CERN-TH.5579/89

The Nonperturbative Unification Scenario[1]

D. Kapetanakis
NRCPS "Democritos", Aghia Paraskevi
Attiki, Greece

S. Theisen
CERN, CH-1211, Geneva 23, Switzerland

N. D. Tracas
Physics Department Nat. Tech. Univ.
Zografou Campus, GR-15773, Athens, Greece

and

G. Zoupanos [2]
CERN, CH-1211, Geneva 23, Switzerland

ABSTRACT

We present the consequences of the assumption that the low energy gauge couplings of the standard model are the nearly infrared stable fixed points of an asymptotically non-free gauge theory. The minimal physics beyond the standard model that is favoured by this assumption is eight fermionic families and a similar number of Higgs doublets or a supersymmetric standard model with five families.

---

(1) Invited talk at XII Warsaw Symposium on Elementary Particle Physics. Presented by G. Zoupanos.

(2) A. v. Humbolt Fellow. Partially supported by a Research Grant of the Greek Secretariat of Science and Technology. On leave from Physics Department, Nat. Tech. Univ., Athens, Greece.

CERN-TH.5579/89
October 1989

In the last few years unification ideas, mostly represented by the minimal SU(5) GUT of Georgi and Glashow (G.G.)[1], had the possibility of being checked experimentally [2]. As a result of this confrontation of theory with experiment one had to rule out the G.G. SU(5) model. Certain assumptions of the model, especially the desert one, suffered from comparison with experiment. Despite the disappointment one can take a positive view and welcome the fact that physics beyond the standard model is not ruled out as would be the case if the "desert hypothesis" of the G.G. model was proved to be correct. In any case, theorists are very intrigued by unification ideas and have now happily jumped to even more ambitious unification scenarios involving more than four dimensions, such as superstring [3] and Kaluza-Klein type [4] theories. Phenomenologically these theories are in no better shape than the G.G. model.

Here we are going to present another unification scheme, first suggested by Maiani, Parisi and Petronzio (M.P.P.) [5], which can confront successfully the low energy data while possibly being part of a more general theory. To motivate it, let us recall some weak points of the perturbative unification of the standard model in GUT's, like the G.G. model.

The behavior of the coupling constants of $SU(3)_C xSU(2)_L xU(1)$ is dictated by the renormalization group equations, which to one loop order are:

$$\frac{d\alpha_i}{d\ln E} = \frac{B_i}{2\pi}\alpha_i^2 \tag{1}$$

where $\alpha_i = g_i^2/4\pi$, i=1,2,3, are the U(1), SU(2) and SU(3) fine structure constants. For n_G number of generations and n_H number of Higgs doublets the B_i's are given by:

$$B_1 = \frac{20}{9}n_G + \frac{1}{6}n_H; \quad B_2 = \frac{4}{3}n_G + \frac{1}{6}n_H - \frac{22}{3}; \quad B_3 = \frac{4}{3}n_G - 11 \tag{2}$$

From eq.(1) we obtain:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\Lambda)} - \frac{B_i}{2\pi}\ln\left(\frac{\mu}{\Lambda}\right) \tag{3}$$

$B_{2,3}<0$ for asymptotically free theories and μ, Λ are two different scales. There are two disturbing facts that we want to emphasize. The first is that couplings at the electroweak scale result from a delicate balance between two large numbers if Λ is the unification scale. Another point is that the $\alpha_i(\mu)$'s depend crucially on the detailed relation between $\alpha_i(\Lambda)$'s at the unification point Λ, as for example in SU(5)

$$\frac{5}{3}\alpha_1(\Lambda) = \alpha_2(\Lambda) = \alpha_3(\Lambda) \tag{4}$$

Therefore, given that despite of our efforts the details of the unified theory are unknown, the perturbative unification is not very attractive. The picture changes drastically if the whole theory is asymptotically non-free in which case all $B_i>0$. Then the couplings are monotonically increasing with energy up to a scale Λ beyond which

we can no longer trust perturbation theory. In that case $\alpha_i(\Lambda)^{-1}$ in eq.(3) is small and can be neglected in a first approximation in which $\alpha_i(\mu)$ does not depend on the values of $\alpha_i(\Lambda)$. The fact that makes this picture very interesting is that it leads to low energy couplings of the standard model close to their experimental values. For example, assuming that $\alpha_i(\Lambda)=0.5$, i=1,2,3, $\Lambda=10^{16}$ GeV and using $n_G=9$ and $n_H=1$ in order to make $B_{2,3}>0$, one obtains

$$\alpha_{em}(M_w)=0.00735; \quad \alpha_3(M_w)=0.139; \quad \sin^2\theta_w(M_w)=0.210 \tag{5}$$

which have to be compared with the experimental values [6]:

$$\alpha_{em}(M_w)=0.00772; \quad \alpha_3(M_w)=0.122\pm0.016; \quad \sin^2\theta_w(M_w)=0.228\pm0.0044 \tag{6}$$

Therefore the M.P.P. scenario, which assumes that all coupling constants become large at a common large scale, predicts at one loop level almost the correct values for the low energy couplings, which can be considered as the infrared fixed points of the asymptotically non-free theory. Of course one can do better than that by taking into account two-loop renormalization group equations. This is in fact required for the cases considered below, where the running of the SU(3) coupling is slow and two loop effects are important. (Three loop contributions turn out to be small in comparison.) Also, given that since the theory is asymptotically non-free the couplings become large at high energies and two-loop contributions become important. Moreover since at two-loop level the renormalization group equations are coupled, one expects that the strongest coupling drives the others to become also large at a common scale. Indeed the two loop renormalization group equations for the $SU(3)_C xSU(2)_L xU(1)$ are, neglecting Yukawa couplings:

$$\frac{d\alpha_i}{d\ln E} = \frac{B_i}{2\pi}\alpha_i^2 + \sum_k \frac{\alpha_i^2\alpha_k}{8\pi^2}B_{ik} \tag{7}$$

where B_i, i=1,2,3, are given by eq.(2) and

$$B_{ik} = \begin{pmatrix} \frac{95}{27} & 1 & \frac{44}{9} \\ \frac{1}{3} & \frac{49}{3} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_G + \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}$$

Then for eight fermionic families and one Higgs doublet, if $\alpha_i(\Lambda)=1$, i=1,2,3, $\Lambda=4\times10^{17}$ GeV, the prediction for the low energy couplings is

$$\alpha_{em}(M_w)=0.00773;\quad \alpha_3(M_w)=0.119;\quad \sin^2\theta_w(M_w)=0.1973$$

which is in better agreement with the experimental values than eq.(5).

When the M.P.P. suggestion is applied to the standard model it automatically requires physics beyond, given that even the simplest way to make the whole theory asymptotically non-free is to assume at least eight fermionic families. From the first LEP results it is clear that the extra neutrinos must be heavy.

Given the M.P.P. philosophy of non-perturbative unification the fact that it works well, but not perfectly well when applied to the standard model even with eight fermionic families, provides a window for testing various suggestions of the physics beyond the standard model due to their influence on the low energy couplings. Then one can guess the right physics beyond the standard model by comparing the predictions of the various schemes for low energy couplings with their experimental values.

A number of studies have already been made within the framework of the M.P.P. scenario which include new physics in the form of new families of fermions [5] and Higgs fields [7,8] but, also introducing supersymmetry N=1 [9], N=2 [10], horizontal interactions [7] or technicolour [11] in the standard model or by extending the electroweak group to $SU(2)_L xSU(2)_R xU(1)$ [12] or $SU(4)xU(1)_{B-L}$ [10]. Due to lack of space we can only present two possible scenarios of physics beyond the standard model, which lead to low energy coupling constants in agreement with the experimental values. They are also the most economical in introducing new parameters in the theory.

Multi-Fermion-Higgs families in the standard model [7,8].

In this case one introduces not only more fermionic generations but also new Higgs doublets; the renormalization group equations and the relevant coefficients can be found in eqs. (2) and (7). Some of the results, starting with $\alpha_i(\Lambda)=1$, i=1,2,3, when the number of fermionic generations is kept equal to eight and the number of Higgs doublets n_H varies, are shown in the table.

n_H	Λ	α_1	α_2	α_3	α_{em}	$\sin^2\theta_w$
1	$4x10^{17}$	0.0096	0.0391	0.119	0.00773	0.1973
2	$2.5x10^{17}$	0.0096	0.0381	0.121	0.00771	0.2021
4	$8x10^{16}$	0.0097	0.0364	0.122	0.00772	0.2116
6	$3x10^{16}$	0.0099	0.0350	0.125	0.00772	0.2201
7	$2x10^{16}$	0.0099	0.0343	0.126	0.00771	0.2243
8	$1.2x10^{16}$	0.0100	0.0337	0.128	0.00772	0.2284
9	$7.2x10^{15}$	0.0100	0.0331	0.128	0.00772	0.2326
10	$4x10^{15}$	0.0101	0.0328	0.130	0.00776	0.2363

Table: The predictions for the standard model low energy couplings α_1, α_2, α_3, α_{em} and $\sin^2\theta_w$ for various numbers of Higgs doublets n_H and eight fermionic families

From the table we find that a large number of Higgs families close to the number fermion generations is favoured and gives very good agreement with eq.(6).

Supersymmetric standard model [9]

In this case N=1 supersymmetry is introduced in the standard model, which has to be broken at some scale M_S given that the low energy spectrum is not supersymmetric. For E less than the supersymmetry breaking scale M_S, the B_i's and B_{ik}'s of eq.(7) are:

$$B_1=\frac{20}{9}n_G+\frac{1}{2}n_H;\qquad B_2=\frac{4}{3}n_G+\frac{1}{2}n_H-\frac{22}{3};\qquad B_3=\frac{4}{3}n_G-11 \qquad (8)$$

and

$$B_{ik}=\begin{pmatrix}\frac{95}{27} & 1 & \frac{44}{9}\\ \frac{1}{3} & \frac{49}{3} & 4\\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3}\end{pmatrix}n_G+\begin{pmatrix}\frac{3}{4} & \frac{9}{4} & 0\\ \frac{3}{4} & \frac{25}{4} & 0\\ 0 & 0 & 0\end{pmatrix}n_H+\begin{pmatrix}0 & 0 & 0\\ 0 & \frac{-136}{3} & 0\\ 0 & 0 & -102\end{pmatrix} \qquad (9)$$

For $E>M_S$ they are:

$$B_1=\frac{10}{3}n_G+\frac{1}{2}n_H;\qquad B_2=2n_G+\frac{1}{2}n_H-6;\qquad B_3=2n_G-9 \qquad (10)$$

and

$$B_{ik}=\begin{pmatrix}\frac{190}{27} & 2 & \frac{88}{9}\\ \frac{2}{3} & 14 & 8\\ \frac{11}{9} & 3 & \frac{68}{3}\end{pmatrix}n_G+\begin{pmatrix}\frac{1}{2} & \frac{3}{2} & 0\\ \frac{1}{2} & \frac{7}{2} & 0\\ 0 & 0 & 0\end{pmatrix}n_H+\begin{pmatrix}0 & 0 & 0\\ 0 & -24 & 0\\ 0 & 0 & -54\end{pmatrix} \qquad (11)$$

The results one finds are: for $\alpha_i(\Lambda)=1$, i=1,2,3, at $\Lambda=10^{17}$ GeV and $M_S=10^4$ GeV with five fermionic generations the low energy couplings are:

$$\alpha_3(M_w)=0.1313;\quad \alpha_{em}(M_w)=0.00771;\quad \sin^2\theta_w(M_w)=0.234$$

in quite good agreement with eq.(6).

A point which should be emphasized is the stability of the low energy predictions against changes in their values at Λ allowing also $\alpha_1(\Lambda)\neq\alpha_2(\Lambda)\neq\alpha_3(\Lambda)$.

In conclusion, we have discussed the assumption that the theory which describes the interactions of elementary particles is asymptotically non free and that coupling constants become large at some common large scale Λ. This is a very interesting assumption since it can predict reasonably well the observed low energy parameters without any fine tuning. It would be part of a more general and truly unified theory which is not specified. As opposed to the G.G. GUT model it cannot only accommodate physics beyond the standard model but it can also provide the framework to test any suggestion for new physics. So far the best solutions for the low energy couplings are provided by a) minimal extensions of the standard model, with eight fermionic families and a similar number of Higgs doublets and b) by the supersymmetric standard model with five fermionic families and supersymmetry breaking scale at 10^4 GeV.

One of us (G.Z.) would like to thank the organizers for their hospitality.

References

1. H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438.
2. See e.g. : F. Raupach, Proc. of the XXIV International Conf. on HEP, Munich 1988, R. Kotthaus and J. H. Kühn (eds.) Springer-Verlag 1989.
3. See e.g. : M. B. Green, J. H. Schwarz and E. Witten "Superstring theory" Vols I and II, Cambridge Univ. Press 1987; D. Lüst and S. Theisen "Lectures on String Theory", Springer Notes in Physics, Vol.346,1989.
4. See e.g. : G. Zoupanos et al. Proc. of the XXIV International Conf. on HEP, Munich 1988, R. Kotthaus and J. H. Kühn (eds.) Springer-Verlag 1989 and references therein.
5. L. Maiani, G. Parisi and R. Petronzio Nucl. Phys. **B136** (1978) 115; G. Grunberg, Phys. Rev. Lett. **58** (1987) 1180.
6. G. Altarelli, Proc. of the HEP-EPS Conf., Uppsala 1987, O. Botner (ed.) Uppsala Univ. 1987; P. Langacker, Proc. of the XXIV International Conf. on HEP, Munich 1988, R. Kotthaus and J. H. Kühn (eds.) Springer-Verlag 1989
7. D. Kapetanakis, S. Theisen and G. Zoupanos Phys. Lett **229B** (1989) 248
8. C. Alacoque, C. Deom and J. Pestieau Phys. Lett **228B** (1989),370
9. N. Cabibbo and G. R. Farrar, Phys. Lett. **110B** (1982) 107; L. Maiani and R. Petronzio, Phys. Lett. **176B** (1986) 12; S. Theisen, N. D. Tracas and G. Zoupanos Z. Phys. **C37** (1988) 597.
10. J. P. Derendinger, R. Kaiser and M. Roncadelli Phys. Lett. **220B** (1989) 164.
11. G. Grunberg Phys. Lett. **B203** (1988) 413; P. Q. Hung Phys. Rev. **D38** (1988) 377.
12. P. Q. Hung and S. Mohan Phys. Rev. Lett. **61** (1988) 31.