TWO-LOOP CALCULATIONS IN QCD AND THE $\Delta I = 1/2$ RULE IN NON-LEPTONIC WEAK DECAYS

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Two-gluon-exchange corrections to the effective operators of the Wilson expansion of the product of two currents have been calculated in order to find the g^4-term of the γ-function. Next to leading order contribution increases the ratio of $\Delta I = 1/2$ to $\Delta I = 1/2, 3/2$ amplitudes, obtained from leading order.

It was suggested by Wilson [1] that the observed $\Delta I = 1/2$ rule in non-leptonic weak interactions might be understood in terms of anomalous dimensions caused by short distance ($x \sim Mw^{-1}$) effects.

The one-loop calculations in QCD by Gaillard and Lee [2] showed that the pure $\Delta I = 1/2$ transitions are enhanced while the mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions are suppressed. The ratio of the two amplitudes is [2]:

$$R \simeq 5\,, \tag{1}$$

for $Mw = 100$ GeV and $g^2/4\pi \simeq 1$ ($\mu \simeq 1$ GeV).

The experimental value of the same ratio is of the order of 20. The question of higher order corrections was open and tempting [3]. Altarelli et al. [4] have computed the two-loop corrections, using a dimensonal reduction scheme [5] to regularize the integrals, and found small subleading corrections. We present here a different technique in evaluating the Wilson coefficients of the two operators O^+ amd O^- in two-loop order.

The anomalous dimensions are controlled by the infinite part of the two-gluon corrections to the four-quarts, dimension-six operators O^+ and O^- defined by

$$O^{\pm} = (I)\cdot(I) \pm (I) \times (I)$$

$$(\bar{s}\gamma^{\mu}(1-\gamma_5)I u)\,(\bar{u}\gamma_{\mu}(1-\gamma_5)\,d)$$

$$\pm\,(\bar{s}\gamma^{\mu}(1-\gamma_5)I d)\,(\bar{u}\gamma_{\mu}(1-\gamma_5)u)\,, \tag{2}$$

where I is the unit colour matrix. By the symbol "X" we mean the Fierz transformed operator. The group G_s of strong interactions is taken to be the SU(3) colour group, while the G_{WEAK} is the SU(2) × U(1) Weinberg–Salam model extended to hadrons. O^+ induces the transitions with the mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$ (u and d are in a symmetric combination), while O^- induces the pure $\Delta I = 1/2$ transition (u and d are in an antisymmetric combination).

There are 34 different types of diagrams we have to consider. Fig. 1 shows these diagrams with the number of equivalent ones.

The dimensional regularization scheme was used. All particles are taken to be massless (external momenta were used to avoid infrared difficulties).

Since the subleading corrections come from the order $1/\epsilon$ terms in two loops it is essential to perform the γ-algebra in n-dimensions. This problem was avoided in Altarelli's work since the dimensional reduction scheme treats the γ-matrices in four-dimensions and introduces a compensating "ϵ-scalar field". Previous two loop calculations [6] have also had to do this, but the calculations always involved the contraction of γ-matrices on a *single* fermion line. The novel feature of the present calculation is that it involves contractions of γ-matrices on *different* fermion lines. After performing the symmetric momentum integrations we encounter expressions like:

$$[\gamma^{\rho}\gamma^{\sigma}\gamma^{\mu}(1-\gamma_5)]_{\alpha\beta}[\gamma_{\mu}\gamma_{\sigma}\gamma_{\rho}(1-\gamma_5)]_{\gamma\delta} \tag{3}$$

Table 1

N_f	Λ' (GeV) 0.3	0.5
3	3.31	3.02
4	5.88	4.10
6	11.87	10.36

In order to absorb ($\ln 4\pi - \gamma$) terms in Λ (in other words using the MS scheme) we rescale Λ:

$$\Lambda \to \Lambda' = \Lambda \exp[1/2(4\pi - \gamma)] \tag{9}$$

where γ is the Euler constant (0.577512...).

Finally we write in table 1 the values of the desired ratio $R = C^-/C^+$, where $C^\pm$ are the coefficients of the $O^\pm$ operators in the effective hamiltonian ‡2.

The results seem reasonable compared with the one-loop corrections. Nevertheless they are not small, so presumably three-loops etc. are not negligible. This means that the enhancement *might* be a short distance effect but we still cannot calculate it with certainty.

The last thing we would like to stress at is that, since the main "theoretical" problem in the calculation is the γ-algebra, another way to avoid that problem is to find a group of diagrams with the same momentum integral in which adding the γ-algebra and using only the anticommutation relation of the γ's we get the answer of the form $(\gamma^\mu(1-\gamma_5))(\gamma_\mu(1-\gamma_5))$ [7]. If this possibility is true, it proves our assumption in eq. (4). However it is not clear that it is possible ‡2.

‡1 When Λ' grows bigger, and therefore $\bar{g}^2$ gets bigger too, we expect R to increase, while in table 1 we see the opposite tendency (in the one-loop correction the ratio R has the desired tendency). We have nothing to propose in order to explain this behaviour.

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‡2 Another approach to this problem, which one of us (N. Tracas) checks is the isolation of the terms in which the ϵ-terms of the γ-matrix expansion (following our procedure) contribute. In case where a complete cancellation of these terms appears, γ-algebra can be done in four dimensions.

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