# SUPERSYMMETRIC RADIATIVE CORRECTIONS AT THE Z ${ }^{0}$ PEAK 

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#### Abstract

We give compact and explicit formulae for the corrections to asymmetry measurements on the $Z^{0}$ peak induced by vacuum polarization due to supersymmetric particles. We investigate their magnitudes in realistic minimal models based on supergravity and inspired by the superstring. We find that the corrections to the forward-backward asymmetry are unobservably small, whilst corrections to the left-right polarization asymmetry are observable if particles have masses in the range accessible to LEP 200.


Among the highlights of physics at the SLC and LEP, there are expected to be precision studies at the $\mathrm{Z}^{0}$ peak [1]. In addition to providing the most precise determinations of Standard Model parameters such as $\sin ^{2} \theta_{\mathrm{w}}$, such measurements may be sensitive probes of physics beyond the Standard Model. The greatest sensitivity to many of these phenomena could be provided by measurements of the left-right polarized beam asymmetry at the $Z^{0}$ peak, as has been emphasized in particular in ref. [2]. Polarized beams are planned for the SLC [3], and there is now renewed interest in them for LEP [4]. A key question is whether such polarization asymmetry measurements provide information which is unobtainable in any other way, or at least more difficult to obtain. In this paper we answer this question from the point of view of phenomenological supersymmetric models [5].

Sample calculations of radiative corrections to the left-right asymmetry induced by supersymmetric particles have already been presented in ref. [2]. However, the sparticle spectra assumed in those calculations were not typical of the spectra found in realistic models. For example, the possibility of large splitting between isospindoublet partner sparticles (e.g. $m_{\bar{v}} \ll m_{\overline{\mathrm{eL}}}$ ) was entertained, but no splitting was included between the spartners of isodoublet left-handed particles and those of isosinglet right-handed particles (e.g. $m_{\hat{e_{L}}} \neq m_{\hat{e}_{\mathrm{R}}}$ ). However, in realistic models [5], the soft supersymmetry-breaking scalar mass terms are isospin-invariant, but different for left- and right-sparticles, whilst D-terms split left- and right-sparticles at the same time as they split isodoublet sparticles. In this paper we present representative calculations of the radiative corrections in phenomenological supersymmetric models based on supergravity [6] and inspired by the superstring [7]. Our first model is the Minimal Supersymmetric Standard Model (MSSM) with soft supersymmetry-breaking mass parameters renormalized from bare values of the scalar and gaugino masses ( $m_{0}, m_{1 / 2}$ ) [5,6]. The second model is a Minimal Superstring-Inspired Model (MSIM) in which there is an $\operatorname{SU}(3)_{\mathrm{c}} \times \operatorname{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{E}}$ gauge group, and the only bare soft supersymmetry-breaking mass parameter is a universal gaugino mass $m_{1 / 2}$ [7]. We calculate the deviations from the Standard Model predictions for the left-right and forward-backward asymmetries in both these models and compare them with $Z^{0} / Z_{\mathrm{E}}$ mixing effects [8,9] in the case of the MSIM.

We find that these supersymmetric radiative corrections at the $Z^{0}$ peak are generally quite small. Sparticles with masses in the range accessible to LEP 200 [10] would have observable effects on the left-right asymmetry at the SLC or at LEP 100, but would not have observable effects on the forward-backward asymmetry. These results do, however, mean that in the MSIM a large deviation of the left-right asymmetry from the Standard Model prediction could be interpreted unambiguously as due to $\mathrm{Z}^{0} / \mathrm{Z}_{\mathrm{E}}$ mixing.

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Radiative corrections in $\mathrm{e}^{+} \mathrm{e}^{-}$collisions can be divided into two categories: pure vacuum polarization effects in gauge-boson propagators, called in ref. [2] oblique corrections; and the rest. The vacuum polarization effects are universal for all the different final-state fermion flavours, whereas the rest are not. Hence the full statistics of all the final states can be included when using the left-right polarization asymmetry $A_{\mathrm{LR}}$ to probe vacuum polarization effects. Moreover, the complete calculation [2] of other radiative corrections in a supersymmetric theory introduces additional model-dependence. Therefore we restrict ourselves in this paper to a calculation of vacuum polarization effects (oblique corrections).

For completeness and convenience, we now give a capsule deviation of the standard result for one-loop oblique corrections to $A_{\mathrm{LR}}$. Using the tree-level couplings $g_{\mathrm{L}}^{\mathrm{c}}=\left(e / \sin \theta_{\mathrm{w}}\right)\left(-\frac{1}{2}+\sin ^{2} \theta_{\mathrm{w}}\right)$ and $g_{\mathrm{R}}^{\mathrm{e}}=e \sin \theta_{\mathrm{w}}$, and denoting $\xi=1-4 \sin ^{2} \theta_{\mathrm{w}}$, we have the tree-level result [1]
$A_{\mathrm{LR}}=2 \xi /\left(1+\xi^{2}\right)$.
In our calculations at the one-loop level we use the following conventional definition [2] of $\theta_{\mathrm{w}}$ :

$$
\begin{equation*}
\theta_{\mathrm{w}}: 4 \sin ^{2} \theta_{\mathrm{w}} \cos ^{2} \theta_{\mathrm{w}}=4 \pi \alpha / \sqrt{2} G_{\mu} m_{\mathrm{Z}}^{2}(1+\Delta r) \tag{2}
\end{equation*}
$$

which is the tree-level formula modified by the inclusion of trivial electromagnetic radiative corrections. Thus the muon decay constant $G_{\mu}$ is that obtained by applying internal QED radiative corrections and bremsstrahlung corrections to the observed muon decay rate, and $\Delta r=0.0711 \pm 0.0013$ expresses the running of $\alpha$ between $Q^{2}=0$ and $m_{2}^{2}$. [Note that we use the time-like metric diag $(1,-1,-1,-1)$. ]

The vacuum polarization corrections to the tree-level formula (1) for $A_{\mathrm{LR}}$ can be divided into two parts:
$\delta A_{\mathrm{LR}}=\delta A_{\mathrm{LR}}^{(1)}+\delta A_{\mathrm{LR}}^{(2)}$,
where the first part, $\delta A_{\mathrm{LR}}^{(1)}$, involves the $Z^{0}$ pole alone, and the second part $\delta A_{\mathrm{LR}}^{(2)}$ arises from $\gamma / Z^{0}$ mixing. The first part $\delta A_{\mathrm{LR}}^{(1)}$ is due solely to the shift in $\xi$ occasioned by radiative corrections to the definition (2) of $\sin ^{2} \theta_{\mathrm{w}}$. Thus we have
$\delta A_{\mathrm{LR}}^{(1)}=\left[2\left(1-\xi^{2}\right) /\left(1+\xi^{2}\right)^{2}\right] \delta \xi$,
where
$\delta \xi=-4 \delta \sin ^{2} \theta_{\mathrm{w}}$,
with eq. (2) yielding
$\delta \sin ^{2} \theta_{\mathrm{w}}\left(1-2 \sin ^{2} \theta_{\mathrm{w}}\right)=\left[\pi \alpha / \sqrt{2} G_{\mu} m_{\mathrm{Z}}^{2}(1-\Delta r)\right]\left(\delta \alpha / \alpha-\delta G_{\mu} / G_{\mu}-\delta m_{\mathrm{Z}}^{2} / m_{\mathrm{Z}}^{2}\right)$.
The individual contributions to eq. (5b) are well known to be
$\delta \alpha / \alpha=\Pi_{\mathrm{AA}}^{\prime}(0)$,
where $\left.\Pi_{\mathrm{AA}}^{\prime}(0) \equiv\left(\partial / \partial Q^{2}\right) \Pi_{\mathrm{AA}}\left(Q^{2}\right)\right|_{Q^{2}=0}$, with $\Pi_{\mathrm{AA}}\left(Q^{2}\right)$ the photon-photon vacuum polarization function,
$\delta G_{\mu} / G_{\mu}=-\Pi_{\mathrm{ww}}(0) / m_{\mathrm{w}}^{2}$
and
$\delta m_{Z}^{2} / m_{Z}^{2}=\operatorname{Re} \Pi_{Z Z}\left(m_{Z}^{2}\right) / m_{Z}^{2}$.
Combining eqs. (4)-(6), we finish up with
$\delta A_{\mathrm{LR}}^{(1)}=\left[-64 \sin ^{4} \theta_{\mathrm{w}} \cos ^{2} \theta_{\mathrm{w}} /\left(1+\xi^{2}\right)^{2}\right]\left[\Pi_{\mathrm{AA}}^{\prime}(0)+\Pi_{\mathrm{ww}}(0) / m_{\mathrm{w}}^{2}-\operatorname{Re} \Pi_{\mathrm{ZZ}}\left(m_{\mathrm{Z}}^{2}\right) / m_{\mathrm{Z}}^{2}\right]$,
to be inserted into eq. (3). The second term from $\gamma / Z^{0}$ mixing is easily found to be
$\delta A_{\mathrm{LR}}^{(2)}=\left[-32 \sin ^{3} \theta_{\mathrm{w}} \cos \theta_{\mathrm{w}} /\left(1+\xi^{2}\right)^{2}\right]\left[2 \cos 2 \theta_{\mathrm{w}} \operatorname{Re} \Pi_{\mathrm{ZA}}\left(m_{\mathrm{Z}}^{2}\right) / m_{\mathrm{Z}}^{2}\right]$,
so that finally
$\delta A_{\mathrm{LR}}=\left[-64 \sin ^{4} \theta_{\mathrm{w}} \cos ^{2} \theta_{\mathrm{w}} /\left(1+\xi^{2}\right)^{2}\right]$

$$
\begin{equation*}
\times\left(\frac{\cos 2 \theta_{\mathrm{w}}}{\sin \theta_{\mathrm{w}} \cos \theta_{\mathrm{w}}} \frac{\operatorname{Re} \Pi_{\mathrm{ZA}}\left(m_{\mathrm{Z}}^{2}\right)}{m_{\mathrm{Z}}^{2}}+\Pi_{\mathrm{AA}}^{\prime}(0)+\frac{\Pi_{\mathrm{Ww}}(0)}{m_{\mathrm{w}}^{2}}-\frac{\operatorname{Re} \Pi_{\mathrm{ZZ}}\left(m_{\mathrm{Z}}^{2}\right)}{m_{\mathrm{Z}}^{2}}\right), \tag{9}
\end{equation*}
$$

to be evaluated in our chosen models.
We now compute the squark and slepton loop corrections to the vacuum polarization functions $\Pi_{a b}$ of (9). There are two Feynman diagrams for each sparticle, involving trilinear and quadrilinear vertices, respectively. Their contributions are
$\Pi_{a b}^{(1)}\left(Q^{2}\right)=\frac{-2 g_{i j}^{a} g_{j i}^{b}}{16 \pi^{2}} \int_{0}^{1} \mathrm{~d} x R_{i j}^{2}\left(-\ln R_{i j}^{2}\right), \quad \Pi_{a b}^{(2)}\left(Q^{2}\right)=\frac{G_{i}^{a b}}{16 \pi^{2}} m_{i}^{2}\left(-\ln m_{i}^{2}\right)$,
where the indices $(i, j)$ denote the particles in the loop with trilinear vertices $g_{i j}^{a}$ and quadrilinear vertices $G_{i}^{a b}$, and
$R_{i j}^{2} \equiv m_{i}^{2}-Q^{2} x(1-x)-\left(m_{i}^{2}-m_{j}^{2}\right) x$.
Assuming that left-right sparticle mixing is negligible (an assumption we return to later), the trilinear vertices are
$g_{i i}^{\mathrm{A}}=e Q_{i}, \quad g_{i l}^{\mathrm{Z}}=\frac{\dot{e}}{\sin \theta_{\mathrm{w}} \cos \theta_{\mathrm{w}}}\left(T^{3}-Q \sin ^{2} \theta_{\mathrm{w}}\right)_{i^{\prime}}, \quad g_{i j}^{\mathrm{W}}=\frac{e}{\sqrt{2} \sin \theta_{\mathrm{w}}}$,
and the non-zero quadrilinear vertices are
$G_{i}^{\mathrm{AA}}=2 e^{2} Q_{i}^{2}, \quad G_{i}^{\mathrm{ww}}=e^{2} / 2 \sin ^{2} \theta_{\mathrm{w}}$,
$G_{i}^{\mathrm{ZZ}}=\frac{2 e^{2}}{\sin ^{2} \theta_{\mathrm{w}} \cos ^{2} \theta_{\mathrm{w}}}\left(T^{3}-Q \sin ^{2} \theta_{\mathrm{w}}\right)_{i}^{2}, \quad G_{i}^{\mathrm{ZA}}=\frac{2 e^{2} Q_{i}}{\sin \theta_{\mathrm{w}} \cos \theta_{\mathrm{w}}}\left(T^{3}-Q \sin ^{2} \theta_{\mathrm{w}}\right)_{i}$.
Using eqs. (10)-(12), we can calculate the different vacuum polarization terms appearing in $\delta A_{\text {LR }}$ [eq. (9)]:
$\frac{\Pi_{\mathrm{ZA}}\left(m_{\mathrm{Z}}^{2}\right)}{m_{\mathrm{Z}}^{2}}=\sum_{i} \frac{2 e^{2} Q_{i}}{\sin \theta_{\mathrm{w}} \cos \theta_{\mathrm{w}}} \frac{\left(T^{3}-Q \sin ^{2} \theta_{\mathrm{w}}\right)_{i}}{16 \pi^{2}}\left(c_{i}-\hat{m}_{i}^{2} \ln \hat{m}_{i}^{2}\right)$,
$\Pi_{A A}^{\prime}(0)=-\sum_{i} \frac{e^{2} Q_{i}^{2}}{48 \pi^{2}}\left(\ln \hat{m}_{i}^{2}+1\right)$,
$\frac{\Pi_{\mathrm{ww}}(0)}{m_{\mathrm{W}}^{2}}=\sum_{\text {doublecs }\left(\frac{1}{\natural},\right.} \frac{e^{2}}{32 \pi^{2} \sin ^{2} \theta_{\mathrm{w}}}\left(\frac{m_{\mathrm{h}}^{2} m_{l}^{2} \ln \left(m_{\mathrm{l}}^{2} / m_{\mathrm{h}}^{2}\right)}{\left(m_{\mathrm{l}}^{2}-m_{\mathrm{h}}^{2}\right) m_{\mathrm{W}}^{2}}-\frac{m_{\mathrm{h}}^{2}+m_{\mathrm{l}}^{2}}{2 m_{\mathrm{W}}^{2}}\right)$,
$\frac{\Pi_{\mathrm{Zz}}\left(m_{\mathrm{Z}}^{2}\right)}{m_{\mathrm{Z}}^{2}}=\sum_{i} \frac{2 e^{2}}{\sin ^{2} \theta_{\mathrm{w}} \cos ^{2} \theta_{\mathrm{w}}} \frac{\left(T^{3}-Q \sin ^{2} \theta_{\mathrm{w}}\right)_{i}^{2}}{16 \pi^{2}}\left(c_{i}-\hat{m}_{i}^{2} \ln \hat{m}_{i}^{2}\right)$,
where $\hat{m}_{i}^{2} \equiv m_{i}^{2} / m_{Z}^{2}$ and
$c_{i} \equiv \frac{5}{18}-\frac{4}{3} \hat{m}_{i}^{2}+\left(-\frac{1}{6}+\hat{m}_{i}^{2}\right) \ln \hat{m}_{i}^{2}+\frac{1}{6}\left(1-4 \hat{m}_{i}^{2}\right)^{3 / 2} \ln \frac{-1+\sqrt{1-4 \hat{m}_{i}^{2}}}{1+\sqrt{1-4 \hat{m}_{i}^{2}}}$.
Inserting the results (13) into the general expression (9) for $\delta A_{\mathrm{LR}}$, we finally obtain the formula

$$
\begin{aligned}
& \delta A_{\mathrm{LR}}=-\frac{32 \sin ^{2} \theta_{\mathrm{w}}}{\left(1+\xi^{2}\right)^{2}} \frac{\alpha}{\pi} \sum_{\text {gener }}\left\{\sum_{i} \theta\left(\hat{m}_{i}^{2}-\frac{1}{4}\right)\left[T_{3 i}\left(Q-T_{3}\right)_{i}-Q_{i}^{2} \sin ^{2} \theta_{\mathrm{w}} \cos ^{2} \theta_{\mathrm{w}}\right]\right. \\
& \times\left[\frac{5}{18}-\frac{4}{3} \hat{m}_{i}^{2}-\frac{1}{6}\left(4 \hat{m}_{i}^{2}-1\right)^{3 / 2}\left(2 \arctan \sqrt{4 \hat{m}_{i}^{2}-1}-\pi\right)\right] \\
& +\sum_{i} \theta\left(\frac{1}{4}-\hat{m}_{i}^{2}\right)\left[T_{3 i}\left(Q-T_{3}\right)_{i}-Q_{i}^{2} \sin ^{2} \theta_{\mathrm{w}} \cos ^{2} \theta_{\mathrm{w}}\right]\left(\frac{5}{18}-\frac{4}{3} \hat{m}_{i}^{2}-\frac{1}{6}\left(1-4 \hat{m}_{i}^{2}\right)^{3 / 2} \ln \frac{1+\sqrt{1-4 \hat{m}_{i}^{2}}}{1-\sqrt{1-4 \hat{m}_{i}^{2}}}\right) \\
& -\frac{1}{12} N_{\mathrm{c}} \ln \frac{\hat{m}_{\hat{\mathrm{El}}}^{2}}{\hat{m}_{\hat{\mathrm{d} .}}^{2}}+\frac{1}{24} \ln \frac{\hat{m}_{\hat{\mathrm{EL}}}^{2}}{\hat{m}_{\mathrm{el}}^{2}}-\frac{1}{6} \sin ^{2} \theta_{\mathrm{w}} \cos ^{2} \theta_{\mathrm{w}} \sum_{i} Q_{i}^{2}
\end{aligned}
$$

where $N_{\mathrm{c}}=3$ and $\sum_{i} \equiv N_{\mathrm{c}} \sum_{i=\tilde{q}_{\mathrm{q}}, \tilde{\mathrm{c}}_{\mathrm{k}}}+\sum_{i=\tilde{\mathrm{c}}_{\mathrm{R}}, \tilde{\mathrm{e}}_{1}, \mathrm{v}_{\mathrm{L}}}$, which is the basis of the numerical results in the rest of this paper. Formula (15) is not novel [2], but it has not previously been derived and exhibited so explicitly.

In order to use eq. (15) one must formulate models for the spin-0 sparticle masses, of which we will present two. The squared masses of the spin- 0 sparticles generally contain three contributions: a soft supersymmetrybreaking term, a D-term, and an F-term equal to the square of the corresponding spin- $1 / 2$ particle mass:
$m_{\tilde{f}_{n}}^{2}=\tilde{m}_{\mathrm{f}_{\mathrm{n}}}^{2}+m_{\mathrm{f}_{\mathrm{n}}}^{\mathrm{D}}{ }^{2}+m_{\mathrm{f}}^{2}$,
where $h=\mathrm{L}, \mathrm{R}$. We can neglect the last term in eq. (16) and off-diagonal terms mixing $\tilde{f}_{\mathrm{L}, \mathrm{R}}$ if
$m_{\mathrm{f}}^{2} \ll\left(\tilde{m}_{\tilde{f}_{\mathrm{L}}}^{2}-\tilde{m}_{\tilde{\mathrm{T}}_{\mathrm{R}}}^{2}\right)+\left(m_{\tilde{f}_{\mathrm{L}}}^{\mathrm{D}}-m_{\tilde{T}_{\mathrm{R}}}^{\mathrm{D}^{2}}\right)$,
as is generally the case. In supergravity and superstring-inspired models [5-7], even if the bare values of the spin- 0 soft supersymmetry-breaking masses $\tilde{m}_{X_{\mathrm{k}}}^{2}$ are equal, radiative corrections resummed using the renormalization group make their physical values significantly different. Moreover, the D-term contributions $m_{\bar{T}_{1}}^{\mathrm{D}^{2}}$ are generally unequal, with equality being achieved in the MSSM only if $v \equiv\langle 0| H|0\rangle=\bar{v} \equiv\langle 0| \bar{H}|0\rangle$. The only sparticle for which L-R mixing has any chance to be important is the stop squark $\mathfrak{t}$ [11], but we do not investigate this possibility here for the following reasons. It is quite model-dependent, likely to be small if $m_{\mathrm{t}}$ is close to the present experimental limit of 40 GeV , and the squark contributions to $\delta A_{\mathrm{LR}}$ [eq. (15)] will in any case turn out to be considerably smaller than the slepton contributions in the models we study.

For numerical studies in the MSSM, we use the following approximate values of the renormalized soft super-symmetry-breaking parameters [5]:
$\tilde{m}_{\mathrm{Q}_{\mathrm{L}},}^{2} \approx m_{0}^{2}+0.50 m_{1 / 2}^{2}, \quad \tilde{m}_{\mathrm{Q}_{\mathrm{R}}}^{2} \approx m_{0}^{2}+0.15 m_{1 / 2}^{2}, \quad \tilde{m}_{\mathrm{q} L \mathrm{k}}^{2} \approx m_{0}^{2}+7.0 m_{1 / 2}^{2}$,
where $m_{0}$ and $m_{1 / 2}$ are the bare spin- 0 and gaugino mass parameters, and the D-terms are [6]
$m_{\mathrm{e} 1 .}^{\mathrm{D} 2}=\cos 2 \theta\left(-1+\tan ^{2} \theta_{\mathrm{w}}\right) m_{\mathrm{w}}^{2} / 2, \quad m_{\mathrm{er}}^{\mathrm{D} 2}=\cos 2 \theta\left(-\tan ^{2} \theta_{\mathrm{w}}\right) m_{\mathrm{w}}^{2}$,
$m_{\mathrm{el}_{2}^{2}}=\cos 2 \theta\left(1+\tan ^{2} \theta_{\mathrm{w}}\right) m_{\mathrm{w}}^{2} / 2$,
$m_{\mathrm{UI} .}^{\mathrm{D}_{\mathrm{T}}}=\cos 2 \theta\left(\frac{1}{2}-\frac{1}{6} \tan ^{2} \theta_{\mathrm{w}}\right) m_{\mathrm{W}}^{2}, \quad m_{\mathrm{d} .}^{\mathrm{D} 2}=\cos 2 \theta\left(-\frac{1}{2}-\frac{1}{6} \tan ^{2} \theta_{\mathrm{w}}\right) m_{\mathrm{w}}^{2}$,
$m_{\mathrm{u} \mathrm{R}}^{\mathrm{D} 2}=\cos 2 \theta\left(2+\frac{1}{3} \tan ^{2} \theta_{\mathrm{w}}\right) m_{\mathrm{W}}^{2}, \quad m_{\mathrm{dR}}^{\mathrm{D} 2}=\cos 2 \theta\left(-\frac{1}{3} \tan ^{2} \theta_{\mathrm{w}}\right) m_{\mathrm{W}}^{2}$,
where $\cos 2 \theta=-\left(1-v^{2} / v^{2}\right) /\left(1+\bar{v}^{2} / v^{2}\right)$. We will later be plotting contours of $\delta A_{\mathrm{LR}}$ in the parameter space ( $m_{0}$, $m_{1 / 2}$ ) for suitable discrete choices of $\overline{v / v}$. These results can be compared with those in a MSIM which has a


Fig. 1. Contours of the oblique radiative corrections $\delta A_{\text {LR }}$ [eq. (15)] due to (a) squarks and (b) sleptons in the ( $m_{0}, m_{1 / 2}$ ) plane of the MSSM for the optimal case $\bar{v} / v=\frac{1}{4}$. Note that the squark effects are generally smaller.
unique rank-5 low-energy gauge group, and in which we believe $m_{0} \ll m_{1 / 2}$ [7]. In this model, the renormalization factors in eqs. (18) are modified to become [7]:
$\tilde{m}_{\overline{\bar{Q}_{-}}}^{2} \approx 0.50 m_{1 / 2}^{2}, \quad \tilde{m}_{\mathrm{eR}}^{2} \approx 0.14 m_{1 / 2}^{2}, \quad \tilde{m}_{\overline{\mathrm{qL}}}^{2} \approx 3.80 m_{1 / 2}^{2}$,
$\tilde{m}_{\hat{\mathrm{u} R}}^{2} \approx 3.50 m_{1 / 2}^{2}, \quad \tilde{m}_{\mathrm{d} \mathrm{K}}^{2} \approx 3.50 m_{1 / 2}^{2}$,
whilst there are additions to the D-terms associated with the additional $\mathrm{U}(1)$ factor in the low-energy gauge group [7]:
$\delta m_{\overline{1} 1}^{\mathrm{D}{ }^{2}}=-\frac{1}{60} F \cdot m_{\mathrm{W}}^{2}, \quad \delta m_{\hat{\mathrm{ER}}}^{D{ }^{2}}=\frac{1}{30} F \cdot m_{\mathrm{W}}^{2}, \quad \delta m_{\mathrm{Q1} .}^{\mathrm{D}^{2}}=\frac{1}{30} F \cdot m_{\mathrm{W}}^{2}$,
$\delta m_{\mathrm{uk}}^{\mathrm{D} 2}=\frac{1}{30} F \cdot m_{\mathrm{W}}^{2}, \quad \delta m_{\mathrm{dk}}^{\mathrm{D}^{2}}=-\frac{1}{60} F \cdot m_{\mathrm{W}}^{2}$,
where
$F=\frac{10}{3}\left[\tan ^{2} \theta_{\mathrm{w}} /\left(1+\bar{v}^{2} / v^{2}\right)\right]\left(5 x^{2} / v^{2}-4-\bar{v}^{2} / v^{2}\right)$.
We will also be plotting contours of $\delta A_{\text {LR }}$ in the parameter space ( $x / v, \overline{\bar{v}} / 0$ ) for suitable discrete choices of $m_{1 / 2}$. After all this theoretical foreplay, we can now get down to the numerical nitty-gritty.
Fig. 1 shows that in the MSSM, the contributions to $\delta A_{\text {LR }}$ [eq. (15)] of squarks (a) are much smaller than those of sleptons (b). Although this fact is exhibited explicitly only for one value of $\bar{\sigma} / 0$, it is true for other values of $\overline{\bar{v}} / v$ in the MSSM, and for the MSIM as well. All our subsequent graphs will combine the contributions of three generations of squarks and three generations of sleptons, with the latter being the móst important.
Fig. 2 shows contours of $\delta A_{\text {LR }}$ in the ( $m_{0}, m_{1 / 2}$ ) plane of the MSSM, for values of $\bar{v} / 0$ in the range previously favoured by model-builders [5,6]. The experimental precision in measuring $A_{\text {LR }}$ is likely [ 3,4 ] to be around $3 \times 10^{-3}$, which we take as an indication of the sensitivity to $\delta A_{\mathrm{LR}}$. The values of $\delta A_{\mathrm{LR}}$ are smallest in fig. 2 a for $\bar{v} / v=1$, in which case the D-terms (19) do not contribute to isospin splitting for the left-sfermions with $T^{3}= \pm \frac{1}{2}$. For this value of $\bar{J} / v=1$, which has been discussed most often in the literature [5,6], the present experimental bounds [12] on sparticle masses can be translated into the indicated excluded domain of ( $m_{0}$,


Fig. 2. Contours of $\delta A_{L R}$ due to squarks and sleptons in the MSSM for (a) $\bar{\sigma} / v=1$ and (b) $\bar{\sigma} / v=\frac{1}{4}$. The shaded region around the origin corresponds to the phenomenological bounds on ( $m_{0}, m_{1 / 2}$ ) discussed in the text.
$m_{1 / 2}$ ) close to the origin. The excluded domain around the origins in fig. 2 b are those for which some spin- 0 sparticle mass-squared is negative. We see that in the case $\bar{\sigma} / 0=\frac{1}{4}$ of fig. 2 b which gives the largest effect, radiative corrections to $\delta A_{\mathrm{LR}} \approx 3 \times 10^{-3}$ are sensitive to $m_{0} \leqslant 110 \mathrm{GeV}$ for small values of $m_{1 / 2}$. This can be compared with the physics reach out to $m_{\tilde{e}_{L, R}} \approx 90 \mathrm{GeV}$ available [10] with LEP 200, which is larger than the ranges of $m_{0}$ accessible via measurements of $\delta A_{\mathrm{LR}}$ for larger values of $\overline{0} / 0$.

Fig. 3 shows contours of $\delta A_{\mathrm{LR}}$ in the ( $x / 0, \bar{\nu} / v$ ) plane of the MSIM for values of $m_{1 / 2}$ at the lower end of the range previously favoured [7] by model-builders. Also shown for comparison is a dashed line corresponding to the bound on $x / v$ given by the non-observation of the $\mathrm{Z}_{\mathrm{E}}$ or its anomalous neutral-current effects [7]. At least one of the spin- 0 sparticle mass-squared is negative above the thick lines traversing figs. $3 \mathrm{a}, 3 \mathrm{~b}$. Close to this line, there is large isospin-splitting in the $\bar{\ell}_{\mathrm{L}}$ isodoublets, which is why $\delta A_{\mathrm{LR}}$ is largest in this region. This effect becomes more pronounced for larger values of $m_{1 / 2}$. In general, the value of $\delta A_{\mathrm{LR}}$ is large enough ( $\gtrsim 3 \times 10^{-3}$ ) to be detected only if the $\tilde{\mathrm{e}}_{\mathrm{R}}$ is light enough to be produced at LEP 200.

There is another contribution to $\delta A_{\text {LR }}$ in the MSIM which should be compared with that plotted in fig. 3 . It is the contribution induced by the mixing between the $Z^{0}$ and the extra $Z_{E}$ boson that appears in the rank5 superstring-inspired model [ 8,9 ]. Its magnitude is independent of $m_{1 / 2}$, and has been plotted as the dotted contours in fig. 3b. We see that it is in general considerably larger than the value of $\delta A_{\mathrm{LR}}$ induced by oblique radiative corrections in the MSIM. Thus, in this model $A_{\mathrm{LR}}$ is better regarded as a probe of the $\mathrm{Z}_{\mathrm{E}}$ rather than of the sparticles.

Although we have not exhibited them explicitly, it is easy to see that the oblique supersymmetric radiative corrections to the forward-backward asymmetry $A_{\text {FB }}$ are not large enough to be detectable. We can compare the tree-level formula (1) for $A_{L R}$ with the tree-level formula
$A_{\mathrm{FB}}=3 \xi^{2} /\left(1+\xi^{2}\right)^{2}$,
from which we can infer that $\delta A_{\mathrm{FB}} / \delta A_{\mathrm{LR}} \approx \frac{1}{8}$. This would make $\delta A_{\mathrm{FB}}$ unobservably small in all the parameter spaces of figs. 2 and 3. However, the $\mathrm{Z}_{\mathrm{E}}$ contributions to $\delta A_{\mathrm{FB}}$ in the MSIM are large enough to be observable, as discussed elsewhere [8,9].

In conclusion, we find that although the oblique supersymmetric radiative corrections $\delta A_{\mathrm{LR}}$ to the left-right polarization asymmetry $A_{\mathrm{LR}}$ may be large enough to be observed, and could be sensitive to sparticle masses


Fig. 3. Contours of $\delta A_{\mathrm{LR}}$ due to squarks and sleptons in the ( $x / 0, \overline{\bar{b}} / 0$ ) plane of the MSIM for (a) $m_{1 / 2}=100 \mathrm{GeV}, m_{1 / 2}=200 \mathrm{GeV}$ and (b) $m_{1 / 2}=300 \mathrm{GeV}$. The thick lines bound phenomenological excluded areas of the plane as discussed in the text. The dashed lines correspond to the non-observation of $\mathrm{Z}_{\mathrm{E}}$-boson effects in present data, and the dotted lines in (b) are contours of the $\delta A_{\mathrm{LR}}$ induced by $Z^{\prime \prime} / Z_{\mathrm{E}}$ mixing.
in the LEP 200 range, the values of $\delta A_{\text {LR }}$ we find in "realistic" phenomenological models are smaller than had previously [2] been hoped.

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## References

[1] G. Altarelli et al., in: Physics at LEP, eds. J. Ellis and R. Peccei, Report CERN 86-02 (CERN, Geneva, 1986) p. 1.
[2] B.W. Lynn, M.E. Peskin and L.G. Stuart, in: Physics at LEP, eds. J. Ellis and R. Peccei, Report CERN 86-02 (CERN, Geneva, 1986) p. 90.
[3] D. Blockus et al., SLC Polarization proposal (1986).
[4] G. Alexander et al., Polarized $\mathrm{e}^{+}$and $\mathrm{e}^{-}$beams at LEP, Report CERN/LEP/87-6, LEPC/M81 (1987).
[5] J. Ellis, Superstrings and supergravity, eds. A.T. Davies and D.G. Sutherland (SUSSP Publications, Edinburgh, 1986) p. 399.
[6] H.-P. Nilles, Phys. Rep. 110 (1984) 1.
[7] J. Ellis, K. Enqvist, D.V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A 1 (1986) 57; Nucl. Phys. B 276 (1986) 14.
[8] V.D. Angelopoulos, J. Ellis, D.V. Nanopoulos and N.D. Tracas, Phys. Lett. B 176 (1986) 203.
[9] G. Bélanger and S. Godfrey, Phys. Rev. D 34 (1986) 1309; TRIUMF preprint TRI-PP-86-18 (1986);
I. Bigi and M. Cvetić, Phys. Rev. D 34 (1986) 1651;
M. Cvetić and B.W. Lynn, SLAC-PUB-3900 (1986);
P. Franzini and F. Gilman, Phys. Rev. D 32 (1985) 237; SLAC-PUB-3932 (1986).
[10] H. Baer et al., in: Physics at LEP, eds. J. Ellis and R. Peccei, Report CERN 86-02 (CERN, Geneva, 1986) p. 297;
C. Dionisi, Report Supersymmetry Working Group, LEP 200 Workshop (Aachen, 1986).
[11] J. Ellis and S. Rudaz, Phys. Lett. B 128 (1983) 248.
[12] M. Davier, Invited talk Interrn. Conf. on High-energy physics (Berkeley, 1986).

