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NEXT TO LEADING N CALCULATIONS IN THE GROSS-NEVEU MODEL.

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A systematic algorithm is given for calculating certain classes of diagarams in any order in perturbation theory. We use this method for calculating the next to leading N terms for the β-function in four and five loops.

1.INTRODUCTION

The Gross-Neveu model has been a useful ground for testing dymamical symmetry breaking in field theory. It is an asymptotically free two-dimensional fermion field theory with a quartic interaction. The model has been analyzed in the $1/N$ approximation and found to exhibit dynamical breaking of the discrete chiral symmetry. The dynamical mass acquired by the fermion depends on a non trivial way on the coupling constant. Furthermore a rich bound state spectrum appears in the broken phase.

The Lagrangian of the model has the form

$$L = \bar{\Psi}(i\partial\!\!\!/)\Psi + \frac{\lambda}{2}(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) \qquad (1)$$

or in the so-called σ-formulation, which is more suitable for our calculations

$$L_\sigma = \bar{\Psi}(i\partial\!\!\!/)\Psi - g(\bar{\Psi}\Psi)\sigma + \frac{\sigma^2}{2} \qquad (2)$$

where $g^2 = \lambda$ and in both (Eq.1) and (Eq.2) we have suppressed the summation over the flavour index N of the fermion field Ψ. The field σ serves as an auxiliary one.

2. THE β-FUNCTION.

In perturbation theory the β-function takes the form

$$\beta(\lambda) = \beta_1\lambda^2 + \beta_2\lambda^3 + \beta_3\lambda^4 + \ldots \qquad (3)$$

where β_i's are expected to be polynomials of N. Each factor of N corresponds to a fermion loop (trace). Therefore for any loop order r, the highest term of β_r will be N^r. Nevertheless, for $r > 1$ the only diagram contributing to the highest N can be constructed from r independent simple fermion loops. Clearly this diagram does not require any new counter terms; the one loop counterterm suffices to renormalize it. Furthemore, the equivalence of the G-N to the Thirring model, for N=1, requires the β_i's to vanish for that value of N. With all these cosiderations the general form of the r-coefficient of the β-function is

$$\beta_r = (N-1)(\beta_{r,r-2}N^{r-2} + \beta_{r,r-3}N^{r-3} + \ldots) \qquad (4)$$

The β-function has been calculated up to three loops and equals [1,2]

$$\beta(\lambda) = (N-1)\left(-\frac{4\lambda^2}{4\pi} + \frac{8\lambda^3}{(4\pi)^2} - \frac{420\lambda^4}{(4\pi)^3}\right) \qquad (5)$$

The interesting point is the lack of a N^2-term in the λ^4 coefficient. Let us see carefully what does this vanishing implies.

2.1. The scheme dependence of the β-function.

Under an N-independent analytic rescaling of the coupling

$$\lambda' = \lambda + c_2\lambda^2 + c_3\lambda^3 + c_4\lambda^4 \qquad (6)$$

the coefficients β_1 and β_2 remain scheme independent while β_3 and β_4 become

$$\beta_3' = \beta_3 - c_2\beta_2 + (c_3 - c_2^2)\beta_2 \tag{7a}$$

$$\beta_4' = \beta_4 + 2\beta_1(c_4 + 2c_2^2 - 3c_3c_2) + \beta_2 c_2^2 - 2\beta_3 c_2 \tag{7b}$$

It is then obvious that the highest N term of each β_i, $i > 2$, is scheme independent. Therefore, a N-independent rescaling of the coupling could render β_3 zero only in the case of vanishing highest N coefficient which is exactly what (Eq.4) shows. If this phenomenon, namely the vanishing of the sheme independent parts, were to persist in all orders, it would be a new step towards the understanding of mass generation.

In the following chapter we develope a recurrent technique for the evaluation of the highest N term of the β_i coefficient for arbitrary i.

3. EVALUATION OF THE HIGHEST N TERM

Each term of the β-function can be extracted from the infinite parts of the four- and two-poin functions. Simple considerations, which we present in details in ref (2), show that the only diagrams contributing to the desired highest N term are shown generically in (Fig.1), where the blobs represent corrections to the appropriate order. As far as the two-point function is concerned the only possible diagram is the one shown in (Fig.2a). The diagram in (Fig.1b) can be easily obtained by differentiating the two-point function with respect to m, a fermion mass which plays the role of an infrared regulator. Finally, the corrections to the σ-propagator, (Fig.1c), can be obtained in a similar way by differentiating the vacuum -to-vacuum diagram, shown in (Fig.2b), twice with respect to m.

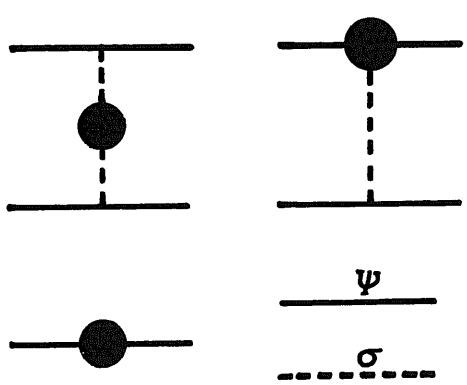

FIGURE 1.
The only diagrams contributing to the highest N term of the β-function.

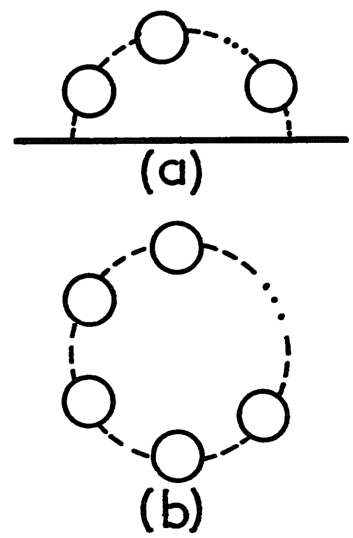

FIGURE 2.
a)The only diagram contributing to the highest N term of the wave function renormalization constant. b) vacuum diagram.

The building block for all calculations is the one-loop correction to the σ-propagator, shown in (Fig.3), which we denote by $\Sigma(k)$. Separating out the infinite part we can write

$$\Sigma(k) = -N\omega(2I - k^2\zeta(k) + 4m^2\zeta(k))g^2 \tag{8a}$$

where

$$I = \int \frac{d^{2\omega}p}{(2\pi)^{2\omega}} \frac{1}{(p^2 - m^2)} \tag{8b}$$

$$\zeta(k) = \int \frac{d^{2\omega}p}{(2\pi)^{2\omega}} \frac{1}{(p^2 - m^2)((p+k)^2 - m^2)} \tag{8c}$$

and 2ω is the dimension of the space-time. The integral I contains the UV divergence for $\omega = 1$ while $\zeta(k)$ is finite. Now the vacuum-to-vacuum diagram shown in (Fig.2b) can be written as

$$V(m) = i^{(j-1)} \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} [\Sigma(k)]^{(j-1)} \tag{9}$$

while (Fig.2a) takes the form

$$\begin{aligned} W(\not{p}, m) = \\ -i^{(j+1)} g^2 \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} [\Sigma(k)]^{(j-1)} \frac{(\not{p} - \not{k}) + m}{(p-k)^2 - m^2} \\ = A\not{p} + Bm \end{aligned} \tag{10}$$

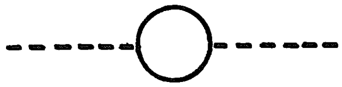

FIGURE 3.
The one-loop correction to the σ-propagator.

The infinite parts of A and B are not independent but related through the simple relation

$$Inf.Part(A) = (1 - 1/\omega) Inf.Part(B)$$

Considering all the above we can reduce all integrals needed to be evaluated into the form

$$K(a,b) = \int \frac{d^{2\omega}p}{(2\pi)^{2\omega}} (p^2)^a \left[\zeta(p)\right]^b, \quad a \geq 0, b \geq 1, a \leq b \tag{11}$$

Now, $K(a,b)$ respects the following powerful recurrent formula

$$K(a+1,b) = \frac{2m^2(2a-b-2\omega)K(a,b)}{(b+1)\omega - 2b + a + 1} + \frac{2b(\omega-1)\ I\ K(a,b-1)}{(b+1)\omega - 2b + a + 1} \tag{12a}$$

with the following initial values

$$K(0,0) = 0; \quad K(0,1) = I^2; \quad K(0,j) = Z^j, j > 1 \tag{12b}$$

where

$$Z^j = \int \frac{d^{2\omega}p}{(2\pi)^{2\omega}} \left[\zeta(p)\right]^j \tag{12c}$$

which are finite for $j > 1$.

Counterterms can be incorporated in that scheme by simply replacing $\Sigma(k)$ by

$$\Sigma(k) - Inf.Part[\Sigma(k)]$$

The only ones which have to be considered separetely are the $(j-1)$-loop wave-function counterterms contained in the j-loop vacuum-to-vacuum diagram.

Now, the desired infinite part of the σ-propagator and the vertex can be taken as

$$\frac{g^2}{2(j-1)} (\partial_m)^2 V(m) \tag{13a}$$

$$g \partial_m W(\not{p}, m) \tag{13b}$$

correspondingly.

The recurrent formula of (Eq.12a) can be manipulated by means of a standard algebraic computer package, in any order. As an example we present the highest N term of the β-function in 4- and 5-loops

$$\text{4-loops:} \quad \frac{\lambda^5}{(4\pi)^4} N^3 \left(-\frac{64}{3}\right) \tag{14}$$

$$\text{5-loops:} \quad \frac{\lambda^6}{(4\pi)^5} N^4 \left(32 + 8m^2 Z^2 + 8m^4 Z^3\right) \tag{15}$$

4.CONCLUSIONS

In this work we attempted to check whether the simple form of the 3-loop β-function persists to higher orders too. The results found do not support the above conjecture. Nevertheless, the algorithm developed can in principle be used for evaluating next to leading N corrections in any order.

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