Modern Physics Letters A Vol. 4, No. 25 (1989) 2429–2435

U(1)'s AND NON-PERTURBATIVE UNIFICATION

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Received 11 August 1989

Revised 25 September 1989

Motivated by the plethora of models, mainly remanant from superstring theories, in which several U(1) factors are present, we consider constraints on these theories from the requirement that non-perturbative, as opposed to perturbative, unification arises close to the Planck scale. We find that non-perturbative unification can be realized with three standard families up to a supersymmetry breaking scale of order 100–500 TeV and six supersymmetric families above that scale.

During the past, there have been two approaches for explaining the low-energy values of the gauge interaction coupling constants of the standard model.

The first well-known approach is perturbative unification,[1] i.e. the assumption that all interactions remain perturbative up to a scale $M_{\rm GUT}$ more or less close to the Planck scale M_P, where they are unified in one way or another. In that case, the values turn out to depend very critically on the value $a_i(M_{\rm GUT}) = a_{\rm GUT}$, a feature which might appear unpleasant if we give a physical meaning to the large scale $M_{\rm GUT}$.

The second alternative approach is the non-perturbative unification,[2] based on the observation[3] that the low-energy coupling constant $a(M_W)$ of an asymptotically divergent interaction becomes more and more insensitive to its value $a(\Lambda)$ at a bigger scale Λ, as Λ gets larger and larger compared to M_W. Λ is expected to be close to M_P, as gravity is supposed to cure the ultraviolet divergent behavior. In that case, the interactions are strong and of comparable strength ($a_i(\Lambda) \approx 0(1)$ at $\Lambda \leq 0(M_P)$). Their low-energy values $a_i(M_W)$ are then essentially determined by the value of Λ only, through renormalization group methods.

The second scenario cannot be easily realized in a non-supersymmetric theory, since there must exist many new states to render the gauge interactions of the standard model asymptotically divergent. Things work better with supersymmetry, where non-perturbative unification can be implemented in a $N = 1$ supersymmetric extension of the standard model with five generations.[4]

On the other hand, string unification is at present the leading candidate for a truly unified theory of all particle interactions.[5] Working in the heterotic string type framework, on which most semi-realistic models are based, below the compactification scale M_C we have an effective four dimensional theory with gauge and gravitational couplings related at tree level by

$$\alpha = \frac{g^2}{4\pi} = 8\, G_N M_S^2 = 8\left(\frac{M_S}{M_P}\right)^2 \tag{1}$$

where $M_S \approx (a')^{-1/2}$ is the string mass scale. Natural and aesthetic arguments then suggest that $M_C \approx M_S \leq M_P$.[6] So, string unification offers the possibility that the standard model gauge couplings become strong and unify with one another (and with gravity) at a single scale M_X close to $M_C \approx M_S \approx 0.1\, M_P$. Predictions of the low-energy parameters of the standard model are then made by solving the two-loop renormalization group equations. Note that computation of string threshold effects,[7] by integrating out the heavy string degrees of freedom, shows that M_X is expected to be

$$M_X \approx g \times 5.3 \times 10^{17}\,\text{GeV}\,. \tag{2}$$

It seems thus that string and non-perturbative unification coexist naturally.

The effective gauge group obtained from compactification of the heterotic type string contains, almost always, many U(1) factors.[5] So, it appears that in the compactification scale the four-dimensional gauge interactions are of the form $SU(3)\times SU(2)\times U(1)^N$, which would correspond to the gauge symmetry of, a superstring vacuum (similar groups with many U(1) factors are also obtained in the four-dimensional formulation of superstrings[8]). In order to make contact with experiment, the coupling constants have to be renormalized from their values at a scale close to the compactification one, down to the weak scale, where the observed gauge group is one of the standard model $SU(3) \times SU(2) \times U(1)_Y$. Gauge coupling renormalization with several U(1) factors have been examined by considering the mixing of the U(1) gauge bosons in the evolution of gauge couplings.[9] In the present work we prefer to describe the mixing by parametrizing the $U(1)_Y$ hypercharge generator normalization constant with $C = (\Sigma c_i^2)^{-1/2}$ obtained through the combination $Y = \Sigma c_i U_i$ of the various U(1)'s.[10]

The purpose of the present work is to discuss the non-perturbative unification scenario in the presence of the parameter C, which reflects the dependence of the $U(1)_Y$ out of many U(1) factors. Our assumptions on the mass scales involved, between M_W and M_C, are the following: above M_W is M_I, an average, approximate scale of supersymmetry breaking, above which the supersymmetric partners contribute to the running coupling constants; and below M_C is M_X, the scale at which the gauge couplings become strong and where the gauge symmetry of the superstring vacuum $SU(3)\times SU(2)\times U(1)^N$ breaks to $SU(3)\times SU(2)\times U(1)_Y$ (we do not complicate the discussion by considering other intermediate mass scales, as these seem not to be favored in most cases[11]). We are going to discuss, in a rather general way, constraints concerning the possible values of C, M_I and M_X for the realization of the non-perturbative unification scenario, so that acceptable values of the low energy parameters are obtained.

Let us now turn to the calculation. The evolution of the gauge coupling constants at two-loops is governed by the renormalization group equations

$$\frac{d}{d\ln E}\alpha_i = \frac{1}{2\pi} b_i \alpha_i^2 + \frac{1}{(8\pi^2)} \sum_j b_{ij} \alpha_i^2 \alpha_j \tag{3}$$

where $i = 1, 2, 3$ stands for the $U(1)_Y$, $SU(2)$ and $SU(3)$ gauge couplings, respectively. We have neglected Yukawa coupling contribution to the above equation since we restrict ourselves to three standard generations between M_W and the next scale M_I. Then, between M_W and M_I, the coefficients of the renormalization group are given by[12]

$$\begin{aligned} b_3 &= -11 + \frac{4}{3} n \\ b_2 &= -\frac{22}{3} + \frac{4}{3} n + \frac{1}{2} n_H \\ b_1 &= C^2 \left(\frac{20}{9} n + \frac{1}{2} n_H \right) \end{aligned} \tag{4}$$

for the one-loop, and

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix} + \begin{pmatrix} \frac{95}{27} C^4 & C^2 & \frac{44}{9} C^2 \\ \frac{1}{3} C^2 & \frac{49}{3} & 4 \\ \frac{11}{18} C^2 & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n + \begin{pmatrix} \frac{3}{4} C^4 & \frac{9}{4} C^2 & 0 \\ \frac{3}{4} C^2 & \frac{25}{4} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H \tag{5}$$

for the two-loops, where n are the number of generations, which we put equal to 3, and n_H the number of Higgs multiplets, which we put equal to 2, given that the least number of Higgs supermultiplets above M_I, which is 2 is also fully active below M_I. Between M_I and M_X, we have $N = 1$ supersymmetry with n' generations and n_H' Higgs supermultiplets. The coefficients of the renormalization group are now given by

$$b_3 = -9 + 2n'$$

$$b_2 = -6 + 2n' + \frac{1}{2}n_H' \qquad (6)$$

$$b_1 = C^2\left(\frac{10}{3}n' + \frac{1}{2}n_H'\right)$$

for the one-loop, and

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + \begin{pmatrix} \frac{190}{27}C^4 & 2C^2 & \frac{88}{9}C^2 \\ \frac{2}{3}C^2 & 14 & 8 \\ \frac{11}{9}C^2 & 3 & \frac{68}{3} \end{pmatrix} n'$$

$$+ \begin{pmatrix} \frac{1}{2}C^4 & \frac{3}{2}C^2 & 0 \\ \frac{1}{2}C^2 & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H' \qquad (7)$$

for the two-loop.

The integration of the above Eq. (3) is performed numerically, starting from M_X and running the coupling constants to lower energies. The known value of $\alpha_{em}(M_W) = 1/128$ is an input in the computation. According to the non-perturbative unification philosophy, the values of $\alpha_i(M_W)$ are insensitive to their values at M_X. To be definite, we report our results for the initial values $\alpha_i(M_X) = g_i^2/4\pi = 1$. We have explicitly checked the lack of sensitivity of our results in a change of this initial value. The relevant parameters are C, M_I, M_X (and n', n_H') and we keep solutions which correspond to acceptable values of $\sin^2\theta_W(M_W)$ and $\alpha_3(M_W)$ [13]

Table

M_X	M_I	C	α_3	$\sin^2\theta_W$
0.1×10^{17}	1.80×10^5	0.75	0.120	0.235
	2.00×10^5	0.65	0.122	0.234
	2.80×10^5	0.52	0.132	0.230
	3.50×10^5	0.47	0.139	0.227
0.2×10^{17}	3.00×10^5	1.42	0.127	0.235
	3.40×10^5	1.00	0.130	0.234
	3.60×10^5	0.88	0.132	0.233
	3.85×10^5	0.80	0.134	0.232
0.3×10^{17}	4.80×10^5	1.70	0.137	0.233
	4.90×10^5	1.60	0.138	0.233
	5.00×10^5	1.40	0.138	0.233
	5.20×10^5	1.20	0.139	0.232

$$0.2226 \le \sin^2\theta_W(M_W) \le 0.2353$$

$$0.106 \le \alpha_3(M_W) < 0.138\,. \tag{8}$$

Our results are presented in the Table and Figs. 1 to 4. The most important feature is that, with three generations of fermions in low energies, there must be six generations above supersymmetry breaking scale M_I, for the scenario of the non-perturbative unification to be realized. The number of Higgs multiplets is two, both below as well as above M_I, which is exactly the minimum number of Higgs needed for supersymmetry breaking. The unification scale M_X is in the range of $(.1 - .3)\times10^{17}$ GeV, while the supersymmetry breaking scale M_I lies in the interval $(1-5)\times10^5$ GeV, its accurate value depending on the parameter C and M_X. This value of M_I is not away from the range relevant to the gauge hierarchy problem. In our numerical calculations we are varying C between 0.5 and 1.6, since we expect it to be around 1. Note that higher unification scales M_X would be possible for larger values of C. In Figs. 1 and 2, $\sin^2\theta_W(M_W)$ and $\alpha_3(M_W)$ are sketched as functions of M_I, for various M_X. For a given M_X, M_I is constrained by the experimentally accepted values of $\alpha_3(M_W)$ and $\sin^2\theta_W(M_W)$. In Figs. 3 and 4, $\sin^2\theta_W(M_W)$ and $\alpha_3(M_W)$ are sketched as functions of C, with similar constraints.

In conclusion, we find it encouraging that the three standard generations, together with a double number of supersymmetric generations and the minimum number of Higgses, suffice to implement the non-perturbative unification scenario, with very reasonable mass scales.

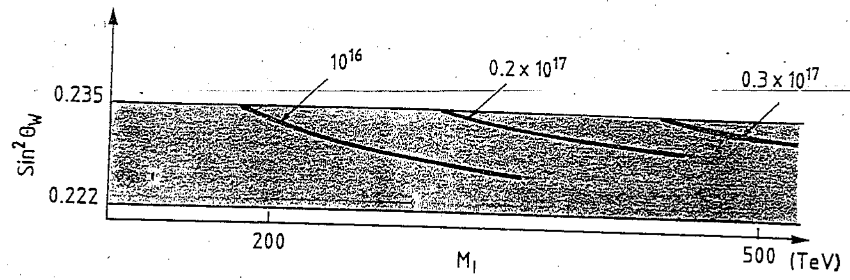

Fig. 1. Plot of $\sin^2\theta_W(M_W)$ as a function of M_I, for $M_X = (0.1, 0.2, 0.3) \times 10^{17}$ GeV. The curves are bounded from below by the experimentally accepted value of $\alpha_3(M_W)$.

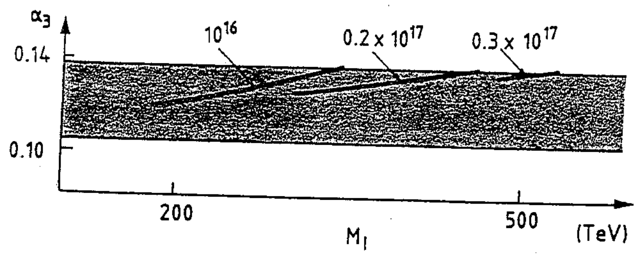

Fig. 2. Same as in Fig. 1 for $\alpha_3(M_W)$ vs M_I. The lower bound now come from $\sin^2\theta_W(M_W)$.

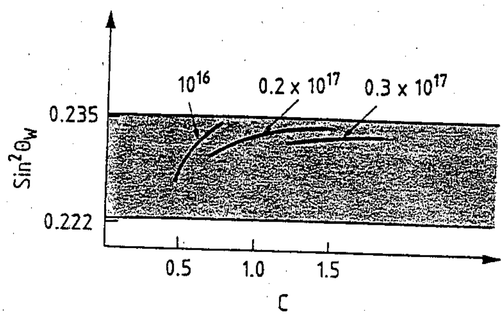

Fig. 3. Same as in Fig. 1 for $\sin^2\theta_W(M_W)$ vs C.

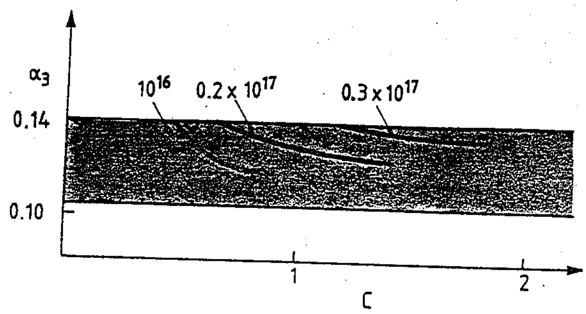

Fig. 4. Same as in Fig. 2 for $\alpha_3(M_W)$ vs C.

Acknowledgments

The authors wish to thank the CERN Theory Division and the Greek Ministry of Research and Technology for support.

References

1. H. Georgi, H. R. Quinn and S. Weinberg, *Phys. Rev. Lett.* **33** (1974) 451.
2. L. Maiani, G. Parisi and R. Petronzio, *Nucl. Phys.* **B136** (1978) 115.
3. L. D. Landau, A. A. Abrikosov and J. M. Khalatnikov, *Dokl. Acad. Nauk* **95** (1954) 773 and 1177; **96** (1954) 261; L. D. Landau, in *Niels Bohr and the Development of Physics*, ed. W. Pauli, (Pergamon Press, 1955); K. Wilson, *Phys. Rev.* **D3** (1971) 1818; G. Parisi, *Phys. Rev.* **D11** (1975) 909.
4. N. Cabibbo and G. Farrar, *Phys. Lett.* **110B** (1982) 107; L. Maiani and R. Petronzio, *Phys. Lett.* **176B** (1986) 120; Erratum **170B** (1986) 457; S. Theisen, N. D. Tracas and G. Zoupanos, *Z. Phys.* **C37** (1988) 597; J.-P. Derendinger, R. Kaiser and M. Roncadelli, *Phys. Lett.* **220B** (1989) 164.
5. M. B. Green, J. Schwarz and E. Witten, *Superstring Theory*, Vols. I, II (Cambridge University Press, 1986).
6. M. Dine and N. Seiberg, *Phys. Rev. Lett.* **55** (1985) 366; V. Kaplunovsky, *ibid.* 1033; R. Petronzio and G. Veneziano, *Mod. Phys. Lett.* **A2** (1987) 707.
7. V. Kaplunovsky, *Nucl. Phys.* **B307** (1988) 145.
8. I. Antoniadis and G. K. Leontaris, *Phys. Lett.* **216B** (1989) 333 and references therein.
9. F. del Aguila, J. A. Gonzalez and M. Quiros, *Nucl. Phys.* **B307** (1988) 571; *Phys. Lett.* **201B** (1988) 315; F. del Aguila, G. D. Coughlan and M. Quiros, *Nucl. Phys.* **B307** (1988) 633; Erratum, *ibid.* **B312** (1989) 751; N. Nakamura, I. Umemura and K. Yamamoto, *Phys. Lett.* **212B** (1988) 198.
10. J. A. Casas and C. Munoz, *Phys. Lett.* **214B** (1988) 543.
11. J. Ellis, K. Enqvist, D. V. Nanopoulos and K. A. Olive, CERN preprint, CERN-TH-5315 (1989) and references therein.
12. D. R. T. Jones, *Nucl. Phys.* **B87** (1975) 127; *Phys. Rev.* **D25** (1982) 581.
13. G. Altarelli, in *Proc. of the HEP-EPS Conf.*, Uppsala 1987, ed. O. Botner (Uppsala University, 1987), p. 1002.