3}$$

where σ is a non-propagating scalar field. This lagrangian gives identical fermion Green functions as the original one. The Feynman rules are shown in fig. 1.

Writing the β-function in the usual form

$$\beta(\lambda)=\beta_0\lambda^2+\beta_1\lambda^3+\beta_2\lambda^4, \tag{4}$$

we expect β_0, β_1 and β_2 to be polynomials in N, vanishing for $N=1$. Indeed it is known that [4]

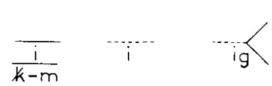

Fig. 1. Feynman rules in the σ formulation.

0370-2693/90/$ 03.50 © Elsevier Science Publishers B.V. (North-Holland)

$$\beta_0 = -4\lambda^2(N-1)/4\pi\,,$$
$$\beta_1 = 8\lambda^3(N-1)/(4\pi)^2. \quad (5)$$

Note that factors of N are associated with the presence of fermionic loops. We therefore expect that β_2 has the form

$$\beta_2 = (N-1)(AN^2+CN+D)$$
$$= AN^3+(C-A)N^2+(D-C)N-D, \quad (6)$$

where A, C and D are constants to be calculated.

As previously stated, A should be associated with three loop graphs containing three fermion loops. In fact there is only one such a graph depicted in fig. 2. It is easy to see, however, that this graph does not require a genuine three loop counterterm (it is only associated with subdivergences) giving therefore no contribution to the β-function at this order. Eq. (6) now simplifies to

$$\beta_2 = CN^2+(D-C)N-D. \quad (7)$$

The physical meaning of this expression is obvious. The constant C is related to graphs containing two fermion loops, $D-C$ to graphs containing one fermion loop and D to graphs containing no fermion loop. This way, the value of D could be deduced from graphs containing two and one fermion loop respectively. It is amusing to realize that this rather trivial observation can be used to prove inductively that any new four-fermion operator appearing in perturbation theory will not require new counterterms.

We now turn on to the actual calculations. The quantities to be computed are the N-dependent infinite parts of the 2- and 4-point functions. The contributing diagrams are shown in fig. 3, where all propagators and vertices are dressed up to the appropriate order. Since the 4-point function is logarithmically divergent, we would not expect any dependence of the infinite part upon dimensional quantities, such as external momenta or mass parameters. In order to keep computational complications down to a minimum, we choose to calculate the 4-point function at zero external momenta and add a fermion mass parameter to be used as an infrared regulator. Mass renormalization effects must now be taken into account, however, the final result does not depend on this particular choice as it should.

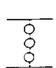

Fig. 2. The only three loop graph proportional to N^3.

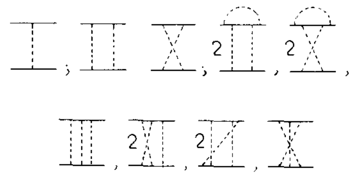

Fig. 3. Skeleton graphs. All propagators and vertices are dressed to the appropriate order.

All calculations have been performed using dimensional regularization and the MS scheme. The general structure of the infinite parts of the relevant diagrams and the corresponding one and two loop counterterms are

three loop graph: $(a+b\epsilon+c\epsilon^2)I^3$, (8a)

two loop counterterm:

$$(A_2/\epsilon^2+B_2/\epsilon)(a_2+b_2\epsilon+c_2\epsilon^2)I, \quad (8b)$$

one loop counterterm:

$$(A_1/\epsilon)(a_1+b_1\epsilon+c_1\epsilon^2)I^2, \quad (8c)$$

where $\epsilon = 1-\frac{1}{2}n$ and $I=\int d^n k/(k^2-m^2)$. Renormalizability of the theory requires the following relations to hold true:

$$3a+a_1A_1=0, \quad a_2A_2+a_1A_1=0,$$
$$3b+a_2B_2+b_2A_2+2b_1A_1=0. \quad (9)$$

These relations also ensure that some unwanted terms, such as $\ln 4\pi$, γ and $\psi(0)$, cancel out between graphs and countergraphs.

The results we finally get are as follows (only the N-dependent residues are shown):

$$Z_2 = 1-\{[\lambda^2/(4\pi)^2]N$$
$$+[\lambda^3/(4\pi)^3](-\tfrac{8}{3}N^2-6N)\}/\epsilon, \quad (10)$$

$$Z_4 = 1-\{[\lambda/(4\pi)]2N$$
$$+[\lambda^3/(4\pi)^3](-\tfrac{16}{3}N^2+58N)]\}/\epsilon. \quad (11)$$

Now eq. (2) can be used to write

$$Z_\lambda = 1 - \{[\lambda/(4\pi)]2N - [\lambda^2/(4\pi)^2]2N + [\lambda^3/(4\pi)^3]70N\}/\epsilon\,, \quad (12)$$

where again only the N-dependent residues are shown.

Finally, the β-function can be computed in terms of the formula

$$\beta(\lambda) = 2\lambda^2 (\mathrm{d}/\mathrm{d}\lambda)[\mathrm{Res}\, Z_\lambda] = 2\lambda^2(\mathrm{d}/\mathrm{d}\lambda)[\mathrm{Res}(Z_4 - 2Z_2)]\,, \quad (13)$$

so that β_2 turns out to be

$$\beta_2 = -[\lambda^4/(4\pi)^3]420(N-1)\,, \quad (14)$$

and the β-function up to this order now reads

$$\beta(\lambda) = (N-1) \times[-4\lambda^2/4\pi + 8\lambda^3/(4\pi)^2 - 420\lambda^4/(4\pi)^3]. \quad (15)$$

The lack of a term proportional to N^2 is interesting and unexpected. It is well known that β_2 is renormalization scheme dependent [5]; upon an analytic rescaling of the coupling, β_2 is modified by a term proportional to $\beta_0(N-1)$. The results found indicate that the whole three loop contribution to the β-function can be eliminated by a suitably chosen rescaling of the coupling which is N-independent and therefore unique for all N.

It would be interesting to speculate on the possibility of eliminating all the coefficients β_n, $n>2$, in an N-independent way. The resulting β-function would then have only the first two terms, a fact that could facilitate the understanding of the mass generation mechanism of the theory. Work towards this direction is in progress.

We acknowledge stimulating discussions with E. Floratos and D. Ross. We would also like to thank the CERN Theory Division, where part of the work was carried out, for hospitality and financial support.

References

[1] D.J. Gross and A. Neveu, Phys. Rev. D 10 (1974) 3235;
R. Dashen, B. Hasslacher and A. Neveu, Phys. Rev. D 12 (1975) 2433;
E.G. Floratos, Phys. Lett. B 154 (1985) 173;
N.D. Vlachos and E.G. Floratos, Phys. Lett. B 189 (1987) 137.

[2] A. Neveu and N. Papanicolaou, Commun. Math. Phys. 58 (1978) 31.

[3] A.B. Zamolodchikov and A.B. Zamolodchikov, Ann. Phys. 120 (1979) 253; Nucl. Phys. B 133 (1978) 525.

[4] W. Wetzel, Phys. Lett. B 153 (1985) 297.

[5] See e.g. D.J. Gross, Applications of the renormalization group to high energy physics, in: Methods in field theory, Les Houches 1975, eds. R. Balian and J. Zinn-Justin (North-Holland, Amsterdam).