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INTERMEDIATE GAUGE COUPLING UNIFICATION IN UNIFIED MODELS OF STRING ORIGIN

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The intermediate scale unification of gauge couplings in models of string origin is investigated. We present useful relations between the β-function coefficients, ensuring unification when Kaluza-Klein excitation are taken into account. Application of these ideas to two models is also discussed.

1 Introduction

As is well known, the minimal supersymmetric standard model (MSSM) spectrum leads to gauge coupling unification at a scale of $M_U \sim 10^{16}$GeV. On the other hand, the possibility that the string and the compactification scale are around the energy determined by the geometric mean of the Planck mass and the electroweak scale, has recently appeared as a viable possibility in Type II string theories [1] with large extra dimensions [2]. In order to reconcile these two approaches, usually power-law running of the gauge couplings is assumed, due to the appearance of the Kaluza–Klein (KK) tower of states above the compactification scale [3,4,5,6,7]. Nevertheless, we should point that the lowering of the unification scale can be achieved also with the presence of extra matter and higgs fields in the context of the standard model group [8]. In the present work, we consider unified models of string origin which break down to the SM group at some intermediate energy. We further assume the existence of a compactification scale M_C (smaller than the would be unification scale if M_C had not existed) above which KK–excitations are considered. In this context, we find that unification can always be ensured whenever certain conditions of the β-function differences are met. In particular, we study models based on the $SU(3)^3$ and $SU(4) \times O(4)$ gauge symmetries. Such models can be derived from strings and possess various properties. They are in principle safe from proton decay operators and they need small higgs representations to break the gauge symmetry. The superpotential possesses various discrete and other symmetries that may prevent undesirable Yukawa couplings, while many unwanted particles are projected out. The original large gauge symmetry breaks down to the intermediate gauge group of the type discussed above owing to the existence of stringy type mechanisms.

2 Inclusion of K-K states and the requirements for unification

In Fig.1 we present the hierarchy of scales as they appear in our present work and which are the following: at the electroweak scale M_W, we use the initial values for the gauge couplings, as they are measured by the experiment. Next we consider $M_S = 1$ TeV, above which the MSSM β-functions are operative; M_G, is the scale above which new physics appears and the β-functions of the specific grand unified model (GUT) are effective; M_C, is the scale where compactification appears and the KK–states start contributing to the β functions, and M_U denotes the scale where the gauge couplings would unify if there were no compactification scale; M_C is taken to be smaller than M_U. Finally, M_{CU} is the scale where the gauge couplings unify when we include the KK–excitations.

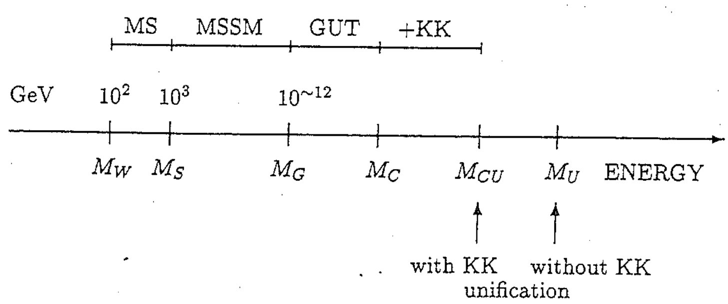

Figure 1: The energy scales appearing in the paper.

We present here a general property of the β–function coefficients. Let $\beta_{ij} = \beta_i - \beta_j$ denote the

β–function differences. We make the following two assumptions:

- There exists an energy scale M_U where the coupling constants α_i's unify, i.e. $\alpha_i(M_U) = \alpha_U$ for all i, assuming conventional logarithmic running (no–compactification scenario). Quantitatively, this is expressed as

$$\frac{\alpha_{ij}^{-1}(M)}{\beta_{ij}} \equiv \frac{\alpha_j^{-1}(M) - \alpha_j^{-1}(M)}{\beta_i - \beta_j} = \frac{\alpha_{ik}^{-1}(M)}{\beta_{ik}} = \frac{1}{2\pi}\log\frac{M_U}{M} > 0, \tag{1}$$

  where M is some initial scale. The positiveness of the ratio ensures the "convergence" (and not "divergence") of the couplings above M. This point becomes essential when we discuss the cases of GUTs.

- The ratios of the differences of the β–functions β_{ij}^{KK} (above the compactification scale M_C) to the corresponding difference β^{ij} (below the compactification scale M_C) have the property:

$$\frac{\beta_{ij}^{KK}}{\beta_{ij}} = \frac{\beta_{ik}^{KK}}{\beta_{ik}} > 0. \tag{2}$$

  Again positiveness ensures "convergence" of the couplings above M_C.

Then, it can be shown that the gauge couplings do unify, whatever energy scale we choose as a compactification scale M_C, above which the massive KK–states contribute to the running.

In the MSSM, we know that the three gauge couplings unify at the scale $\sim 10^{16}$GeV. Now assuming that only the gauge bosons and the higgs acquire KK–states (the matter fields are placed on the fixed points of the heterotic string and therefore no KK–states appear for them), it is easy to check that to a good approximation the two aforementioned requirements are fulfilled and therefore, whatever energy scale we choose as our compactification scale, the three couplings will unify (note here that, since the matter multiplets are complete $SU(5)$ ones, even in the case where they had KK–excitations, the unification would be achieved).

3 The $SU(4) \times SU(2)_L \times SU(2)_R$ model

We first take as an example the $SU(4) \times SU(2)_L \times SU(2)_R$ model, which is assumed to break to the SM–symmetry at some scale M_G. Above M_G, apart from the MSSM matter content, we have the following extra states

$$\begin{aligned} n_6 = (6,1,1), \quad n_4 = (4,1,1), \qquad & n_L = (1,2,1), \quad n_R = (1,1,2) \\ n_{22} = (1,2,2), \qquad & n_H = (4,1,2)/(\bar{4},1,2). \end{aligned}$$

where we show the quantum numbers under the GUT group. The subscript "H" refers to the higgs fields that break the $SU(4)$ and the $SU(2)_R$ groups, while the "22" gives the Standard Model higgs. The relations between the MSSM and the GUT model couplings, at M_G, are

$$\alpha_4 = \alpha_3, \quad \alpha_L = \alpha_2, \quad \alpha_R^{-1} = (5/3)\alpha_1^{-1} - (2/3)\alpha_4^{-1}.$$

The one loop β–functions are

$$\begin{aligned} \beta_R &= -6 + 2n_G + 2n_H + 2n_{22} + n_R/2 \\ \beta_L &= -6 + 2n_G + 2n_{22} + n_L/2 \\ \beta_4 &= -12 + 2n_G + n_H + n_6 + n_4/2. \end{aligned} \tag{3}$$

where n_G is the number of generations.

There exist several particle contents below and above the compactification scale fulfilling the two requirements for unification. In the following table we give one example, where the content below M_C

can, in principle, be reproduced by the string $SU(4)\times SU(2)_L\times SU(2)_R$ model, while we have chosen $M_G=10^{12}$GeV

	n_6	n_4	n_L	n_R	n_H	n_{22}
below M_C	4	8	10	10	4	4
above M_C	0	2	0	0	4	4.

(4)

In Fig. 2 we show the running of the coupling constants for the above content and for several values of M_C.

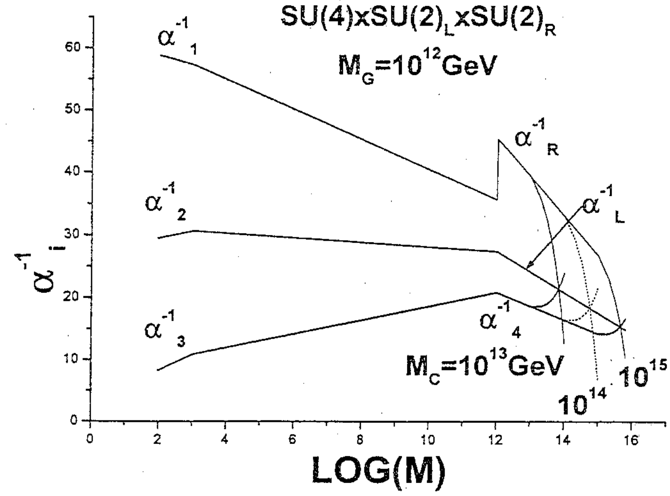

Figure 2: The inverse of the three gauge couplings as a function of energy, for the $SU(4)\times SU(2)_L\times SU(2)_R$ GUT with the specific content appearing in (4). We have chosen $M_G=10^{12}$GeV and three values of the compactification scale $M_C=10^{13},10^{14},10^{15}$GeV.

4 The $SU(3)\times SU(3)_L\times SU(3)_R$ model

Another interesting string derived model, which admits a low (intermediate) unification scale (no dangerous dimension–six operators), is based on the $SU(3)\times SU(3)_L\times SU(3)_R$ symmetry. The MSSM content is found in the 27 representation of the E_6 group

$$27\rightarrow(3,\bar{3},1)+(\bar{3},1,3)+(1,3,\bar{3}). \tag{5}$$

where

$$(3,\bar{3},1)=\begin{pmatrix}u\\ d\\ D\end{pmatrix},(\bar{3},1,3)=\begin{pmatrix}u^c\\ d^c\\ D^c\end{pmatrix},(1,3,\bar{3})=\begin{pmatrix}h^0 & h^+ & e^c\\ h^- & \bar{h}^0 & \nu^c\\ e & \nu & N\end{pmatrix}. \tag{6}$$

At the breaking to the MSSM scale, the following relations hold (some details on the β-functions and the string spectra may be found in [9]).

$$\alpha_L=\alpha_2,\quad \alpha_R^{-1}=(5/4)\alpha_Y^{-1}-(1/4)\alpha_L^{-1}.$$

Apart from the SM states, in the string model, fractionally charged and other exotic states usually appear, belonging to the representations

$(\overset{(-)}{3},1,1)$	$(1,\overset{(-)}{3},1)$	$(1,1,\overset{(-)}{3})$
0	$\pm1/3$ and $\pm2/3$	$\pm1/3$ and $\pm2/3$

(7)

where the second line shows the corresponding (electric) charges. One should not be misled by the values of these charges: the neutral states are coloured, while the others are singlet under the colour group. Therefore, after the symmetry breaking, these states will result in exotic lepton doublets and singlets carrying charges like those of the down and up quarks. Note that such states are not common in GUTs, however, they are generic in string models.

The one-loop β-functions are given by

$$\beta_3 = -9 + \frac{1}{2}(3n_Q + 3n_{Q^c} + n_C) \tag{8}$$

$$\beta_L = -9 + \frac{1}{2}(3n_Q + 3n_L + n_{L'}) \tag{9}$$

$$\beta_R = -9 + \frac{1}{2}(3n_{Q^c} + 3n_L + n_{L''}), \tag{10}$$

where n_Q, n_{Q^c} and n_L are the number of the representations appearing in the complete 27, Eq. (5), while n_C, $n_{L'}$ and $n_{L''}$ are the number of the exotic representations of (7).

As in the case of the previous model, several massless spectra pass the two conditions and provide unification of the three couplings. Although it seems that the $SU(3)^3$ is probably less constrained (giving a lot of possible contents, presumably because of the symmetric form of the β-functions), one should be careful, since the unification coupling could be high enough in some cases and get out of the perturbative region. This of course happens for high matter content, when the β-functions become large and positive. We should note at this point (and it is a general remark not applicable only to the specific GUT) that the value of M_C starts playing a significant role in the case where the unification coupling constant is getting large: if the β-functions between M_G and M_C are already large, M_C cannot be much larger than M_G if we want to avoid a non-perturbative value of the unification coupling.

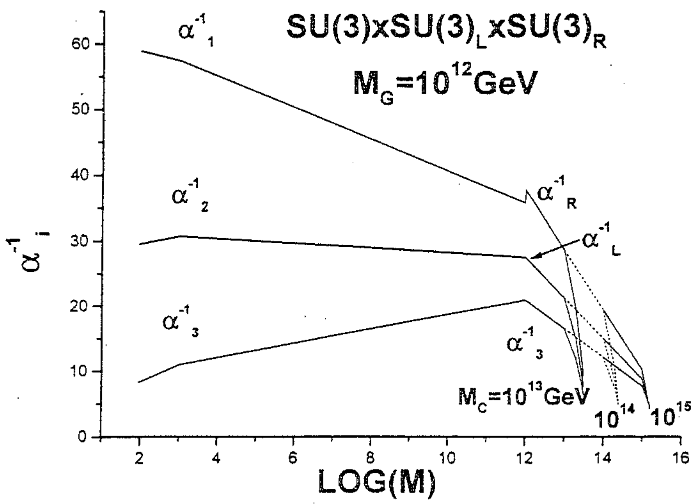

Figure 3: Same as in Fig.2 for the $SU(3) \times SU(3)_L \times SU(3)_R$ model and the specific content of (11).

In the following table, we give, as an example, the content below and above M_C, for the $SU(3)^3$

model, where we have chosen $M_G = 10^{12}$ GeV

$$\begin{array}{lcccccc} & n_Q & n_L & n_{Q^c} & n_C & n_{L'} & n_{L''} \\ \text{below } M_C & 10 & 4 & 6 & 10 & 0 & 2 \\ \text{above } M_C & 6 & 0 & 2 & 10 & 0 & 2. \end{array} \qquad (11)$$

In Fig. 3 we show the running of the couplings for the above content and for several values of M_C.

Concluding, we would like to point out that although the possibility of lowering the unification scale is an exciting fact – allowing experimental tests of its implications – the notorious proton decay problem is present in almost all GUTs. Here, we have presented two models which can overcome this problem and we have showed that there exist numerous cases of massless spectra (which can be derived from the superstrings), implying naturally intermediate scale unification.

This work has been done in collaboration with G.K. Leontaris.

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