Ansatz for Quark, Charged Lepton, and Neutrino Masses in SUSY GUTS

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Abstract

We extend a fermion mass matrix Ansatz by Giudice to include neutrino masses. The previous predictions are maintained. With two additional parameters, a large Majorana neutrino mass and a hierarchy factor, we have seven *further* low energy predictions: the masses of the neutrinos, the mixing angles and the phase in the leptonic sector. We choose a reasonable hierarchy of Majorana masses and fit the overall mass scale according to a solution of the solar neutrino problem via the MSW mechanism, which is in agreement with the ^{37}Cl, Kamiokande, and GALLEX data. We then also obtain a cosmologically interesting tau-neutrino mass.

One of the fundamental problems of particle physics is to understand the observed fermion mass spectrum. In the Standard Model it is described by thirteen parameters: nine masses, three quark mixing angles and a CP violating phase. Ultimately one hopes for a solution in terms of a more fundamental model, *e.g.* string theory, where the structure of the mass matrices is determined by a set of symmetries. Meanwhile any reduction in the set of required parameters could be a guide in finding the appropriate symmetries and also lead to a prediction of yet undetermined fermion masses in terms of experimentally known masses and angles.

There has been much fruitful work on this problem [1,2,3]. Recently, motivated by the observed merging of the Standard Model gauge coupling constants in supersymmetric grand unified theories (SUSY GUTs) [4], Dimopoulos, Hall and Raby (DHR) [2] and Giudice [3] have considered simple fermion mass matrices at the GUT scale and derived the resulting spectrum below the electroweak scale using the renormalization group equations (RGEs) for the case of minimal supersymmetry. In the DHR ansatz the fourteen parameters (including $\tan\beta$, the ratio of Higgs vevs in supersymmetry) are given in terms of eight input parameters; hence six predictions. In the modified ansatz studied by Giudice more structure is imposed on the mass matrices (by hand) leading to one input parameter less and thus one more prediction. Both are in good agreement with the known experimental values [2,5,3]. In a large class of mass matrices these ansätze are the only consistent solutions [6].

In the work of DHR and of Giudice neutrino mass matrices were not considered. Most GUT models. *e.g.* $SO(10)$, $SU(4)\otimes SU(2)^2$, $SU(5)\otimes U(1)$, and E_6, predict the existence of right-handed neutrinos leading to Dirac-neutrino matrix at the GUT scale. In this letter we extend the ansatz by Giudice to include neutrino masses. This modifies the analysis in the following points:

(i) New Higgs representations should provide heavy Majorana masses to the right-handed neutrinos, in order to realize the see-saw mechanism [7].

(ii) The leptonic mixing angles are completely determined by the charged-lepton and up-quark masses. With one additional parameter we obtain the neutrino mass spectrum and the mixing angles in the lepton sector. We thus have seven further low-energy predictions. We obtain a possible solution to the solar neutrino problem via the MSW mechanism [8] in agreement with the combined data of the ^{37}Cl, Kamiokande, and GALLEX experiments [9] and the tau-neutrino mass is cosmologically relevant.

A realistic fit to the desired structure of the mass matrices in SUSY GUT models requires a rather complicated Higgs sector. In a more fundamental theory the structure of the matrices might arise more naturally. However, as an example we consider here the relevant couplings in an $SO(10)$ SUSY GUT, assuming that additional discrete symmetries, as in Harvey *et al.* of Ref.[1], prevent the appearance of specific entries in the mass matrices.

Within an $SO(10)$ SUSY GUT the down-like quark and charged lepton masses arise from the following terms

$$\hat{f}(\underline{16}_1\cdot\underline{16}_2\cdot\underline{10}_{<\bar{5}>})+(2\hat{d})(\underline{16}_2\cdot\underline{16}_3\cdot\underline{10}_{<\bar{5}>})+\hat{d}(\underline{16}_2\cdot\underline{16}_2\cdot\underline{126}_{<\overline{45}>})+\hat{c}(\underline{16}_3\cdot\underline{16}_3\cdot\underline{10}_{<\bar{5}>}), \quad (1)$$

where $\underline{16}_i$ are the fermion families. The simple relation between the second and third coefficient is imposed by hand. The $\underline{126}$ Higgs representation has a non-zero vev along the $<\overline{45}>$ direction of the $SU(5)$. The structure of this vev results in a relative factor (-3) in the (2,2)-entry of the mass matrices M_d and M_e below [10,11] and leads to the successful Georgi-Jarlskog relations $m_b \simeq m_\tau$, $m_s \simeq m_\mu/3$, and $m_d \simeq 3m_e$ at the GUT-scale [11]. The up-quark and the Dirac-neutrino masses arise from

$$\hat{b}(\underline{16}_1\cdot\underline{16}_3\cdot\underline{10}_{<5>}+\underline{16}_2\cdot\underline{16}_2\cdot\underline{10}_{<5>})+\hat{a}\,(\underline{16}_3\cdot\underline{16}_3\cdot\underline{10}_{<5>}), \quad (2)$$

and the heavy Majorana-neutrino mass matrix from

$$\lambda_{\nu_i}\underline{16}_i\cdot\underline{16}_i<\underline{126}>_{<1>}\rightarrow M_i\nu_i^c\nu_i^c. \quad (3)$$

where we have assumed a diagonal Majorana mass matrix, $diag(M_1, M_2, M_3)$. In the following we shall make the simple hierarchical ansatz $M = M_3 = kM_2 = k^2M_1$, $k = 10$, motivated by the known fermion mass hierarchies. λ is a Yukawa coupling and $i = 1, 2, 3$ is a generation index. As we have already pointed out however, these are not the most general couplings allowed, but we assume that other relevant couplings are not present due to some kind of discrete symmetry of the underlying theory.

Thus, in the present case, we obtain the Giudice mass matrices [3] augmented by a simple ansatz for the neutrino masses

$$M_u = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \qquad M_{\nu\nu^c} = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \quad (4)$$

$$M_d = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & d & 2d \\ 0 & 2d & c \end{pmatrix}, \quad M_e = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & -3d & 2d \\ 0 & 2d & c \end{pmatrix}, \tag{5}$$

$$M_{\nu^c\nu^c} = M\, diag(k^{-2}, k^{-1}, 1). \tag{6}$$

The entries in the mass matrices are given by the corresponding Yukawa coupling of Eq.(1-3) multiplied with the appropriate vev. In general these entries are complex. However, the fields can be redefined such that all parameters are real and we only have the shown phases [3]. We have included a single phase in the charged lepton mass matrix, since the neutrinos are no longer degenerate. For simplicity we have chosen this phase to be identical with the phase in M_d. [1] We thus have eight parameters at the GUT scale (including k) in order to provide a total of 20 parameters at low energies: six quark masses; three charged lepton masses; three light neutrino masses; six mixing angles, three for the Cabbibo-Kobayashi-Maskawa mixing matrix and three for the corresponding leptonic mixing matrix; and two phases, one for each mixing matrix. Hence, we have seven predictions for the Standard Model parameters and seven predictions for the extension of the Standard Model due to the massive neutrinos.

Using the results of Ref.[12] we obtain the renormalization group equations for the Yukawa couplings at the one-loop level

$$16\pi^2 \frac{d}{dt}\lambda_U = \left(I \cdot Tr[3\lambda_U\lambda_U^\dagger] + 3\lambda_U\lambda_U^\dagger + \lambda_D\lambda_D^\dagger - I \cdot G_U\right)\lambda_U, \tag{7}$$

$$16\pi^2 \frac{d}{dt}\lambda_N = \left(I \cdot Tr[\lambda_U\lambda_U^\dagger] + \lambda_E\lambda_E^\dagger - I \cdot G_N\right)\lambda_N, \tag{8}$$

$$16\pi^2 \frac{d}{dt}\lambda_D = \left(I \cdot Tr[3\lambda_D\lambda_D^\dagger + \lambda_E\lambda_E^\dagger] + 3\lambda_D\lambda_D^\dagger + \lambda_U\lambda_U^\dagger - I \cdot G_D\right)\lambda_D, \tag{9}$$

$$16\pi^2 \frac{d}{dt}\lambda_E = \left(I \cdot Tr[\lambda_E\lambda_E^\dagger + 3\lambda_D\lambda_D^\dagger] + 3\lambda_E\lambda_E^\dagger - I \cdot G_E\right)\lambda_E, \tag{10}$$

where λ_α, $\alpha = U, N, D, E$, represent the 3x3 Yukawa matrices which are defined in terms of the mass matrices given in Eq.(4-6), and I is the 3x3 identity matrix. We have neglected one-loop corrections proportional to λ_N^2. $t \equiv \ln(\mu/\mu_0)$, μ is the scale at which the couplings are to be determined and μ_0 is the reference scale, in our case the GUT scale. The gauge contributions are given by

$$G_\alpha = \sum_{i=1}^{3} c_\alpha^i g_i^2(t), \tag{11}$$

$$g_i^2(t) = \frac{g_i^2(t_0)}{1 - \frac{b_i}{8\pi^2} g_i^2(t_0)(t - t_0)}. \tag{12}$$

The g_i are the three gauge coupling constants of the Standard Model and b_i are the corresponding beta functions in minimal supersymmetry. The coefficients c_α^i are given by

$$\{c_U^i\}_{i=1,2,3} = \left\{\frac{13}{15}, 3, \frac{16}{3}\right\}, \qquad \{c_D^i\}_{i=1,2,3} = \left\{\frac{7}{15}, 3, \frac{16}{3}\right\}, \tag{13}$$

[1] In general the leptonic mixing matrix will include three phases.

$$\{c_E^i\}_{i=1,2,3} = \left\{\frac{9}{5}, 3, 0\right\}, \qquad \{c_N^i\}_{i=1,2,3} = \left\{\frac{3}{5}, 3, 0\right\}. \tag{14}$$

We find it convenient to redefine the quark and lepton fields such that λ_U *and* λ_N are diagonal

$$\begin{aligned} \lambda_U \to \tilde{\lambda}_U &= K^\dagger \lambda_U K, \qquad \lambda_N \to \tilde{\lambda}_N = K^\dagger \lambda_N K, \\ \lambda_D \to \tilde{\lambda}_D &= K^\dagger \lambda_D K, \qquad \lambda_E \to \tilde{\lambda}_E = K^\dagger \lambda_E K. \end{aligned} \tag{15}$$

The diagonalizing matrix is given by [3]

$$K = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \quad \tan 2\theta = \frac{2b}{a}. \tag{16}$$

For the up-quarks and the corresponding Dirac-neutrinos we obtain at the GUT scale

$$\begin{aligned} m_u &= m_{\nu_{D1}} = m_0 \tan^2\theta, \quad m_c = m_{\nu_{D2}} = m_0 \tan\theta, \\ m_t &= m_{\nu_{D3}} = m_0, \quad m_0 = \frac{a}{1-\tan^2\theta}. \end{aligned} \tag{17}$$

We apply the field redefinitons (15) to the differential equations (7-10) and within the parenthesis on the right hand side only retain the dominant Yukawa coupling $\tilde{\lambda}^2_{U_{33}}$

$$16\pi^2 \frac{d}{dt}\tilde{\lambda}_U = (\tilde{\lambda}^2_{U_{33}} \begin{pmatrix} 3 & & \\ & 3 & \\ & & 6 \end{pmatrix} - G_U(t)\, I)\tilde{\lambda}_U, \tag{18}$$

$$16\pi^2 \frac{d}{dt}\tilde{\lambda}_N = (\tilde{\lambda}^2_{U_{33}} - G_N(t))\, I))\tilde{\lambda}_N, \tag{19}$$

$$16\pi^2 \frac{d}{dt}\tilde{\lambda}_D = (\tilde{\lambda}^2_{U_{33}} \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} - G_D(t)\, I)\tilde{\lambda}_D, \tag{20}$$

$$16\pi^2 \frac{d}{dt}\tilde{\lambda}_E = -G_E(t)\, I\, \tilde{\lambda}_E. \tag{21}$$

The equations for $\tilde{\lambda}_U, \tilde{\lambda}_D$, and $\tilde{\lambda}_E$ are thus unaffected by the neutrinos. We have solved this (approximate) system by first obtaining the exact solution for $\tilde{\lambda}_{U_{33}}$ [13,3]

$$\tilde{\lambda}_{U_{33}}(t) = \tilde{\lambda}_{U_{33}}(t_0)\zeta^6\gamma_U(t) \tag{22}$$

where

$$\begin{aligned} \gamma_\alpha(t) &= \exp(-\int G_\alpha(t)\, dt/(16\pi^2)) && (23) \\ &= \prod_{j=1}^{3} \left(\frac{\alpha_{j,0}}{\alpha_j}\right)^{c^j_\alpha/2b_j} && (24) \\ &= \prod_{j=1}^{3} \left(1 - \frac{b_{j,0}\alpha_{j,0}(t-t_0)}{2\pi}\right)^{c^j_\alpha/2b_j}, && (25) \\ \zeta &= \exp(\frac{1}{16\pi^2}\int_{t_0}^{t} \lambda^2_{U_{33}}\, dt) && (26) \\ &= \left(1 - \frac{\kappa}{8\pi^2}\lambda_{U_{33}}(t_0)\int_{t_0}^{t}\gamma^2_\alpha(t)\, dt\right)^{-1/(2\kappa)}. && (27) \end{aligned}$$

where κ is the constant infront of the $\lambda_{U_{33}}$ in the differential equation for the λ_α Yukawa coupling. We then use this solution for $\tilde{\lambda}_{U_{33}}$ to solve the equations for the other couplings. We reproduce the results of [3], including

$$m_b = \eta_b \frac{\gamma_D}{\gamma_E} \zeta m_\tau, \tag{28}$$

$$m_t = \zeta^3 \frac{\eta_u}{\eta_c^2} \frac{m_c^2}{m_u}. \tag{29}$$

$\eta_q = m_q(1\,GeV)/m_q(m_t)$ for the light quarks and $\eta_q = m_q(m_q)/m_q(m_t)$ for the c and b quark. In our analysis we have taken $\eta_u = \eta_d = \eta_s = 2$, $\eta_c = 1.8$ and $\eta_b = 1.3$. $\gamma_D/\gamma_E = 2.1$ and $\gamma_U/\gamma_N = 2.4$. In order to produce the proper running bottom quark mass [14,15]

$$m_b(m_b) = 4.25 \pm 0.1\,GeV \tag{30}$$

we require $\zeta = 0.81 \pm 0.2$. Following [3] we obtain the range

$$125\,GeV \leq m_t \leq 170\,GeV \tag{31}$$

from bounds on $\tan\beta$ from electroweak breaking as well as on m_u [15].

In order to determine the lepton mixing matrices we must find the matrix V_l such that $(V_l^\dagger \tilde{\lambda}_E^R (\tilde{\lambda}_E^R)^\dagger V_l)$ is diagonal. We parametrize the mixing matrix as

$$V_l = \begin{pmatrix} c_1 c_3 e^{i\phi} - s_1 s_2 s_3 & s_1 c_3 e^{i\phi} + c_1 s_2 s_3 & -c_2 s_3 \\ -s_1 c_2 & c_1 c_2 & s_2 \\ c_1 s_3 e^{i\phi} + s_1 s_2 c_3 & s_1 s_3 e^{i\phi} - c_1 s_2 c_3 & c_2 c_3 \end{pmatrix}, \tag{32}$$

where $c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$, *etc.* Then we find

$$V_{\nu_\mu e} \simeq |s_1| = \sqrt{y}(1 - \frac{1}{2}y) \simeq 6.9 \cdot 10^{-2}, \tag{33}$$

$$V_{\nu_\mu \tau} = |s_2| = \frac{2}{3}x(1 - y - \frac{13}{9}x) \simeq (3.95 \pm 0.01)10^{-2}, \tag{34}$$

$$V_{\nu_e \tau} = |s_3| = \frac{m_c}{\eta_c m_t} \simeq (4.9 \pm 0.2)10^{-3}\left(\frac{145\,GeV}{m_t}\right). \tag{35}$$

where $x = m_\mu/m_\tau$, $y = m_e/m_\mu$ and we have used the values for the charged lepton masses given by the particle data group [16]. In the last equation we have used the constraint on the top quark mass (31) and $m_c = (1.27 \pm 0.05)\,GeV$. Thus we predict the "leptonic Cabbibo angle" (θ_1) to be substantially smaller than the corresponding quark angle. This is different from phenomenological ansätze for the angle in Refs.[17], where the leptonic mixing angles are approximated by the corresponding quark angles. It is directly due to the factor of (-3) in the down-quark mass matrix, which is *necessary* to obtain the proper mass predictions. In ref. [18] a similar value is obtained from a different phenomenological ansatz.

The angle ϕ determines a CP violating quantity J discussed in [3], which is experimentally only poorly determined. The corresponding J_l for the leptonic mixing matirx is smaller by about a factor of 3, corresponding to $s_1^l \simeq \frac{1}{3}s_1^q$.

From the equations (33-35) we obtain the following neutrino mixing angles which we choose to write in the form $\sin^2 2\theta_{ex}$, motivated by the solutions to the solar neutrino problem

$$\sin^2 2\theta_{e\mu} = 1.9 \cdot 10^{-2} \tag{36}$$

$$\sin^2 2\theta_{\mu\tau} = (6.2 \pm 0.1)10^{-3} \tag{37}$$

$$\sin^2 2\theta_{e\tau} = (9.6 \pm 0.4)10^{-5} \tag{38}$$

and we have estimated the last angle by θ_3 since this ansatz does not determine the phase ϕ. We thus obtain a mixing angle $\theta_{e\tau}$ which is smaller than the phenomenological ansatz in [18] and is presumably out of the reach of the NOMAD and CHORUS experiments.

Renormalizing λ_N down to m_t and expressing the eigenvalues in terms of the up-quark masses, the Dirac-neutrino masses are

$$\begin{aligned} m_{\nu_{D1}} &= \frac{\gamma_N}{\gamma_U}\frac{1}{\eta_u\zeta^2}m_u, \\ m_{\nu_{D2}} &= \frac{\gamma_N}{\gamma_U}\frac{1}{\eta_c\zeta^2}m_c, \\ m_{\nu_{D3}} &= \frac{\gamma_N}{\gamma_U}\frac{1}{\zeta^5}m_t. \end{aligned} \tag{39}$$

In order to obtain realistic neutrino masses at the weak-scale we implement the see-saw mechanism [7] via the universal heavy Majorana mass matrix (6). The matrix K, which diagonalizes the Dirac-neutrino mass matrix produces off-diagonal elements in the heavy Majorana mass matrix $M_{\nu^c\nu^c}$. However, to a good approximation the eigenvalues of the complete 6x6 neutrino mass matrix are unaffected by this rotation. For each generation we thus have the neutrino mass matrix

	ν_i	ν_i^c
ν_i	0	$\frac{1}{2}m_{\nu_{Di}}$
ν_i^c	$\frac{1}{2}m_{\nu_{Di}}$	M_i

(40)

Diagonalizing this matrix we obtain the light neutrino masses $m_{\nu_i} = m^2_{\nu_{Di}}/(4M_i)$, which we can express in terms of the up-quark masses using Eqs.(39)

$$m_{\nu_e} = \frac{1}{4}(\frac{\gamma_N}{\gamma_U}\frac{1}{\eta_u\zeta^2})^2\frac{k^2m_u^2}{M} = (2.4 \pm 0.2)\frac{m_u^2}{M}, \tag{41}$$

$$m_{\nu_\mu} = \frac{1}{4}(\frac{\gamma_N}{\gamma_U}\frac{1}{\eta_c\zeta^2})^2\frac{km_c^2}{M} = (3.0 \pm 0.3)10^{-1}\frac{m_c^2}{M}, \tag{42}$$

$$m_{\nu_\tau} = \frac{1}{4}(\frac{\gamma_N}{\gamma_U}\frac{1}{\zeta^5})^2\frac{m_t^2}{M} = (0.35 \pm 0.09)\frac{m_t^2}{M}. \tag{43}$$

Recall that we have made the simple hierarchy ansatz $M = M_3 = kM_2 = k^2M_1$, $k = 10$, which also gives three heavy neutrinos which are inaccessible to experiment. The ratio of the light neutrino masses is predicted to be

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = (10m_u/\eta_u)^2 : (3.1m_c/\eta_c)^2 : (m_t^2/\zeta^3). \tag{44}$$

This is what one would expect in a general GUT see-saw mechanism [7]. It is similar to the phenomenological ansatz discussed in [18]. Due to our restrictive ansatz (4-6) we have the additional constraint (29) and m_t is *not* a free parameter. As seen above, the mixing angles in the leptonic sector are determined through known quantities. So with just one additional parameter, M, and a hierarchy ,k, we fix the scale of the neutrino masses and have a completely determined neutrino sector.

We chose to fix M so as to obtain a solution to the solar neutrino problem via the MSW mechanism [8]. We assume a two generation mixing solution to this problem. From Fig.1 in the second reference [9], which includes the ^{37}Cl, the Kamiokande and the GALLEX data, we have two possible solutions

$$\sin 2\theta_{ex} = (0.39 - 2.2)10^{-2}, \quad \Delta m^2 = (0.29 - 2.0)10^{-5} eV^2, \tag{45}$$

and

$$\sin 2\theta_{ex} = 0.2 - 0.8, \quad \Delta m^2 = (0.31 - 9.1)10^{-5} eV^2. \tag{46}$$

Comparing with Eqs(36-38), we have one possible solution in the range of (45) for $\nu_e - \nu_\mu$ mixing. Using Fig. 1 of ref[9] this requires

$$m_{\nu_\mu} = (1.7 - 1.9)\, 10^{-3} eV. \tag{47}$$

This fixes M to be

$$M = (2.7 \pm 0.5)10^{11}\, GeV \tag{48}$$

Using Eqs(41.43) we then predict the other neutrino masses

$$m_{\nu_e} = (2.3 \pm 0.6)10^{-7} eV \tag{49}$$

$$m_{\nu_\tau} = (27.3 \pm 8.7) eV \left(\frac{m_t}{145\, GeV}\right)^2 \tag{50}$$

The resulting tau-neutrino mass is in a cosmologically interesting range [19]. Using the formula $\Omega_\nu h^2 = m_\nu / 91.5\, eV$, where $h = 0.5 - 1$ is the Hubble parameter [19], for the relic density of the neutrinos we obtain

$$\Omega_\nu h^2 = 0.27 \pm 0.07. \tag{51}$$

This is in the range one would expect from the recent COBE data.

In conclusion we have expanded an ansatz by Giudice for the fermion mass matrices at the GUT scale to include massive neutrinos, since these are predicted by most GUTs. Giudice's predictions for the Standard Model parameters are retained. Without any additional parameters we predict the neutrino mixing angles completely in terms of known fermion masses. In addition with one parameter and a simple hierarchy we predict a neutrino mass spectrum, which gives a possible solution to the solar neutrino problem via the MSW mechanism and gives a significant contribution to the hot dark matter in agreement with recent interpretations of the COBE data.

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Note Added: While this letter was being written we received a copy of Ref.[20] in which DHR extend their original ansatz to include neutrino masses as well. They have one fewer parameter in the neutrino sector and obtain a lighter tau-neutrino mass. Since this is a different mass matrix ansatz our work is complementary to their's.

We also just received a copy of [21] where among other topics the constraint on the Giudice ansatz due to CP-asymmetries in B-meson decays is considered. This leads to stronger bounds on the quark mixing angles and indirectly to a stronger bound on m_t than our Eq.(31): $125 \leq m_t \leq 155\,GeV$ [21].

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