Z. Phys. C – Particles and Fields 37, 597–600 (1988)

Zeitschrift für Physik C Particles and Fields

Unification of coupling constants without a covering GUT

S. Theisen[1], D. Tracas[2,a], G. Zoupanos[1,a,b]

[1] Max-Planck-Institut für Physik und Astrophysik, D-8000 München, Federal Republic of Germany
[2] CERN, CH-1211 Geneva 23, Switzerland

Received 5 October 1987

Abstract. We discuss the possibility of unification of gauge coupling constants of the standard model and its extensions at some high scale without the assumption of the existence of a covering GUT at that scale. In our analysis we examine (i) the standard model, (ii) its supersymmetric extension and (iii) an extension in which the spontaneous symmetry breaking is based on the condensation of high-colour fermions. The latter case is favoured for perturbative coupling constants unification while the supersymmetric extension of the standard model, with five families, is favoured for non-perturbative unification.

Introduction

Attempts to unify the observed low energy interactions at some higher scale have attracted for many years a lot of interest, both theoretical [1] and experimental [2], since it fulfills an esthetic dream of many physicists. In addition, the possibility to predict [3] some of the standard model free parameters has made these attempts even more challenging. Most of the effort has been within the perturbative framework of grand unified theories which predict that the proton is unstable. However, the negative results from proton decay experiments [2] seem to rule out at least the simplest Georgi-Glashow $SU(5)$-model.

Another interesting framework of unification which might or might not require a covering GUT, is the non-perturbative one of Maiani et al. [4]. It requires that the theory is not asymptotically free. Then, at low energies, the values of the coupling constants are insensitive to those at large energy scales.

Finally, a presently popular framework is that the unification might occur in higher dimensions. Superstring theories [5] or higher-dimensional gauge theories [6, 7] offer the possibility to start with a unified group in higher than four dimensions, while the dimensional reduction procedure could lead in four dimensions to gauge theories without a covering GUT.

The negative results from proton decay experiments, the theoretical prospect that a unification involving a covering GUT might take place in higher than four dimensions, as well as the recent formulation of superstrings in four dimensions [8] which does not necessarily require a covering GUT, lead to the question: which theories can have coupling constant unification in four dimensions? In this work we investigate three representative types of theories requiring only coupling constant unification at some high scale. The theories are (i) the standard model with three or four families, (ii) a simple supersymmetric extension of the standard model and (iii) a simple extension of the standard model with dynamical symmetry breaking of the electroweak sector according to the high colour scenario.

In our analysis we write down the renormalization group equations (RGE's) [4, 9] corresponding to each theory, and as an input we require to have [4, 10]:

$$\begin{aligned} \alpha_{em}(M_W) &= 0.00772 \\ \alpha_3(M_W) &= 0.12^{+0.01}_{-0.02} \\ \sin^2\theta_W(M_W) &= 0.230 \pm 0.005. \end{aligned} \tag{1}$$

We then examine numerically whether the three gauge couplings g_3, g_2, g_1 of $SU(3)_C \times SU(2)_L \times U(1)$ can meet at some high scale. It is worth noting here that, since we do not assume the existence of a covering GUT, there is nothing to suggest that we should use the freedom of rescaling the $U(1)$ coupling constant.* This in turn means that at the coupling constant

[a] On leave from Department of Physics, National Technical University, Athens, Greece
[b] Alexander von Humboldt Fellow

* Recall that for instance in the $SU(5)$ GUT we rescale $g_1 : g_{1_{SU(5)}} = \sqrt{\frac{3}{5}} g_{1_{W-S}}$

unification scale the natural value of $\sin^2\theta_W = g_1^2/(g_1^2+g_2^2)$ is 0.5. Then our criterion for a successful coupling constant unification is just the existence of such a scale below M_{Planck}, in order that the renormalizable theory we consider, which does not include gravity, makes sense.

Standard model

The evolution of the gauge couplings of the $SU(3)_C \times SU(2)_L \times U(1)$ standard model is governed by the following two-loop RGE's($\alpha_i = g_i^2/4\pi$):

$$\frac{d\alpha_i}{d\ln E} = \frac{\alpha_i^2}{2\pi} B_i + \sum_k \frac{\alpha_i^2 \alpha_k}{8\pi^2} B_{ik} \tag{2}$$

where

$$\begin{aligned} B_1 &= \tfrac{20}{9} n_G + \tfrac{1}{6} n_H \\ B_2 &= \tfrac{4}{3} n_G + \tfrac{1}{6} n_H - \tfrac{22}{3} \\ B_3 &= \tfrac{4}{3} n_G - 11 \end{aligned} \tag{3a}$$

and

$$B_{ik} = \begin{pmatrix} \frac{95}{27} & 1 & \frac{44}{9} \\ \frac{1}{3} & \frac{49}{3} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_G + \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}. \tag{3b}$$

Studying the evolution of the three coupling constants for three generations ($n_G = 3$) and one Higgs scalar ($n_H = 1$), we find that unification cannot be achieved at any high scale. (Extra generations or Higgs doublets do not improve the situation.) Thus, the standard model fails to give simple coupling constant unification.

Supersymmetric standard model

To the extent that the standard electroweak model $SU(2)_L \times U(1)$ contains elementary Higgs fields, one needs a mechanism to stabilize the low against the high unification scale. A popular way out is to introduce $N = 1$ supersymmetry [11] which has to break at some scale M_S, higher than M_W, given that the low energy particle spectrum is not supersymmetric. For E less than the supersymmetric breaking scale M_S, the B_i's and B_{ik}'s of (2) are:

$$\begin{aligned} B_1 &= \tfrac{1}{2} n_H + \tfrac{20}{9} n_G \\ B_2 &= \tfrac{1}{2} n_H + \tfrac{4}{3} n_G - \tfrac{22}{3} \\ B_3 &= \tfrac{4}{3} n_G - 11 \end{aligned} \tag{4a}$$

and

$$B_{ik} = \begin{pmatrix} \frac{95}{27} & 1 & \frac{44}{9} \\ \frac{1}{3} & \frac{49}{3} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_G + \begin{pmatrix} \frac{3}{4} & \frac{9}{4} & 0 \\ \frac{3}{4} & \frac{25}{4} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}. \tag{4b}$$

For $E > M_S$ the corresponding quantities are:

$$\begin{aligned} B_1 &= \tfrac{10}{3} n_G + \tfrac{1}{2} n_H \\ B_2 &= 2 n_G + \tfrac{1}{2} n_H - 6 \\ B_3 &= 2 n_G - 9 \end{aligned} \tag{5a}$$

and

$$B_{ik} = \begin{pmatrix} \frac{190}{27} & 2 & \frac{88}{9} \\ \frac{2}{3} & 14 & 8 \\ \frac{11}{9} & 3 & \frac{68}{3} \end{pmatrix} n_G + \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix}. \tag{5b}$$

By examining the R.G.E.'s with $n_G = 3$ and $n_H = 2$ (which is the minimum allowed number of Higgs doublets in the supersymmetric standard model) we are unable to find an energy scale where all three couplings have the same value despite the fact that we have varied M_S between 10^3 and 10^6 GeV. For $n_G = 4$ we find the same result. For $n_G = 5$ we have reproduced the very interesting non-perturbative unification result of Maiani and Petronzio [4]. Specifically, starting from values

$$\alpha_{1,2,3}(\Lambda) \simeq 1; \quad \Lambda = 10^{17}\,\text{GeV} \tag{6}$$

we have found the results collected in Table 1:

The values of the low energy coupling constants for the choice $M_S = 10^4$ GeV agree very well with the experimental values as given in (1) above. The other results are given for comparison with the results of Maiani and Petronzio [4].

Therefore, as far as perturbative coupling constants unification is concerned the introduction of supersymmetry does not help to improve the situation of the standard model, if we limit ourselves to three or four fermion families. With five families we obtain the non-perturbative Parisi-Maiani-Petronzio scenario.

Table 1

	$M_S = 10^5$ GeV		$M_S = 10^4$ GeV	
	$E = 80$ GeV	$E = 250$ GeV	$E = 80$ GeV	$E = 250$ GeV
α_1	0.0103	0.0105	0.0100	0.0102
α_2	0.0350	0.0351	0.0330	0.0331
α_3	0.1781	0.1578	0.1313	0.1197
α_{em}^{-1}	125.9	123.5	129.7	127.3
$\sin^2\theta_W$	0.227	0.230	0.234	0.237

Simple extension of the standard model according to the high-colour scenario

Another popular way to avoid the hierarchy problem due to elementary Higgs fields is not to introduce them at all in the theory. Instead we might expect the breaking of $SU(2)_L \times U(1)$ to come from fermion bound states [12] carrying appropriate quantum numbers. Here we shall consider a simple extension of the standard model by introducing a family of colour sextet quarks having the same electroweak couplings as ordinary quarks [13, 14]. The chiral symmetry breaking of these quarks due to colour forces is expected to take place at much higher scale than for ordinary quarks and could in principle be responsible for the spontaneous breaking of $SU(2)_L \times U(1)$ down to $U(1)_{\mathrm{em}}$ [13, 14]. We remind that the high colour scenario found strong support in lattice calculations on chiral symmetry breaking of quarks which belong to different representations [15].

Beyond the threshold of twice the dynamical mass of the sextets, $\mu_6 \sim 250$ GeV, the new quarks contribute to the β-function and the asymptotic freedom of $SU(3)_C$ is lost. Therefore, α_3 starts increasing for $E > 2\mu_6$. Consistency of the theory requires that $\alpha_3(E)$ does not exceed the value $\alpha_3(\mu_6)$ at the chiral symmetry breaking scale [14]. Therefore, after a certain scale new physics has to appear in order for the picture to be consistent. The simplest thing to do is to enlarge $SU(3)_C$ to $G_S = SU(3) \times SU(3)$ at some scale [13, 16] before $\alpha_3(E)$ reaches the value $\alpha_3(\mu_6)$.

Let us then consider in the following a simple consistent extension of the standard model involving sextet quarks [13]. The model is based on the gauge group $G = G_S \times SU(2)_L \times U(1)$ where the strong sector is $G_S = SU(3) \times SU(3)$ and the electroweak is the standard one. The gauge group G_S breaks at a scale M_S to the diagonal $SU(3)_C$. This breaking can be dynamical, but in order to keep the picture as simple as possible let us introduce elementary Higgs fields transforming as $(3, \bar{3})$ under G_S and being singlets under $SU(2)_L \times U(1)$. The fermionic content of the theory before this breaking is (the quantum numbers refer to $(SU(3) \times SU(3))_S \times SU(2)_L \times U(1)$):

Quarks:

$$\left.\begin{matrix} (\bar{3}, \bar{3}, 2, +1/6)_L \\ (3, 3, 1, -2/3)_L \\ (3, 3, 1, +1/3)_L \end{matrix}\right\} \quad n_6 \text{ times}$$

$$\left.\begin{matrix} (3, 1, 2, +1/6)_L \\ (\bar{3}, 1, 1, -2/3)_L \\ (\bar{3}, 1, 1, +1/3)_L \end{matrix}\right\} \quad n_Q \text{ times}$$

$$\left.\begin{matrix} (1, 3, 2, +1/6)_L \\ (1, \bar{3}, 1, -2/3)_L \\ (1, \bar{3}, 1, +1/3)_L \end{matrix}\right\} \quad n_Q \text{ times}$$

Leptons:

$$\left.\begin{matrix} (1, 1, 2, -1/2)_L \\ (1, 1, 1, +1)_L \end{matrix}\right\} \quad n_L \text{ times}$$

Higgs:

$(3, \bar{3}, 1, 0)$ n_H times

where the multiplicities are constrained by requiring anomaly cancellation: $n_L = 2n_Q + 3n_6 = n_G + 2n_6$, where n_G is the number of ordinary quark families at energies below $2\mu_6$. Each exotic quark family gives, when G_S breaks to $SU(3)_C$, one family of ordinary quarks and one family of colour sextet quarks.

The RGE's valid in the various energy regions are:

$E < 2\mu_6$

$$\begin{aligned} B_1 &= \tfrac{11}{9} n_G + n_L \\ B_2 &= n_G + \tfrac{1}{3} n_L - \tfrac{22}{3} \\ B_3 &= \tfrac{4}{3} n_G - 11 \end{aligned} \tag{7a}$$

and

$$B_{ik} = \begin{pmatrix} \frac{137}{108} & \frac{1}{4} & \frac{44}{9} \\ \frac{1}{12} & \frac{49}{4} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_G + \begin{pmatrix} \frac{9}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{49}{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_L + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix} \tag{7b}$$

$2\mu_6 < E < M_S$:

$$\begin{aligned} B_1 &= \tfrac{11}{9} n_G + \tfrac{22}{9} n_6 + n_L \\ B_2 &= n_G + 2n_6 + \tfrac{1}{3} n_L - \tfrac{22}{3} \\ B_3 &= \tfrac{4}{3} n_G + \tfrac{20}{3} n_6 - 11 \end{aligned} \tag{8a}$$

and

$$B_{ik} = \begin{pmatrix} \frac{137}{108} & \frac{1}{4} & \frac{44}{9} \\ \frac{1}{12} & \frac{49}{4} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_G + \begin{pmatrix} \frac{137}{54} & \frac{1}{2} & \frac{220}{9} \\ \frac{1}{6} & \frac{49}{2} & 20 \\ \frac{55}{18} & \frac{15}{2} & \frac{500}{3} \end{pmatrix} n_6 + \begin{pmatrix} \frac{9}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{49}{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_L + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}. \tag{8b}$$

For $E > M_S$, where our gauge group is $(SU(3) \times SU(3))_S \times SU(2)_L \times U_1$, with equal couplings for two $SU(3)$ we have:

$$\begin{aligned} B_1 &= \tfrac{11}{3} n_6 + \tfrac{22}{9} n_Q + n_L \\ B_2 &= 3n_6 + 2n_Q + \tfrac{1}{3} n_L - \tfrac{22}{3} \\ B_3 &= B_4 = 4n_6 + \tfrac{4}{3} n_Q + \tfrac{1}{2} n_H - 11 \end{aligned} \tag{9a}$$

and

$$B_{ik} = \begin{pmatrix} \frac{137}{36} & \frac{3}{4} & \frac{44}{3} & \frac{44}{3} \\ \frac{1}{4} & \frac{147}{4} & 12 & 12 \\ \frac{11}{6} & \frac{9}{2} & 76 & 16 \\ \frac{11}{6} & \frac{9}{2} & 16 & 76 \end{pmatrix} n_6$$

$$+\begin{pmatrix} \frac{274}{108} & \frac{1}{2} & \frac{44}{9} & \frac{44}{9} \\ \frac{1}{6} & \frac{49}{2} & 4 & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} & 0 \\ \frac{11}{18} & \frac{3}{2} & 0 & \frac{76}{3} \end{pmatrix} n_Q$$

$$+\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 11 & 8 \\ 0 & 0 & 8 & 11 \end{pmatrix} n_H + \begin{pmatrix} \frac{9}{4} & \frac{3}{4} & 0 & 0 \\ \frac{1}{4} & \frac{49}{12} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} n_L$$

$$+\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 & 0 \\ 0 & 0 & -102 & 0 \\ 0 & 0 & 0 & -102 \end{pmatrix} \quad (9b)$$

where for $i, j = 4$ we mean the second $SU(3)$ factor.

For $n_6 = n_Q = 1$ (i.e. $n_L = 5$) we find that for $M_S = 3.5 \cdot 10^5$ GeV there exists a unification of the coupling constants at the scale $E_U \sim 10^{18}$ GeV with the value

$$\alpha_1(E_U) = \alpha_2(E_U) = \alpha_3(E_U) = 0.030$$

which corresponds at low energies to

$$\alpha_1(80\,\text{GeV}) = 0.0100 \quad \alpha_2(80\,\text{GeV}) = 0.030$$
$$\alpha_3(80\,\text{GeV}) = 0.1094 \quad \alpha_{\text{em}}^{-1}(80\,\text{GeV}) = 129.6$$
$$\sin^2\theta_W = 0.233$$

Conclusions

Motivated by the negative results of proton decay experiments we have examined here unification at some high scale without assuming a covering GUT at that scale. We found that such a unification of coupling constants is not possible in the standard model and in its supersymmetric extensions with three or four families. On the other hand, nonperturbative coupling constant unification is achieved in a supersymmetric extension of the standard model with five families and perturbative unification in the high colour scheme for dynamical electroweak symmetry breaking. The coupling constant unification scale is in both latter cases closer to the Planck scale as compared to GUT's which might suggest that coupling constant unification is more relevant than GUT's unification in theories with intrinsic Planck scale. Such theories could be unified theories in higher dimensions or string theories, especially those formulated in four dimensions.

Acknowledgements. We thank J.-M. Gérard, Dieter Lüst and Roberto Petronzio for useful discussions.

References

1. J.C. Pati, A. Salam: Phys. Rev. D8 (1973) 1240; H. Georgi, S.L. Glashow: Phys. Rev. Lett. 32 (1974) 438; H. Fritzsch, P. Minkowski: Ann. Phys. 93 (1975) 193, F. Gursey, P. Ramond, P. Sikivie: Phys. Lett. 60B (1975) 177
2. For a review on the experimental status see: B. Degrange: Plenary talk at EPS 1987 Conf. Uppsala, to be published in the proceedings
3. H. Georgi, H. Quinn, S. Weinberg: Phys. Rev. Lett. 33 (1974) 451; A. Buras et al.: Nucl. Phys. B135 (1978) 66
4. L. Maiani, G. Parisi, R. Petronzio: Nucl. Phys. B136 (1978) 115; N. Cabibbo, G.R. Farrar: Phys. Lett. 110B (1982) 353; L. Maiani, R. Petronzio: Phys. Lett. 176B (1986) 120; G. Grunberg: Phys. Rev. Lett. 58 (1987) 1180
5. M. B. Green, J. H. Schwarz: Phys. Lett. 149B (1984) 117; for a review see: M.B. Green, J.H. Schwarz, E. Witten: Superstring theory, Vols I, II: Cambridge, Cambridge University Press 1987
6. E. Cremmer, J. Scherk: Nucl. Phys. B103 (1976) 399; Z. Horvath, L. Palla, E. Cremmer, J. Scherk: Nucl. Phys. B127 (1977) 57; J. F. Luciani: Nucl. Phys. B135 (1978) 111; S. Ramdjbar-Daemi, A. Salam, J. Strathedee: Nucl. Phys. B214 (1983) 491; P.H. Frampton, P.J. Moxhay, K. Yamamoto: Phys. Lett. 114B (1984) 354; Ch. Wetterich: Nucl. Phys. B244 (1984) 355; A.N. Schellekens: Nucl. Phys. B248 (1984) 706
7. P. Forgacs, N.S. Manton: Commun. Math. Phys. 72 (1980) 15; G. Chapline, N.S. Manton: Nucl. Phys. B209 (1982) 461; F.A. Bais, K.J. Barnes, P. Forgacs, G. Zoupanos: Nucl. Phys. B263 (1986) 557; For a review see: D. Lüst, P. Forgacs, G. Zoupanos: Physics from multi-dimensional gauge theories. Proceedings of 2nd Hellenic School (1985). Singapore: World Scientific 198
8. K. Narain: Phys. Lett. 169B (1986) 41; H. Kawai, D. Lewellen, S. Tye: Phys. Rev. Lett. 57 (1986) 1832; Nucl. Phys. B288 (1987) 1; W. Lerche, D. Lüst, A. N. Schellekens: Nucl. Phys. B287 (1987) 477; I. Antoniadis, C. Bachas, C. Kounnas: Nucl. Phys. B289 (1987) 87; D. Lüst, S. Theisen, G. Zoupanos: Nucl. Phys. B296 (1988) 500
9. D.R.T. Jones: Phys. Rev. D25 (1982) 581; M.E. Machacek, M.T. Vaughn: Nucl. Phys. B222 (1983) 83; L.E. Björkman, D.R.T. Jones: Nucl. Phys. B259 (1985) 533
10. U. Amaldi et al.: A comprehensive analysis of data pertaining to the weak neutral current and the intermediate vector boson masses. Pennsylvania University preprint UPR-0331T (1987)
11. L. Maiani: Proc. Sum. School Gil-sur-Yvette (1980); M. Beltman: Acta Phys. Pol. B12 (1981), 437; S. Dimopoulos, S. Raby, F. Wilczek: Phys. Rev. D24 (1981) 1681
12. For reviews see: E. Fahri, L. Susskind: Phys. Rev. C74 (1981) 277; R.K. Kaul: Rev. Mod. Phys. 55 (1983) 449
13. G. Zoupanos: Phys. Lett. 129B (1983) 315
14. W. Marciano: Phys. Rev. D21 (1980) 315; P.Q. Hung: Fermilab report FERMILAB-PUB-80/78-THY (unpublished); D. Lüst, E. Papantonopoulos, G. Zoupanos: Phys. Lett. 158B (1985) 55; D. Lüst, E. Papantonopoulos, K.H. Streng, G. Zoupanos: Nucl. Phys. B268 (1986) 49; F. Braaten, A.R. White, C.R. Wilcox: J. Mod. Phys. 1A (1986) 693; K. Kang, A. White: J. Mod. Phys. 2A (1987) 409; W. Bardeen: Proc. of Rencontres de Moriond (1986); T.E. Clark, C.N. Leung, S.T. Love, T.L. Rosner: Phys. Lett. 177B (1986) 413
15. J.B. Kogut et al.: Phys. Rev. Lett. 48 (1982) 1140; ibid 50 (1983) 353; J.B. Kogut, J. Shigemitsu, D.K. Sinclair: Phys. Lett. 138B (1984) 283; ibid 145B (1984) 239
16. For similar recent speculations see: P.H. Frampton, S.L. Glashow: Phys. Rev. Lett. 58 (1987) 2168; P.H. Frampton, S.L. Glashow: Phys. Lett. 190B (1987) 157